# Problèmes quantiques

## Développements mathématiques

Circuits et développements mathématiques de problèmes quantiques.


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Automne 2022

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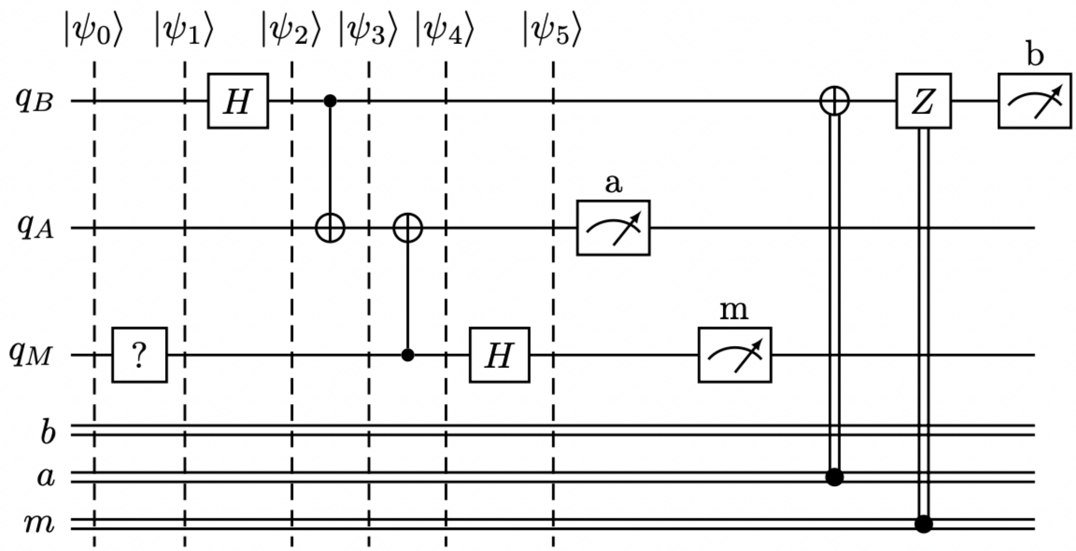
## Teleportation quantique




$$\begin{aligned}
|\psi_0\rangle &= |000\rangle \\
&= |0\rangle\,|0\rangle\,|0\rangle
\end{aligned}$$

$$\begin{aligned}
|\psi_1\rangle &= \hat{?}\,|\psi_0\rangle \\
&= (a\,|0\rangle + b\,|1\rangle)\,|00\rangle \\
&= a\,|0\rangle\,|0\rangle\,|0\rangle + b\,|1\rangle\,|0\rangle\,|0\rangle
\end{aligned}$$

$$\begin{aligned}
|\psi_2\rangle &= \hat{H}_0\,|\psi_1\rangle \\
&= a\,|0\rangle\,|0\rangle\left(\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\right) \quad + \quad b\,|1\rangle\,|0\rangle\left(\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\right) \\
&= \frac{a}{\sqrt{2}}\,|0\rangle\,|0\rangle\,|0\rangle + \frac{a}{\sqrt{2}}\,|0\rangle\,|0\rangle\,|1\rangle \quad + \quad \frac{b}{\sqrt{2}}\,|1\rangle\,|0\rangle\,|0\rangle + \frac{b}{\sqrt{2}}\,|1\rangle\,|0\rangle\,|1\rangle
\end{aligned}$$

$$
\begin{aligned}
|\psi_3\rangle &= \hat{CX}_{01}|\psi_2\rangle \\
&= \tfrac{a}{\sqrt{2}}|0\rangle|0\rangle|0\rangle + \tfrac{a}{\sqrt{2}}|0\rangle|1\rangle|1\rangle + \tfrac{b}{\sqrt{2}}|1\rangle|0\rangle|0\rangle + \tfrac{b}{\sqrt{2}}|1\rangle|1\rangle|1\rangle
\end{aligned}
$$

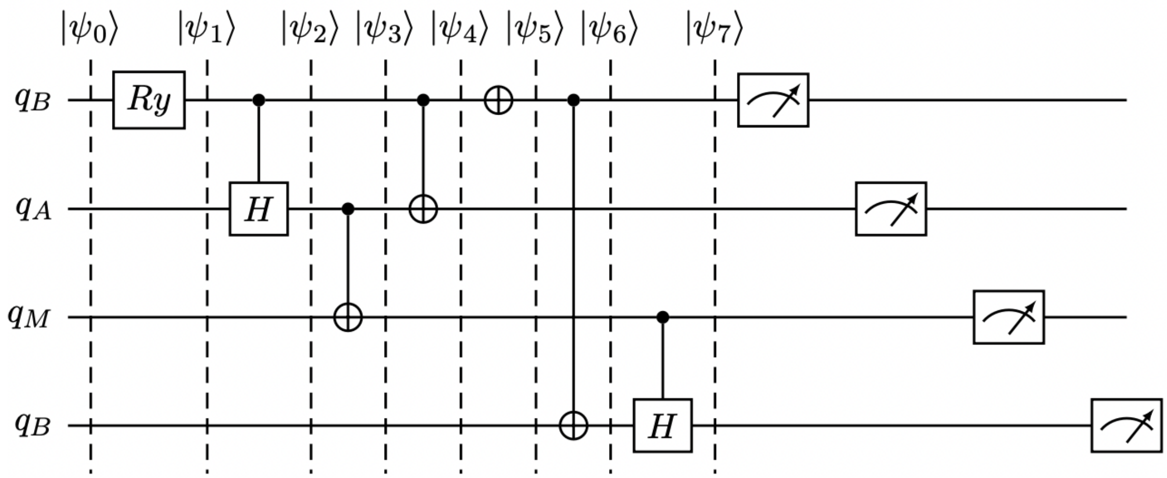
$$
\begin{aligned}
|\psi_4\rangle &= \hat{CX}_{21}|\psi_3\rangle \\
&= \tfrac{a}{\sqrt{2}}|0\rangle|0\rangle|0\rangle + \tfrac{a}{\sqrt{2}}|0\rangle|1\rangle|1\rangle + \tfrac{b}{\sqrt{2}}|1\rangle|1\rangle|0\rangle + \tfrac{b}{\sqrt{2}}|1\rangle|0\rangle|1\rangle
\end{aligned}
$$

$$
\begin{aligned}
|\psi_5\rangle &= \hat{H}_2|\psi_4\rangle \\
&= \tfrac{a}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right)|0\rangle|0\rangle + \tfrac{a}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right)|1\rangle|1\rangle + \\
&\quad \tfrac{b}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle\right)|1\rangle|0\rangle + \tfrac{b}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle\right)|0\rangle|1\rangle \\
&= \tfrac{a}{2}|0\rangle|0\rangle|0\rangle + \tfrac{a}{2}|1\rangle|0\rangle|0\rangle + \tfrac{a}{2}|0\rangle|1\rangle|1\rangle + \tfrac{a}{2}|1\rangle|1\rangle|1\rangle + \\
&\quad \tfrac{b}{2}|0\rangle|1\rangle|0\rangle - \tfrac{b}{2}|1\rangle|1\rangle|0\rangle + \tfrac{b}{2}|0\rangle|0\rangle|1\rangle - \tfrac{b}{2}|1\rangle|0\rangle|1\rangle \\
&= \tfrac{a}{2}|000\rangle + \tfrac{b}{2}|001\rangle + \tfrac{a}{2}|100\rangle - \tfrac{b}{2}|101\rangle + \\
&\quad \tfrac{a}{2}|011\rangle + \tfrac{b}{2}|010\rangle + \tfrac{a}{2}|111\rangle - \tfrac{b}{2}|110\rangle \\
&= \tfrac{1}{2}|00\rangle(a|0\rangle + b|1\rangle) + \tfrac{1}{2}|10\rangle(a|0\rangle - b|1\rangle) + \\
&\quad \tfrac{1}{2}|01\rangle(a|1\rangle + b|0\rangle) + \tfrac{1}{2}|11\rangle(a|1\rangle - b|0\rangle)
\end{aligned}
$$

En mesurant $M$ et $A$ on obtient que:

$$\begin{array}{lllcl}
\text{si } \ket{00} & \ket{\psi} & a\ket{0}+b\ket{1} & \rightarrow & a\ket{0}+b\ket{1} \\
\text{si } \ket{01} & \ket{\psi} & a\ket{1}+b\ket{0} & \xrightarrow{\hat{X}} & a\ket{0}+b\ket{1} \\
\text{si } \ket{10} & \ket{\psi} & a\ket{0}-b\ket{1} & \xrightarrow{\hat{Z}} & a\ket{0}+b\ket{1} \\
\text{si } \ket{11} & \ket{\psi} & a\ket{1}-b\ket{0} & \xrightarrow{\hat{X} puis \hat{Z}} & a\ket{0}+b\ket{1}
\end{array}$$

## Énigme 004 - Problème du Monty Hall




$$
\begin{aligned}
|\psi_0\rangle &= |0000\rangle \\
&= |0\rangle\,|0\rangle\,|0\rangle\,|0\rangle
\end{aligned}
$$

$$
\begin{aligned}
|\psi_1\rangle &= \hat{R_{y_0}}\,|\psi_0\rangle \\
&= \left(\frac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\right) + \left(\frac{\sqrt{2}}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_2\rangle &= C\hat{H}_{01}\,|\psi_1\rangle \\
&= \left(\frac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\right) + \left(\frac{\sqrt{2}}{\sqrt{3}}\,|0\rangle\,|0\rangle\left(\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\right)|1\rangle\right) \\
&= \left(\frac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\right) + \left(\frac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right) + \left(\frac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|1\rangle\,|1\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_3\rangle &= C\hat{X}_{12}\,|\psi_2\rangle \\
&= \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|1\rangle\,|1\rangle\,|1\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_4\rangle &= C\hat{X}_{01}\,|\psi_3\rangle \\
&= \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|1\rangle\,|1\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|1\rangle\,|0\rangle\,|1\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_5\rangle &= \hat{X}_{0}\,|\psi_4\rangle \\
&= \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|1\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|1\rangle\,|0\rangle\,|0\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_6\rangle &= C\hat{X}_{03}\,|\psi_5\rangle \\
&= \left(\tfrac{1}{\sqrt{3}}\,|1\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|1\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|1\rangle\,|0\rangle\,|0\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_7\rangle &= C\hat{H}_{23}\,|\psi_6\rangle \\
&= \left(\tfrac{1}{\sqrt{3}}\,|1\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|1\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\left(\tfrac{1}{\sqrt{2}}\,|0\rangle + \tfrac{1}{\sqrt{2}}\,|1\rangle\right)|1\rangle\,|0\rangle\,|0\rangle\right) \\
&= \left(\tfrac{1}{\sqrt{3}}\,|1\rangle\,|0\rangle\,|0\rangle\,|1\rangle\right) + \left(\tfrac{1}{\sqrt{3}}\,|0\rangle\,|0\rangle\,|1\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{6}}\,|0\rangle\,|1\rangle\,|0\rangle\,|0\rangle\right) + \left(\tfrac{1}{\sqrt{6}}\,|1\rangle\,|1\rangle\,|0\rangle\,|0\rangle\right) \\
&= \tfrac{1}{\sqrt{3}}\,|1001\rangle + \tfrac{1}{\sqrt{3}}\,|0010\rangle + \tfrac{1}{\sqrt{6}}\,|0100\rangle + \tfrac{1}{\sqrt{6}}\,|1100\rangle
\end{aligned}
$$