浙江大学

2024～2025学年离散数学第二次小测验

测试方式：闭卷 测试时间：45分钟 卷面总分：100分

姓名：____________ 学号：_____________ 分数：____________.

1.How many Strings of length 11 consist of 4 As,3 Cs,2 Us,2 Gs, and end with CAA?(15')

2.Solve the recurrence relation $a_n = a_{n-1}^2 a_{n-2}^3$ if $a_0 = 2$ and $a_1 = 2$ (15')

3.A dynamic programming algorithm for solving the problem of finding a subset S of items chosen from a set of n items where item i has a weight w_i ,which is a positive integer ,so that the total weight of the items in S is a maximum but does not exceed a fixed weight limit W .Let $M(j,w)$ denote the maximum total weight of the items(a subset of the first j items) such that this total weight does not greater than w .This problem is known as the knapsack problem.
For example,n items with weight $w_1, w_2, \ldots, w_n$(w_i is a positive integer),Let S_a denote the set of the selected items ,$S_a = \{x_1, x_2, \ldots, x_m\}$,and $S_a \subseteq \{1, 2, 3, \ldots, n\}$
$W(S)$ denote the total weight of S ,$W(S) = \sum_{a \in S} w_a$
$M(j,w)$ denote the maximum $W(S)$,$W(S) \le w$,$S \subseteq \{1, 2, 3, \ldots, j\}$
a)Show that if $w_j > w$,then $M(j,w) = M(j-1,w)$.(10')
b)Show that if $w_j < w$,then $M(j,w) = max(M(j-1,w), w_j + M(j-1, w - w_j))$(10')

4.Show that the reflexive closure of the symmetric closure of a relation is the same as the symmetric closure of its reflexive closure.(15')

5.Suppose that $S_1, S_2, \ldots, S_n$ is a collection of subsets of a set S where n is a positive integer.**A system of distinct representatives (SDR)** for this family is an ordered n-tuple($a_1, a_2, \ldots, a_n$) with the property that $a_i \in S_i$ for $i = 1, 2, \ldots, n$ and $a_i \neq a_j$ for all $i \neq j$
a)Find a SDR for the set
$S_1 = \{a, c, m, e\}, S_2 = \{m, a, c, e\}, S_3 = \{a, p, e, x\}, S_4 = \{x, e, n, a\}, S_5 = \{n, a, m, e\}, S_6 = \{e, x, a,$
(6')
b)Use Hall's Marriage theorem to show that a collection of finite subsets $S_1, S_2, \ldots, S_n$ of a set S has a SDR ($a_1, a_2, \ldots, a_n$) if and only if $\forall I \subseteq \{1, 2, \ldots, n\}, |U_{i \in I} S_i| \ge |I|$.(14')

6.Show that if G is a simple graph with at least 11 vertices ,then either G or $\overline{G}$ (The complementary graph of G),is nonplanar.The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G.Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.(15')