$$\psi_{nk}(\vec r) = \frac{1}{\sqrt V} e^{i\vec k\cdot\vec r}\, u_{nk}(\vec r)$$

$$// \cancel{\delta_{nn'}\delta_{kk'}}$$

$$\langle \psi_{n'k'}(\vec r)|\psi_{nk}\rangle = \int_V d^3r\, \frac{1}{V}\, e^{-i\vec k'\cdot\vec r + i\vec k\cdot\vec r}\, u^*_{n'k'}(\vec r)\, u_{nk}(\vec r)$$

$$= \frac{1}{V} \sum_{R_l} \int_\Omega e^{i(\vec k-\vec k')\cdot(\vec r+\vec R_l)}\, u^*_{n'k'}(\vec r+\vec R_l)\, u_{nk}(\vec r+\vec R_l)$$

$$= \frac{1}{V} \sum_{R_l} e^{i(\vec k-\vec k')\cdot\vec R_l} \int_\Omega e^{i(\vec k-\vec k')\cdot\vec r}\, u^*_{n'k'}(\vec r)\, u_{nk}(\vec r)$$

$$= \frac{1}{V} \cancel{\Sigma}\, N \delta_{k,k'} \int_\Omega u^*_{n'k}(\vec r)\, u_{nk}(\vec r)$$

$$= \frac{N}{V} \delta_{k,k'} \int_\Omega u^*_{n'k'}(\vec r)\, u_{nk}(\vec r)$$

$$= \delta_{kk'}\, \underline{\frac{1}{\Omega} \int_\Omega u^*_{n'k'}(\vec r)\, u_{nk}(\vec r)} \quad = \delta_{nn'}$$

or $\quad \delta \langle \psi_{n'k'}|\psi_{nk}\rangle = \delta_{nn'}\, \delta(k-k')$

$$= \frac{1}{V}\, \frac{N(2\pi)^3}{\Omega}\, \delta(\vec k-\vec k') \int_\Omega u^*_{n'k}(\vec r)\, u_{nk}(\vec r)$$

$$= \boxed{\frac{(2\pi)^3}{\Omega^2}}\, \delta(k-k') \int_\Omega u^*_{n'k}(\vec r)\, u_{nk}(\vec r)$$

$$= \delta_{nn'}$$

$\cancel{|\Psi(t)\rangle = \Sigma}$

$$\langle \psi_{n'k'} | \vec{p} | \psi_{nk} \rangle = \int_V \psi^*_{n'k'} (-i\hbar \nabla_r)\, \psi_{nk}(\vec{r})\, d^3r$$

$$= \int_V e^{-i\vec{k}'\cdot\vec{r}}\, u^*_{n'k'}(\vec{r})\, (-i\hbar\nabla_r)\, e^{i\vec{k}\cdot\vec{r}}\, u_{nk}(\vec{r})\, d^3r$$

$$= \int_V e^{-i\vec{k}'\cdot\vec{r}}\, u^*_{n'k'}(\vec{r}) \Big\{ \hbar\vec{k}\, e^{i\vec{k}\cdot\vec{r}}\, u_{nk}(\vec{r}) + e^{i\vec{k}\cdot\vec{r}} \left[-i\hbar\nabla_r u_{nk}(\vec{r}) \right] \Big\}\, d^3r$$

$$= \hbar\vec{k}\, \delta_{nn'}\, \delta(\vec{k}-\vec{k}') + \int_V e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}\, u^*_{n'k'}(-i\hbar\nabla_r u_{nk})\, d^3r$$

$$= \hbar\vec{k}\, \delta_{nn'}\, \delta(\vec{k}-\vec{k}') + \delta(\vec{k}-\vec{k}') \int_\Omega u^*_{n'k}(\vec{r})\, (-i\hbar\nabla_r)\, u_{nk}(\vec{r})\, d^3r$$

$$= \underbrace{\hbar\vec{k}}_{\text{처음해로}}\, \delta_{nn'}\, \delta(\vec{k}-\vec{k}') + \delta(\vec{k}-\vec{k}')\, \vec{P}_{n'n}(\vec{k})$$

$$\vec{P}_{n'n}(\vec{k}) = \int_\Omega u^*_{n'k}(\vec{r})\, (-i\hbar\nabla_r)\, u_{nk}(\vec{r})\, d^3r$$

$$\langle \psi_{n'k'} | \vec{r} | \psi_{nk} \rangle = \int_V \psi^*_{n'k'}(\vec{r})\, \vec{r}\, \psi_{nk}(\vec{r})\, d^3r$$

$$= \int_V e^{-i\vec{k}'\cdot\vec{r}}\, u^*_{n'k'}(\vec{r})\, \vec{r}\, e^{i\vec{k}\cdot\vec{r}}\, u_{nk}(\vec{r})\, d^3r$$

$$\cancel{= \sum_{R_l} \int_V e^{i(\vec{k}-\vec{k}')(\vec{R}_l+\vec{r})}\, u^*_{n'k'}(\vec{r})\, (\vec{R}_l+\vec{r})\, u_{nk}(\vec{r})\, d^3r}$$

$$= \int_V e^{-i\vec{k}'\cdot\vec{r}}\, u^*_{n'k'}(\vec{r}) \left[(-i\nabla_k)\, e^{i\vec{k}\cdot\vec{r}} \right] u_{nk}(\vec{r})\, d^3r$$

$$= \int_V e^{-i\vec{k}'\cdot\vec{r}}\, u^*_{n'k'}(\vec{r}) \Big\{ -i\nabla_k \left[e^{i\vec{k}\cdot\vec{r}}\, u_{nk}(\vec{r}) \right] + i\, e^{i\vec{k}\cdot\vec{r}} \left[\nabla_k u_{nk}(\vec{r}) \right] \Big\}\, d^3r$$

$$= -i\nabla_k \int_V e^{-i\vec{k}'\cdot\vec{r}}\, u^*_{n'k'}(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}\, u_{nk}(\vec{r}).$$

$$+ \int_V e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}\, u^*_{n'k'}(\vec{r}) \left[i\nabla_k u_{nk}(\vec{r}) \right] d^3r$$

$$= -i\nabla_k\left[\delta_{nn'}\,\delta(\vec k-\vec k')\right] + \delta(\vec k-\vec k')\int_\Omega u^*_{n'k}(\vec r)\left[i\nabla_k u_{nk}(\vec r)\right]$$

$$= -i\nabla_k\,\delta(\vec k-\vec k')\,\delta_{nn'} + \delta(\vec k-\vec k')\,\vec X_{n'n}(\vec k) \qquad \vec A_n(\vec k) = \vec X_{nn}(\vec k)$$

$$= \delta_{nn'}\left\{-i\nabla_k + \vec A_n(\vec k)\right\}\delta(\vec k-\vec k') + \delta(\vec k-\vec k')\,\vec X_{n'n}(\vec k)(1-\delta_{nn'})$$

Berry connection

$$[\hat{H},\hat{r}] = \left[\frac{\vec p^{\,2}}{2m}+V(\vec r),\ \vec r\right] = \left[\frac{\vec p^{\,2}}{2m},\ \vec r\right]$$

$$= \frac{-i\hbar}{m}\,\vec p$$

$$\frac{-i\hbar}{m}\langle\psi_{n'k'}|\vec p|\psi_{nk}\rangle = \langle\psi_{n'k'}|\hat H\vec r-\vec r\hat H|\psi_{nk}\rangle$$

$$= \mathcal{E}_{n'}(\vec k')\langle\psi_{n'k'}|\vec r|\psi_{nk}\rangle - \langle\psi_{n'k'}|\vec r|\psi_{nk}\rangle\,\mathcal{E}_n(\vec k)$$

$$= \left[\mathcal{E}_{n'}(\vec k')-\mathcal{E}_n(\vec k)\right]\langle\psi_{n'k'}|\vec r|\psi_{nk}\rangle$$

$$= \left[\mathcal{E}_{n'}(\vec k')-\mathcal{E}_n(\vec k)\right]\Big\{\delta_{nn'}\left[-i\nabla_k+\vec A_n(\vec k)\right]\delta(\vec k-\vec k') + \delta(\vec k-\vec k')\,X_{n'n}(\vec k)\,\delta(1-\delta_{n'n})\Big\}$$

$$= \delta_{nn'}\left[\mathcal{E}_{n'}(\vec k')-\mathcal{E}_n(\vec k)\right]\left[-i\nabla_k\,\delta(\vec k-\vec k')\right]$$
$$+ \left[\mathcal{E}_n(\vec k')-\mathcal{E}_n(\vec k)\right]A_n(\vec k)\,\delta(\vec k-\vec k')$$
$$+ \delta(\vec k-\vec k')\left[\mathcal{E}_{n'}(\vec k)-\mathcal{E}_n(\vec k)\right]X_{n'n}(\vec k)(1-\delta_{n'n})$$

$$= \delta_{nn'}(-i\nabla_k)\left\{\left[\mathcal{E}_n(\vec k')-\mathcal{E}_n(k)\right]\delta(\vec k-\vec k')\right\}$$
$$+ \left\{i\nabla_k\left[\mathcal{E}_n(\vec k')-\mathcal{E}_n(\vec k)\right]\right\}\delta(\vec k-\vec k')$$
$$+ \delta(\vec k-\vec k')\left[\mathcal{E}_{n'}(\vec k)-\mathcal{E}_n(\vec k)\right]X_{n'n}(\vec k)(1-\delta_{n'n})$$

$$= -i\left[\nabla_k \mathcal{E}_n(\vec{k})\right] \delta(\vec{k}-\vec{k}')\,\delta_{nn'} + \delta(\vec{k}-\vec{k}')\left[\mathcal{E}_{n'}(\vec{k})-\mathcal{E}_n(\vec{k})\right] X_{n'n}(\vec{k})\,(1-\delta_{n'n})$$

of $n' \neq n$

$$-\frac{i\hbar}{m}\vec{P}_{n'n}(\vec{k}) = \left[\mathcal{E}_{n'}(\vec{k}) - \mathcal{E}_n(\vec{k})\right]\vec{X}_{n'n}(\vec{k})$$

$$\vec{X}_{n'n}(\vec{k}) = \frac{i\hbar}{m}\frac{\vec{P}_{n'n}(\vec{k})}{\mathcal{E}_n(\vec{k}) - \mathcal{E}_{n'}(\vec{k})} \Leftarrow \text{ of } \mathcal{E}_n \neq \mathcal{E}_{n'}$$

对于非简并的带

$$\vec{X}_{n'n} \sim \frac{i\hbar\vec{P}_{n'n}}{m E_g} \ll 1$$

of $n = n'$

$$\langle \psi_{n'k}|\vec{p}|\psi_{nk}\rangle = \left[\hbar\vec{k} + \vec{P}_{n'n}(\vec{k})\right]\delta(\vec{k}-\vec{k}')$$

$$-\frac{i\hbar}{m_e}\langle \psi_{nk}|\vec{p}|\psi_{nk}\rangle = \left[-i\nabla_k \mathcal{E}_n(\vec{k})\right]\delta(\vec{k}-\vec{k}')$$

$$= -i\nabla_k \frac{\hbar^2\vec{k}}{2m^*} = -\frac{i\hbar^2\vec{k}}{m^*}$$

$$\therefore \langle \psi_{nk}|\vec{p}|\psi_{nk}\rangle = \frac{m_e\hbar}{m^*}\vec{k}$$

$$\vec{P}_{nn}(\vec{k}) = \boxed{\frac{m_e\hbar}{m^*}\vec{k} - \hbar\vec{k}}$$

§3 ~~Wave packet~~ motion of wave-packet

$$\vec{k}\,|\psi_{nk}\rangle = k\,|\psi_{nk}\rangle$$

$$|\psi(t)\rangle = \sum_{nk} g_{nk}(t)\,|\psi_{nk}\rangle$$

$$\vec{k}_c(t) = \sum_{n'k'nk} g^*_{n'k'}(t)\, g_{nk}(t)\, \underbrace{\langle\psi_{n'k'}|\vec{k}|\psi_{nk}\rangle}_{\vec{k}\,\delta_{nn'}\,\delta(\vec{k}-\vec{k}')}$$

$$= \sum_{nk} \vec{k}\,|g_{nk}(t)|^2$$

$$\vec{r}_c(t) = \sum_{n'k'nk} g^*_{n'k'}(t)\, g_{nk}(t)\,\langle\psi_{n'k'}|\vec{r}|\psi_{nk}\rangle$$

$$= \sum_{n'nk} \int dk'dk\; g^*_{n'k'}(t)\, g_{nk}(t)\,\{\delta_{nn'}[-i\nabla_k + \vec{A}_n(\vec{k})]\,\delta(\vec{k}-\vec{k}') + \underbrace{(1-\delta_{n'n})\,\vec{X}_{n'n}(\vec{k})\,\delta(\vec{k}-\vec{k}')}_{\simeq 0}\}$$

$$= \sum_n \int d^3k\,d^3k'\; g^*_{n'k'}(t)\,[i\nabla_k g_{nk}(t) + \vec{A}_n(\vec{k})\,g_{nk}(t)]\,\delta(\vec{k}-\vec{k}')$$

$$= \sum_n \int d^3k\; g^*_{nk}(t)\,[i\nabla_k + \vec{A}_n(\vec{k})]\,g_{nk}(t)$$

$$= \sum_n \int d^3k\; g^*_{nk}(t)\,(i\vec{D}_k)\,g_{nk}(t)$$

$$\vec{D}_k = \nabla_k - i\vec{A}_n(\vec{k}).$$

$$g_{nk}(0) = C\, e^{-i\vec{k}_0\cdot\vec{r}_c(0) - \frac{(\vec{k}-\vec{k}_c(0))^2}{\Delta^2}}$$

Time evolution

$$i\hbar \partial_t |\Psi(t)\rangle = \mathcal{H} |\Psi(t)\rangle$$

$$\mathcal{H} = \mathcal{H}_0 + e\vec{E}\cdot\vec{r} = \mathcal{H}_0 - \vec{F}\cdot\vec{r}$$

$$i\hbar\partial_t |\Psi(t)\rangle = (\mathcal{H}_0 - \vec{F}\vec{r}) |\Psi(t)\rangle$$

$$\sum_{nk} i\hbar\partial_t g_{nk}(t) |\Psi_{nk}\rangle = (\mathcal{H}_0 - \vec{F}\vec{r}) \sum_{nk} g_{nk}(t) |\Psi_{nk}\rangle$$

$$= \sum_{nk} \varepsilon_n(\vec{k})\, g_{nk}(t) |\Psi_{nk}\rangle - \sum_{nk} g_{nk}(t) \vec{F}\cdot\vec{r} |\Psi_{nk}\rangle$$

$\langle\Psi_{n'k'}|$

$$\Rightarrow i\hbar\partial_t g_{n'k'}(t) = \varepsilon_{n'}(\vec{k}')\, g_{n'k'}(t) - \sum_{nk} g_{nk}(t) \langle\Psi_{n'k'}|\vec{F}\vec{r}|\Psi_{nk}\rangle$$

$$= \varepsilon_{n'}(\vec{k}')\, g_{n'k'}(t) - \sum_n \int dk\, g_{nk}(t)\, \vec{F}\cdot\{(-i\nabla_k + d_n)\,\delta_{nn'}\,\delta(\vec{k}-\vec{k}') + (1-\delta_{n'n})\, X_{n'n}(\vec{k})\,\delta(\vec{k}-\vec{k}')\}$$

$$= \varepsilon_{n'}(\vec{k}')\, g_{n'k'}(t) - \vec{F}\cdot[i\nabla_k + \vec{d}_n(\vec{k}')]\, g_{n'k'}(t)$$

$$- \sum_n (1-\delta_{n'n}) \underbrace{\vec{F}\cdot\vec{X}_{n'n}(\vec{k})}_{\ll 1}\, g_{nk}(t)$$

$$i\hbar\partial_t g_{nk}(t) = [\varepsilon_n(\vec{k}) - i\vec{F}\cdot\vec{D}_k]\, g_{nk}(t)$$

$$\cancel{\tilde{g}_{nk}(t) = e^{-i\varepsilon_n(k)t/\hbar}\, \tilde{g}_{nk}(t)}$$

$$\cancel{i\hbar\partial_t \tilde{g}_{nk}(t)}$$

$$-i\hbar\partial_t g^*_{nk}(t) = \varepsilon_n(k)\, g^*_{nk}(t) - \vec{F}[-i\nabla_k + \vec{d}_n(\vec{k})]\, g^*_{nk}(t)$$

$$= \varepsilon_n(k)\, g^*_{nk} + i\vec{F}\cdot\vec{D}^*_{nk}\, g^*_{nk}$$

$$i\partial_t |g_{nk}|^2 = i g^*_{nk}\partial_t g_{nk} + i g_{nk}\partial_t g^*_{nk}$$

$$= g^*_{nk}[\varepsilon_{nk} - i\vec F\cdot\vec D_{nk}]\, g_{nk} - g_{nk}[\varepsilon_{nk} + i\vec F\cdot\vec D^*_{nk}]\, g^*_{nk}$$

$$= -i\vec F\, g^*_{nk}\cdot\vec D_{nk}\, g_{nk} - i\vec F(\vec D^*_{nk}\, g^*_{nk})\, g_{nk}$$

$$i\frac{d}{dt}\vec k_c(t) = \sum_{nk}\int i\partial_t(|g_{nk}|^2)\,\vec k\, d\vec k$$

$$= -i\vec F\sum_n\int \left[g^*_{nk}(\vec D_{nk}\, g_{nk}) + (\vec D^*_{nk}\, g^*_{nk})\, g_{nk}\right]\vec k\, d\vec k$$

$$= -i\vec F\sum_n\int g^*_{nk}(\vec D_{nk}\, g_{nk})\,\vec k + [(\nabla_k + i\vec A_n)\, g^*_{nk}]\, g_{nk}\,\vec k\, d\vec k$$

$$= -i\vec F\sum_n\int g^*_{nk}(\vec D_{nk}\, g_{nk})\,\vec k + [g^*_{nk}\underbrace{(-\nabla_k + i\vec A_n)}_{=-\vec D_{nk}}(g_{nk}\vec k)]$$

$$= -i\vec F\sum_n\int\{g^*_{nk}(\vec D_{nk}\, g_{nk})\,\vec k + g^*_{nk}[(-\nabla_k + i\vec A_n)\, g_{nk}]\,\vec k$$

$$+ g^*_{nk}(-g_{nk}\vec k)\}$$

$$= +i\vec F\sum_n\int |g_{nk}|^2\, \cancel{k}\, d^3\vec k$$

$$= i\vec F$$

$$\frac{d}{dt}\vec k_c(t) = \vec F$$

$$i\hbar \frac{d}{dt} \vec{r}_c(t) = \sum_{nk} \int \left[\cancel{i\partial_t}\, |g_{nk}|^2\right] \left[-i \nabla_k g\right]$$

$$= i \frac{d}{dt} \sum_k \int dk \quad g^*_{nk}(\vec{k},t)\; i\, \vec{D}_{nk}\; g_{nk}(\vec{k},t)$$

$$= \sum_k \int dk \Big\{ \left[i\partial_t g^*_{nk}(t) \right] \left(i \vec{D}_{nk} \right) g_{nk}$$

$$+ g^*_{nk}(t) \left(i \vec{D}_{nk} \right) \underline{i\partial_t g_{nk}} \Big\}$$

$$= \sum_n \int dk \; \Big\{ -\varepsilon_n(\vec{k}) \overset{g^*_{nk}}{\cancel{+}} \underline{- i \vec{F} \cdot \vec{D}^{\dagger}_{nk}\; g^*_{nk} \Big\} \; i \vec{D}_{nk} \cdot g_{nk}}$$

$$+ \left\{ g^*_{nk} \right\} \underline{\left(i \vec{D}_{nk} \right)} \left\{ \varepsilon_n(\vec{k}) \overset{g_{nk}}{\,} - i \vec{F} \cdot \vec{D}_{nk}\; g_{nk} \right\}$$

$$\shortparallel$$

$$i \left[\vec{D}_{nk}, \varepsilon_n(\vec{k}) \right] g_{nk} + i \varepsilon_n(\vec{k})\, \vec{D}_{nk}\, g_{nk}$$

$$+ \vec{D}_{nk} \cdot \vec{D}_{nk} \cdot \vec{F} \quad g_{nk}.$$

$$\int \vec{F} \left(\vec{D}^{*}_{nk}\, g^*_{nk} \right) \; \vec{D}_{nk}\; g_{nk}$$

$$= -\vec{F} \cdot g^{\dagger}_{nk}\; \vec{D}_{nk}\; \vec{D}_{nk}\; g_{nk}$$

$$= \sum_n \int dk \; \Big\{ -\vec{F}\; g^*_{nk} \; \left[\vec{D}_{nk}, \vec{D}_{nk} \right] g_{nk} \; + i \; g^{\dagger}_{nk} \left[\vec{D}_{nk}, \varepsilon_n(\vec{k}) \right] g_{nk} \Big\}$$

$$\hbar \dot{\vec{r}}_c(t) = \sum_n \int dk \quad g^*_{nk} \left[\nabla_k \varepsilon_n(\vec{k}) \right] g_{nk} \quad + g^*_{nk}\, \vec{F}\; i \left[\vec{D}_{nk}, \vec{D}_{nk} \right] \; g_{nk}$$

$$= \sum_n \int dk \; g^*_{nk} \quad \left[\nabla_k \varepsilon_n(\vec{k}) - \vec{F} \times \vec{\Omega}_n(\vec{k}) \right] g_{nk}$$

Berry curvature

$$\cancel{\left[\vec{D}_{\ell}, \vec{D}_{m} \right] = \left[\partial_\ell - i A_\ell,\; \partial_m - i A_m \right]} \quad \Bigg| \quad \cancel{\partial_\ell \partial_m - i \partial_\ell A_m} \; -(\partial_m \partial_\ell$$

$$[D_{nk}^{\alpha}, D_{nk}^{\beta}] = [\partial_k^{\alpha} - i A_n^{\alpha}(\vec{k}),\ \partial_k^{\beta} - i A_n^{\beta}(\vec{k})]$$

$$= -i\,[\partial_k^{\alpha}, A_n^{\beta}(\vec{k})] - i\,[A_n^{\alpha}(\vec{k}), \partial_k^{\beta}]$$

$$= -i\,\partial_k^{\alpha} A_n^{\beta}(\vec{k}) + i\,\partial_k^{\beta} A_n^{\alpha}(\vec{k})$$

$$\vec{F} \cdot i\,[\vec{D}_{nk}, \vec{D}_{nk}] = i F^{\alpha} [-i\,\partial_k^{\alpha} A_n^{\beta}(\vec{k}) + i\,\partial_k^{\beta} A_n^{\alpha}(\vec{k})]$$

$$= F^{\alpha} \underline{[\partial_k^{\alpha} A_n^{\beta}(\vec{k}) - \partial_k^{\beta} A_n^{\alpha}(\vec{k})]}$$

$$= F^{\alpha}\, \Omega_n^{\alpha\beta}(\vec{k}) \quad //$$

$$\hbar \dot{\vec{r}}_c(t) = \sum_n \int dk\; g_{nk}^{*} [\nabla_k \varepsilon_n(\vec{k}) + \vec{F} \cdot \overleftrightarrow{\Omega}_n(\vec{k})]$$

$$\approx \sum_n \nabla_k \varepsilon_n(\vec{k}_c) + \vec{F} \cdot \overleftrightarrow{\Omega}_n(\vec{k}_c)$$

in 3D

$$\vec{A} \times (\vec{B} \times \vec{C}) = \epsilon_{ijk} A_j \epsilon_{klm} B_l C_m \;\hat{e}_i$$

$$= \epsilon_{ijk} \epsilon_{lmk} A_j B_l C_m \vec{e}_i$$

$$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) A_j B_l C_m \vec{e}_i$$

$$= \underbrace{A_j B_i C_j}\, \vec{e}_i - \underbrace{A_j B_j}\, C_i\, \vec{e}_i$$

$$= \underbrace{A \vec{B} C} - \underbrace{A B}\, \vec{C}$$

$$F^{\alpha} \Omega_n^{\alpha\beta}(\vec{k}) = \underbrace{F^{\alpha} \partial_k^{\alpha} A_n^{\beta}(\vec{k})} - \underbrace{F^{\alpha} \partial_k^{\beta} A_n^{\alpha}(\vec{k})}$$

$$= -\vec{F} \times [\nabla \times \vec{A}_n(\vec{k})] = -\vec{F} \times \vec{\Omega}_n(\vec{k}).$$

$$\hbar \dot{\vec{r}}_c(t) \approx [\nabla_k \varepsilon_n(\vec{k}_c) - \vec{F} \times \vec{\Omega}_n(\vec{k}_c)]$$ Berry curvature

with magnetic field

$$\nabla \times \vec{A}(\vec{r}) = \vec{B}(\vec{r})$$

$$\vec{p} \to \vec{p} + e\vec{A}(\vec{r})$$

$$\mathcal{H} = \frac{(\vec{p} + e\vec{A})^2}{2m} + V(\vec{r}) + e\vec{E}\cdot\vec{r}$$

$$\vec{F} = -e\{\vec{E}(\vec{r}) - \vec{B}(\vec{r}) \times \vec{v}(\vec{r})\}$$

$$\Rightarrow -e\{\vec{E}(\vec{r}_c) + \dot{\vec{r}}_c(t) \times \vec{B}(\vec{r}_c)\}$$

$$= -e\{\vec{E}(\vec{r}_c) + \dot{\vec{r}}_c(t) \times [\nabla \times \vec{A}(\vec{r}_c)]\}$$

$$\hbar\dot{\vec{k}}_c(t) = \vec{F} = -e\{\vec{E}(\vec{r}_c) + \dot{\vec{r}}_c(t) \times \vec{B}(\vec{r}_c)\}$$

$$= -e\{\vec{E}(\vec{r}_c) + \dot{\vec{r}}_c(t) \times [\nabla_r \times \vec{A}(\vec{r}_c)]\}$$

$$\hbar\dot{\vec{r}}_c(t) = \nabla_k \varepsilon_n(\vec{k}_c) - \hbar\dot{\vec{k}}_c \times \vec{\Omega}_n(\vec{k}_c)$$

$$= \nabla_k \varepsilon_n(\vec{k}_c) - \hbar\dot{\vec{k}}_c \times [\nabla_k \times \vec{A}_n(\vec{k}_c)]$$