MARKING SCHEME

BSJE JOINT EXAMINAT-2024 -

Kenya Certificate of Secondary Education

121/2 Mathematics Alt. A Paper 2

July, 2024 TIME: 2½ Hrs

Name: .... Index No: ....

School: ....Signature: ....

121/2 - Mathematics
Tuesday, 16^{TH} July, 2024
Morning
8.00am-10.30am

Instructions to candidates

(a) Write your name and index number in the spaces provided above
(b) Sign and write the date of examination in the spaces provided above
(c) This paper consists of TWO sections: Section I and only five questions form Section II
(d) Answer ALL the questions in Section I and only five questions from Section II
(e) All answers and workings must be written on the question paper in the spaces provided below each question.
Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.
(f) Marks may be given for correct working even if the answer is wrong
(g) Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.
(h) This paper consists of 18 printed pages
(i) Candidates should check the question paper to ascertain that all the pages are printed as indicted and that no questions are missing.

For Examiner's use only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	19	19	20	21	22	23	24	Total

Grand Total

SECTION I (50 marks)

1. Use logarithms to evaluate

(4marks)

$$\sqrt{\left(\frac{\log 9\times 0.954}{0.301\times 4.3}\right)^{-2}} = \sqrt{\left(\frac{0.301\times 4.3}{\log 9\times 0.954}\right)^{2}} = \frac{0.301\times 4.3}{\log 9\times 0.954}$$

NO	Std	Log
0.301	3.01×10^{-1}	$\bar{1}.4786$ +
4.3	4.3×10^{0}	0.6335
		0.1121
log 9 (0.9542)	9.542	$\bar{1}.9796$ +
0.954	9.54	$\bar{1}.9795$
		$\bar{1}.9591$

0.1121
$-\bar{1}.9591$
0.0530
$10^{0}\times 1.422$
$\Rightarrow$ ~~1.4230~~
$\Rightarrow 1.422$

M1 - Correct logarithms
M1 - Correct Addition & subtraction
M1 - Antilogarithm
A1 - C.A.O

2. AB is the diameter of the circle. Given that A(2, -3) and B(4, -7). Find the equation of the circle in the form $x^2+y^2-2ax+2by+c=0$ (3 marks)

Centre $=\left(\frac{2+4}{2}, \frac{-3+-7}{2}\right)$
$=(3,-5)$

Radius $=\sqrt{(1)^2+(-2)^2}$
$=\sqrt{5}$

$(x-3)+(y+5)^2=(\sqrt{5})^2$

$x^2-6x+9+y^2+10y+25=5$

$x^2+y^2-6x+10y+29=0$

M1 - For both Centre & radius
M1 - Expression $(x-a)^2+(y-b)^2=r^2$
A1
03

3. A quantity P varies partly as the cube of Q and partly varies inversely as the square of Q. When Q = 2, P = 108 and when Q =3, P = 259. find the value of P when Q = 6 (4 rmarks)

$P\alpha Q^3+\frac{1}{Q^2}$
$P=aQ^3+\frac{b}{Q^2}$
$108=8a+\frac{b}{4}$
$259=27a+\frac{b}{9}$
$432=32a+b$ ---(i)
$2331=243a+b$ --(ii)

$211a=1899$
$a=9$
$432=288+b$
$b=144$
$P=9Q^3+\frac{144}{Q^2}$
$P=9\times216+\frac{144}{36}$
$P=1948$

M1 Equation (i) & (ii)
M1 Expression for value a b
M1 Expression for P
A1 C.A.O
04

4. (a) expand $(a-b)^5$ (2 marks)

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$a^5 \quad a^4 \quad a^3 \quad a^2 \quad a^1 \quad a^0$

$-b^0 \quad -b^1 \quad -b^2 \quad -b^3 \quad -b^4 \quad -b^5$

B1

$a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

B1

(b) Use the first three terms of the expansion in (a) the above to find the value of 1.97^5 to two decimal place (2 marks)

$a^5 - 5a^4b + 10a^3b^2$

$(2-0.03)^5 = (a-b)^5$

$a = 2$

$b = 0.03$

$2^5 - (5 \times 2^4 \times 0.03) + 10(2^3 \times 0.03^2)$

$32 - 2.4 + 0.0072$

$\Rightarrow 29.672$

≈ 29.67

M1 substitution for values a & b

A1 C.A.O

5 . Simplify without using tables or calculator

$\mathrm{Log}_7 49 \times \log_3 27$ (2 marks)

$\mathrm{Log}_7 7^2 \times \log_3 3^3$

$2\log_7 7 \times 3\log_3 3$

2×3

$\Rightarrow 6$

M1 simplified Expression for log 49 & log 27

A1

02

6. Water flows from a tap. At the rate 27cm^3 per second, into a rectangular container of length 60cm, breath 30 cm and height 40 cm. If at 6.00 p.m. the container was half full, what will be the height of water at 6.04 pm? (3 marks)

$4 \times 60 = 240$ sec

$\text{Vol}_3 = \dfrac{27 \times 240}{1}$

$= 6480\text{cm}^3$

$V = 60 \times 30 \times h = 6480$

$h = 3.6\text{cm}$

height of water. $= 20 + 3.6$

$\approx 23.6\text{cm}.$

M1 Expression for vol. 6480cm^3

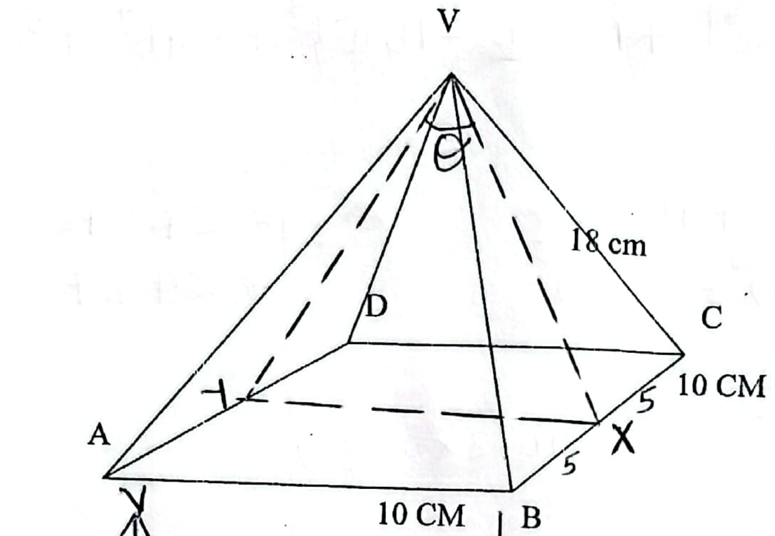
M1 Expression for H

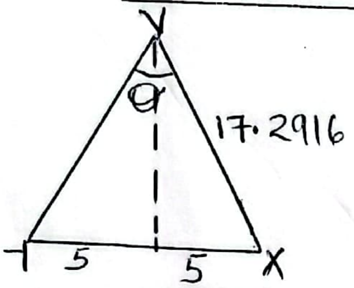
A1 C.A.

03

7.) Figure below shows a square based pyramid ABCD .AV=BV=CV=DV=18 cm.

AB= 10 cm. Calculate the angle between the plane BVC and AVD. (3 marks)





$VX = \sqrt{18^2 - 5^2} = 17.2916$

$\theta = \sin^{-1}\left(\frac{5}{17.2916}\right)$

$\theta = 16.81^\circ$

$\therefore \angle XVY = 16.81^\circ \times 2$
$= 33.62^\circ$

M1	Expression for VX
M1	Expression for θ
A1	C.A.O
03	

8. The roots of a quadratic equation are -3 and ½ . Form the quadratic equation in the form $ax^2 + bx + c = 0$ Where a, b, and c are constants (3 marks)

$(x+3)(2x-1)=0$

$2x^2 - x + 6x - 3 = 0$

$2x^2 + 5x - 3 = 0$

Accept; $x^2 + 2.5x - 1.5 = 0$

B1	For the roots in the form $(x+3)$ & $(2x-1)$
M1	Expression.
A1	

9. Triangle ABC is mapped onto triangle A'B'C' by a transformation given by the matrix N= $\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$. If A'B' = 4 cm , find AB

(3 marks)

A.S.F = Det = 3 − (−1)
= 4

L.S.F = $\sqrt{4}$ = 2

$\frac{A'B'}{AB} = \frac{2}{1}$

$\frac{4}{AB} = \frac{2}{1}$

AB = 2cm

M1 - Determinant

M1 Expression for L.S.F

A1

03

10. Using log 2 = 0.3010 and log 3 = 0.4771 . Evaluate log 45

(3 marks)

log 4.5 = log $\left(\frac{45}{10}\right)$ ± log $\left(\frac{9}{2}\right)$
= log $\left(\frac{3^2}{2}\right)$
= 2log3 − log 2
⇒ 2(0.4771) − 0.3010)
⇒ 0.6532

11. The position of two towns A and B on the earth's surface are (36°N,49°E) and (36°N, 131°W) respectively.

(a) Find the local time at A if the time at B is 12.35pm on Sunday.

(1mk)

$\frac{180 \times 4}{60}$ = 12hrs ∴ +1235
1200
2435

Monday ; 12:35 a.m

B1

(b) Using 6370km as the radius of the earth, calculate the shortest distance between town A and B

(2mks)

θ = 180 − (2×36)
= 108°

D = $\frac{108}{360} \times 2 \times \frac{22}{7} \times 6370$

⇒ 12,012km

M1 - Expression for Distance

A1

12. The first, the third and the seventh terms of an increasing arithmetic progression are three consecutive terms of a geometric progression. In the first term of the arithmetic progression is 10, find the common difference of the arithmetic progression.(3Mrks)

$$\frac{a+6d}{a+2d} = \frac{a+2d}{a}$$

$$\frac{10+6d}{10+2d} = \frac{10+2d}{10}$$

$$100+20d+20d+4d^2 = 100+60d$$

$$4d^2 = 20d$$

$$d = 5$$

M1 - Expression $\frac{10+6d}{10+2d} = \frac{10+2d}{10}$

M1 - Simplified Expression $4d^2 = 20d$

A1 value of d

13. A two digit number is made by combining any of two digits 1,2,3,4, and 5 at random. Find the probability that the number formed is even (2marks)

X	1	2	3	4	5
1	X	1,2	1,3	1,4	1,5
2	2,1	X	2,3	2,4	2,5
3	3,1	3,2	X	3,4	3,5
4	4,1	4,2	4,3	X	4,5
5	5,1	5,2	5,3	5,4	X

$$P = \frac{8}{20}$$

$$\Rightarrow \frac{2}{5} \text{ or } 0.4$$

14. The deviation d from the mean X of the set of data x are given below;

X	15	18	M	23	M+6	28	30	
d	-8	-5	Y	0	-y	5	7	
a) $\|d\|$	8	5	3	0	3	5	7	$\Sigma\|d\| = 31$

b) Find the value of m and y (2 marks)

$m-y=23$
$m+6+y=23$
$m+y=17$

$m-y=23$
$m+y=17$
$2m=40$
$m=20$

$20-y=23$
$y=-3$

A1 - Expression for m

B1 value for y

02

c) Calculate the mean a bsolute deviation of the data (2 marks)

$$\frac{\Sigma|d|}{N} = \frac{31}{7}$$

$$= 4.4286$$

$$\cong 4.429$$

M1 $\frac{\Sigma|d|}{N}$

A1 - C.A Accept 4.429

15. Make y the subject o f the formula (3 marks)

$$q = m\sqrt{\frac{r^2-y^2}{y^2-3}}$$

$(q)^2 = (m)^2\left(\sqrt{\frac{r^2-y^2}{y^2-3}}\right)^2$

$q^2 = m^2\left(\frac{r^2-y^2}{y^2-3}\right)$

$\frac{q^2}{m^2} = \frac{r^2-y^2}{y^2-3}$

$q^2y^2 - 3q^2 = m^2r^2 - m^2y^2$

$q^2y^2 + m^2y^2 = m^2r^2 + 3q^2$

$y^2 = \frac{m^2r^2+3q^2}{q^2+m^2}$

$y = \pm\sqrt{\frac{m^2r^2+3q^2}{q^2+m^2}}$

M1	squares.
M1	Expression for value of y^2
A1	Value of y
03	

16. Solve the equation: $3\cos X = 2\sin^2 X$ where $0 \leq X \leq 360$ (3marks)

$3\cos X = 2(1-\cos^2 X)$

$3\cos x = 2 - 2\cos^2 x$

$2\cos^2 x + 3\cos x - 2 = 0$

Let $\cos x$ be y

$2y^2 + 3y - 2 = 0$

$2y^2 + 4y - y - 2 = 0$

$2y(y+2) - 1(y+2) = 0$

$(2y-1)(y+2) = 0$

$y = 0.5$ or -2

$\cos x = 0.5$

$x = 60°, 300°$

M1	Expression $2y^2+3y-2=0$
M1	Values of y
A1	Values of $x = 60°, 300°$
03	

Section II (50 marks) . Answer only five questions in this section in the spaces provided

17.The table below shows the analysis of examination marks scored by 160 candidates

Marks (%)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
No of candidates	2	6	15	22	36	34	20	15	6	4

a)Using an assumed mean of 45.5. calculate

I. The mean (3 marks)

Marks	x	f	$d=x-A$	fd	fd^2	Cf
1-10	5.5	2	-40	-80	3200	2
11-20	15.5	6	-30	-180	5400	8
21-30	25.5	15	-20	-300	6000	23
31-40	35.5	22	-10	-220	2200	45
41-50	45.5	36	0	0	0	81
51-60	55.5	34	10	340	3400	115
61-70	65.5	20	20	400	8000	135
71-80	75.5	15	30	450	13500	150
81-90	85.5	6	40	240	9600	156
91-100	95.5	4	50	200	10,000	160
		$\Sigma f=160$		$\Sigma fd = 850$	$\Sigma fd^2 = 61300$	

$$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 45.5 + \frac{850}{160}$$

$$\Rightarrow 50.8125$$

B1 - Σfd

M1 - subst

A1

II. The standard deviation (4 marks)

ii) Standard deviation;

$$\sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$\sqrt{\frac{61300}{160} - \left(\frac{850}{160}\right)^2}$$

$$\Rightarrow 18.8389$$

B1 - Σfd^2

B1

M1

A1

b) Calculate the minimum mark for grade A if 40 student got grade A (3 marks)

$$60.5 + \left(\frac{120-115}{20}\right)10$$

$\Rightarrow$ 63 marks

B1 - Cf

M1

A1

18 .A and B are two points on latitude 40^0.The two points lie on the longitude 80^0W and 100^0 E respectively.(taking $\pi=\frac{22}{7}$ and R =6370 KM) .

(a) Calculate;

(i) The distance from A to B along the parallel of latitude (3mks)

$$\frac{180^0}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 40$$

$$= 15{,}336.21 \text{ km}.$$

M1 M1 - Expression including Longitudinal difference.

A1

03

(ii)The distance from A to B along the great circle (~~4~~ 2 mks)

$$\theta = 180 - (2 \times 40) = 100^\circ$$

$$\frac{100}{360} \times 2 \times \frac{22}{7} \times 6370$$

$$\Rightarrow 1122\tfrac{2}{9} \text{ km} \approx 1122.22 \text{ km}$$

M1

A1

02

(b) Two planes P and Q left A for B at 400 knots and 600 knots respectively. If P flew along the great circle and Q along the parallel of latitude ,which one arrived earlier and by how much? Give your answer to the nearest minute. (3mks)

AB in Great circle

$(100 \times 60) = 6000$ nm

Time Taken by P

$\Rightarrow \frac{6000}{400} \Rightarrow 15$ hours.

Distance in small circle; Q

$60 \times 180 \cos 40$

$\Rightarrow 8273.28$ nm

Time by Q $\Rightarrow \frac{8273.28}{600}$

$= 13.79$ hrs

≈ 13 hr 47 min

Q arrived earlier by;

1.211 hours

or 1 hr. 13 min//

73 mins.

Accept: 1 hr 12 min 36 sec

M1

M1

M1

M1

A1

05

19. a).Complete the table below by filling in the blank spaces for the functions

y = sin(x +30) and $y = \cos\frac{1}{2}x$ (2 marks)

X	0	30	60	90	120	150	180	210
Y=sin $(x+30^0)$	0.5	0.87	1	0.87	0.5	0.00	-0.5	-0.87
$Y = \cos\frac{1}{2}x$	1.00	0.97	0.87	0,71	0.5	0.26	0.00	-0.26

b.) Using the scale,x-axis 1cm =30 ,y-axis 1cm=0.5 units, draw their graphs of the functions on the same axes (5marks)

c)Use your graph to solve

i. $\sin(x + 30) - \cos\frac{1}{2}x = 0$ (1 mark)

$X = 120^\circ \pm 6^\circ$

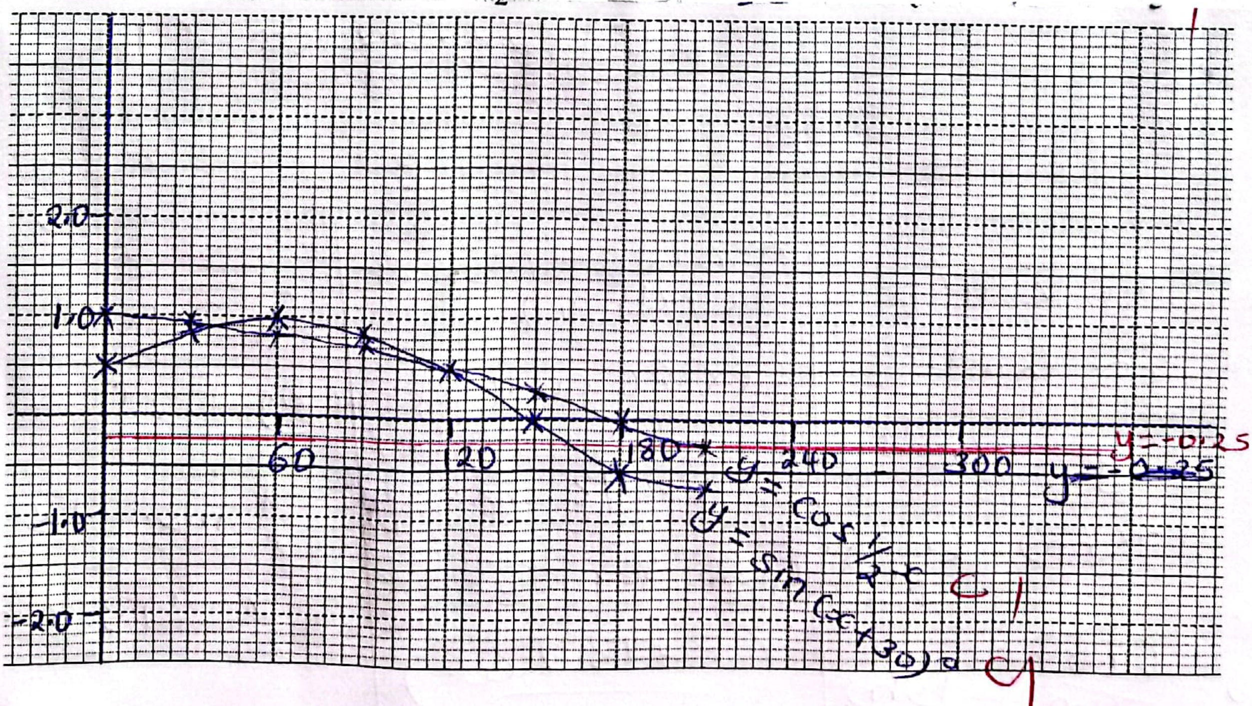
$X = 36^\circ$

ii. $\sin(x + 30) = 0$ (1 mark)

$x = 156^\circ \pm 6^\circ$

iii) $\cos\frac{1}{2}x = -0.25$ (1mark)

$x = 210^\circ \pm 6^\circ$



S1
P1
P1
10

20. The equation of a curve is $y=3x^2-4x+1$

a) Find the gradient function of the curve when $x=2$

(2 marks)

$$\frac{dy}{dx} = 6x-4$$
$$\Rightarrow 6x-4$$
$$= 6(2)-4$$
$$= 8$$

M1

A1

02

b) Determine

i) The equation of the tangent of the curve at the point (2, 5)

(2marks)

$$G = 6(2)-4 = 8$$
$$\frac{y-5}{x-2} = 8$$
$$y = 8x-16$$

M1

A1

ii) The angle which the tangent to the curves at the point (2,5) makes with the horizontal line

$$y = 8x-11$$
$$\text{Gradient} = 8$$
$$\tan\theta = 8$$
$$\theta = 82.87^\circ$$

(2mks)

M1

A1

02

iii) The equation of the line through the point (2, 5) which is perpendicular to the tangent in (b) i above

(4marks)

$$M_2 = -\frac{1}{8}$$
$$\frac{y-5}{x-2} = -\frac{1}{8}$$
$$8y-40 = -x+2$$
$$8y = -x+42$$
$$x+8y-42=0$$
or
$$y = -\frac{1}{8}x+\frac{21}{4}$$

B1 – M_2

M1 – Expression $\frac{y-5}{x-2} = -\frac{1}{8}$

M1 $8y-40=-x+2$

A1 Equation in any form.

04

21. The table shows income tax rates for the year 2023

Monthly income in Kenya shillings	Tax Rate in each shilling
0 – 10164	10%
10165 – 19740	15%
19741 – 29316	20%
29317 – 38892	25%
38893 and above	30%

In a certain month of that year an employee's taxable income in the fourth band was K£ 334.2

Tax relief of Ksh 1162 pm. Was allowed.

a)Calculate

i) The employees total taxable income in that month (3 marks)

$29316 + (334.2 \times 20)$ M1

$\Rightarrow$ Ksh. 36,000 A1

ii) The P.A.Y.E. in that month (5 marks)

1st band $= \frac{10}{100} \times 10164 =$ Ksh. 1016.40 M1

2nd band $= \frac{15}{100} \times 9576 =$ Ksh. 1436.40 M1

3rd band $= \frac{20}{100} \times 9576 =$ Ksh. 1915.20 M1

4th band $= \frac{25}{100} \times 6684 =$ Ksh. 1671 M1

G.T = Ksh. 6039

Less Tax Relief;

$$\begin{array}{r} 6039 \\ -\ 1162 \\ \hline \end{array}$$

Ksh. 4877 A1

05

b)The employees income included taxable allowances a mounting to Kshs. 14000. The employees contributed 6% of the basic salary to a co- operative society. Calculate the employee net income for that month . (3 marks)

Basic salary $= 36{,}000 - 14000$ M1

= Ksh. 22,000

Co-operative $= \frac{6}{100} \times 22{,}000 \Rightarrow$ Ksh. 1320 M1

Total deduction $\Rightarrow 1320 + 4877$

$\Rightarrow$ Ksh. 6197 A1

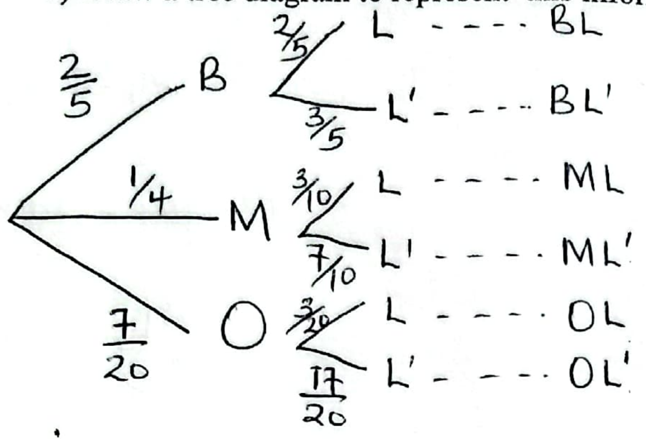
Net Income $= 36{,}000 - 6197$

= Ksh. 29,803.

03

22. The probability that Zora goes to school by boda boda is $\frac{2}{5}$ and by a matatu is $\frac{1}{4}$. If she uses a boda boda , the probability that she will be late is $\frac{2}{5}$ and $\frac{3}{10}$ if she uses a matatu . If she uses other means of transport the probability of being late is $\frac{3}{20}$

a) Draw a tree diagram to represent this information (3 marks)



B1B1B1

03

b)Find the probability that she will be late for school (2 marks)

P(BL or ML or OL)

$\left(\frac{2}{5}\times\frac{2}{5}\right)+\left(\frac{1}{4}\times\frac{3}{10}\right)+\left(\frac{7}{20}\times\frac{3}{20}\right)$ M1

$\frac{4}{25}+\frac{3}{40}+\frac{21}{400}$

$\Rightarrow \frac{115}{400}$ or $\frac{23}{80}$ A1

02

c)Find the probability that she will be late for school if she does not use a matatu (3 marks)

P(BL or Other means) M1

$=\left(\frac{2}{5}\times\frac{2}{5}\right)+\left(\frac{7}{20}\times\frac{3}{20}\right)$ M1

$\Rightarrow \frac{85}{400}$ or $\frac{17}{80}$ A1

03

d)Find the probability that she will not be late for school (2 marks)

$\left(\frac{2}{5}\times\frac{3}{5}\right)+\left(\frac{1}{4}\times\frac{7}{10}\right)+\left(\frac{7}{20}\times\frac{17}{20}\right)$ M1

$\Rightarrow \frac{6}{25}+\frac{7}{40}+\frac{119}{400} \Rightarrow \frac{285}{400}$ or $\frac{57}{80}$ A1

OR

$1-\frac{23}{80}$

$=\frac{57}{80}$

02

23. The acceleration of a particle after passing a fixed point P is given by a = 3t-3. Given that the velocity of the particle when t = 2 seconds is 5m/s, find:

a). Its velocity:

i). In terms of t (3 marks)

$V = \int (3t-3)\,dt$

$V = \frac{3t^2}{2} - 3t + C$

$5 = \frac{3}{2}(2)^2 - 3(2) + C$

$C = 5$

$V = \frac{3}{2}t^2 - 3t + 5$

M1 - Integral
M1 - Expression with C
A1 - Velocity function
03

ii). When t = 4 (2 marks)

$V = \frac{3}{2}(4)^2 - 3(4) + 5$

$\Rightarrow 17\text{ m/s}$

M1 - substitution
A - C.A.O
02

b). The maximum velocity attained by the particle. (2 marks)

at max; velocity ; $a = 0$

$3t - 3 = 0$

$t = 1$

$V = \frac{3}{2}(1)^2 - 3(1) + 5$

$= 3.5\text{ m/s} \approx 3\frac{1}{2}\text{ m/s}.$

M1 - substitution for t = 1
A1 - C.A.O
02

c). Its displacement during the third second. (3 marks)

$S = \int_2^3 \left(\frac{3}{2}t^2 - 3t + 5\right)$

$S = \left. \frac{3t^3}{6} - \frac{3t^2}{2} + 5t \right|_2^5$

$S = \left[\left(\frac{3(3)^3}{6} - \frac{3}{2}(3)^2 + 5(3)\right] - \left[\frac{3(2)^3}{6} - \frac{3}{2}(2)^2 + 5(2)\right)\right]$

$S = 15 - 8$

$\Rightarrow \underline{\underline{8\text{ m}}}$

M1 - Integral to give s
M1 - substitution for t = 3 & 2
A1 - C.A.O (8m)
03

24.A triangle ABC with vertices A(2,0), B(4,-2) and C(4,4) is mapped onto its image $A^1B^1C^1$ under a transformation represented by the matrix $\begin{pmatrix}1 & 1/2\\ 0 & 1\end{pmatrix}$

a)i) State the coordinates of $A^1B^1C^1$ (2 Marks)

$$\begin{pmatrix}1 & \frac{1}{2}\\ 0 & 1\end{pmatrix}\begin{array}{c}\begin{array}{ccc}A & B & C\end{array}\\ \begin{pmatrix}2 & 4 & 4\\ 0 & -2 & 4\end{pmatrix}\end{array} = \begin{array}{c}\begin{array}{ccc}A' & B' & C'\end{array}\\ \begin{pmatrix}2 & 3 & 6\\ 0 & -2 & 4\end{pmatrix}\end{array} \Rightarrow A'(2,0)\quad B'(3,-2)\quad C'(6,4)$$

ii) On the grid provided plot triangle ABC and triangle $A^1B^1C^1$ (2 marks)