MIRROR JET

Kenya Certificate of Secondary Education
2024 KCSE PREPARATION SERIES

121/1 MATHEMATICS Paper 1

–Alt. A–

Term 2. 2024 – 2½ hours

Name .... **Index Number**...............................

Class .... **Admission Number** ....**Date**...............................

Instructions to candidates

a) *Write your name and index number in the spaces provided above.*
b) *Write your class, admission number and the date of examination in the spaces provided above.*
c) *This paper consists of **two** sections; **Section I** and **Section II**.*
d) *Answer all the questions in **Section I** and only **five** questions from **Section II**.*
e) ***Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.***
f) *Marks may be given for correct working even if the answer is wrong.*
g) ***Non – programmable** silent electronic calculators **and** KNEC Mathematical tables may be used, except where stated otherwise.*
h) ***This paper consists of 14 printed pages.***
i) ***Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.***
j) ***Candidates should answer the questions in English.***

For Examiner's Use Only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

SECTION I (50 Marks)

*Answer **all** the questions in this section in the spaces provided.*

1. Express $2.\dot{8}\dot{1}$ as a fraction hence evaluate $\left[\frac{2}{11} of\, 26\frac{2}{5} + \left(-4\frac{1}{4}\right)\right] \div \left(-11 \times 2.\dot{8}\dot{1}\right)$ (3 marks)

$r = 2.8181...$

$100r = 281.8181...$

$99r = 279$ B1

$r = \frac{279}{99} = \frac{31}{11} \checkmark = 2\frac{9}{11}$

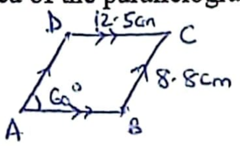
$\frac{2}{11} \times \frac{132}{5} = \frac{24}{5} = 4\frac{4}{5}$

$4\frac{4}{5} - 4\frac{1}{4} = \frac{16-5}{20} = \frac{11}{20}$

$-11 \times \frac{31}{11} = -31$

$\frac{11}{20} \div -31 = \frac{11}{20} \times \frac{1}{-31}$ ✓M1 $= \frac{-11}{620}$ ✓A1

2. A parallelogram ABCD is such that side BC = 8.8 cm, DC = 12.5 cm and $\angle BAD = 60°$. Find the exact area of the parallelogram. (3 marks)



Area = $12.5 \times 8.8 \times \sin 60°$

$= 110 \times \frac{\sqrt{3}}{2}$

$= 55\sqrt{3}\ cm^2$

3. Solve for x in the equation $81^{x-3} \times 27^{x+4} = \frac{\sqrt{6561}}{3^x}$ (3 marks)

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\sqrt{6561} = \sqrt{3^8} = 3^4 = 81$

$3^{4(x-3)} \times 3^{3(x+4)} = \frac{3^4}{3^x}$

$3^{4x-12} \times 3^{3x+12} = 3^{4-x}$

$3^{4x-12+3x+12} = 3^{4-x}$

$7x = 4 - x$

$8x = 4$

$x = 4/8 = 0.5$

2

4. Simplify $\dfrac{-3mn-(m-2n)(m+2n)}{1\frac{1}{2}m+6n}$ (3 marks)

Numerator $= -3mn - (m^2 - 4n^2) = -3mn - m^2 + 4n^2$

$= 4n^2 - 3mn - m^2$

$= 4n^2 - 4mn + mn - m^2 = 4n(n-m) + m(n-m)$

$= (n-m)(4n+m)$

$\dfrac{(n-m)(4n+m)}{1.5(4n+m)} = \dfrac{n-m}{1.5} = \underline{\underline{\dfrac{2n-2m}{3}}}$

5. A cone is formed from sector of circle diameter 21 cm that subtends an angle of 75° at the centre. Find the volume of the cone correct to 1decimal place. (3 marks)

Arc length $= \frac{75}{360} \times 2 \times \frac{22}{7} \times 10.5 = 13.75$ cm

$C = 2\pi r$

$13.75 = 2 \times \frac{22}{7} \times r$

$r = \dfrac{13.75 \times 7}{2 \times 22} = 2.1875$ cm

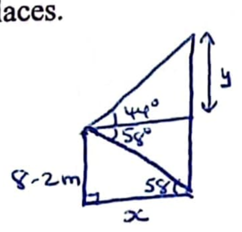
$h = \sqrt{10.5^2 - 2.1875^2} = 10.27$ cm

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 2.1875^2 \times 10.27$

$= 51.48 = \underline{51.5 \text{ cm}^3}$

6. A building is 8.2 m tall. A man standing on the top of the building elevates the top of a tree at 44° and depresses the bottom of the tree at 58°. Find the height of the tree giving your answer to 2 decimal places. (3 marks)



$\tan 58° = \dfrac{8.2}{x}$

$x = \dfrac{8.2}{\tan 58°} = 5.124$ m

$\tan 44° = \dfrac{y}{5.124}$

$y = 5.124 \tan 44° = 4.948$

height $= 4.948 + 8.2 = 13.148 = \underline{\underline{13.15 \text{ m}}}$

7. Two similar buckets A and B have capacities 12 litres and 40.5 litres respectively. If the vertical height of B is 24 cm more than the vertical height of A, determine the vertical height of B. (2 marks)

$$V.s.f = \frac{40.5}{12} = 3.375$$

$$L.s.f = \sqrt[3]{3.375} = 1.5$$

~~Ans~~ - Vertical height of B $= x$

Vertical height of A $= \frac{x}{1.5} = \frac{2x}{3}$

$$x - \frac{2x}{3} = 24$$

$$\frac{1}{3}x = 24$$

$$x = 24 \times 3 = 72$$

height of B $=$ 72 cm

8. Tree seedlings are planted on each side of a street such that the first pair of tree seedlings are opposite each other. The seedlings are planted at intervals of 9 m on one side and 12.6 m on the other side. Calculate the number of tree seedlings planted by the time another pair of seedlings are opposite each other. (3 marks)

LCM =

2	900, 1260
2	450, 630
3	225, 315
3	75, 105
5	25, 35
5	5, 7
7	1, 7
	1, 1

$$= 2^2 \times 3^2 \times 5^2 \times 7 = 6300\,cm = 63\,m$$

Number of seedlings $= \left(\frac{63}{9} + 1\right) + \left(\frac{63}{12.6} + 1\right)$

$$= 8 + 6 = 14 \text{ seedlings}$$

*9. Three of the interior angles of an irregular polygon measure ~~63°~~ 126 each. The remaining interior angles measure ~~73.5°~~ 147 each. Find the number of sides of the polygon hence name the polygon. (3 marks)

$$(63 \times 3) + 73.5(n-3) = 180(n-2)$$

$$189 + 73.5n - 220.5 = 180n - 360$$

$$73.5n - 31.5 = 180n - 360$$

$$= 106.5n$$

$$(126 \times 3) + 147(n-3) = 180(n-2)$$

$$378 + 147n - 441 = 180n - 360$$

$$147n - 63 = 180n - 360$$

$$297 = 33n$$

$$n = 9 \text{ sides}$$

10. By selling 40 exercise books ~~at~~ for sh. 640, a sales man realizes a loss of 20%. How many books should be sold at sh. 182 to realize a profit of 30%. (3 marks)

S.P per book $= \frac{640}{40} = sh\ 16$

B.P per book $= \frac{100}{80} \times 16 = sh\ 20$

$\frac{100}{130} \times 182 = sh\ 140$

Number of books $= \frac{140}{20}$

$= 7$ books

11. Use mid ordinate rule with 4 strips to estimate the area enclosed by the line $y = x$ and the curve $y = 9x - x^2$. (4 marks)

$x = 9x - x^2$

$8x - x^2 = 0$

$x(8 - x) = 0$

$x = 0$ or 8

$h = \frac{8-0}{4} = 2$

$0 + \frac{2}{2} = 1$

x	1	3	5	7
$y=9x-x^2$	8	18	20	14
$y=x$	1	3	5	7
y_1-y_2	7	15	15	7

Area $= 2(7+15+15+7) = 2 \times 44 = 88$ square units

12. A translation T maps a point P(2,1) onto $P'(-1,2)$. Given that $Q'(5,1)$ is the image of Q under the same translation, calculate the distance between P and Q. (3 marks)

$T = \begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix}$

Object + translation vector = Image

Object = Image − translation vector

$= \begin{pmatrix}5\\1\end{pmatrix} - \begin{pmatrix}-3\\1\end{pmatrix} = \begin{pmatrix}8\\0\end{pmatrix}$

$Q(8,0)$

Distance PQ $= \sqrt{(8-2)^2 + (0-1)^2} = \sqrt{37} = 6.083$ units

13. Daniel bought 3 pens, 5 exercise books and 2 sets. A pen costs sh. 15, an exercise books cost sh. 100 and a set cost sh. 250

(a) Write a 1×3 matrix to represent items bought by Daniel. (1 mark)

$$\begin{pmatrix} 3 & 5 & 2 \end{pmatrix}$$

(b) Write a 3×1 matrix to represent the price of the items bought by Daniel (1 mark)

$$\begin{pmatrix} 15 \\ 100 \\ 250 \end{pmatrix}$$

(c) Use the matrix above to find Daniel's total expenditure (2 marks)

$$\begin{pmatrix} 3 & 5 & 2 \end{pmatrix} \begin{pmatrix} 15 \\ 100 \\ 250 \end{pmatrix}$$

$$= \begin{pmatrix} 45 + 500 + 500 \end{pmatrix} = \begin{pmatrix} 1045 \end{pmatrix}$$

Total expenditure = sh 1045

14. Find the equation of the normal to the curve $y = 2x^2 - 3x + 4$ at $x = 2$ (3 marks)

$$\frac{dy}{dx} = 4x - 3$$

When $x = 2$, $\frac{dy}{dx} = 4(2) - 3 = 5$

Gradient of normal $= -\frac{1}{5}$

When $x = 2$, $y = 2(2)^2 - 3(2) + 4 = 8 - 6 + 4 = 6$ $(2, 6)$

$$\frac{y - 6}{x - 2} = -\frac{1}{5}$$

$$5y - 30 = -x + 2$$

$$5y = -x + 32$$

$$y = -\frac{1}{5}x + 6\frac{2}{5}$$

15. Solve the inequalities $2x - 1 \leq 7x + 12 \leq 5x + 18$ and represent the solution on a number line. (3 marks)

$$2x - 1 \leq 7x + 12$$

$$-5x \leq 13$$

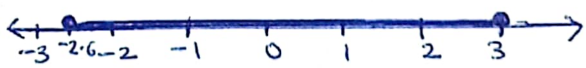
$$x \geq -2.6$$

$$7x + 12 \leq 5x + 18$$

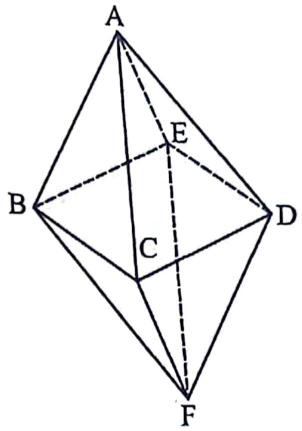
$$2x \leq 6$$

$$x \leq 3$$

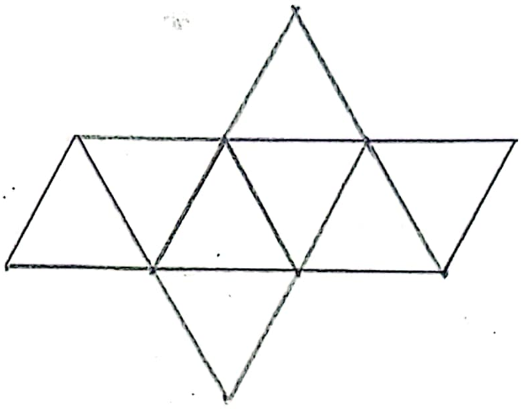
$$-2.6 \leq x \leq 3$$



16. The figure below shows a regular octahedron of side 2 cm.



(a) Sketch a net of the solid. (2 marks)



(b) Hence or otherwise, show that the total surface area of the solid is $8\sqrt{3}\ \text{cm}^2$. (2 marks)

Total surface area $= \left(\frac{1}{2} \times 2 \times 2 \times \sin 60^\circ\right) \times 8$

$= 16 \sin 60^\circ$

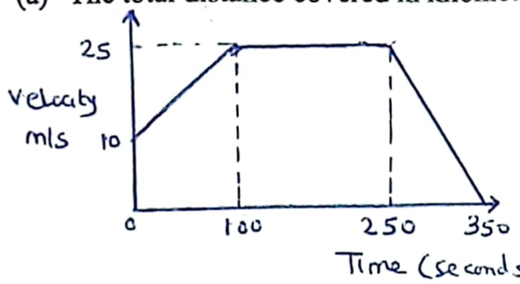
$= 16 \times \frac{\sqrt{3}}{2} \quad = \underline{\underline{8\sqrt{3}\ \text{cm}^2}}$

SECTION II (50 Marks)

*Answer only **five** questions from this section in the spaces provided.*

17. A car travelling at 10 m/s accelerate uniformly in 100 seconds to velocity of 25 m/s. It maintains this velocity for another 150 seconds before decelerating uniformly to rest after 100 seconds.
Calculate

(a) The total distance covered in kilometers (3 marks)



$$\text{Distance} = \left(\frac{10+25}{2}\right)100 + (150 \times 25) + \left(\frac{1}{2} \times 100 \times 25\right)$$

$$= 1750 + 3750 + 1250 = 6750\,m = \underline{\underline{6.75\,km}}$$

(b) The average speed in the first 200 seconds. (3 marks)

$$\text{Distance} = \left(\frac{10+25}{2}\right)100 + (100 \times 25)$$

$$= 1750 + 2500 = 4250\,m$$

$$\text{Average Speed} = \frac{4250\,m}{200\,s}$$

$$= \underline{\underline{21.25\,m/s}}$$

(c) The initial acceleration (1 mark)

$$a = \frac{25-10}{100}$$

$$= \underline{\underline{0.15\,m/s^2}}$$

(d) Time taken to travel the last half of the journey. (3 marks)

$$\tfrac{1}{2} \times 6750 = 3375\,m$$

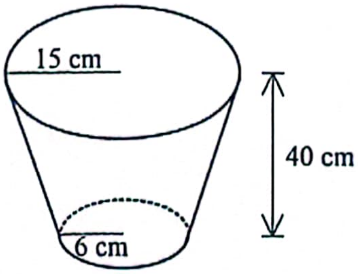
$$3375 - 1250 = 2125\,m$$

$$25 \times t = 2125$$

$$t = \frac{2125}{25} = 85 \text{ seconds}$$

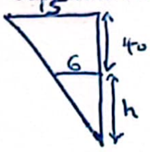
$$\text{Time} = 100 + 85 = \underline{\underline{185 \text{ seconds}}}$$

18. The figure below shows a bucket in the shape of a frustum whose top radius is 15 cm and base radius is 6 cm. The height of the bucket is 40 cm.



Taking $\pi = 3.142$

a) Calculate correct to 4 significant figures the surface area of the bucket. (4 marks)



By similarity,

$$\frac{h+40}{h} = \frac{15}{6}$$

$$15h = 6h + 240$$

$$9h = 240$$

$$h = 26\tfrac{2}{3}\text{ cm}$$

$$H = 40 + 26\tfrac{2}{3} = 66\tfrac{2}{3}\text{ cm}$$

$$l = \sqrt{(26\tfrac{2}{3})^2 + 6^2} = 27.33\text{ cm}$$

$$L = \sqrt{(66\tfrac{2}{3})^2 + 15^2} = 68.33\text{ cm}$$

Curved surface area

$$= \pi RL - \pi rl$$

$$= (3.142 \times 15 \times 68.33) - (3.142 \times 6 \times 27.33)$$

$$= 3220.3929 - 515.22516$$

$$= 2705.16774 = 2705\text{ cm}^2$$

Area of bottom circle $= 3.142 \times 6^2 = 113.112$

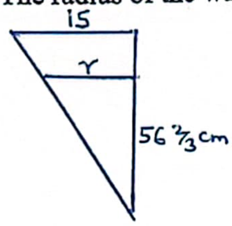
Surface area $= 2705.16774 + 113.112$

$$= 2818.27974$$

$$= \underline{\underline{2818\text{ cm}^2}}$$

b) If the bucket is filled to a height of 30 cm, calculate:

i. The radius of the water surface (3 marks)



$$26\tfrac{2}{3} + 30 = 56\tfrac{2}{3}\text{ cm}$$

By similarity,

$$\frac{56\tfrac{2}{3}}{66\tfrac{2}{3}} = \frac{r}{15}$$

$$r = \frac{56\tfrac{2}{3} \times 15}{66\tfrac{2}{3}} = \frac{170}{3} \times 15 \times \frac{3}{200} = \underline{\underline{12.75\text{ cm}}}$$

ii. The volume of water inside the bucket correct to 1 decimal place. (3 marks)

$$\text{Volume} = \left(\frac{1}{3} \times 3.142 \times 12.75^2 \times 56\tfrac{2}{3}\right) - \left(\frac{1}{3} \times 3.142 \times 6^2 \times 26\tfrac{2}{3}\right)$$

$$= 9647.90375 - 1005.44$$

$$= 8642.46375$$

$$= \underline{\underline{8642.5\text{ cm}^3}}$$

19. The data below shows the masses of 40 students in a class

45	65	66	67	72	79	69	57
65	58	65	80	66	67	49	50
51	70	79	69	84	52	55	65
85	90	87	69	81	68	69	58
74	70	49	82	58	52	64	60

a) Starting with a class of 45-54 and using , a uniform class width prepare a frequency distribution table. (2 marks)

Class	Tally	Frequency	x	fx	c.f
45-54	~~////~~ //	7	49.5	346.5	7
55-64	~~////~~ //	7	59.5	416.5	14
65-74	~~////~~ ~~////~~ ~~////~~ //	17	69.5	1181.5	31
75-84	~~////~~ /	6	79.5	477	37
85-94	///	3	89.5	268.5	40
		Σf 40		Σfx 2690	

b) From the frequency distribution table above estimate.

i. Mean mass (3 marks)

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{2690}{40} = \underline{67.25}$$

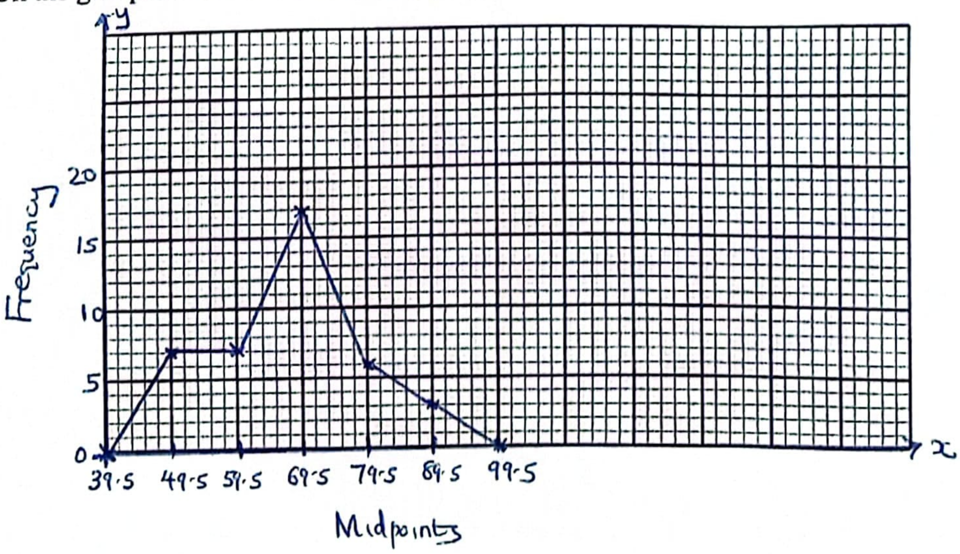
ii. The median mass (3 marks)

$$\text{Median} = L + \frac{\left(\frac{N}{2} - c.f\right) i}{f}$$

$$= 64.5 + \frac{(20-14)\,10}{17}$$

$$= \underline{68.03}$$

c) On the grid provided draw a frequency polygon (2 marks)



30. A parent has two children whose age difference is 5 years. Twice the sum of the ages of two children is equal to the age of the parent.

(a) Taking x to be the age of the elder child, write an expression for:

(i) The age of the younger child (1 mark)

$x-5$

(ii) The age of the parent. (1 mark)

$= 2(x-5+x)$

$= 4x-10$

(b) In twenty years', time, the product of the children's age will be 15 times the age of their parents

(i) Form an equation in x and hence determine the present possible ages of the elder child. (4 marks)

In 20 years time,

elder child $= x+20$

younger child $= x-5+20 = x+15$

Parent $= 4x-10+20 = 4x+10$

$(x+20)(x+15) = 15(4x+10)$

$x(x+15)+20(x+15) = 60x+150$

$x^2+35x+300 = 60x+150$

$x^2-25x+150 = 0$

$x^2-10x-15x+150=0$

$x(x-10)-15(x-10)=0$

$(x-10)(x-15)=0$

$x = $ 10 years or 15 years

(ii) Find the present possible ages of the parent (2 marks)

$= 4(10)-10$ or $4(15)-10$

$=$ 30 years or 50 years

(iii) Find the possible sum of ages of the children in 20 years' time (2 marks)

Sum of the ages $= x+20+x+15$

$= 2x+35$

$= 2(10)+35$ or $2(15)+35$

$=$ 55 years or 65 years

21. The displacement S metres of a bouncing particle after t seconds is given by $s = t^3 - 5t^2 + 7t + 3$.

Determine

a) The displacement of particle during the 4[th] second. (2 marks)

When $t = 3$, $S = 3^3 - 5(3)^2 + 7(3) + 3$

$= 27 - 45 + 21 + 3 = 6\,m$

When $t = 4$, $S = 4^3 - 5(4)^2 + 7(4) + 3$

$= 64 - 80 + 28 + 3 = 15\,m$

Displacement during the 4th second

$= 15 - 6 = \underline{9\,m}$

b) The velocity of the after 4 seconds. (3 marks)

$V = \frac{ds}{dt} = 3t^2 - 10t + 7$

When $t = 4$,

$V = 3(4)^2 - 10(4) + 7$

$= 48 - 40 + 7$

$= \underline{15\,m/s}$

c) The time when the particle is momentarily at rest (3 marks)

At rest, $V = 0$

$3t^2 - 10t + 7 = 0$

$3t^2 - 3t - 7t + 7 = 0$

$3t(t-1) - 7(t-1) = 0$

$(t-1)(3t-7) = 0$

$t = $ 1 second or $2\frac{1}{3}$ seconds

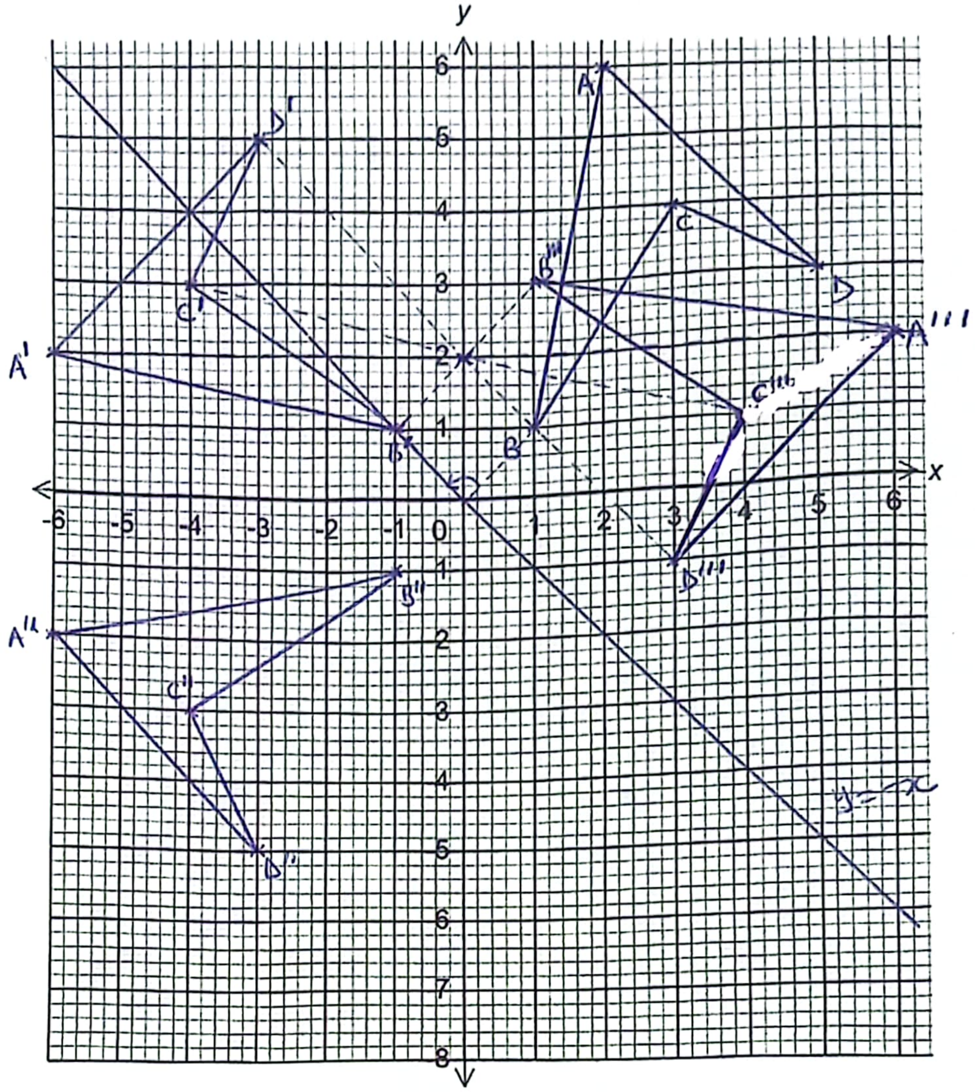
d) The acceleration of the particle when $t = 3$ seconds (2 marks)

$a = \frac{dv}{dt} = 6t - 10$

When $t = 3$, $a = 6(3) - 10$

$= \underline{8\,m/s^2}$

22. (a) The points $A(2,6)$, $B(1,1)$, $C(3,4)$ and $D(5,3)$ are vertices of quadrilateral ABCD. Plot the quadrilateral ABCD. (1 mark)



(b) Locate and write down the coordinates of the points A′B′C′D′ , the images of ABCD under a rotation of positive 90° about the origin. Draw A′B′C′D′ (2 marks)

A′(−6, 2) B′(−1, 1) C′(−4, 3) D′(−3, 5)

(c) A″B″C″D″ is the image of A′B′C′D′ under a reflection in the x – axis. Draw the quadrilateral A″B″C″D″ and write down its coordinates. (2 marks)

A″(−6, −2) B″(−1, −1) C″(−4, −3) D″(−3, −5)

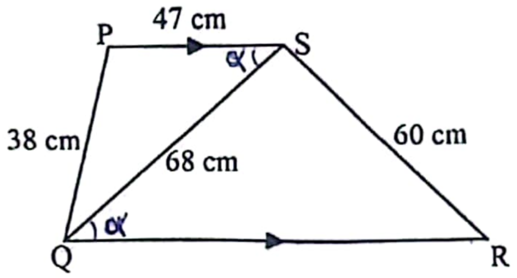
(d) A‴B‴C‴D‴ is the image of A′B′C′D′ under enlargement . Scale factor −1 centre $(0,2)$. On the grid draw A‴B‴C‴D‴ (3 marks)

A‴(6, 2) B‴(1, 3) C‴(4, 1) D‴(3, −1)

(e) Describe a transformation that would map triangle ABCD onto A″B″C″D″ (2 marks)

Reflection in the line $y = -x$

23. The figure PQRS is a trapezium in which PS is parallel to QR. PQ = 38 cm, PS = 47 cm, QS = 68 cm and RS = 60 cm



Calculate to 2 d.p

i. The size of angle QPS (3 marks)

$68^2 = 38^2 + 47^2 - 2 \times 38 \times 47 \times \cos\theta$

$4624 = 1444 + 2209 - 3572\cos\theta$

$3572\cos\theta = -971$

$\cos\theta = -0.2718$

$\theta = \underline{105.77^\circ}$

ii. The size of angle SQR (2 marks)

$\frac{68}{\sin 105.77} = \frac{38}{\sin\alpha}$

$\sin\alpha = \frac{38\sin 105.77}{68} = 0.5378$

$\alpha = 32.53^\circ$

$\angle SQR = \underline{\underline{32.53^\circ}}$

iii. Area of triangle QRS (3 marks)

$\frac{60}{\sin 32.53} = \frac{68}{\sin\beta}$

$\sin\beta = \frac{68\sin 32.53}{60} = 0.6094$

$\beta = 37.55^\circ$

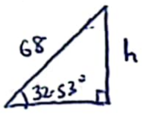
$180 - (37.55 + 32.53) = 109.92^\circ$

Area of Δ QRS

$= \frac{1}{2} \times 68 \times 60 \times \sin 109.92$

$= \underline{\underline{1917.95\ cm^2}}$

iv. Perpendicular height of the trapezium (2 marks)

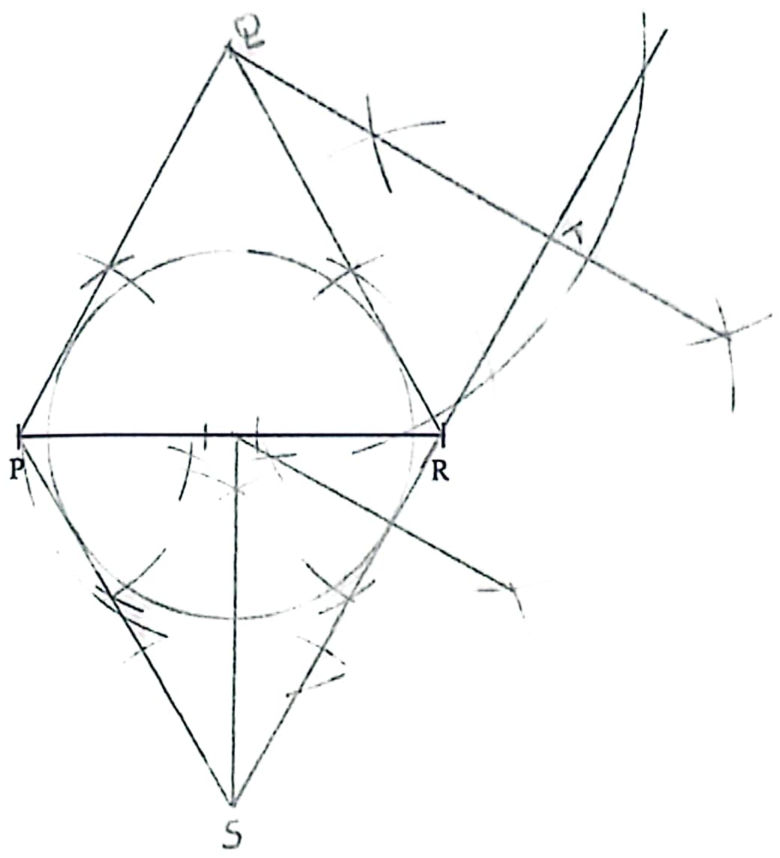


$\sin 32.53^\circ = \frac{h}{68}$

$h = 68\sin 32.53$

$= \underline{\underline{36.57\ cm}}$

24. PR represents the diagonal of a rhombus PQRS in which $\angle PRS = 60^{\circ}$. Complete the figure. (2 marks)



i. Measure QS (1 mark)

QS = 10cm

ii. Construct a perpendicular from Q to meet SR produced at T. Measure QT. (2 marks)

QT = 5cm

iii. Construct a circle to touch the sides of the rhombus (2 marks)

iv. Find the area of rhombus that is outside the circle (3 marks)

Area of the rhombus $= \frac{1}{2} \times 5.6 \times 10 = 28\,cm^2$

Area of the circle $= 3.142 \times 2.4^2$

$= 18.098\,cm^2$

$28 - 18.098 = \underline{9.902\,cm^2}$

THIS IS THE LAST PRINTED PAGE.