LHS = RHS ∴ hence proved.

(1.1)

Verify the following identities.

1. $\|a+b\|^2 + \|a-b\|^2 = 2(\|a\|^2 + \|b\|^2)$.

$\|a-b\|^2 = \langle a-b, a-b \rangle$

$= \langle a,a \rangle - 2\langle a,b \rangle + \langle b,b \rangle$

$= \|a\|^2 - 2\langle a,b \rangle + \|b\|^2$ —— (a)

$\|a+b\|^2 = \langle a+b, a+b \rangle$

$= \langle a, a+b \rangle + \langle b, a+b \rangle$

$= \langle a,a \rangle + \langle a,b \rangle + \langle b,a \rangle + \langle b,b \rangle$

$= \langle a,a \rangle + 2\langle a,b \rangle + \langle b,b \rangle$

$= \|a\|^2 + 2\langle a,b \rangle + \|b\|^2$ —— (2)

adding (a) & (b) we get

$= 2(\|a\|^2 + \|b\|^2)$

hence proved.

2. $(a+b)^T(a-b) = \|a\|^2 - \|b\|^2$.

if we represent $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$ in R^n by column vectors, then their eucledian inner product is given by

$$\boxed{\langle u, v \rangle = u^T v = v^T . u}$$

$$(a+b)^T (a-b)$$

$$(a^T + b^T)(a-b)$$

$$a^T a - a^T b + b^T a - b^T b.$$

$$\langle a, a \rangle \underline{- \langle a, b \rangle + \langle a, b \rangle} - \langle b, b \rangle$$

$$\langle a, a \rangle - \langle b, b \rangle$$

$$\|a\|^2 - \|b\|^2.$$

graphically

1.2

A matrix B is symmetric if $B = B^T$. Prove that for any square matrix B, $B + B^T$ is symmetric and that if A is invertible then $(A^{-1})^T = (A^T)^{-1}$.

given:- A matrix B is symmetric if $B = B^T$

to prove : $(B + B^T) = (B + B^T)^T$

.... (expansion by property of transporse.)

$$(B + B^T) = (B)^T + \underline{\underline{(B^T)^T}}$$

.... $((A)^T)^T = A)$

$$(B + B^T) = (B)^T + (B)$$

.... (for addition matrix is commutative)

$$(B + B^T) = B + (B)^T$$

LHS = RHS.

hence proved.

To prove : if A invertible then $(A^{-1})^T = (A^T)^{-1}$

$$\Rightarrow (A^{-1})^T = (A^T)^{-1}$$

multiply by $(A)^T$

$$(A^T)(A^{-1})^T = \underline{\underline{(A^T)(A^T)^{-1}}}$$

$$\therefore \boxed{(A)^T (A^{-1})^T = I}$$

$(A \cdot A^{-1} = I)$

.... by converse of distribution of transpose.

$(A \cdot A^{-1})^T = I$

but $(A \cdot A^{-1} = I)$

$(I)^T = I$

$I = I$

LHS = RHS : hence proved.

(1.1)

1.3 For finite dimensional vector space, prove that l_1 and l_2 norms are equivalent. Specifically, there exist constants $c_1, c_2 \in \mathbb{R}$ such that $0 < c_1 \leq c_2$ and

$$c_1 \|x\|_2 \leq \|x\|_1 \leq c_2 \|x\|_2 \quad \forall x.$$

$\Rightarrow$ consider vector x with linear combination of $\{x_1, x_2, x_3 \dots x_n\}$ & $\{\alpha_1, \alpha_2, \alpha_3 \dots \alpha_n\}$ scalar

$$\therefore \|x\|_1 = \sum_{i=1}^{n} |\alpha_i| \quad \text{———— (1)}$$

$$\|x\|_2 = \|\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 \dots \alpha_n x_n\|_2$$

$$\|x\|_2 \leq |\alpha_1| \|x_1\| + |\alpha_2| \|x_2\|_2 + \alpha_3 \|x_3\|$$

$$\|x\|_2 \leq \beta \sum_{i=1}^{n} |\alpha_i| \quad \text{———— (2)}$$

... as $\{x_1, x_2, x_3 \dots\}$ linear independent & triangular inequality

from.

$$\boxed{\frac{1}{\beta} \|x\| \leq \|x\|_1} \quad \text{———— (3)}$$

Now interchange the role of $\|\cdot\|$ and $\|\cdot\|_1$ in (3) we obtain.

$$\|x\|_1 \leq \alpha \|x\| \quad \text{———— (4)}$$

from 3 & (4)

$$\frac{1}{\beta} \|x\| \leq \|x\|_1 \leq \alpha \|x\|$$

β & α are constant

$$c_1 \|x\| \leq \|x\|_1 \leq c_2 \|x\|$$ hence proved

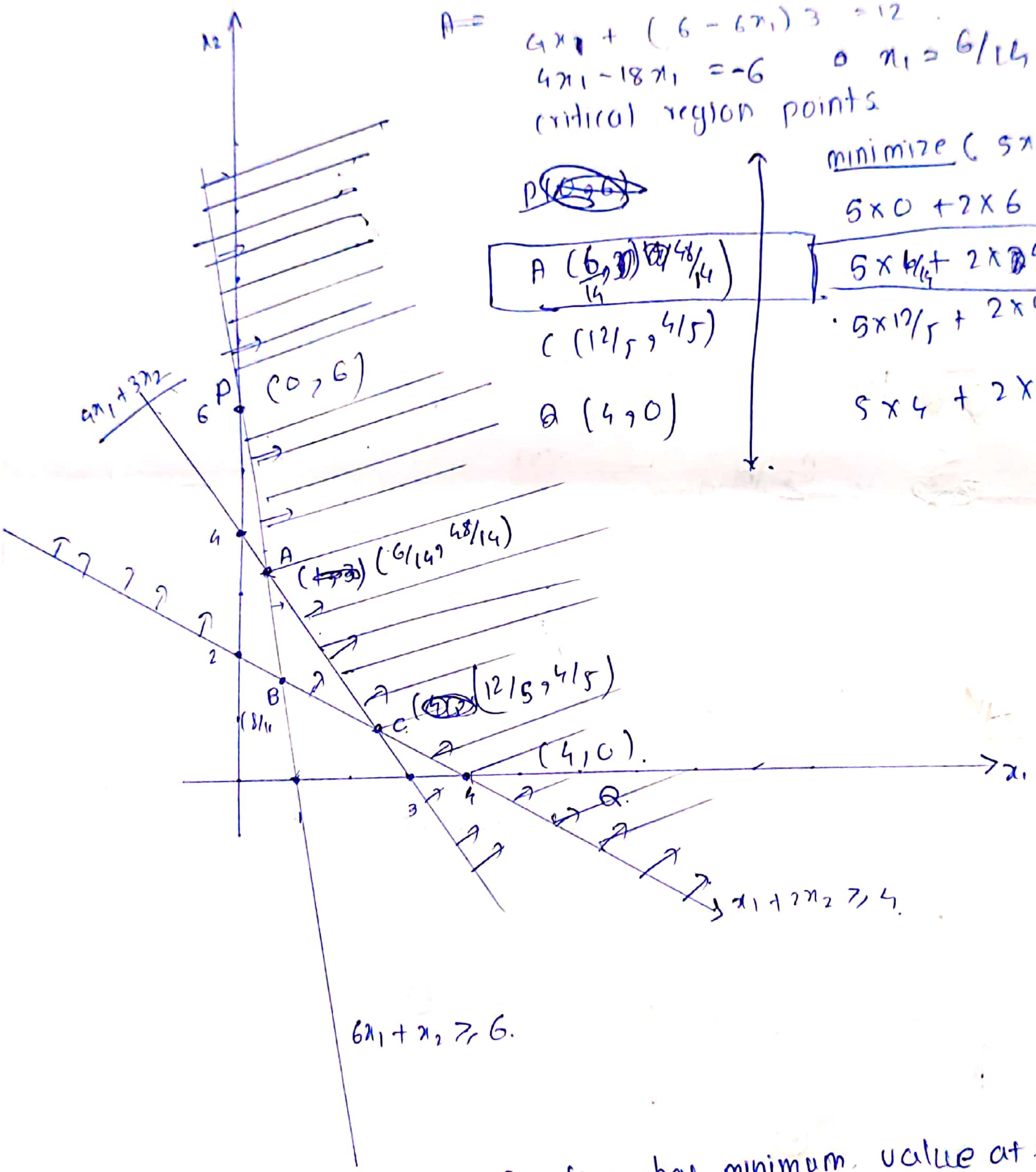
2.1 Solve the following problem graphically

<u>minimize</u> $z = 5x_1 + 2x_2$

subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$



A = $4x_1 + (6 - 6x_1)3 = 12$

$4x_1 - 18x_1 = -6 \quad \Rightarrow x_1 = 6/14$

critical region points

<u>minimize</u> $(5x_1 + 2x_2)$

P (0, 6) : $5 \times 0 + 2 \times 6 = 12$

A $(6/14, 48/14)$: $\boxed{5 \times 6/14 + 2 \times 48/14 = 9}$

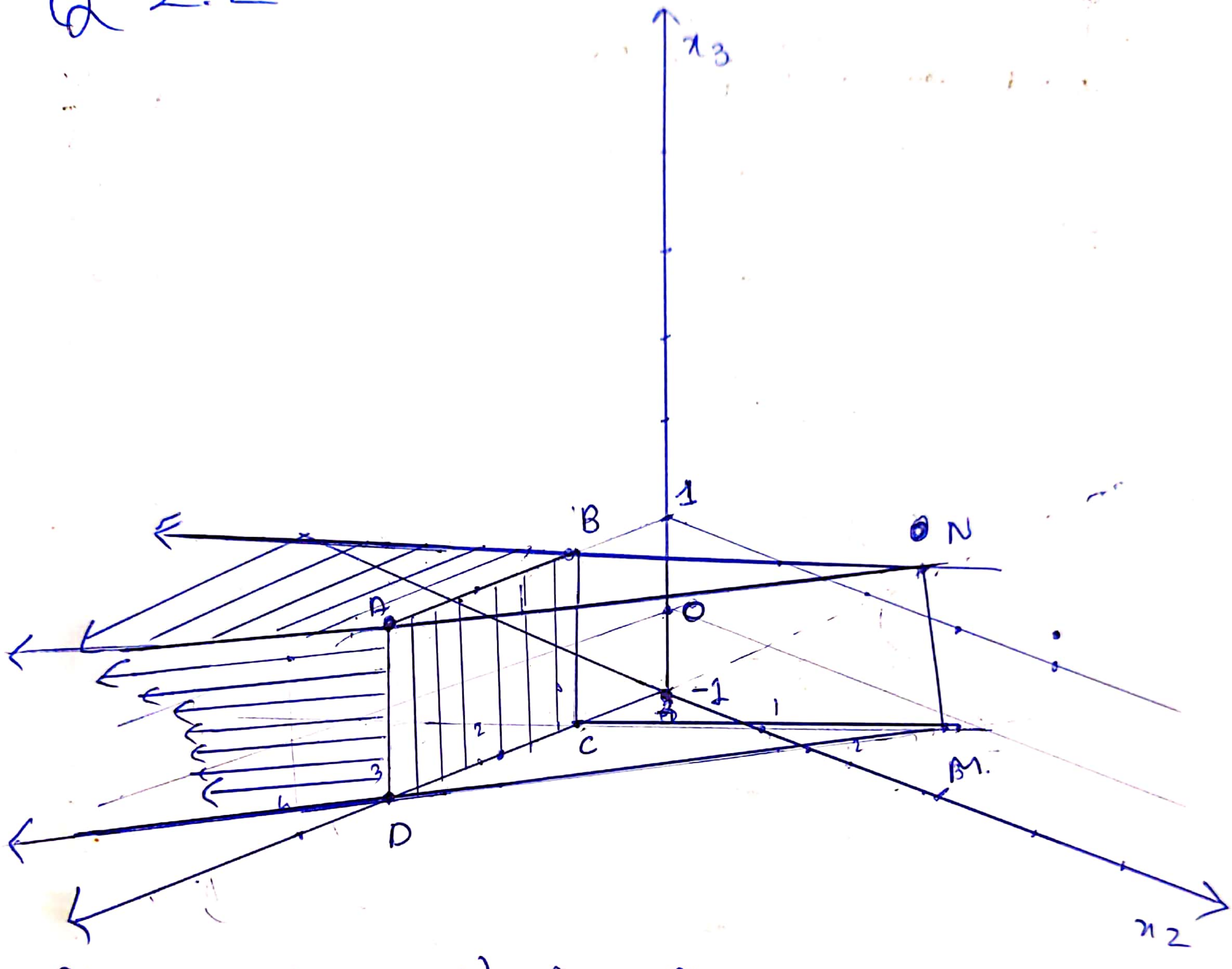
C $(12/5, 4/5)$: $5 \times 12/5 + 2 \times 4/5 = 12 + 8/5$

Q (4, 0) : $5 \times 4 + 2 \times 0 = 20$

hence given objective function has minimum value at A$\boxed{(6/14, 48/14)}$

minimum value is $\boxed{9}$

Q 2.2



A (3, 0, 1)
B (1, 0, 1)
C (1 0, -1)
D (3, 0, -1)

corner ~~and~~ points of feasible region.

Scanned by CamScanner

(i) $c = (-1, 0, 1)$

$z = -x_1 + x_3$

		z value
A	$(3, 0, 1)$	-2
B	$(+1, 0, 1)$	0
C	$(1, 0, -1)$	-2
D	$(3, 0, -1)$	-4

hence we get minimum value of z at -4

as $-x_1 + x_3 \le -4$ has no point in feasible region hence z has optimal minimum value as -4

(ii) $(0, 1, 0)$

$z = x_2$

		(z value)
A	$(3, 0, 1)$	0
B	$(1, 0, 1)$	0
C	$(1, 0, -1)$	0
D	$(3, 0, -1)$	0

~~same values~~ minimum

no ~~maximum~~ value of objective function can be defined

minimum ~~pers~~ value of z at A, B, C, D is 0

As region is un-bounded x_2 ~~is~~ minimum value is $-\infty$ as ans $x_2 < 0$

Line is common to feasible region.

(iii) $(3, 0, -1)$

$z = -x_3$

		z value
A	$(3, 0, 1)$	-1
B	$(1, 0, 1)$	-1
C	$(1, 0, -1)$	1
D	$(3, 0, -1)$	1

minimum value of z at infinite points on line AB is -1

Q.3 transportation problem

Given:- cannery 1 can transport maximum 250 & 2 will transport at max 450

x_{ij} will be decision variable.

& $i \in \{1,2\}$, $j \in \{a,b,c\}$ where x_{ij} represent number of cases transported by ith cannery to jth warehouse.

minimization ~~maximization~~ of transportation required

hence

$$\min \left(z = x_{1a} \cdot 3.4 + 2.2\, x_{1b} + 2.9\, x_{1c} + 3.4\, x_{2a} + 2.4\, x_{2b} + 2.5\, x_{2c} \right)$$

constraints.

$x_{1a} + x_{1b} + x_{1c} \leq 250$ (maximum capacity 1 cannery)

$x_{2a} + x_{2b} + x_{2c} \leq 450$ (maximum capacity 2 cannery).

$x_{1a} + x_{2a} \leq 200$

$x_{1b} + x_{2b} \leq 200$

$x_{1c} + x_{2c} \leq 200$

(maximum demand of warehouse

non-negative constraint. $x_{ij} \geq 0$

Scanned by CamScanner

HW 3.1

consider K_7 as complete graph of having weight of each edge as 1 to find path between vertex so adjacency matrix will be used to count number of edges. and value at cell (i,j) denotes number of walks

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 6 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 6 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 6 & 5 & 5 & 5 \\ 6 & 5 & 5 & 5 & 6 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 6 & 5 \\ 5 & 5 & 5 & 6 & 5 & 5 & 6 \end{bmatrix}$$

$$A^2 \cdot A^2 = \begin{array}{c|ccccccc} & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \\ \hline 1 & 186 & 185 & 216 & 186 & 185 & 185 & 153 \\ 2 & 185 & 186 & 216 & 186 & 189 & 188 & 153 \\ 3 & 189 & 189 & 216 & 186 & 186 & 185 & 153 \\ 4 & 185 & 185 & 216 & 186 & 185 & 186 & 153 \\ 5 & 185 & 185 & 216 & 186 & 185 & 185 & 154 \\ 6 & 175 & 185 & 216 & 186 & 185 & 185 & 154 \\ 7 & 105 & 185 & 216 & 186 & 185 & 185 & 154 \end{array}$$

$$A^5 = \begin{array}{c|ccccccc} & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \\ \hline 1 & 1110 & 1111 & 1111 & 1111 & 1111 & 1111 & 1111 \\ 2 & 1111 & 1110 & 1111 & 1111 & 1111 & 1111 & 1111 \\ 3 & 1111 & 1111 & 1110 & 1111 & 1111 & 1111 & 1111 \\ 4 & 1111 & 1111 & 1111 & 1110 & 1111 & 1111 & \boxed{1111} \\ 5 & 1111 & 1111 & 1111 & 1111 & 1110 & 1111 & 1111 \\ 6 & 1111 & 1111 & 1111 & 1111 & 1111 & 1110 & 1111 \\ 7 & 1111 & 1111 & 1111 & 1111 & 1111 & 1111 & 1110 \end{array}$$

As we mentioned earlier. cell(4,7) will represent number of path from vertex 4 to 7 of ~~length~~ edges 5

hence ans is 1111

3.2

let f and g be unbounded montonically increasing function on R. dose the following implication hold?

$f \in O(g) \Rightarrow \log f \in O(\log g)$

~~Log f~~

$\Rightarrow$ since $f(n) = O(g(n))$ which means

~~$\exists c$ and n_0~~ so

$f(n) \leq c\, g(n)$ for $\forall n \geq n_0$.

taking log of both side.

$\log(f(n)) \leq \log c + \log(g(n)) \quad \forall n \geq n_0$

$$\log f(n) \leq \log(g(n)) \left[\frac{\log c}{\log(g(n))} + 1 \right]$$

let $n = n_0$ $\frac{\log c}{\log(g(n_0))} + 1$ is constant

∴ there will be some constant K.

$$K \geq \frac{\log c}{\log(g(n_0))} + 1$$

$$\boxed{\log f(n_0) \leq K \cdot \log(g(n_0))}$$

$f(n)$ & $g(n)$ is strictly increasing then for $\forall n \geq n_0$ this will be true.

$\boxed{\log f(n) \leq K \log g(n)}$ hence given identity is valid broadcasting

label encod

3.3. $\log(n!) \in \Theta(n \log n)$

to prove.

$c_1 n \log n \leq \log(n!) \leq c_2 n \log n$.

where $n > n_0$.

Proof

$\log n! = \log(n \cdot (n-1) \cdot (n-2) \cdots)$

$\log n! = \log n + \log(n-1) + \log(n-2) \cdots$

$\log n! \leq \log n + \log n + \log n \cdots$

... (upperbound of each term)

$\boxed{\log n! \leq n \log n}$

~~$C_1 n \log n$~~

$\log n! = \log 1 + \log 2 \cdots \log n/2 + \cdots + \log n.$

$\log n! \geq \Theta\left(\log(n/2) + (\log n/2 + 1) + \cdots \log(n)\right)$

take lower bound of each term.

$\log n! \geq \Theta\left(\log(n/2) + \log n/2) + \cdots + \log(n/2)\right).$

$\log n! \geq \Theta\ n/2(\log(n/2))$

$\log n! \geq \Theta\left[\frac{n}{2}\log n - \frac{n}{2}\log 2\right] \rightarrow (1)$

Scanned by CamScanner

now show that $\frac{n}{2}\log n - \frac{n}{2} \geq c\, n\log n$

$\log n \geq 2$

$\frac{1}{4}\log n \geq \frac{1}{2}$.

$\frac{1}{4} n\log n \geq \frac{1}{2} n$

$\frac{1}{4} n\log n - \frac{1}{2} n \geq 0$.

$$\boxed{\frac{1}{2} n\log n - \frac{n}{2} \geq \underbrace{\frac{1}{4}}_{\text{constant}} n\log n.} \longrightarrow (2)$$

from (1) & (2)

$\log n! \geq c\, n\log n$

hence proved.

Scanned by CamScanner