

浙江大学

2024~2025学年离散数学第一次小测验

测试方式：闭卷 测试时间：45分钟 卷面总分：100分

姓名：_____ 学号：_____ 分数：_____.

1.Are these system specifications consistent?

"The router can send packets to the edge system only if it supports the new address space.

For the router to support the new address space it is necessary that the latest software release be installed.

The router can send packets to the edge system if the latest software release is installed.

The router does not support the new address space."(10')

2.Translate each of these nested quantifications into an English statement that expresses a mathematic fact.The domain in each case consists of all real numbers.(10')

a) $\forall x \forall y (((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$

b) $\exists x \exists y (((x \leq 0) \wedge (y \leq 0)) \wedge (x - y > 0))$

3.Show that these statements about the integer x are equivalent:(i) $3x + 2$ is even,(ii) $x + 5$ is odd,(iii) x^2 is even(12')

4. Let $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{1, 2, 3, 4\}$; List the Set represented below using the notation $\{x_1, x_2, \dots\}$. (12')

a) $A \cap B$

b) $A \cup B$

c) $A \times B$

d) $(C \setminus A) \cap B$

e) $\mathcal{P}(A) \cup B$

f) $\emptyset \cup \mathcal{P}(\emptyset)$

5. Determine whether each of these functions from \mathbb{Z} to \mathbb{Z} is one-to-one. (12')

a) $f(n) = n - 1$

b) $f(n) = n^2 + 1$

c) $f(n) = n^3$

6. Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive

integers and that set.(12')

a)the integers less than 100

b)the real numbers between 0 and $\frac{1}{2}$

c)the set $\mathbb{A} \times \mathbb{Z}$ where $\mathbb{A} = \{2, 3\}$

7.List all steps used to search for 9 in the sequence 1,3,4,5,6,7,8,9,11 using binary search.
(12')

8.Show that $x \log(x)$ is $\mathcal{O}(x^2)$ but that x^2 is not $\mathcal{O}(x \log(x))$.(10')

9.a)Use pseudocode to describe the algorithm that puts the first four terms of a list of real numbers of arbitrary length in increasing order.

b)Show that this algorithm has time complexity $\mathcal{O}(1)$ in terms of the number of comparisons used.(10')