HW Set ~~[illegible]~~.

Q.1 solve the following algo using bala's algorithm.

max $9x_1 + 5x_2 + 6x_3 + 4x_4$.

S.T

$6x_1 + 3x_2 + 5x_3 + 2x_4 \le 10$

$x_3 + x_4 \le 1$

$x_3 \le x_1$

$x_4 \le x_2$

x_j is binary for $j = 1 \dots 4$

$\Rightarrow$ all x_j's are binary

$y_i = (1 - x_i)$

min $(9y_1 + 5y_2 + 6y_3 + 4y_4)$

S.T

$6 - 6y_1 + 3 - 3y_2 + 5 - 5y_3 + 2 - 2y_4 \le 10$

$2 - y_3 - y_4 \le 1$

$1 - y_3 \le 1 - y_1$

$1 - y_4 \le 1 - y_2$.

Subject $-6y_1 - 3y_2 - 5y_3 - 2y_4 \le -6$

$\Rightarrow 6y_1 + 3y_3 + 5y_3 + 2y_2 \geq 6$

$0y_1 + 0y_2 - y_3 - y_4 \le -1$

$\Rightarrow 0y_1 + 0y_2 + y_3 + y_4 \geq 1$

$-y_1 + y_3 \geq 0$

$-y_2 + y_4 \geq 0$

hence converted objective f^n is

$$\min \left(\underset{④}{9y_1} + \underset{②}{5y_2} + \underset{③}{6y_3} + \underset{①}{4y_4} \right)$$

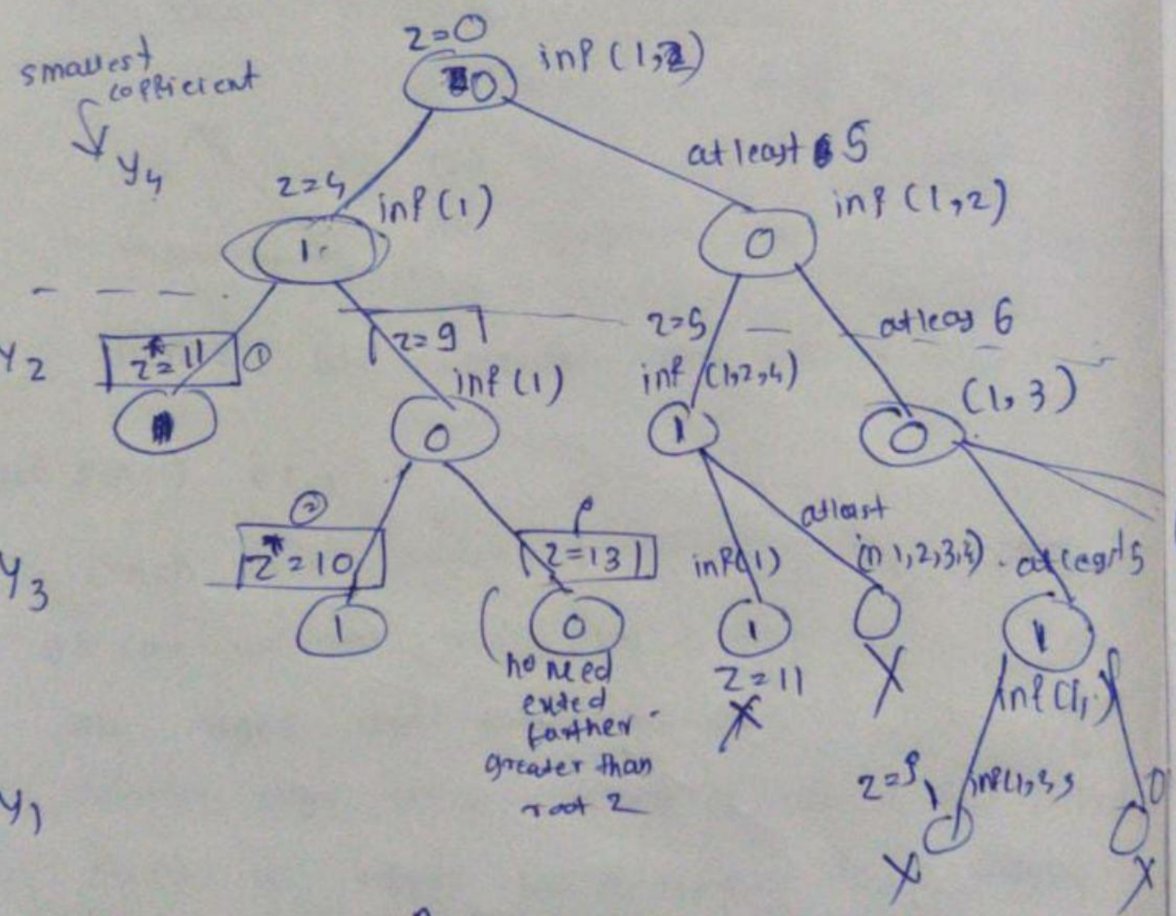
S-T

$$6y_1 + 3y_2 + 5y_3 + 2y_4 \geq 6 \quad \text{—— (1)}$$

$$0y_1 + 0y_2 + y_3 + y_4 \geq 1 \quad \text{—— (2)}$$

$$-y_1 + y_3 \geq 0 \quad \text{—— (3)}$$

$$-y_2 + y_4 \geq 0 \quad \text{—— (4)}$$



hence final $sol^n \rightarrow y_1 = 0,\ y_2 = 0,\ y_3 = 1,\ y_4 = 1$

i.e

$$\boxed{x_1 = 1,\ x_2 = 1,\ x_3 = 0,\ x_4 = 0}$$

a) let x^* be solution of ILP
y^* = LP relaxation

y^* satisfies constraint of ILP ie
$x_i + x_j \geq 1 \quad \forall \{i,j\} \in E$
and

x_i's are real number bet' 0 & 1 for LP relax version

a feasible region of LP relax is always inside ILP.

$$OP_{LP} < OP_{ILP}$$

Also we know that LP soln is inferior.
for minimization problem $OPT_{LP} \leq OP_{ILP}$

b) $x_u + x_v \geq 1$
at least one should be ½ and it picked in the vertex cover. i.e one of u & v belongs to S.
hence LPP formulation gives valid vertex cov

c) $OPT_{\text{revised}} = \sum_{i \in V} x_i^* \quad x_i^* \begin{cases} 1 & \text{if } x_i \geq 1/2 \\ 0 & \text{if } x_i < 1/2 \end{cases}$

$\leq \sum_{i \in V} 2x_i$ where x_i is fractional value.

$= 2\,OPT_{LP}$ (solution of LP relax version)

$\leq 2\,OPT_{ILP}$ (Revised (a))

$$\frac{OPT_{\text{revised}}}{OPT_{ILP}} \leq 2$$

hence max of this is $\underline{\underline{2}}$

hence LPR version is 2-approximation of ILP classmate

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Given the value of λ

As we know that if

$2x_1 + \cancel{2x_2 + 7x_3 = 7}$

$\cancel{3x_1} +$

$$[A|B] \quad \left[\begin{array}{ccc:c} 2 & 2 & 7 & 7 \\ 3 & \lambda+9 & \lambda+11 & 10 \\ 3 & \lambda+4 & 11 & 10 \end{array}\right]$$

$R_2 \longrightarrow R_2 - 3/_2 R_1$

$R_3 \longrightarrow R_3 - 3/_2 R_1$

$$\left[\begin{array}{ccc:c} 2 & 2 & 7 & 7 \\ 0 & \lambda+6 & \lambda+1/_2 & -1/_2 \\ 0 & \lambda+1 & 1/_2 & -1/_2 \end{array}\right]$$

$$\left[\begin{array}{cccc} 2 & 2 & 0 & \\ 0 & 1 & \frac{\lambda+1/_2}{\lambda+6} & \frac{-1}{2(\lambda+6)} \\ 0 & \lambda+1 & 1/_2 & -1/_2 \end{array}\right]$$

$R_3 \longrightarrow R_3 - (\lambda+1) R_1$

$$\left[\begin{array}{ccc:c} 2 & 2 & 2 & 7 \\ 0 & 1 & \frac{\lambda+1/2}{\lambda+6} & -\frac{1}{2(\lambda+6)} \\ 0 & 0 & \frac{1}{2} - \frac{(\lambda+1)(\lambda+1/2)}{(\lambda+6)} & -\frac{1}{2} + \frac{(\lambda+1)}{2(\lambda+6)} \end{array}\right]$$

if $\frac{1}{2} - \frac{(\lambda+1)(\lambda+1/2)}{(\lambda+6)} = 0$

& $\lambda \neq -6$

$$\frac{(\lambda+1)(\lambda+1/2)}{(\lambda+6)} = \frac{1}{2}$$

$$(2\lambda+2)(\lambda+1/2) = \lambda+6$$

$$2\lambda^2 + \lambda + 2\lambda + 1 = \lambda + 6$$

$$2\lambda^2 + 2\lambda - 5 = 0$$

$$\lambda = \frac{-2 \pm \sqrt{44}}{4} = \frac{-1 \pm \sqrt{11}}{2}$$

$$-\frac{1}{2} + \frac{(\lambda+1)}{2(\lambda+6)} \neq 0$$

$$(\lambda+1) \neq (\lambda+6)$$

that means this matrix will never be zero

~~hence for~~ $\cancel{\lambda = \frac{-1 \pm \sqrt{81}}{2}}$ ~~give equation have no solution~~ & for any value of λ do not have infinitely many solation because [A | B] never have $\rho(A|B)$ less than 3

$F(x) : 10x_1 + 2x_2 + 11x_3 + Mx_4 + Mx_5 = \min$

$2x_1 + 7x_4 + x_3 + x_4 = 4$

$5x_1 + 8x_2 - 2x_3 + x_5 = 17$

			x_1	$x_2 \downarrow$	x_3	x_4	x_5	Q.
B	cb	P	10	2	11	M	M.	
$x_4 \leftarrow$	M	4	2	$\boxed{7}$	1	1	0	0.57.
x_5	M	17	5	8	-2	0	1	2.13

Min 21M.

Iteration 2.

			$x_1 \downarrow$	x_2	x_3	x_4	x_5	Q.
B	cb	P	10	2	11	M	M.	
$x_2 \leftarrow$	2	0.57	0.79	1	0.14.	0.14	0	2.
x_3.	M	12.43	2.71	0	-3.14	-1.14	1	4.58

min 17.43M+1.14

~~minimize $\| C(x-x_0) \|_2$~~

~~subject to $Ax = b$~~

			x_1	x_2	x_3	x_4	x_5
B	Cb	P	10	2	11	M	M
x_1	10	2	1	3.5	0.5	0.5	0
x_5	M	7	0	-9.5	-4.5	-2.5	1
		7M+10	0	-9.5M+33			0

system has no solation or ney many solution.

cholesky decomposition.

$A = LL^T$, every symmetric ~~pos~~ PD matrix A can be decomposed into product of unique lower triangular matrix and its transporse.

$\Rightarrow$ Here Matrix is symmetric
let us check for PD.

$$\begin{bmatrix} 4 & 6 & 2 & -6 \\ 6 & 34 & 3 & -9 \\ 2 & 3 & 2 & -1 \\ -6 & -9 & -1 & 8 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 3/2 R_1$
$R_3 \leftarrow R_3 - 1/2 R_1$
$R_4 \leftarrow R_4 + 1.5 R_1$

$$\begin{bmatrix} 4 & 6 & 2 & -6 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ -6 & -9 & -1 & 8 \end{bmatrix}$$

$R_4 \leftarrow R_4 - 2R_3$

$$\begin{bmatrix} 4 & 6 & 2 & -6 \\ 0 & 25 & 0 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

pivot are the first non zero element in each of this eliminated matrix

here -5 is not positive hence matrix matrix is not PD

hence cholesky decomposition is not possi show the calculation

$$\begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 5 \\ 3 & 9 & 4 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 65 & 35 & 75 \\ 35 & 26 & 29 \\ 75 & 29 & 106 \end{bmatrix}$$

eigen vector of AA^T

$$|AA^T - \lambda I| = 0$$

$$\begin{vmatrix} (65-\lambda) & 35 & 75 \\ 35 & (26-\lambda) & 29 \\ 75 & 29 & (106-\lambda) \end{vmatrix}$$

$(76-\lambda)\ (76-\lambda) \times \left((106-\lambda) - 29\times 29\right)$

$- 35\left(35 \times (106-\lambda) - 19\times 25\right) + 75\ \left(35\times 29 - (76-\lambda)\right.$ $\times 35$

$= 0$

$(65-\lambda)\left(\left((\cancel{286}\,2756 - 132\lambda + \lambda^2) - 841\right) - 35\,(3716 - 35\lambda - 2175)\right.$

$+ 75\,(1015 - (1950 - \ldots$

$= 0$

$\lambda^3 - 197\lambda^2 + 364\lambda - 625 = 0$

$\lambda = 176.351102,\ 20.478,\ 0.173086$

+1 Eigenvectors for $\lambda = 176.351102$

$V_1 = \begin{bmatrix} 0.792133 \\ 0.37281 \\ 1 \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} -0.613546 \\ -1.367353 \\ 1 \end{bmatrix}$

$V_3 = \begin{bmatrix} -2.052217 \\ 1.65855 \\ 1 \end{bmatrix}$

$$A^T \cdot A = \begin{bmatrix} 13 & 37 & 24 \\ 37 & 107 & 71 \\ 24 & 71 & 77 \end{bmatrix}$$

Find eigen vector of $\begin{bmatrix} (13-\lambda) & 37 & 24 \\ 37 & (107-\lambda) & 71 \\ 24 & 71 & (77-\lambda) \end{bmatrix}$

$$= (\lambda^3 - 197\lambda^2 + 364\lambda - 625) = 0$$

$$V_1 = \begin{bmatrix} 0.430684 \\ 1.21966 \\ 1 \end{bmatrix} \qquad V_2 \begin{bmatrix} -0.273047 \\ -0.703818 \\ 1 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 50.134769 \\ -18.028864 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/d_1 & 0 & 0 \\ 0 & 1/d_2 & 0 \\ 0 & 0 & 1/d_3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/d_1 & 1/d_1 & 1/d_1 \\ 1/d_2 & 1/d_1 & 1/d_2 \\ 1/d_3 & 1/d_3 & 1/d_3 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_2/d_1 & d_3/d_1 \\ d_1/d_2 & 1 & d_3/d_1 \\ d_1/d_3 & d_2/d_3 & 1 \end{bmatrix} = B$$

$$|B - \lambda I| = 0$$

$$\begin{vmatrix} (1-\lambda) & d_2/d_1 & d_3/d_1 \\ d_1/d_2 & (1-\lambda) & d_3/d_1 \\ d_1/d_3 & d_2/d_3 & (1-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)^2 - \frac{d_3}{d_2} \cdot \frac{d_2}{d_3}\right]$$

$$+ \frac{d_2}{d_1}\left[\cancel{\frac{d_3}{d_2} \cdot \frac{d_1}{d_3}} - \frac{d_1}{d_2}(1-\lambda)\right]$$

$$+ \frac{d_3}{d_1}\left[\frac{d_1}{d_2} \cdot \frac{d_2}{d_3} - \frac{d_1}{d_3}(1-\lambda)\right]$$

(1)

$(1-\lambda)^3 - (1-\lambda) + [1-(1-\lambda)] + [1-(1-\lambda)] =$

$\cancel{1} - \cancel{3}\lambda + 3\lambda^2 - \lambda^3 - \cancel{1} + \cancel{3}\lambda$

$$3\lambda^2 - \lambda^3 = 0$$

$$\lambda^2(3-\lambda) = 0$$

$$\boxed{\lambda = 0, 0, 3}$$

$Ax = b$

minimize $\|x - x_0\|_2^2$

s.t. $\quad Ax = b$

In least norm we solve $\min \|x\|$

subject to $Ax = b \quad m < n.$

hence we can move origin to x_0

$$x' = x - x_0$$

$$\boxed{x' + x_0 = x}$$

$$\min \|x'\|$$

subject to $\quad Ax' = b$

$$A(x - x_0) = b$$

$$Ax - Ax_0 = b$$

$$Ax = \underbrace{b + Ax_0}_{\text{constant}}$$

$$Ax = b'$$

$$(x')^T(x') - \lambda(Ax - b')$$

$$2x + A^T\lambda = 0$$

$$Ax - b' = 0$$

$$x = \frac{-A^T\lambda}{2}$$

$$\frac{A(-A^T\lambda)}{2} - b' = 0$$

$$\lambda = -2(AA^T)^{-1}b'$$

$$x = A^T(AA^T)^{-1}b'$$

we know that $b' = (b + Ax_0)$

$$x = A^T(AA^T)^{-1}(b + Ax_0)$$

minimize $z = 6x_1 + 7x_2 + 3x_3$

subject to: $5x_1 + 6x_2 + x_3 \leq 9$

$-2x_1 - x_2 - 4x_3 = 3$

$13x_1 + 0x_2 - 8x_3 \leq 0$

$x_1, x_2, x_3 \leq 0$

Revised primal

$z = 6x_1 + 7x_2 + 3x_3$

Subject to:

y_1 $\quad -5x_1 - 6x_2 - x_3 \geq 9$

y_2 $\quad -2x_1 - x_2 - 4x_3 \geq 3$

y_3 $\quad 2x_1 + x_2 + 4x_3 \geq -3$

y_4 $\quad -13x_1 \cancel{+0} - 0x_2 + 8x_3 \geq 0$

$x_3, x_1, x_2 \geq 0$

$\uparrow$

z^*

Dual of revised problem. (Not a final dual)

maximise $z^* = 9y_1 + \underline{3y_2 - 3y_3} + 0y_4$

$-5y_1 - \underline{2y_2 + 2y_3} - 13y_4 \leq 6$

$-6y_1 - \underline{y_2 + y_3} \leq 7$

$-y_1 - \underline{4y_2 + 4y_3} + 8y_4 \leq 3$

$y_1, y_2, y_3, y_4 \geq 0$

substitute $y' = (y_2 - y_3)$ where y' is unrestricted

~~maximize~~ $\cancel{z^* = -16 y_2 + 30 y' + 10 y_4}$
~~subject to~~ $\cancel{-2y_1 + y' + y_4 \leq 1}$

maximize $z^* = 9y_1 + 3y'$

$-5y_1 - 2y' - 13y_4 \leq 6$
$-6y_1 - y' \leq 7$
$-y_1 - 4y' + 8y_4 \leq 3$
$y_1, y_4 \geq 0$

y' unrestricted in sign.