

Functional and Logic Programming - Home Assignment 5

28.5.2019

Due: Thursday, 6.6.2019 - 23:55

Instructions:

- Please create a PDF file which contains all the answers, numbered.
- Please make sure the answers are tidy and readable.

Exercises - λ -Calculus

1

Given the following definitions:

$$S = \lambda x. \lambda y. \lambda z. (x\ z)\ (y\ z)$$

$$K = \lambda x. \lambda y. x$$

$$I = \lambda x. x$$

$$C = \lambda f. \lambda x. \lambda y. f\ y\ x$$

reduce the following expressions to Normal Form:

a) $C\ I\ 10\ (\lambda z. *\ 5\ z)$

b) $S\ K\ K$

c) $S\ (K\ S)\ K$

Make sure you use α -renaming when needed to avoid name collisions.

2

Perform **normal order** reduction (to normal form) of the following expressions:

- a) $(\lambda f. \lambda x. f (f x)) (\lambda y. + y 1) (+ 2 3)$
- b) $(\lambda x. (\lambda z. z x) (\lambda x. x)) y$
- c) $(\lambda x. + ((\lambda y. ((\lambda x. * x y) 2)) x) y)$
- d) $(\lambda x. (\lambda y. + x y) 5) ((\lambda y. - y 3) 7)$
- e) $(\lambda x. \lambda y. x) (\lambda f. f (f 1)) ((\lambda x. x x x) (\lambda x. x x x)) (\lambda y. * 2 y)$

What would have happened if the above expression (e) was reduced in **applicative order**?

3

Give a **non-recursive** denition of the function **mcc**, using the **Y combinator**:

$$\mathbf{mcc} = \lambda n. \mathbf{mcc} (> n 100) (- n 10) (\mathbf{mcc} (\mathbf{mcc} (+ n 11)))$$

Exercises - Graph Reduction

4

For each of the following expressions:

- a) $(\lambda x. \lambda y. x) ((\lambda x. x) (\lambda x. * x 2)) 2$
- b) $(\lambda f. \lambda y. f y) (\lambda x. * 5 x) 5$
- c) $(\lambda p. \lambda x. \mathbf{if} (p x) x (+ 5 x)) (== 5) 5$
- d) $(\lambda x. x x) (\lambda f. \lambda x. f (f x)) (\lambda x. + 5 x) 1$
- e) $(\lambda z. * ((\lambda x. x) z) z) ((\lambda y. y) 5)$

1. Draw the graph representing the expression.
2. Reduce the graph to **WHNF** using **normal order reduction**.
3. Mark the redex by an **asterix** (*) at every reduction stage.
4. Show the subgraph that go to garbage collection.