Machine Learning Exercise 3

June 12, 2019

This exercise is composed of 2 parts :

1. Probability Theory Part
2. Coding Part

Probability Theory Questions

1. In a bar in Tel Aviv there are 15 Goldstar beer bottles and 20 Stella beer bottles. If you ask for a random beer at the bar there is $\frac{15}{35}$ probability that you will get a Goldstar.
   In the bar storage, there are 4 boxes (each box has 6 beers) of Goldstar and 7 boxes of Stella (also 6 in each box). A random box is moved from the storage to the bar. You ask for a random beer at the bar and get a Goldstar. What is the probability that the box that was moved from storage was Stella?

   Denote A as the event that you got a Goldstar after a random box was moved.
   Denote B as the event that a box of Stella was moved from storage.

   - $P(A, B) = \frac{7}{11} \cdot \frac{15}{41} = 0.2328$
   - $P(A) = \frac{7}{11} \cdot \frac{15}{41} + \frac{4}{11} \cdot \frac{21}{41} = 0.419$

   $$P(B|A) = \frac{P(A, B)}{P(A)} = \frac{0.2328}{0.419} = 0.555$$

2. A radar at the beach is used to detect ships. Ships are located in 1 of four zones : A, B C and D. The probability of detection per zone is 0.8, 0.7, 0.6, 0.5 for A, B, C and D respectively. The probability of being at a specific zone is 0, 0.2, 0.3, 0.5 for A, B, C and D respectively.

   (a) What is the probability that a ship will be detected.
   (b) Given that a ship is detected, what is the probability that it was in Zone C?
   (c) Given that a ship is detected, what is the probability that it was in Zone B?

   (a) $P(Discovery) = 0.8 \cdot 0 + 0.2 \cdot 0.7 + 0.3 \cdot 0.6 + 0.5 \cdot 0.5 = 0.57$
   (b) $P(C|Discovery) = \frac{0.3 \cdot 0.6}{0.57} = 0.31$
   (c) $P(B|Discovery) = \frac{0.2 \cdot 0.7}{0.57} = 0.24$

3. Find 3 random variables X, Y, C such that:

   (a) $X \perp Y|C$ - meaning X and Y are independent given C.
   $*X \perp Y|C \;\; if \;\; \forall x, y, c \; P(X = x, Y = y|C = c) = P(X = x|C = c) \cdot P(Y = y|C = c)$

(b) $X \not\perp Y$ - meaning X and Y are not independent.

(c) X, Y, C are all binary.

(d) The following conditions hold:

i. $P(X=0) = 0.3$

ii. $P(Y=0) = 0.3$

iii. $P(C=0) = 0.5$

You need to specify the value of $P(X=x, Y=y, C=c)$ (there are 8 of them)

$C=0$

-	X=0	X=1
Y=0	0.09	0.81
Y=1	0.01	0.09

$C=1$

-	X=0	X=1
Y=0	0.25	0.25
Y=1	0.25	0.25

4. The probability of having a descent meal in Karnaf is 0.7.

(a) What is the proability of having 3 descent meals in a week (5 days)

(b) What is the probability of having at least 2 descent meals in a week.

(c) A class of 100 students recorded the number of descent meal they had during a specific week. They averaged their results, what do you expect the value of that average to have been?

Let X be a random variable denoting the number of time you had a descent meal in the karnaf.

(a) $P(X=3) = \binom{5}{3} \cdot 0.3^2 \cdot 0.7^3 = 0.308$

(b) $P(X \geq 2) = \sum_{i=2}^{5} \binom{5}{i} \cdot 0.3^{5-i} \cdot 0.7^i = 0.969$

(c) $E[X] = 3.5$

5. Let $U = \{(x,y) | 0 \leq x, y \leq 1\}$ and let $C = \{(x,y) | x^2 + y^2 < 1\}$.
Suppose we sample 50 points from U, denoted as D, and let
$X = |D \cap C|$, meaning, X count the number of sampled points from C.

(a) How is X distributed?

(b) Using python, plot the CDF of X from 1 to 50.
This means you'll need to compute $P(X \leq i)$ where i ranges from 1 to 50. Your x axis is i and the y axis $P(X \leq i)$. You can use whichever library you wish.
Hand in only the plot.

(a) $X \sim Binom(50, \frac{\pi}{4})$

(b)