Programming Assignment 2

Description

This assignment will cover chapter 3. Heads up, this requires tons of programming. In Chapter 3, we covered interpolating polynomials. Polynomial interpolation is the interpolation of a given data set by the polynomial of lowest possible degree that passes through the points of the dataset.

This helps us approximate points on complicated curves. The methods require tons of operations on different points along the curve and introduces matrix operations.

Please do not procrastinate and get a head start on this assignment.

- Replit:

Turn in a program in Python for each question. Screenshots of the output should be captured and attached after each question.

- GitHub:

For full credit on this assignment, the following is required:

- Create expected structure.
- ensure there's a readme with instructions on how tocompile.
- output for each question is correct.

Constraints

Standard Structure

For this assignment, you will need the following structure:

Top Level

```
|-- src/
|  |-- main/
|  |  |-- __init__.py
|  |  |--assignment_2.py
|  |-- test/
|  |  |-- __init__.py
|  |  |-- test_ assignment_2.py
|-- requirements.txt
|-- README.md
```

Please ensure your repository is named “cot-4500-as2”.

Compilation Instructions

Keeping true to adding industry likeness to each assignment, it is required that you need to include a README per repository. A README is a file used to describe a repo's purpose, include compilation purposes, or both. A proper README.md is typically filled out with sections, akin to an essay.

For this assignment, a README needs to be included and filled out. The amount of info you put into the README is up to you, but you will need the following at least:

- Mention of requirements.txt
  - No need to include this UNLESS you plan on using a library other thanNumPy
- Mention of running python
  - Since we require instructions on to run, you will need to include the command to run the script from command line.

Questions

1. Using Neville's method, find the 2nd degree interpolating value for f(3.7) for the following set of data
    a.

x	f(x)
3.6	1.675
3.8	1.436
3.9	1.318

i	x_i	$x - x_i$	Q	$Q_{i,1}$	$Q_{j,2}$
0	3.6	.1	1.675 P_0	0	0
1	3.8	-.1	1.436 P_1	1.5555	0
2	3.9	-0.2	1.318 P_2	1.554	$\approx$ 1.555

$$f(3.7) \approx 1.555$$

$$\frac{1}{x_1 - x_0}\left((x - x_0)(P_1) - (x - x_1)(P_0)\right)$$

Program Output:

```
1.5549999999999995
```

2. Using Newton's forward method, print out the polynomial approximations for degrees 1, 2, and 3 using the following set of data
   a. Hint, create the table first
   b.

x	f(x)
7.2	23.5492
7.4	25.3913
7.5	26.8224
7.6	27.4589

3. Using the results from 2, approximate f(7.3)?

7.2	23.5492	0	0	0
			0	0
7.4	25.3913	9.2105		
			$\frac{10201}{600}$	0
7.5	26.8224	14.311		$-\frac{34039}{240}$
7.6	27.4589	6.365	-34.73	

$$P_1(x) = 23.5492 + (9.2105)(x-7.2)$$
$$= 24.47025$$
$$P_2(x) = P_1(x) + \frac{10201}{600}(x-7.4)(x-7.2)$$
$$= 24.30023333$$
$$P_3(x) = P_2(x) - \frac{34039}{240}(x-7.5)(x-7.4)(x-7.2)$$
$$= 24.016575$$

Program Output:

```
24.016574999999992

9.210500000000001
17.00166666666675
-141.82916666666722
```

4. Using the divided difference method, print out the Hermite polynomial approximation matrix

x	f(x)	f'(x)
3.6	1.675	-1.195
3.8	1.436	-1.188
3.9	1.318	-1.182

$$
\begin{array}{c|c}
i \backslash j & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
3.6 & 1.675 & 0 & 0 & 0 \\
3.6 & 1.675 & -1.195 & 0 & 0 \\
3.8 & 1.436 & -1.195 & \approx 0 & 0 \\
3.8 & 1.436 & -1.188 & 0.035 & 0.175 \\
3.9 & 1.318 & -1.18 & 0.08 & 0.15 \\
3.9 & 1.318 & -1.182 & -0.02 & -1
\end{bmatrix}
\end{array}
$$

Program Output:

```
[ 3.60000000e+00,  1.67500000e+00,  0.00000000e+00,  0.00000000e+00,  0.00000000e+00]
[ 3.60000000e+00,  1.67500000e+00, -1.19500000e+00,  0.00000000e+00,  0.00000000e+00]
[ 3.80000000e+00,  1.43600000e+00, -1.19500000e+00, -9.99200722e-15,  0.00000000e+00]
[ 3.80000000e+00,  1.43600000e+00, -1.18800000e+00,  3.50000000e-02,  1.75000000e-01]
[ 3.90000000e+00,  1.31800000e+00, -1.18000000e+00,  8.00000000e-02,  1.50000000e-01]
[ 3.90000000e+00,  1.31800000e+00, -1.18200000e+00, -2.00000000e-02, -1.00000000e+00]
```

5. Using cubic spline interpolation, solve for the following using this set ofdata:

x	f(x)
2	3
5	5
8	7
10	9

a. Find matrix A
b. Vector b
c. Vector x

$h_0 = X_1 - X_0$ $\quad a_0 \quad 3$

$\vdots \qquad\qquad\qquad \vdots = 5$

$\qquad\qquad\qquad\qquad 7$

$h_n = X_{n+1} - X_n \quad a_3 \quad 9$

$h_0 = 3$
$h_1 = 3$
$h_2 = 2$

$$b = \begin{bmatrix} 0 \\ \frac{3}{h_{n-1}}(a_{n+1} - a_n) - \frac{3}{h_{n-1}}(a_n - a_{n-1}) \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{3}{2}(9-7) - \frac{3}{3}(7-5)$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ h_0 & 2(h_0+h_1) & h_1 & 0 \\ 0 & h_1 & 2(h_1+h_2) & h_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Solve $Ax=b$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 3 & 12 & 3 & 0 & 0 \\ 0 & 3 & 10 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 10/3 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/37 \\ 0 & 0 & 1 & 0 & 4/37 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 12 & 3 & 0 \\ 0 & 3 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{X} = \begin{bmatrix} 0 \\ -1/37 \\ 4/37 \\ 0 \end{bmatrix}$$

Program Output:

```
[ 1.0,  0.0,  0.0,  0.0]
[ 3.0, 12.0,  3.0,  0.0]
[ 0.0,  3.0, 10.0,  2.0]
[ 0.0,  0.0,  0.0,  1.0]
[0. 0. 1. 0.]
[0 -0.027027027 0.10810811 0]
```

Expected Output

```
1.5549999999999995

9.210500000000001
-0.7183802816901438
-0.12461196085345332

24.47718457889519

[ 3.60000000e+00  1.67500000e+00  0.00000000e+00  0.00000000e+00 0.00000000e+00]
[ 3.60000000e+00  1.67500000e+00 -1.19500000e+00  0.00000000e+00 0.00000000e+00]
[ 3.80000000e+00  1.43600000e+00 -1.19500000e+00 -9.99200722e-15 0.00000000e+00]
[ 3.80000000e+00  1.43600000e+00 -1.18800000e+00  3.50000000e-02 1.75000000e-01]
[ 3.90000000e+00  1.31800000e+00 -1.18000000e+00  8.00000000e-02 -1.28571429e-02]
[ 3.90000000e+00  1.31800000e+00 -1.18200000e+00 -2.00000000e-02 -1.00000000e+00]

[ 1.  0.  0.  0.]
[ 3. 12.  3.  0.]
[ 0.  3. 10.  2.]
[ 0.  0.  0.  1.]
[0. 0. 1. 0.]
[0. -0.02702703  0.10810811  0.]
```