Electric Charges and field

Electrostatic :- The branch of Physics which deals with charge at rest.

⇒ Electric charge :- Electric charge is an intrinsic property of matter of the fundamental particles that govern how the particles are affected by an electric or magnetic field.

Types of Charge :- Charges are of two types.
(i) Positive charge (ii) Negative charge.

* To show that there are two kinds of charges let us perform the following experiment -
If two glass rods rubbing with silk cloth are brought to close to each other, they repel each other. However if a glass rod rubbed with silk is brought near an ebonite rod rubbed with fur, they get attract to each other.
From this experiment we conclude that :-
(i) Like charges repel each other.
(ii) Unlike charges attract each other.

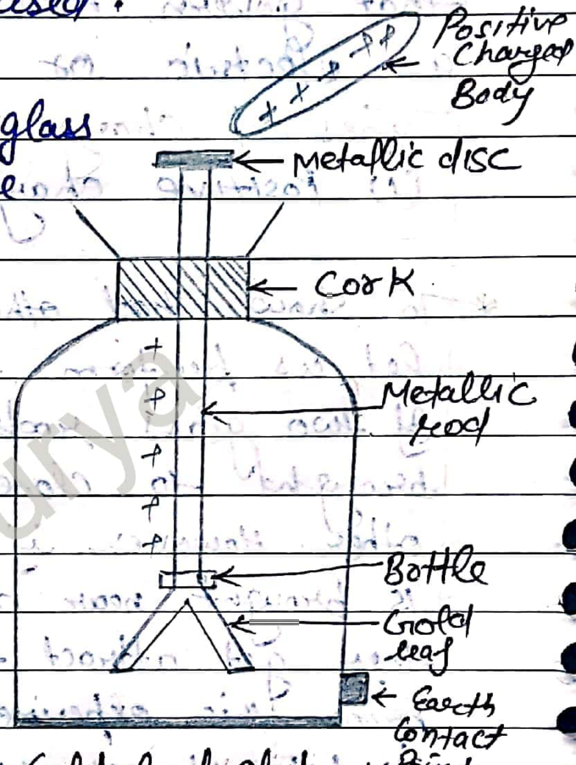
The pairs of objects which get charge on rubbing against each other are listed in table :-

Positive charge	Negative charge
Glass rod	Silk cloth
Fur or Cat skin	(i) Ebonite rod (ii) Plastic rod
Woollen Cloth	(i) Amber (ii) Ebonite

Unit of Charge :- The SI Unit of Charge is coulomb (C). It is represented by C.

⇒ Gold leaf Electroscope :- To detect the presence of charge on a body, a simple device called gold leaf electroscope is used.

According to figure there is a glass jar in a gold leaf electroscope in which a metallic rod is placed vertically. There is a metallic disc on the upper end of the rod whereas there are two gold strips tied at the lower end. when a charged body is taken to be in contact with metallic disc then some of charge get transfer to the gold strip and the gold strip move away from each other. If the body and electroscope have unlike charges, then the gold strip attract toward each other.



Gold leaf Electroscope

⇒ Conductor and Insulator :-

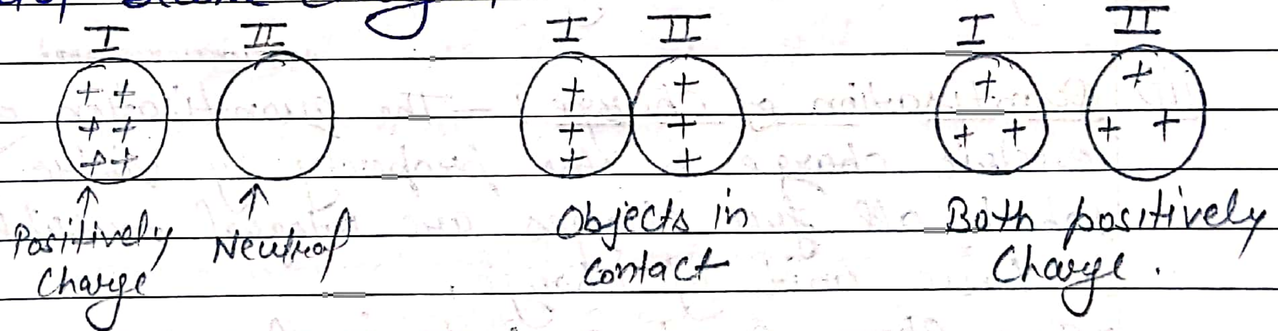
Conductor :- Those substances which allow electricity to pass through them easily are called Conductors.
ex - Metals, human, animals and earth.

Insulators :- Those substances which does not allow electricity to pass through them are called insulators. ex - Plastics, glass etc.

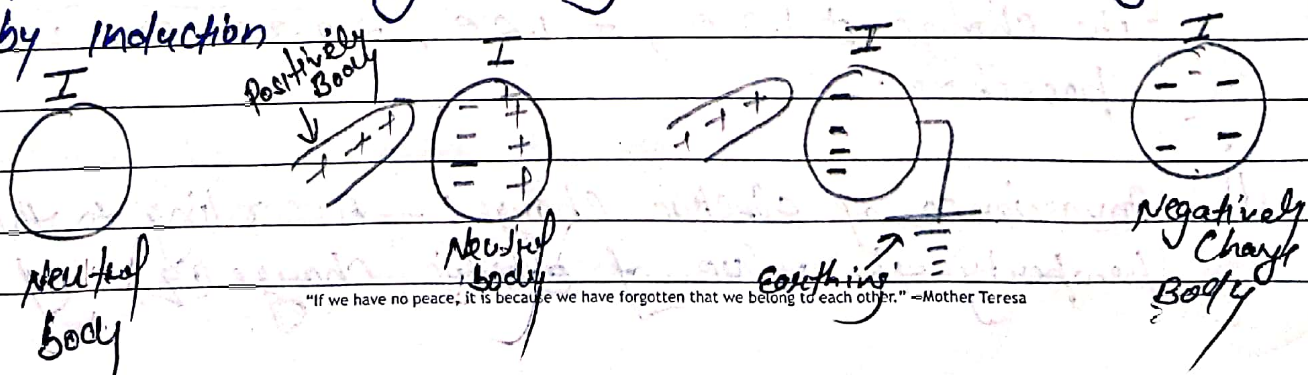
⇒ Methods to Charge the objects :-

(i) Charging by friction :- when two objects are rubbed against each other, then the generating electricity is called frictional electricity. In this method electrons transfered from one object to another.
The material which loses electron gets a positive charge and which gains electron gets an equal negative charge.

(ii) Charging by Conduction (Contact) :- when two objects are in contact, the charge transfers from one to another directly.
Let us consider a positively charge conducting plate be touched to a neutral body, then neutral metal become charged.



(iii) Charging by induction :- The process in which an uncharged body develops opposite kind of charge on it without in contact with charge body. Then the uncharged body is said to be charged by induction.



⇒ Basic Properties of Electric Charge :-

(I) **Additivity of Electric Charges :-** Additivity of charge is a property by virtue of which total charge of a system is obtained by adding algebraically all the charges present anywhere on the system.
For example if a system contains charge $+q$, $-2q$, $+3q$ and $+5q$ then total charge of the system is $= \boxed{+q-2q+3q+5q = +7q}$

(II) **Conservation of charge :-** Conservation of charge is the property by virtue of which total charge of an isolated system always remain constant.
The electric charge can neither be created nor be destroyed. It can only be transfered from one body to other body..

(III) **Quantisation of charge :-** The quantisation of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge.
Thus charge q of a body is always given by
$$\boxed{q = \pm ne}$$
Here n is the no. of electron / Proton
& e is the charge on electron / Proton.
Charge on electron is $\boxed{1.6 \times 10^{-19} \text{ Coulomb}}$
The charge $\pm 1.2e$, $\pm 1.6e$, $\pm 2.3e$ are not possible.

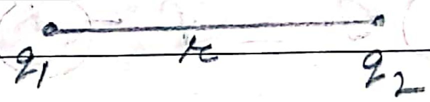
(IV) **Invariance of electric charge :-** According to this property the value of electric charge (q) of a

particle not depend on the speed of charge.
Charge at rest = Charge in motion.

$$\boxed{q_{rest} = q_{motion}}$$

⇒ <u>Coulomb's Law</u> :— According to Coulomb's Law, The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

suppose two point charge q_1 & q_2 are separated by a distance r.



then by coulomb's law

$$F \propto q_1 q_2 \quad \text{———} (i)$$

$$F \propto \frac{1}{r^2} \quad \text{———} (ii)$$

from eqn (i) & (ii) we get

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{F = K \frac{q_1 q_2}{r^2}}$$

Here K is electrostatic Constant.

$$\boxed{K = 9 \times 10^9 \, Nm^2 C^{-2}}$$

For vacuum or air

$$K = \frac{1}{4\pi\epsilon_0}$$

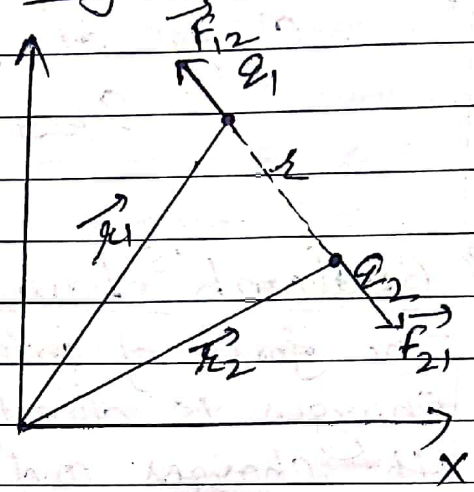
Here ϵ_0 (Epsilon not) is the permittivity of free space.

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \, C^2/Nm^2}$$

Thus for vacuum or air

$$\boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}}$$

⇒ Coulomb's Law in vector form :-



→ As shown in figure in XY plane we have taken two charges q_1 and q_2.

→ $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of charge q_1 and q_2 with respect to origin.

By coulomb law we know force between two charges

$$\vec{F} = K\frac{q_1 q_2}{r^2}\hat{r}$$

[Here $\hat{r}$ is unit vector $\boxed{\hat{r} = \frac{\vec{r}}{|r|}}$

Force on q_1 due to q_2

$$\vec{F}_{12} = \frac{K q_1 q_2}{|r_{12}|^2}\hat{r}_{12}$$

$$= \frac{K q_1 q_2}{|r_{12}|^3}\vec{r}_{12} \quad \left[\because \hat{r}_{12} = \frac{\vec{r}_{12}}{\vec{r}_{12}}\right]$$

$$\vec{F}_{12} = \frac{K q_1 q_2(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \quad \text{—— (i)}$$

Force on q_2 due to q_1

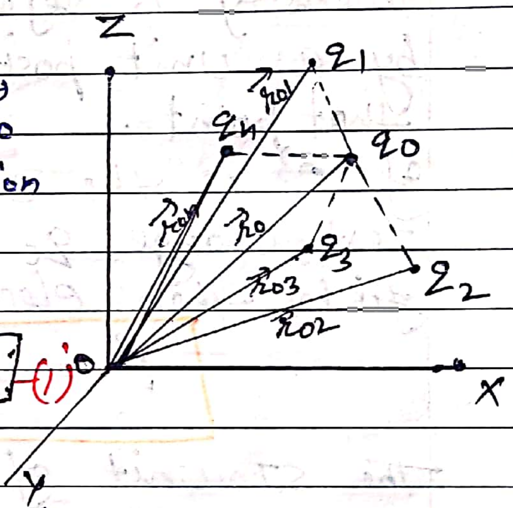
$$\vec{F}_{21} = \frac{K q_1 q_2}{|r_{21}|^2}\hat{r}_{21} \Rightarrow \frac{K q_1 q_2}{|r_{21}|^3}\vec{r}_{21} \Rightarrow \frac{K q_1 q_2(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3} \quad \text{—— (ii)}$$

From eqn (i) & (ii) we get

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Thus coulomb law obey's Newton third law of motion.

⇒ **Force between multiple charges OR Principle of superposition :** — According to superposition principle, total force on any charge due to a number of other charges at rest is the vector sum of all the force on that charge due to other charges.



In general form total force $\vec{F}_0$ on a test charge q_0 at position $\vec{r}_0$ due to all the n discrete charges can be written as

$$\boxed{F_0 = F_{01} + F_{02} + F_{03} + \cdots + F_{0n}} \quad (1)$$

According to Coulomb law

$$\vec{F}_{01} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{r_{01}^2}\hat{r}_{01}$$

$$\vec{F}_{02} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_2}{\vec{r}_{02}^{\,2}}\hat{r}_{02}$$

$$\vdots$$

$$\vec{F}_{0n} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_n}{r_{0n}^2}\hat{r}_{0n}$$

Putting these value in eq^n (1) we get

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0 q_1}{r_{01}^2}\hat{r}_{01} + \frac{q_0 q_2}{r_{02}^2}\hat{r}_{02} + \cdots + \frac{q_0 q_n}{r_{0n}^2}\hat{r}_{0n}\right]$$

$$\boxed{\vec{F}_0 = \frac{q_0}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{0i}^2}\hat{r}_{0i}}$$

⇒ Electric field :- A region around a charged particle within which a force would be exerted on other charged particles.

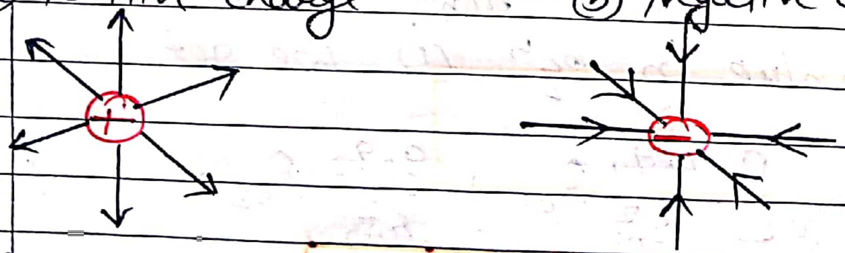
Electric field intensity (E) :- Electric field intensity is defined as the force experienced by a unit positive test charge placed at that point.
If $\vec{F}$ is the electrostatic force acting on a test charge q_0 at a point in electric field then electric field intensity at that point

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

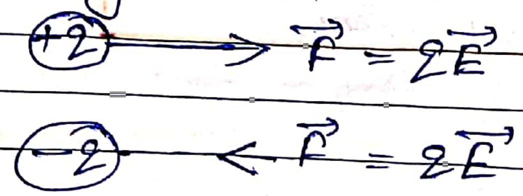
The SI unit of $\vec{E}$ is NC^{-1} (Newton per Coulomb)

Important facts :-
(i) Due to positive test charge the direction of electric field is radially outward from the charge and due to negative charge it is radially inwards.

(a) Positive charge (b) Negative Charge

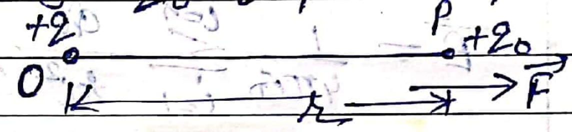


(ii) The direction of force on positive and Negative point charges will be according to the figure



(iii) The magnitude of force F on charge q due to Charge Q depends only on the distance r of the Charge q from the Charge Q. the magnitude of $\vec{E}$ will also depend only on the distance r.

⇒ Electric field intensity due to a point charge :– Consider a point charge q at point O as shown in figure.
suppose a test charge q_0 kept at point P which



is at distance r from O. we have to calculate intensity of Electric field at point P.
According to coulomb's law, force acting on charge q_0 is

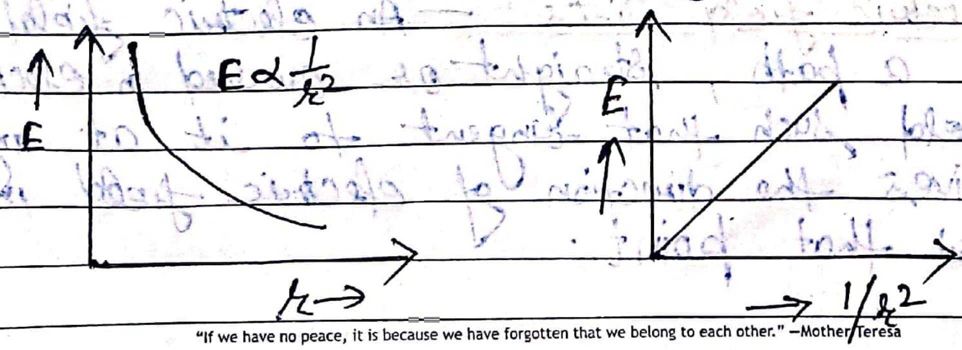
$$F = \frac{1}{4\pi\epsilon_0}\frac{q\,q_0}{r^2}$$

Therefore electric field intensity

$$E = \frac{F}{q_0} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q\,q_0}{r^2}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Hence $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\ N/C}$

Graph b/w electric field intensity and distance r



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⇒ Electric field intensity due to a group of charges:–
The electric field intensity at any point due to a group of point charges at the same point. For this, we first calculate the electric field intensity at the given point P due to individual charges and then add them vectorially i.e.

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \cdots + \vec{E_n}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{i=n} \frac{Q_i}{r_i^2} \hat{r}_i}$$

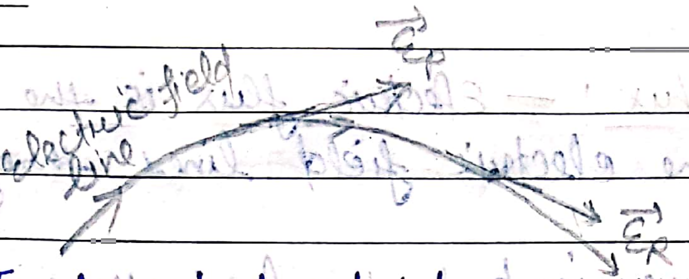
Here r_i is the distance of the point P from the ith charge Q_i and $\hat{r}_i$ is unit vector directed from Q_i to the point P.

⇒ Physical Significance of electric field intensity:–
From the knowledge of electric field intensity $\vec{E}$ at any point $\vec{r}$, we can readily calculate the magnitude and direction of force experienced by any charge q_0 held at that point.

$$\boxed{\vec{F}(r) = q_0 \vec{E}(r)}$$

This is the physical significance of electric field.

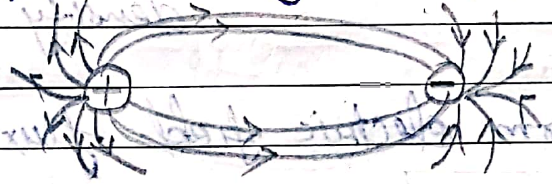
⇒ Electric field lines:– An electric field line as a path, straight or curved in electric field such that tangent to it at any point gives the direction of electric field intensity at that point.



The tangent at a point given the direction of Electric field at that point.

⇒ Properties of Electric field lines: —

(i) Electric field lines are continuous curve they begin from positive charge and terminate at negative charge.



(ii) No electric lines of force exist inside the charged body. Thus electric field lines are continous curve but do not form closed loops.

(iii) Tangent to the electric field line at any point give the direction of electric field intensity at that point.

(iv) No two electric lines of force can intersect each other. Because at the point of intersection if we draw tangents then they shows two direction of Electric field lines which is not possible.

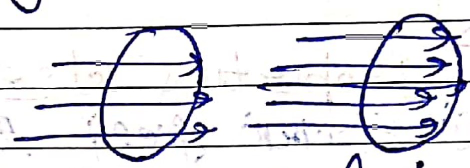
(v) The electric field lines are always normal to the surface of conductor.

(vi) The electric field lines exert lateral pressure due to repulsion between like charge.

⇒ **Electric flux :—** Electric flux is the rate of flow of the electric field lines through a given area.

Electric flux is proportional to the number of electric field lines going through a virtual surface.

For uniform electric field flux (ϕ_E)



$$\boxed{\phi_E = ES\cos\theta}$$

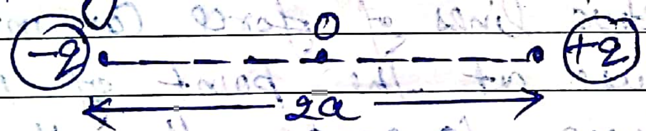
Flux is proportional to density of flow

For non uniform electric field flux is

$$\boxed{d\phi_E = E.ds}$$

SI unit of Electric flux is volt meters (Vm).

⇒ **Electric dipole :—** An electric dipole consist of a pair of equal and opposite point charges separated by some small distances.



Here -q & +q are the two point charges.
2a is the small distance of dipole.
The total charge on the electric dipole
⇒ -q + q = 0

This does not means that the field of the electric dipole is zero.
The line joining these charges is called axial line.

⇒ **Dipole moment ($\vec{P}$)** :— Dipole moment is measured the strength of electric dipole.

OR

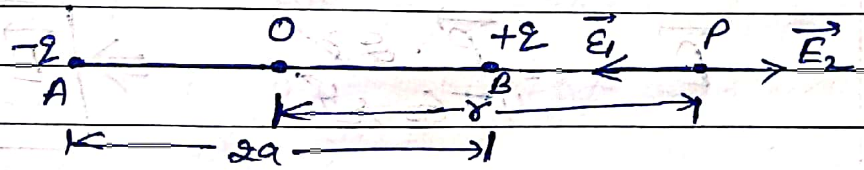
It is the product of the magnitude of either Charge and the distance between them.

$$\boxed{\vec{P} = q(2a)}$$

The direction of $\vec{P}$ is always from −ve Charge to +ve.
The SI unit of $\vec{P}$ is Coulomb-metre (C-m).

Dipole field :— It is the space around the dipole in which the electric effect of the dipole can be experienced.

⇒ **Electric field intensity on axial line of electric dipole** :— Consider an electric dipole Consisting of two point Charge $-q$ and $+q$ separated by small distance $2a$. Let we have calculate electric field intensity E at a point P on the axial line of the dipole.



If E_1 is the electric field intensity at P due to $-q$.

$$|\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} \text{ along } \vec{PA}$$

If E_2 is the $\vec{E}$ at P due to $+q$

$$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(BP)^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} \text{ along } \vec{BP}$$

Hence the resultant electric field intensity is

$$|\vec{E}| = |\vec{E_2}| - |\vec{E_1}|$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{4ar}{(r^2-a^2)^2} \Rightarrow \frac{q \times 2a \times 2r}{4\pi\epsilon_0 (r^2-a^2)^2}$$

But $q \times 2a = \vec{P}$

Hence

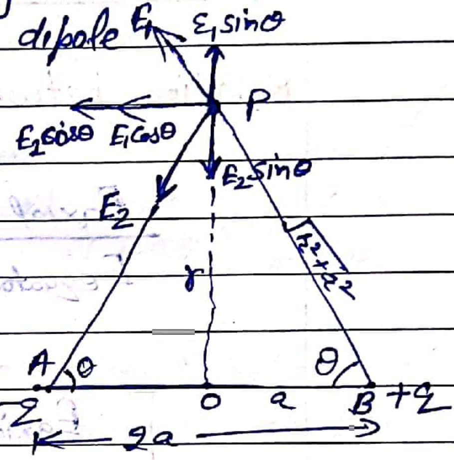
$$\boxed{\vec{E} = \frac{\vec{P} \cdot 2r}{4\pi\epsilon_0 (r^2-a^2)^2}}$$

If dipole is short then $2a <<< r$

Hence a can be neglect

$$\vec{E} = \frac{\vec{P} \cdot 2r}{4\pi\epsilon_0 r^4} \Rightarrow \boxed{\vec{E} = \frac{2\vec{P}}{4\pi\epsilon_0 r^3}}$$

⇒ Electric field intensity of Equatorial line of Electric dipole:-
Consider an electric dipole consist of two point charge $-q$ and $+q$ separated by small distance $2a$.
We have to find electric field intensity $\vec{E}$ at a point P on the equatorial line of the dipole



electric field intensity at point P due to charge $+q$

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \text{ along } BP$$

Electric field intensity at point P due to charge $-q$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2+a^2} \text{ along } PA$$

Resultant electric field at point P is obtained by resolving the component $\vec{E_1}$ and $\vec{E_2}$ we see that component normal to AB ie $E_1 \sin\theta$ and $E_2 \sin\theta$ will cancel each other while component $E_1 \cos\theta$ and $E_2 \cos\theta$ will add up. Thus resultant $\vec{E}$

$$\vec{E} = E_1 \cos\theta + E_2 \cos\theta$$

$$\Rightarrow E = E_1 \cos\theta + E_1 \cos\theta \qquad [\because E_1 = E_2]$$

$$E = 2E_1 \cos\theta$$

we know $\cos\theta = \frac{OA}{AP} = \frac{a}{\sqrt{r^2+a^2}}$

$$\vec{E} = 2E_1 \cos\theta = 2 \times \frac{1}{4\pi\epsilon_0} \frac{q}{r^2+a^2} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$\boxed{\vec{E} = \frac{2aq}{4\pi\epsilon_0 (r^2+a^2)^{3/2}} = \frac{\vec{P}}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}} \qquad [\because \vec{P} = 2qa]$$

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If dipole is short $2a << r$
then

$$\boxed{\vec{E} = \frac{|\vec{P}|}{4\pi\epsilon_0 r^3}}$$

[$\because$ a can be neglect]

Relation between Eaxil and Eequatorial

$$\frac{E_{axial}}{E_{equatorial}} = \frac{\frac{2P}{4\pi\epsilon_0 r^3}}{\frac{P}{4\pi\epsilon_0 r^3}} = 2$$

$$\boxed{\frac{E_{axial}}{E_{equatorial}} = 2}$$

(i) Hence the electric field intensity placed at axis is twice the electric field intensity at point on the equatorial line of the electric dipole.

(ii) The direction of electric field at a point on the axial line of the dipole is in same direction whereas it is opposite direction in case when the point is at equatorial line of the electric dipole.

(iii) In both the cases for distant point ($r >>> 2a$) the electric field

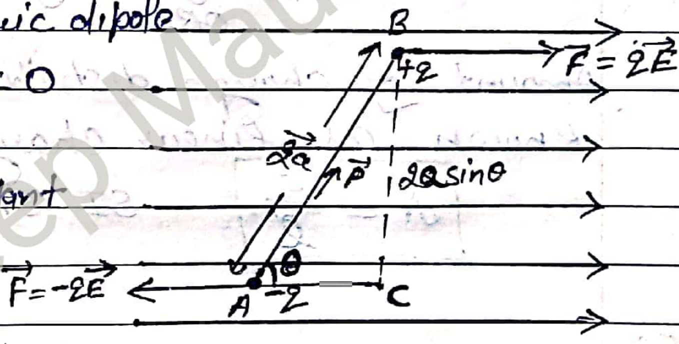
$$\boxed{E \propto \frac{1}{r^3}}$$

⇒ Physical significance of dipoles :— In most molecules, the centre of positive charge and Negative charge lies at the same place. Therefor their dipole moment is zero. Such molecules are known as Non-polar molecule. CO_2 and CH_4 are these type of molecules.
But in some molecules the centre of positive Charge and negative charge do not coincide, Therefore they have permanent dipole moment. Such molecules are known as Polar molecules. H_2O is a polar molecule.

⇒ Torque on a dipole in a uniform Electric field :—
Let an electric dipole AB placed in a uniform electric field at an angle θ. The force $\vec{F} = q\vec{E}$ acts on $+q$ charge of electric dipole in the direction of electric field. The force $\vec{F} = -q\vec{E}$ acts on $-q$ charge of electric dipole in the direction opposite to electric field.

Thus resultant force on electric dipole

$$\vec{F}_{total} = q\vec{E} + (-q\vec{E}) = 0$$



This means that the resultant force is zero and the dipole does not move.

But torque acts on the electric dipole.
Thus Torque = force on either of the charges × Perpendicular distance

$$\tau = qE(BC) \qquad \left[\because \sin\theta = \frac{BC}{2a}\right]$$
$$\tau = qE(2a\sin\theta)$$
$$\tau = pE\sin\theta \qquad [\because p = q.2a]$$

In vector form

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

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Special Conditions :-

(i) When $\theta = 0^\circ$

$\sin\theta = 0$

then $\tau = PE\sin 0^\circ = 0 \Rightarrow \boxed{\tau = 0}$

Hence electric dipole in stable state.

(ii) When $\theta = 180^\circ$

Then $\tau = PE\sin 180^\circ = 0 \Rightarrow \boxed{\tau = 0}$

In this dipole opposite direction of $\vec{E}$. Hence electric dipole in unstable state.

(iii) When $\theta = 90^\circ$

The $\tau_{max} = PE\sin 90^\circ \Rightarrow \boxed{\tau_{max} = PE}$

⇒ Continuous Charge distribution :-

(i) Linear charge density (λ) :- When the charge is distributed uniformey along a line

OR

Amount of charge distributed per unit length is known as linear charge density.

$\boxed{\lambda = \frac{q}{l}}$ SI unit is coulomb per metre (Cm^{-1})

(ii) Surface charge density (σ) :- Amount of charge distributed per unit area is called surface charge density (σ).

$\boxed{\sigma = \frac{q}{A}}$ SI unit is Cm^{-2}.

→ It is used in expressions of charged disc and infinite charged plate.

→ For sharp surface σ is maximum while for plane surface it is minimum.

(iii) Volume charge distribution (ρ) :- Amount of charge distributed per unit volume is called volume charge density.

$$\boxed{\rho = \frac{q}{v}}$$ SI unit is Cm^{-3}.

It is used in the expression of spherical charge distribution.

⇒ Electric field due to continuous charge distributions :-

(i) In case of Linear charge distribution

we know $\lambda = \frac{dq}{dl} \Rightarrow dq = \lambda dl$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \Rightarrow d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2}$$

$$\boxed{\vec{E} = \frac{\lambda}{4\pi\epsilon_0} \int_L \frac{dl}{r^2}}$$

(ii) In case of surface charge distribution

we know $\sigma = \frac{dq}{ds} \Rightarrow dq = \sigma ds$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \Rightarrow d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\sigma ds}{r^2}$$

$$\boxed{\vec{E} = \frac{\sigma}{4\pi\epsilon_0} \int_S \frac{ds}{r^2}}$$

(iii) In case of volume charge distribution

we know $\rho = \frac{dq}{dv} \Rightarrow dq = \rho dv$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \Rightarrow d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\rho dv}{r^2}$$

$$\boxed{\vec{E} = \frac{\rho}{4\pi\epsilon_0} \int_V \frac{dv}{r^2}}$$

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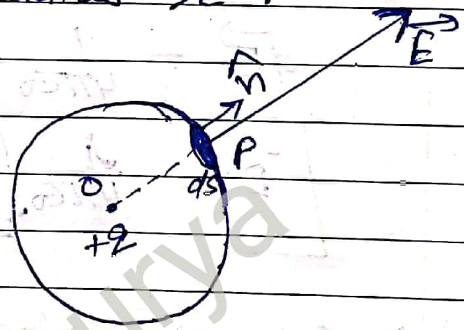
⟹ Gauss's Theorem OR Gauss's law in Electrostatics :—

According to Gauss's law, the surface integral of electric field over an imaginary closed surface is $\frac{1}{\epsilon_0}$ times of the total charge enclosed in surface.

$$\boxed{\phi = \oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\epsilon_0}}$$

The surface chosen to calculate the surface integral is called Gaussian surface.

Proof of Gauss's theorem :— suppose an isolated positive point charge q is situated at the centre O of a sphere of radius r.



According to coulomb law electric field intensity at any point P on the surface of the sphere is

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

Consider a small element ds of the sphere around P.

$$d\phi_E = \vec{E}\cdot\vec{ds} = \frac{q}{4\pi\epsilon_0 r^2}\vec{ds}$$

integrating over the closed surface area of the sphere, we get

$$\phi_E = \int\vec{E}\cdot\vec{ds} = \frac{q}{4\pi\epsilon_0 r^2}\oint ds$$

$$= \frac{q}{4\pi\epsilon_0 r^2} \times \text{total surface area of sphere}$$

$$= \frac{q}{\cancel{4\pi\epsilon_0 r^2}} \times \cancel{4\pi r^2} = \frac{q}{\epsilon_0}$$

Hence $\boxed{\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}}$

Some important points regarding this ~~ohm's~~ law:—

(i) Q represents the algebraic sum of charges in the closed surface.

(ii) Total Electric flux is the algebraic sum of electric flux leaving the surface (+ve sign) and the electric flux entering the surface (−ve sign).

(iii) The surface selected to apply Gauss law is called gaussian surface. It is an imaginary closed surface.

(iv) Gauss's law depends only upon the total charge through the closed surface.

(v) If total charge through a closed surface is zero, then electric flux associated with the surface will be zero.

$$\phi_{total} = \phi_{entering} + \phi_{leaving} = 0$$

(vi) Gauss's law is valid only for those vector field that follow inverse square law.

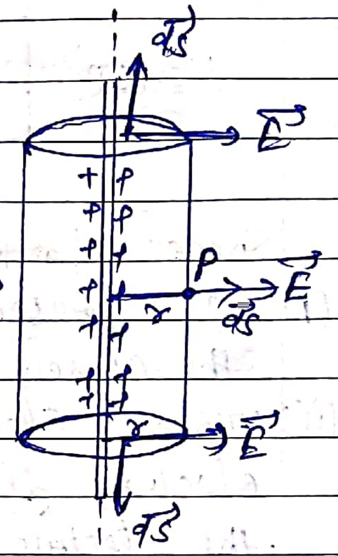
⟹ Application of Gauss law:—

(1) Electric field due to infinite long straight uniformely charged wire.

Consider an infinitely long thin wire with uniform linear charge density λ.

Let us consider a right circular closed cylinder of radius r and length l with the infinitely long line of charge at its axis.

Electric flux over the curved surface of cylinder



$$\phi = \oint \vec{E}\cdot\vec{ds} = \oint E\,ds\cos 0^\circ + \oint E\,ds\cos 90^\circ + \oint E\,ds\cos 90^\circ$$

$$= \oint E\,ds\cos 0^\circ + 0 + 0$$

$$\phi = \oint E\,ds = E\oint ds = E(2\pi r l)$$

$$\phi = E(2\pi r l) \quad \text{———— (i)}$$

We known that

Charge enclosed in the cylinder (q) = λl — (ii)

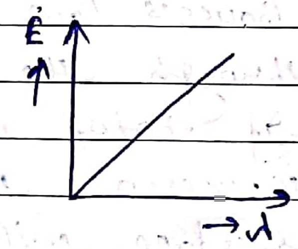
According to Gauss theorem

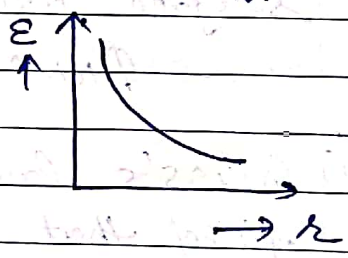
$$\phi_E = \frac{q}{\epsilon_0}$$

Using (i) and (ii) we get

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$





(2) Electric field intensity due to a thin infinite plane sheet of charge :-

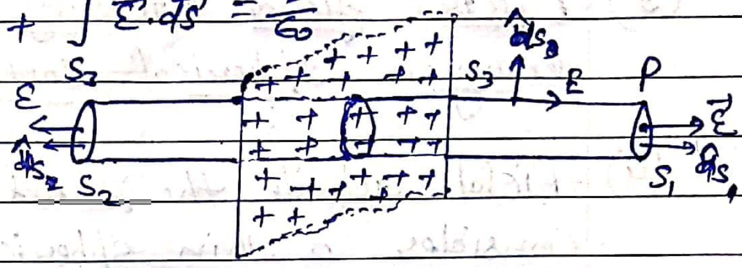
Consider a thin infinite plane sheet of charge. Let σ be the surface density of the charge on the sheet. We have find the $\vec{E}$ at any point P.

Let us imagine a cylinder of cross section of area ds around P and length 2r. At the two edges of cylinder P, Q, the $\vec{E}$ & $\hat{n}$ are parallel to each other

See it for the first time as a newborn child that has no name."

Chitra

$$\phi = \int_{S_1} \vec{E}\cdot\vec{ds} + \int_{S_2} \vec{E}\cdot\vec{ds} + \int_{S_3} \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$



$$\phi = \int_{S_1} E\,ds\cos 0^\circ + \int_{S_2} E\,ds\cos 0^\circ + \int_{S_3} E\,ds\cos 90^\circ = \frac{q}{\epsilon_0}$$

$$\phi = \int E\,ds + \int E\,ds = \frac{\sigma S}{\epsilon_0} \qquad \left[\because \sigma = \frac{q}{S}\right]$$

$$E\int_{S_1} ds + E\int_{S_2} ds = \frac{\sigma S}{\epsilon_0}$$

$$ES + ES = \frac{\sigma S}{\epsilon_0}$$

$$2E\cancel{S} = \frac{\sigma \cancel{S}}{\epsilon_0}$$

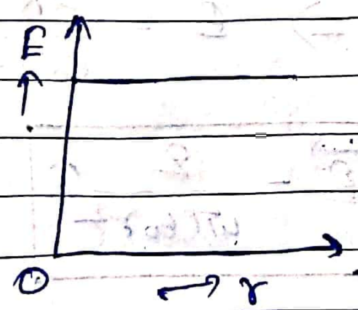
$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}}$$

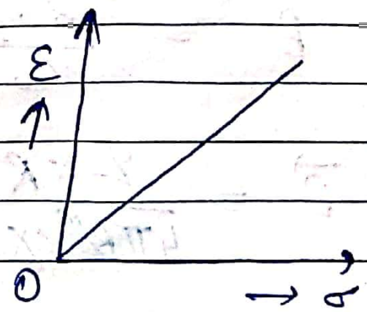
Important points :-

(I) The Electric field due to infinite uniformly charged Sheet does not depend upon distance.

(II) If $\sigma > 0$ then $\vec{E}$ outward from the sheet uniformly

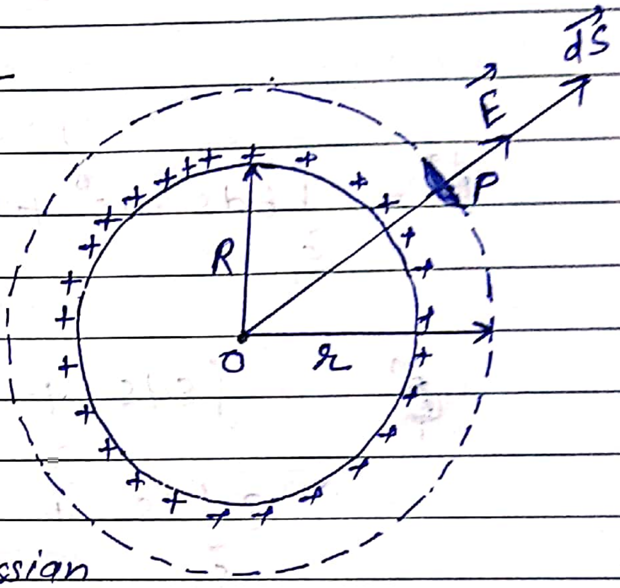
If $\sigma < 0$ then $\vec{E}$ inward normal to the plane sheet





(3) Electric field intensity due to a uniformly charged spherical shell :-

(a) Field outside the shell :-



Consider a thin spherical shell of radius R with centre O. Let a +q charge be distributed uniformly over the surface of the shell. To calculate electric field intensity at any point P. Let imagine a spherical Gaussian surface of radius r. According to Gauss theorem

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

If σ is surface density of charge on the shell, then

$$\sigma = \frac{q}{A} \Rightarrow \sigma = \frac{q}{4\pi R^2} \Rightarrow q = \sigma \cdot 4\pi R^2$$

$$\phi = \oint E\cdot ds = \frac{q}{\epsilon_0}$$

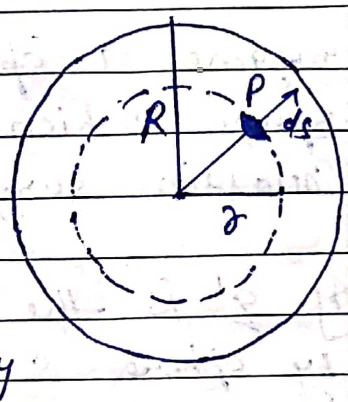
$$E \oint ds = \frac{\sigma \cdot 4\pi R^2}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{\sigma \cdot 4\pi R^2}{\epsilon_0}$$

$$E = \frac{\sigma \cdot \cancel{4\pi} R^2}{\epsilon_0 \times \cancel{4\pi} r^2} \Rightarrow \vec{E} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

$$\vec{E} = \frac{q \times \cancel{R^2}}{4\pi \cancel{R^2} \epsilon_0 \times r^2} \Rightarrow \boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}} \quad \left[\because \sigma = \frac{q}{4\pi R^2}\right]$$

(b) Field inside the shell :-



When the point is inside the shell. $(r<R)$. If the point at which electric field intensity is to be determined is inside the charged shell. Then the charge enclosed by the Guassian surface will be zero because charge is situated only on the surface of shell.

Hense $\oint \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}$

$$\oint \vec{E}\cdot d\vec{s} = 0 \qquad [\because q = 0]$$

$$\boxed{\vec{E} = 0}$$

(C) When the point is on the surface of sphere $(r=R)$

In this case $r = R$

We know that when point P outside the shell then

$$\vec{E} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

In this case

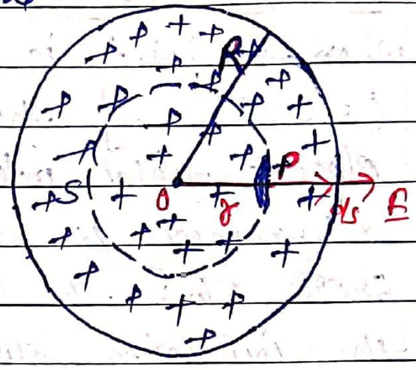
$$\vec{E} = \frac{\sigma \cancel{R^2}}{\epsilon_0 \cancel{R^2}} \qquad [\because r = R]$$

$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}}$$

(4) Electric field intensity due to a non-conducting charged solid sphere :-

suppose a non conducting solid sphere of radius R and centre O has a uniform volume charge density ρ. we have calculate electric field intensity $\vec{E}$ at any point P inside the solid sphere.

Imagine a sphere S inside the sphere which is act as Guassian surface of radius r.



If q' is the charge enclosed by sphere S then according to Gauss law

$$\oint \vec{E}\cdot\vec{ds} = \frac{q'}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q'}{\epsilon_0} \Rightarrow E = \frac{q'}{4\pi\epsilon_0 r^2} \quad \text{—(1)}$$

Charge inside S ie $q' = \rho V$

$$q' = \rho \times \frac{4}{3}\pi r^3$$

Put these value in (1) we get

$$\vec{E} = \frac{\rho\, 4 \times \pi r^{\cancel{3}}}{3 \times 4\pi\epsilon_0 \cancel{r^2}} = \frac{\rho r}{3\epsilon_0}$$

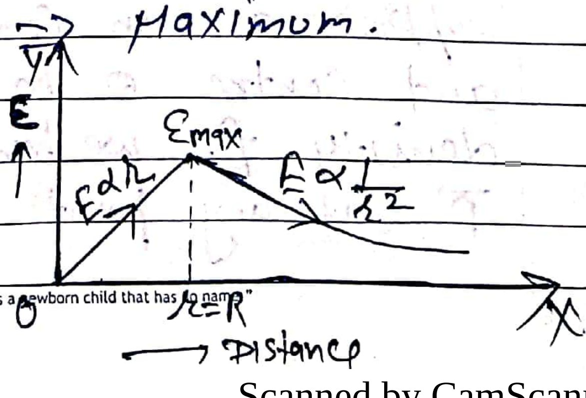
$$\boxed{\vec{E} = \frac{\rho r}{3\epsilon_0}}$$

At the centre of sphere $r = 0$

Hence $\vec{E} = 0$

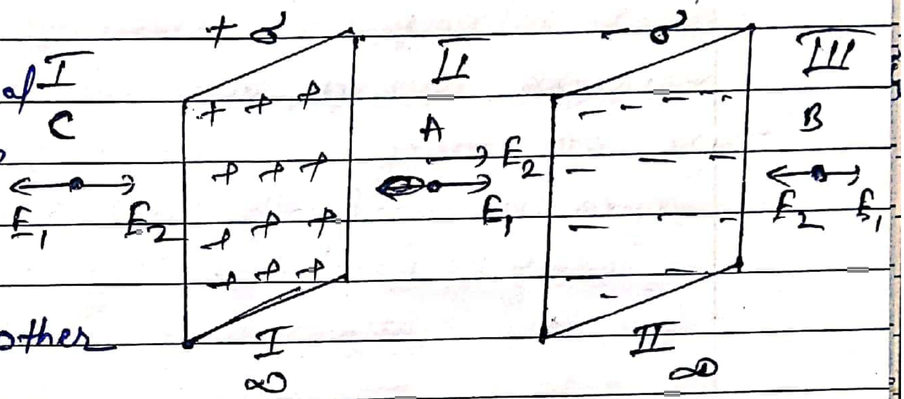
At the surface of sphere $r = R$

Hence $\boxed{\vec{E} = \frac{\rho R}{3\epsilon_0}} \Rightarrow$ Maximum.



5) Electric field intensity due to two thin infinite parallel sheet of charge :—

Two thin infinite plane sheet with equal and opposite uniform surface densities of charge are held parallel to each other



We know that $E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$

Hence
$$E_I = E_1 - E_2 = 0$$
$$E_{II} = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$
$$E_{III} = E_1 - E_2 = 0$$

Hence Resultant $\vec{E}_R = E_I + E_{II} + E_{III} = 0$
$$= 0 + \frac{\sigma}{\epsilon_0} + 0$$

$$\boxed{\vec{E}_R = \frac{\sigma}{\epsilon_0}}$$

$\vec{E}$ does not depend upon the distance between the two sheets.