HW set 4 2019201049

1.1

Solve $Ax = b$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

(i) is this system consistent?

$$[Ab] \begin{bmatrix} 0 & 1 & \vdots & 6 \\ 1 & 1 & \vdots & 0 \\ 2 & 1 & \vdots & 0 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - \frac{1}{2} R_3$

$$[Ab] \begin{bmatrix} 0 & 1 & \vdots & 6 \\ 0 & 1/2 & \vdots & 0 \\ 2 & 1 & \vdots & 0 \end{bmatrix}$$

$R_1 \longrightarrow R_1 - 2 R_2$

$$= \begin{bmatrix} 0 & 0 & \vdots & 6 \\ 0 & 1/2 & \vdots & 0 \\ 2 & 1 & \vdots & 0 \end{bmatrix}$$

$\because$ Rank(A) < Rank(Ab)

hence inconsistent solution.

Finding least square solution to a system

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$Ax = b$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

Solve using $A^T A x = A^T b$

$$\cancel{A^T A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}}$$

$$\cancel{= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}}$$

$$A^T b = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3} \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

$$= \begin{bmatrix} 0 \\ 6 \end{bmatrix}_{2\times 1}$$

$$A^T A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$$5x_1 + 3x_2 = 0$$
$$3x_1 + 3x_2 = 6$$

$$3x_1 - 5x_1 = 6$$

$$-2x_1 = 6$$

$$\& \boxed{\begin{aligned} x_1 &= -3 \\ x_2 &= 5 \end{aligned}}$$

b) solution is not exact it is least square

c) if the solution is not exact what is quantity being minimized.

$Ax = b$

$r = b - Ax$ i.e minimize $||b - Ax||$

we try to minimize r. length

i.e $\underline{r^T r}$

2.1

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(i) $A = \begin{bmatrix} 2 & -1 & 1 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix}_{2\times 4}$.

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4\times 1}$ $b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_{2\times 1}$

$m = 2$
$n = 4$

(i) since $m<n$ it is an undetermined

mean underconstrained

(ii) minimization is $\Rightarrow$ norm of x.

ie system can be written as

minimise $\|x\|$.
subject to $Ax = b$.

2.1

underconstrained. 2019201049

(ii) least norm problem

hence we say that soln will be

$$A^T (AA^T)^{-1} b$$

$$A = \begin{bmatrix} 2 & -1 & 1 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$AAT = \begin{bmatrix} 7 & -1 \\ -1 & 3 \end{bmatrix} \Rightarrow AA^T = \begin{bmatrix} 3/20 & 1/20 \\ 1/20 & 7/20 \end{bmatrix}$$

$$A^T (AA^T)^{-1} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 3/20 & 1/20 \\ 1/20 & 7/20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3/10 & 1/10 \\ -1/10 & 3/10 \\ 1/10 & -3/10 \\ -1/5 & -2/5 \end{bmatrix}$$

$$x = \begin{bmatrix} 3/10 & 1/10 \\ -1/10 & +3/10 \\ 1/10 & -3/10 \\ -1/5 & -2/5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = x$$

$\|x\|$ will be $\sqrt{2}$

$\boxed{\text{for least norm } \sqrt{2} \text{ a is solution}}$

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a)

~~(3)~~ simplex pivot theory (3.1)

a) Ans true

~~let given statement is true~~

let us consider basic variable x_m

set of non basic variable $= x_s$

$z = \bar{z}_0 + \bar{c}_s x_s$

$x_B = \bar{b} - \bar{A}_s x_s$

here we choose x_s as $c_s < 0$ and we make z small by maxing x_s as large as possible.

but the value of x_s is bounded so as to retain feasibility of $x_B \geq 0$.

Basic feasible soln choosen such that ratio b_i/a_{is} is smallest ie

$$x_s = \frac{\bar{b}_r}{a_{rs}} = \min b_i / a_{is}$$

in case of tie outgoing variable will be zero and hence feasible set obtained after this will have atleast one basic variable zero hence soln be degenerate

simplex pivot theory (3.1

(a)

2.5

b) false in general

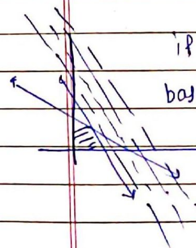
→ solution is different after every pivot step

→ degenerate soln occur. it is possible $b_r = 0$ & in that situation value z will not decrease

→ some selection of s and r happened if iteration are not discontinued for each iteration some basic set could recover and this recurrence can take at every place

→ consecutive feasible solution may same.

c) false



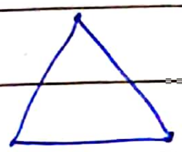
if the objective is parallel to one of basis (i.e line) on which optimal soln lies then then optimal soln may lie on one basis giving infinite many solution

4.1

4 convexity

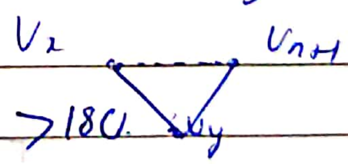
A planar polygon is convex if it contain all the line segments connecting any pair of its points

proof by contradiction & mathematical induction
base case $n = 3$



all angle are less than 180 we can say that all segment are with poly

hence we consider that $p(n)$ is true let $p(n+1)$ be
which means adding one vertices to graph
as we consider that $p(n)$ is true means all the edges are within polygon consider edge $V_x V_{n+1}$ which is outside polygon, $V_x V_y$ is edges which lies right side of $V_x V_{n+1}$



that means $\angle V_x V_y V_{n+1} > 180$.
which contradicts the given that all angles are less than 180° hence whenevery the angle is ~~greater~~ less than so polygon is convex

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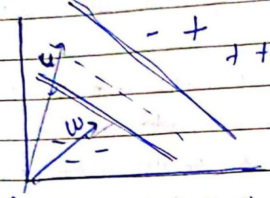
SVM 5.3

infinitely many hyperplanes



If take projection u on w then we can say that

$\bar{w}\cdot\bar{u} + b \geq 0$ then + —— (1)

now let us consider any constant C. which represent

$\bar{w}\cdot\bar{x} + b \geq C$ +ve class

$\bar{w}\cdot\bar{x} + b \leq -C$ -ve class. —— (2)

Q. if add or substract constant t_1 & t_2 from eq (1)

$\bar{w}\cdot\bar{x} + \underline{b - t_1} = 0$ for + class

$\bar{w}\cdot\bar{x} + \underline{b + t_2} = 0$ for - class

hence each value between (t_1, t_2) will cut point in two classes means $(b+C)$ such that $C \in (t_1, t_2)$ will cut classes into two parts

but as infinite points lies bet' (t_1, t_2) there are many constant ~~which~~ ~~represent~~ for hyperplane which seperate points

we can write any value of c in eqn (2)

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how do we judge which is better ?

As we know that greater the width of margin that solution is more better

more is margin better result on user data.

hence we formulate ~~width~~ objective function to maximise the width

and derive the hyperplane which minise the cost function due to outliers

Primal formulation.

$$\text{width} = (\underbrace{\bar{x}_+}_{(1-b)} - \underbrace{\bar{x}_-}_{(1+b)})\cdot\frac{\bar{w}}{\|w\|} = \frac{2}{\|w\|}$$

primal formulation $\max \frac{2}{\|w\|}$ i.e $\min \frac{1}{2}\|w\|$

s.t $y_i(w^T x_i + b) \geq 1$ $i = 1, \dots, N$

we construct lagrangian for optimization

$$\frac{1}{2}\|w\|^2 + \sum_{i=1}^{N} \alpha_i \left(1 - d_i(w^T x_i + b)\right)$$

subject to $\alpha_i \geq 0 \ \forall i$

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why it is undesirable but not forbidden to have points of one class on opposite side of hyperplane.

→ we can allow some misclassified points to make our classifying hyperplane more general to avoid overfitting
so that it can classify new points better as will have good margine.

The primal formulation changes as follows

$$\min \frac{\|w\|}{2} + \frac{C}{n} \sum \epsilon_i$$

$$\text{s.t} \quad y_i(w x_i + b) \geq 1 - \epsilon_i$$

$$\forall i \quad \& \quad \epsilon_i \geq 0$$

where i is the parameter that allow some of missclassification for maximizing the margin

6.2

6. KKT b)

what are the KKT conditions ? & for what cases are these constraints sufficient optimality

KKT conditions: The Karush-kuhn Tucker condition

1) first derivative test for a solution in non linear programing to be optimal provided that some regularity condition are satisfied

Necessary conditions

suppose that objective function. $f: R^n \rightarrow R$ and the constrain function $g_i : R^n \rightarrow R$ and $h_j\ R^n \rightarrow R$ are continuously differenciable at a pt x^* if x^* is local optima and the optimisation problem satisfies some regular condition. then there exist constants $\mu_i\ (i=1,\dots m)$ & $\lambda_j\ (j=1,\dots,l)$ called KKT multiplies such that following condition hold

(i) $f(x)$: $\nabla f(x^*) \mp \sum_{i=1}^{M} \mu_i \nabla g_i (x^*) - \sum_{j=1}^{l} \lambda_i \nabla h_j (x^*) = 0$

for maximizing (-) & minimizing (+)

this is stationary

primary feasibly $g(x^*) \leq 0$ for $i=1 \cdots m$.

$K(x^*)$ 0 for $j=1, \dots l$

(ii) dual feasibilty $\mu_i \geq 0$ for $i=1 \cdots m$.

v) complementory slackess

$\mu_i g_i (x^*) = 0$ for $i=1, \dots M$.

sufficient for optimality

objective function. z of a maximation problem is a concave function. the equality constant g_i are continuously differenciable f^n g_i convex f^n & the equility constraint k_j are affine function simulary if the objective f^n of minimization problem is convex f^n necessary condition are sufficient for optimality

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7.2 (alloted in shiksha portal)

min $c^T x$.

$1^T x \le k$

$0 \le x_i \le 1$ i =

~~optima~~ primal function → X

dual → Y

x^* is optimal and y^* is optimal of dual

if and only if

$x_j(c^T A_j - c_j) = 0 \quad \forall j$

$y_i(b_i - A_i x_i) = 0 \quad \forall i$

x^* is primal optimal iff y^* exist

& x^* and y^* should be complement of each other

min $c^T x$

s.t $1^T x \le b$

$x \le 1$

$0 \le x$

Let y & α be vectors of dual variable

corresponding to $1^T x \le k$ and $x \le 1$ respectively

max $k^T y + 1^T \alpha$

$y^T + \alpha^T \ge c^T$

$y \ge 0 \quad \alpha \ge 0$

vector x & $(y^T, \alpha^T)^T$ is optimal to primal dual respectively iff

$$x_j \left(y^T 1 + \alpha_j - c_j \right) = 0 \quad \forall j$$

$$y_i \left(k - 1^T x \right) = 0 \quad \forall i$$

$$\alpha_j \left(1 - x_j \right) = 0 \quad \forall j$$

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8.1

8.1

$$\max z = 2x_1 + 3x_2 + 0S_1 + 0S_2 - MA_1 - MA_2.$$

S.T

$$5x_1 + 25x_2 + S_1 = 40$$

$$x_1 + 3x_2 - S_2 + A_1 = 20$$

$$x_1 + x_2 + A_2 = 20$$

and $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

B	C_j	2	3	0	0	-M	-M	X_B

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iteration-1

B	C_B	C_j / x_B	2 / x_1	3 / x_2	0 / S_1	0 / S_2	-M / A_1	-M / A_2	x_B/x_2
S_1	0	40	5	(25)	1	0	0	0	$40/25 = 1.6$
A_1	-M	20	1	1	0	0	1	0	$20/1 = 20$
A_2	-M	20	1	3	0	-1	0	1	$20/3 = 6.6667$
$z = -40M$		z_j	-2M	-4M	0	+M	-M	-M	
		$z_j - c_j$	-2M-2	-4M-3	0	M	0	0	

Negative minimum $z_j - c_j$ is $-4M-3$ and its column index 2 so the entering variable is x_2

Min ratio is 1.6 and its row index is 1 so leaving basis variable is S_1

∴ The pivot element is 25.

$\boxed{\text{iteration 2}}$

B	C_B	C_j / x_B	2 / x_1	3 / x_2	0 / S_1	0 / S_2	-M / A_1	-M / A_2	x_B/x_1
x_2	3	1.6	(0.2)	1	0.04	0	0	0	8
A_1	-M	18.4	0.8	0	-0.04	0	1	0	23
A_2	-M	15.2	0.4	0	0.12	-1	0	1	38
$z = -33.6M + 4.8$		z_j	$-1.2M + 0.6$	3	0.16M	M	-M	-M	
		$z_j - c_j$	$-1.2M - 1.4$	0	0.16M +0.12	M	0	0	

~~Neg~~ negative min $z_j - c_j$ is $-1.2M - 1.4$ and its colum index is 1 so, the ending variable is x_1

minimum ratio is 8 and its row index. is 1

so the leaving basis variable is x_2

the pivot element is 0.2

$\boxed{\text{iteration 3}}$

B	C_B	C_j / X_B	X_1	X_2	S_1	S_2	A_1	A_2
x_1	2	8	1	5	0.2	0	0	0
A_1	-M	12	0	-4	-0.2	0	1	0
A_2	-M	12	0	-2	-0.2	-1	0	1
$z = -24M + 16$		z_j	2	6M+10	0.4M +0.4	M	-M	-M
		$z_j - c_j$	0	6M+7	0.4M +0.4	M	0	0

Since all $z_j - c_j \geq 0$

hence optimal soln is arrived with value of variable as:

$x_1 = 8$, $x_2 = 0$

Max $z = 16$

But the soln is not feasible

becoz the final soln violates the 2nd constraint $x_1 + x_2 \geq 2$

and the A_1 appears in the basis with +ve 12

9.3

we know that

product of eigen values of matrix is equal to determinant of ~~A~~ matrix,

$\therefore$ ~~to~~ consider matrix of $n \times n$ ~~of~~ lonel As A who has eigen value $\lambda_1, \lambda_2 \ldots \lambda_n$

hence we can write as

$$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \ldots \lambda_n = |A|$$

to make this product zero at least one eigen value should be zero

hence from this property we can conclude that, ~~product of eig~~ ~~at~~ for determinant of matrix become zero at least one. eigen value should be zero

10.1

Q.10 What can we say about matrix B if $AB = BC$ and matrices A and C have no common eigenvalues.

Let us consider that A and C have common eigen value λ

$$\boxed{Au = \lambda u.}$$

Let u be a right eigenvector for A

and let v be left eigenvector for C that is

$$\boxed{v^T C = \lambda v^T}$$

We have $AB = BC$

$AB = Auv^T = \lambda uv^T = \lambda B$

$\cancel{\lambda u^T B} = \cancel{v^T A B \lambda u}$

$BC = uv^T C = \lambda uv^T = \lambda B$

$\cancel{v^T B} = \cancel{v^T B u}$

means if A and C have at least one common eigen value λ then $B = uv^T$ which is non-zero matrix.

As their is no common eigen value ~~B is no~~ $B = 0$