ASSIGNMENT - 2

① $f(n) = n - 10$ $\qquad$ $g(n) = n + 10$

$$f(n) = \Theta(g(n))$$

Big-O:

$$f(n) \leq c_1 g(n)$$

$$n - 10 \leq c_1 (n + 10)$$

when $c_1 = 1$, the equation is satisfied.

$\Rightarrow$ $f(n) = O(g(n)) \longrightarrow$ True.

Omega:

$$f(n) \geq c_2 \cdot g(n)$$

$$n - 10 \geq c_2 \cdot (n + 10)$$

If $n = 100$ and $c_2 = 1/2$ then,

$$100 - 10 \geq \frac{1}{2}(100 + 10)$$

$$90 \geq 55$$

$\Rightarrow$ $f(n) = \Omega(g(n)) \longrightarrow$ True.

$\Rightarrow$ Therefore, $f(n) = \Theta(g(n))$

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⑤ $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

Big-O: $f(n) \leq c_1 \cdot g(n)$

$$128^{\log_2 n} \cdot n^2 = n^9 \cdot c_1$$

$$n^{\log_2 128} \cdot n^2 = n^9 \cdot c_1$$

$$n^{\log_2 2^7} \cdot n^2 = n^9 \cdot c_1$$

$$n^{7\log_2 2} \cdot n^2 = n^9 \cdot c_1$$

$$n^7 \cdot n^2 = n^9 \cdot c_1$$

$\Rightarrow \boxed{c_1 = 1} \rightarrow$ True.

$\Rightarrow$ $f(n) = O\,g(n)$

Omega: $f(n) \geq c_2 \cdot g(n)$

$$128^{\log_2 n} \cdot n^2 = c_2 \cdot n^9$$

$$n^7 \cdot n^2 = c_2 \cdot n^9$$

$\Rightarrow \boxed{c_2 = 1} \rightarrow$ True.

$\Rightarrow$ $f(n) = \Omega(g(n))$

$\Rightarrow$ Therefore, $f(n) = \Theta(g(n))$

② $f(n) = n$ $\qquad g(n) = n$

$$f(n) = \Theta(g(n))$$

Big-O :

$$f(n) \leq O(g(n))$$

$$n \leq c_1 \cdot n$$

$$\Rightarrow \boxed{C_1 = 1} \longrightarrow \underline{\underline{\text{True}}}$$

$$\Rightarrow \underline{\underline{f(n) = O(g(n))}}$$

Omega :

$$f(n) \geq C_2 \cdot (g(n))$$

$$n \geq c_2 \cdot n$$

$$\Rightarrow \boxed{C_2 = 1} \longrightarrow \underline{\underline{\text{True}}}$$

$$\Rightarrow \underline{\underline{f(n) = \Omega(g(n))}}$$

$\Rightarrow$ Therefore, $\underline{\underline{f(n) = \Theta(g(n))}}$

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(3) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

Big-O: $f(n) \le c \cdot g(n)$

$2^{6\log_2 n} \cdot 2^{5\log_2 n} = c \cdot n^5$

$2^{\log_2 n^6} \cdot 2^{\log_2 n^5} = c \cdot n^5$

$(n^6)^{\log_2 2} \cdot (n^5)^{\log_2 2} = c \cdot n^5$

$n^6 \cdot n^5 = c \cdot n^5$

$\Rightarrow \boxed{c = n^6} \longrightarrow$ False.

$\Rightarrow$ Therefore, $f(n) = O(g(n))$.

(4) $\frac{4^n}{2^n} = O(2^n)$

Big-O: $f(n) \le c \cdot g(n)$

$\frac{4^n}{2^n} = c \cdot 2^n$

$\frac{2^n \cdot \cancel{2^n}}{\cancel{2^n}} = c \cdot 2^n$

$\Rightarrow \boxed{c = 1} \longrightarrow$ True.

$\Rightarrow$ Therefore, $f(n) = O(g(n))$.