ECEg 3142:

Network Analysis and Synthesis

Chapter 7:

Filters Types and Specifications

Chapter 7: Filters

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7.1. Introduction

- Filters are frequency selective electric networks.
- In other words, a filter is required to separate unwanted signal from a mixture of unwanted and wanted signals.
- Qualitatively, the tr. ch. of a filter is shaped so that the ratio of unwanted to wanted signal at the o/p of the filter is minimized.
- Filter tr. ch. are usually given in terms of filter specs.
- The filter spec. are then given in terms of cut-off , pass-band and stop-band frequency.

7.1. Introduction …

- **Pass-band**: - the frequency band of wanted signals.
- **Stop-band**: - the frequency band of unwanted signals.
- **Cut-off frequency**: -
  - associated with the boundary b/n stop band and an adjacent pass-band.
  - Freq. at which the o/p is **0.707** times the max. value in the pass band.
- Electric filters are ubiquitous and modern electronic devices do employ and electric filter.
- Filters have many practical applications.
  - A simple, single-pole, LPF (the integrator) is often used to stabilize amplifiers by rolling off the gain at higher frequencies where excessive phase shift may cause oscillations.
  - A simple, single-pole, HPF can be used to block dc offset in high gain amplifiers or single supply circuits.

7.1. Introduction …

- Filters can be used to separate signals, passing those of interest, and attenuating the unwanted frequencies.
  - An example of this is a radio receiver, where the signal you wish to process is passed through, typically with gain, while attenuating the rest of the signals.
  - In data conversion, filters are also used to eliminate the effects of aliases in A/D systems.
  - They are used in reconstruction of the signal at the output of a D/A as well, eliminating the higher frequency components, such as the sampling frequency and its harmonics, thus smoothing the waveform.

7.1. Introduction …

- According to their pass-band and stop-band frequencies, filters are categorized as:
  - Low pass filters (LPF)
  - Band pass filters (BPF)
  - High pass filters (HPF)
  - Band reject filters (BRF)

7.1. Introduction …

- An ideal filter
  - should pass wanted signals with no attenuation and
  - provide infinite attenuation for unwanted signals.
- The attenuation of a filter is determined by the i/p to o/p voltage ratio, $\frac{V_2(s)}{V_1(s)}$, called the filter gain, at pass-band and stop-band frequencies.
- This filter gain is usually given in terms of attenuation.

$$\alpha(w) = -20\log\left|\frac{V_2(j\omega)}{V_1(j\omega)}\right|$$

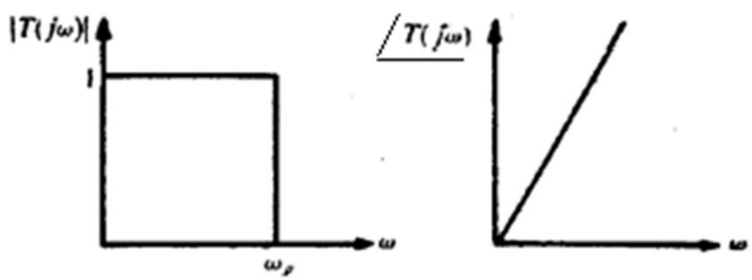
- Two important attenuation values are the maximum pass-band attenuation, α_p, and the minimum stop-band attenuation, α_s.

7.2. Filter Types

7.2.1. Low pass filters

- A LPF passes low frequency signals but attenuates signals with frequencies higher than the cutoff frequency.
- The magnitude and phase of an ideal low pass filter is

$$|T(j\omega)| = \begin{cases} 1 & |\omega| \leq \omega_p \\ 0 & otherwise \end{cases}$$



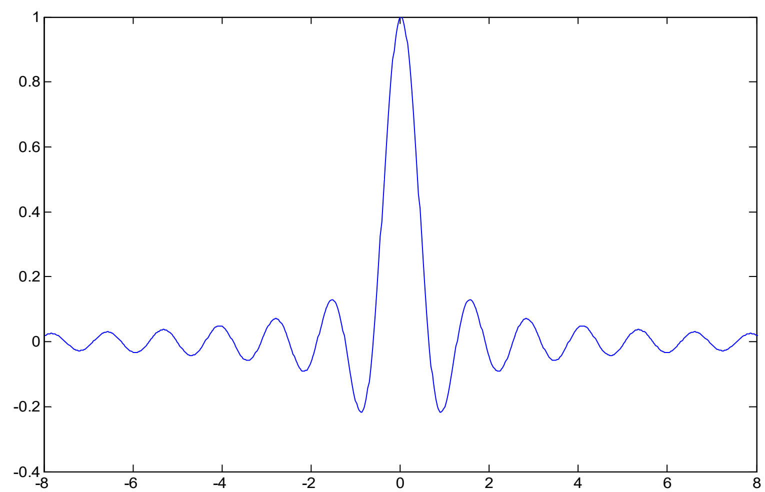
- ω_p is the pass band frequency.

7.2. Filter Types

7.2.1. Low pass filters …

- The impulse response of the ideal filter is

$$h(t) = \frac{\sin t}{t}$$

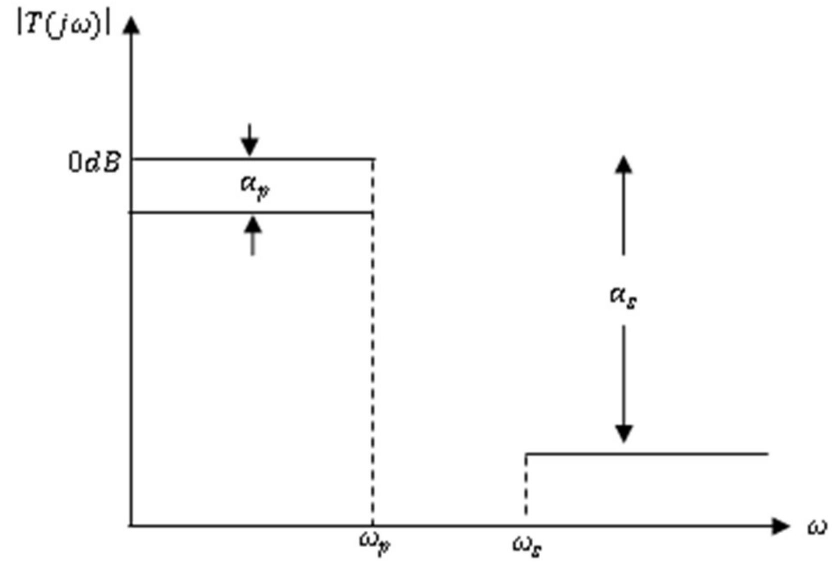


- We see this impulse response is not causal and unrealizable.
- According to the Paley-Wiener criterion, a band limited frequency response leads to non-causal impulse response.

7.2. Filter Types

7.2.1. Low pass filters …

- To circumvent this limitation, filter specs are given as follows



- $\omega_p \rightarrow PB\ edge\ freq.$
- $\omega_s \rightarrow SB\ edge$ freq.
- $\omega_p < \omega < \omega_s \rightarrow transtion\ band$
- $\alpha_p \ \rightarrow Maximum\ PB\ attenuation\ in\ dB$
- $\alpha_s \ \rightarrow Minimum\ SB\ attenuation\ in\ dB$

7.2. Filter Types

7.2.1. Low pass filters …

- That is,

$$\alpha \leq \alpha_p, \qquad \omega \leq \omega_p$$
$$\alpha \geq \alpha_s, \qquad \omega \geq \omega_s$$

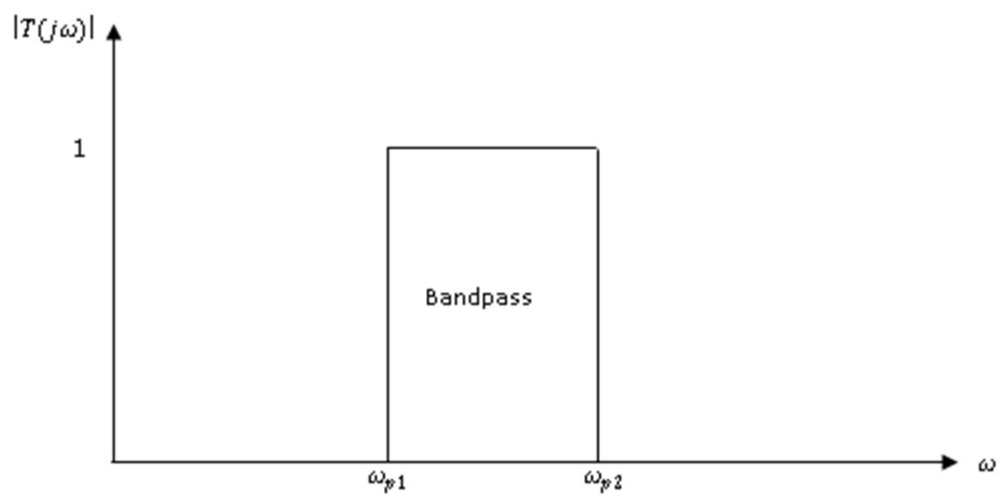
where α is attenuation o f filter in dB

In words:

- The attenuation should be less than α_p in the PB.
- The attenuation should be greater than α_s in the SB.
- The attenuation can be anything in the transition band.

7.2.Filter Types

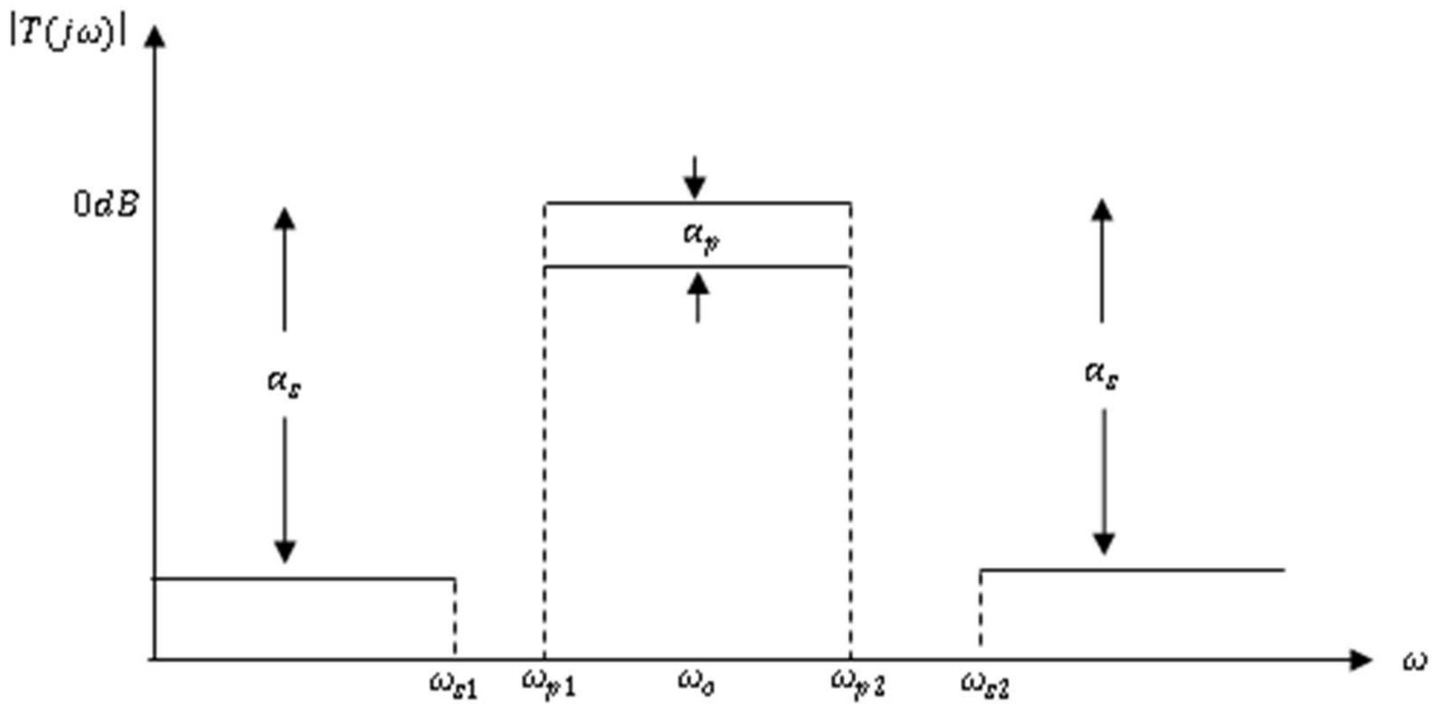
- **7.2.2. Band pass filters**
- A **BPF** passes signals of frequencies within a certain range and rejects (attenuates) frequencies outside that range.
- It passes signals between two cutoff frequencies but attenuates (reduces the amplitude of) the rest.
- The ideal spec



7.2. Filter Types

7.2.2. Band pass filters …

- Again this ideal BPF is not realizable.
- A realizable BPF spec is:



7.2. Filter Types

7.2.2. Band pass filters …

where

$$\omega_{p1}, \omega_{p2} \quad \text{are the cutoff frequencies}$$

$$\omega_0 = \sqrt{\omega_{p1}\omega_{p2}} \quad \text{is the center frequency}$$

$$\omega_{s1}, \omega_{s2} \quad \text{are the lower and upper stop - band frequencies}$$

$$\alpha \leq \alpha_p \qquad \omega_{p1} \leq \omega \leq \omega_{p2}$$

$$\alpha \geq \alpha_s \qquad \omega \leq \omega_{s1} \quad \text{or} \quad \omega \geq \omega_{s2}$$

$$BW = \omega_{p2} - \omega_{p1}$$

- For a geometrically symmetric BPF, the following eqn is satisfied.

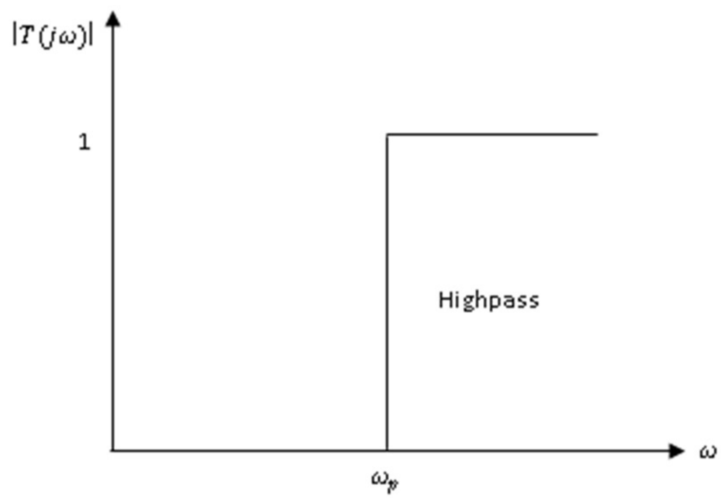
$$\frac{\omega_{s1}}{\omega_{p1}} = \frac{\omega_{p2}}{\omega_{s2}}$$

- Hence

$$\omega_0 = \sqrt{\omega_{p1}\omega_{p2}} = \sqrt{\omega_{s1}\omega_{s2}}$$

7.2. Filter Types

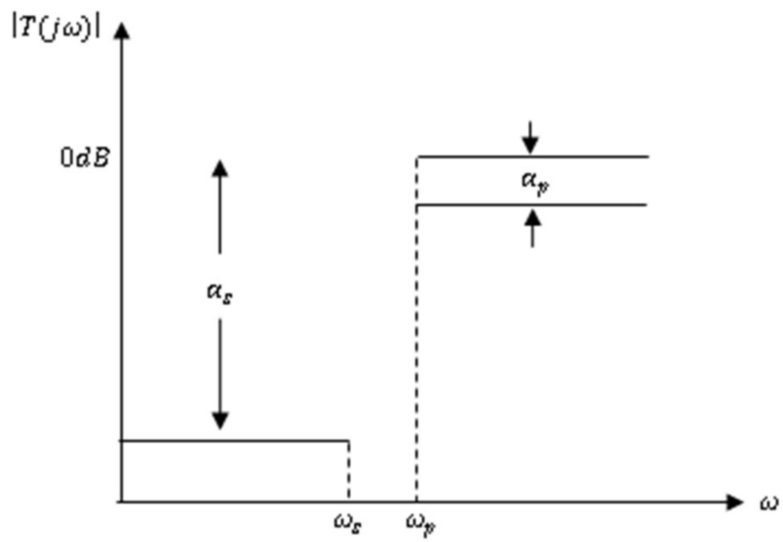
- **7.2.3. High pass filters**
- A HPF that passes HF signals but attenuates signals with frequencies lower than the cutoff frequency.
- An ideal HPF has the following ch.



- This is not realizable frequency response however.

7.2. Filter Types

- **7.2.3. High pass filters ..**
- A realizable HPF spec is given as follows:



- where

ω_s is the SB frequency

ω_p is the cutoff frequency

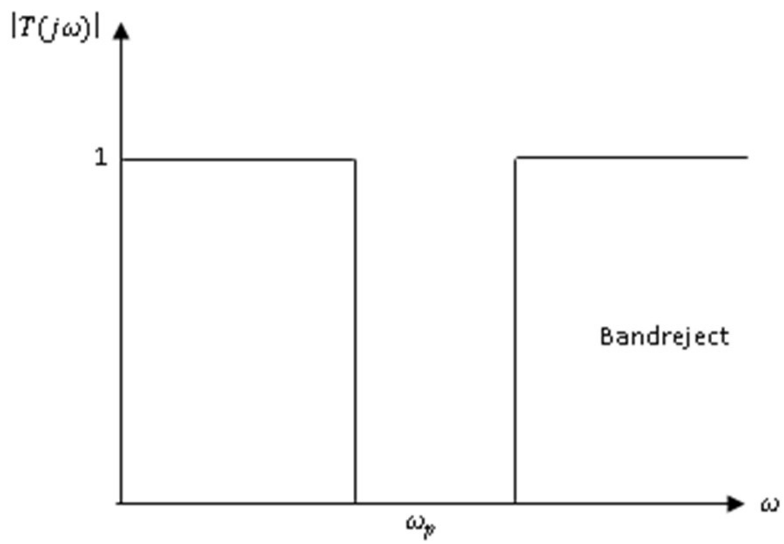
$\alpha \le \alpha_p \qquad \omega \ge \omega_p$

$\alpha \ge \alpha_s \qquad \omega \le \omega_s$

7.2. Filter Types

7.2.4. Band reject filters

- BRFs are the exact opposite of BPF.
- They attenuate signals b/n two freq. ranges but pass the rest.



- They can be obtained by the following equation

$$T_{BR}(s) = 1 - T_{BP}(s)$$

$T_{BR}(s)$ is the transfer function of band reject

$T_{BP}(s)$ is the transfer function of band pass

7.3. Frequency transformation

- Most of the widely used passive filter design and synthesis techniques (discussed in next ch) are applicable for LPFs only.
- Frequency transformation is used to synthesize BP, HP and BR filters.

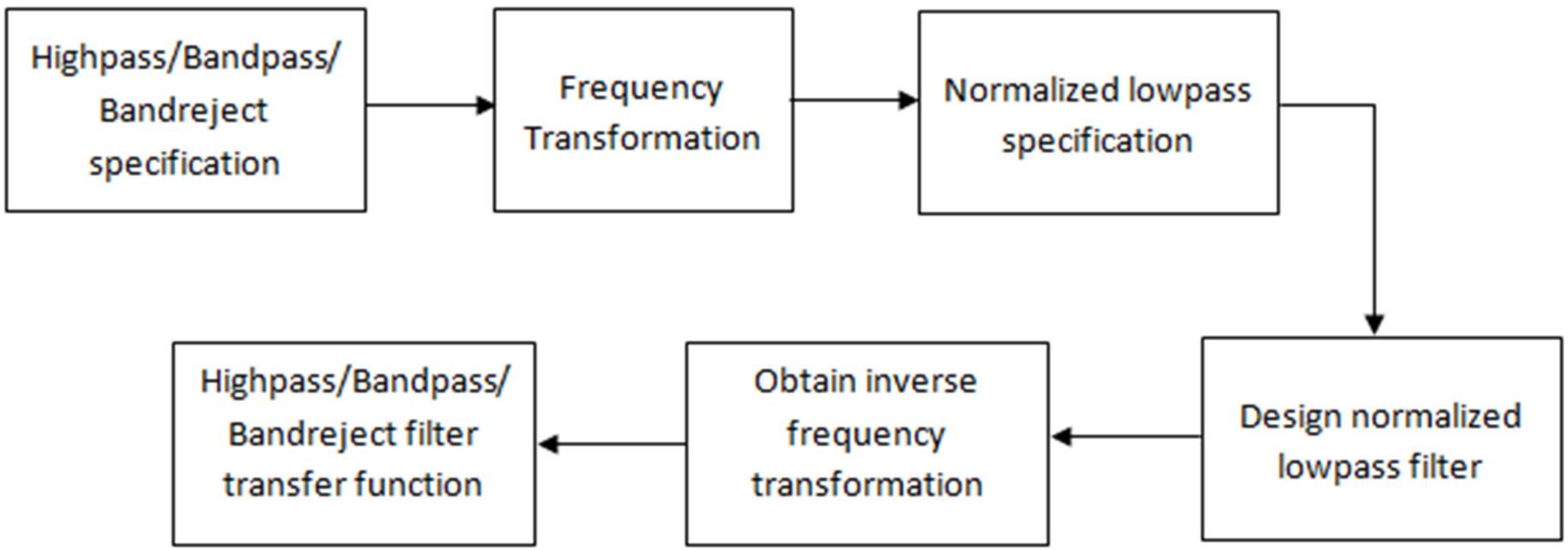


Figure: Frequency transformation

7.3. Frequency transformation . ..

By FT we mean:

1. Transform the given filter spec. to normalized LPF spec.
2. Obtain the normalized LPF tr. function using modern filter synthesis methods.
3. Transform the LPF tr. fn back to the required type of filter.
4. Synthesize the filter.

Note: Steps 3 and 4 can be exchanged by synthesizing the normalized LPF and then transforming the synthesized network back to the desired filter

7.3.1. Highpass to lowpass

- The HPF tr. fn. is converted to LPF by scaling the tr. fn. by 1/s.
- In practice, this amounts to C's becoming L's with value 1/C and L's becoming C's with a value of 1/L.
- To convert it to normalized LPF, the cutoff frequency is introduced.
- This leads to the following transformation equation.

$$s_n = \frac{\omega_0}{s}$$

s_n is the normailize low - pass filter variable

s is the high pass filter

ω_0 is the cutoff frequency of the high pass filter

7.3.1. Highpass to lowpass . . .

- Note that in this transformation:

$$\lim_{s\to 0} s_n = \infty$$
$$\lim_{s\to \infty} s_n = 0$$
$$\lim_{s\to \pm j\omega_0} s_n = \mp 1$$

- That is, the frequency range $[\omega_p, \infty]$ is transformed to [0,1]. Therefore, in this frequency range the filter is a LPF.

Procedure for designing HPF

- Transform the given spec. to normalized LPF spec. with $\omega_n = \frac{\omega_0}{\omega}$
- Design the normalized LPF with the appropriate method
- Transform the obtained normalized LPF tr. fn back to HPF with

$$s_n = \frac{\omega_0}{s}$$

7.3.2. BP to LP Frequency Transformation

- The FT from BP to LP tr.fn is a little more complicated.
- The FT from BP to LP is given by the following formula.

$$s_n = \frac{\omega_0}{BW}\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)$$

s_n is the normailized LPF freq. variable

s is the BPF freq. variable

ω_0 is the center frequency of the BPF

BW is the band width of the BPF

$BW = \omega_{p2} - \omega_{p1}$

- Note that in this transformation,

$$\lim_{s\to 0} s_n = \infty$$

$$\lim_{s\to \infty} s_n = -\infty$$

$$\lim_{s\to \omega_0} s_n = 0$$

$$\lim_{s\to \omega_{p1}} s_n = -1$$

$$\lim_{s\to \omega_{p2}} s_n = 1$$

7.3.2. BP to LP Freq. Transformation …

- This means the frequency range $[\omega_{p1}, \omega_{p2}]$ in the BP is transformed to [-1, 1] in the normalized LP.
- That is the frequency range $[\omega_o, \omega_{p2}]$ *is* transformed to [0, 1], which is a LPF spec.

Procedure for designing BPF

- Transform the given spec. to normalized LPF spec. with

$$\omega_n = -\frac{1}{BW}\frac{\omega_0^{\ 2} - \omega^2}{\omega}$$

- Design the normalized LPF with the appropriate method.
- Transform the obtained normalized LPF tr. fn back to BPF with

$$s_n = \frac{1}{BW}\frac{s^2 + \omega_0^{\ 2}}{s}$$

7.3.3. Band reject to low pass FT

- The FT from BRF to normalized LPF is given by the formula:

$$s_n = \frac{s}{\omega_0^2 + s^2}$$

s_n is the normalized LPF freq. variable

s is the BRF freq variable

ω_0 is the center frequency of the BRF

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Procedure for designing BPF

- Transform the given spec. to normalized LPF spec. with

$$\omega_n = \frac{\omega}{\omega_0{}^2 - \omega^2}$$

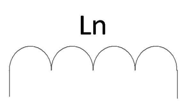
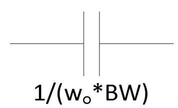
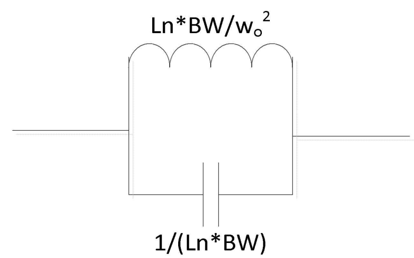
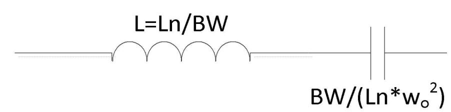
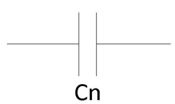
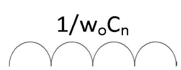
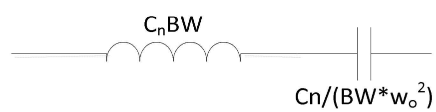
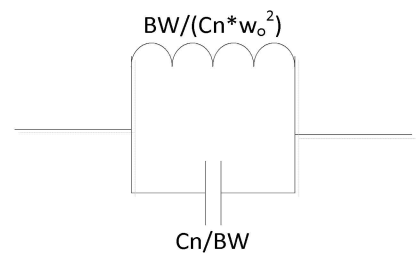
- Design the normalized LPF with the appropriate method
- Transform the obtained normalized LPF tr. fn back to BPF with

$$s_n = \frac{s}{\omega_0^2 + s^2}$$

7.3.3. Band reject to low pass FT . . .

Add Example

- If steps 3 and 4 were interchanged, the following element substitutions can be used to transform the synthesized normalized LP network to the required type of filter.

Lowpass	Highpass	Bandreject	Bandpass
 Ln	 $1/(w_o*BW)$	 $Ln*BW/w_o^2$; $1/(Ln*BW)$	 $L=Ln/BW$; $BW/(Ln*w_o^2)$
 Cn	 $1/w_oC_n$	 C_nBW; $Cn/(BW*w_o^2)$	 $BW/(Cn*w_o^2)$; Cn/BW

- **Exercise:** Derive the above element substitution formulas.