Quiz for Discrete Mathematics and Its Application (2024, April 7)

Register No. ______________ Name: ______________ Score: ______________

1. (20 points) Determine whether the following statements are true or false. If it is true write a √ otherwise a × in the blank before the statement.

( ) (1) The proposition "if he is absent, then he is present" is equivalent to "he is present".

( ) (2) $p \to (q \to p)$ and $\neg p \to (p \to q)$ are equivalent .

( ) (3)If $\forall x(P(x) \lor Q(x))$ are true, then $\forall xQ(x) \lor \forall xP(x))$ is also true, where the domains of all quantifiers are the same.

( ) (4)There is a countable infinite set A with a bijection: $A \to A \times A$.

( )(5) If $P(A) \in P(B)$, then $A \in B$. ($P(S)$ is the power set of S).

( ) (6) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

( )(7) For all positive real numbers x and y, $\lfloor x \cdot y \rfloor \le \lfloor x \rfloor \cdot \lfloor y \rfloor$.

( ) (8)The function $f(n) = 3\lfloor n/3 \rfloor$ from Z to Z is a one-to-one function (injection), where Z is the set of integers.

( )(9) $\dfrac{3n-8-4n^3}{2n-1}$ is $O(n^2)$..

( )(10)If $f(x)$ is $O(g(x))$, then $2^{f(x)}$ is $O(2^{g(x)})$

2. (20points) Fill in the blanks.

( )(10)If $f(x)$ is $O(g(x))$, then $2^{f(x)}$ is $O(2^{g(x)})$

2. (20points) Fill in the blanks.

(1) On the island of knights and knaves you encounter two people, A and B. Person A says, "B is a knave." Person B says, "We are both knights." Determine whether each person is a knight or a knave.

____

(2) Suppose $A = \{a,b,c,d,e\}$ and $B = \{\{\phi\}\}$, then $|P(A \times B)| =$ ______________________.

(3) Write English statement using the following predicates and any needed quantifiers.

Suppose the variable x represents students and y represent courses, and :

$M(y)$: y is a math course $A(x)$: x is a part-time student $T(x,y)$: student x is taking course y.

There is a part-time student who is not taking any math course.

____

(4) Find a recurrence relation with initial condition(s) satisfied by the following sequence $\{a_n\}$ ($n =$ 1,2,3,...).

$$a_n = n^2$$

____

(5) Give a big-O estimate for the following function. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function g of smallest order.

$$(n^3+n^2 \log n)(\log n+1) + (17 \log n+19)(n^3+2)$$

____

3. (10 points) 5. Show that the following argument is valid.

Each natural number is either even number or odd number. A natural number is an even number if and only if it is divided by 2. Not all natural numbers are divided by 2. Then some natural numbers are odd numbers.

4.(10points) Convert the following formula into logically equivalent formula in full conjunctive normal form. Determine whether it is tautology, contradiction or contingence. Find the assignments of p, q and r for which the formula is true.

$$p \wedge (q \leftrightarrow r)$$