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ME 639: Assignment-2

Q1.
Ans:

$R \in SO(3)$ & $a \in \mathbb{R}^3$.
let take $b \in \mathbb{R}^3$ be arbitrary vector.

$$
\begin{aligned}
R \cdot S(a) \cdot R^T \cdot b &= R \cdot (a \times R^T b) \quad \text{------} \quad (S(a) \cdot P = a \times P) \\
&= Ra \times R \cdot R^T b \\
&= (Ra) \times b \\
&= S(Ra)\, b
\end{aligned}
$$

$$\therefore \quad R \cdot S(a) \cdot R^T \cdot b = S(Ra)\, b$$

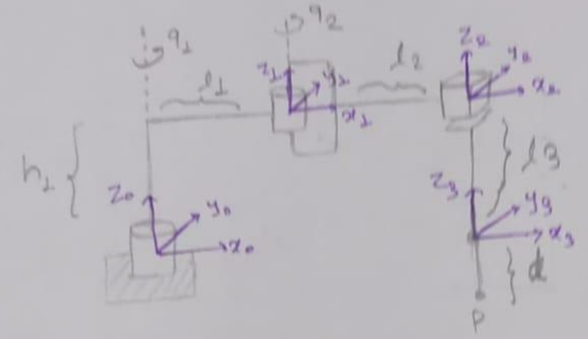
$$\therefore \quad R \cdot S(a) \cdot R^T = S(Ra)$$

Q2.

Ans:

Q.2

$\longrightarrow$ <u>SCARA</u> :-



$$P_3 = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$$R_0^1 = R_{z,q_1} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ h_1 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & l_1 \cos q_1 \\ \sin q_1 & \cos q_1 & 0 & l_1 \sin q_1 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = R_{z,q_2} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_2 \cos q_2 \\ l_2 \sin q_2 \\ 0 \end{bmatrix}$$

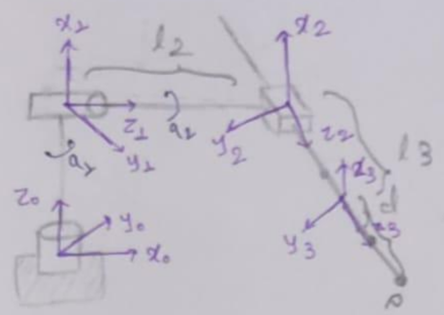
$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \cos q_2 \\ \sin q_2 & \cos q_2 & 0 & l_2 \sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}, \quad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \implies P_0 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1+q_2) \\ l_2 \sin q_1 + l_2 \sin(q_1+q_2) \\ h_1 - l_3 - d \end{bmatrix}$$

Q4.
Ans:

Q.4.
→ Stanford



$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$$

$$R_0^1 = R_{y,\pi/2} \cdot R_{z,\pi} \cdot R_{x,q_1} = \begin{bmatrix} 0 & \sin q_1 & \cos q_1 \\ 0 & -\cos q_1 & \sin q_1 \\ 1 & 0 & 0 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_1^2 = R_{x,-\pi/2} \cdot R_{y,q_2} = \begin{bmatrix} \cos q_2 & 0 & \sin q_2 \\ -\sin q_2 & 0 & \cos q_2 \\ 0 & -1 & 0 \end{bmatrix} \qquad d_2 = \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ l_3 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \sin q_1 & \cos q_1 & 0 \\ 0 & -\cos q_1 & \sin q_1 & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & 0 & \sin q_2 & 0 \\ -\sin q_2 & 0 & \cos q_2 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

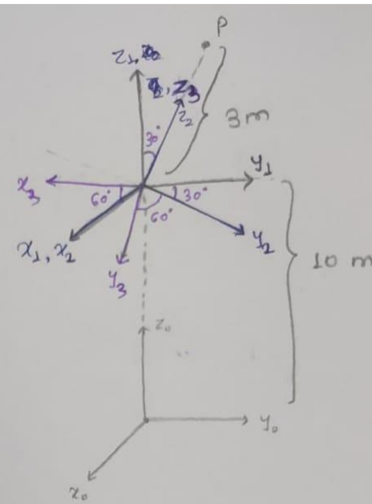
$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \Longrightarrow P_0 = \begin{bmatrix} l_2 \cos q_1 + (l_3 + d) \sin q_1 \cos q_2 \\ l_2 \sin q_1 - (l_3 + d) \cos q_1 \cos q_2 \\ l_1 + (d + l_3) \sin q_2 \end{bmatrix}$$

Q5.

Ans:

5)

$\longrightarrow \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$



$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = R_{x,30^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$R_2^3 = R_{z,60^\circ} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \cdot \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\therefore \quad P_0 = \begin{bmatrix} 0 \\ -3/2 \\ 10 + \frac{3\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \end{bmatrix}$$

$$P_0 = [0, -1.5, 12.598]^T$$

Q6.

Ans: Different types of gear boxes are as follows:

1. **Spur gearbox:**
   Pros: Simple and compact design, efficient for low speed and high torque applications.
   Cons: Greater vibration, less efficient at high speed, high maintenance.
   Applications: clock, industrial machinaries, aircraft engines.

2. **Planetary gearbox:**
   Pros:high precision and efficiency, high torque density, compact design, low backlash.
   Cons:complex and expensive design, high friction, high maintainance.
   Applications: Engines, wind turbines, robots, CNC machine.

3. **Worm gearbox:**
   Pros: High reduction ratio, self locking that prevents back driving, high torque.
   Cons:sliding friction, more heat generation, limited speed range.
   Applications: Conveyor lifts, box tower ladder, heavy duty machines.

4. **Helical gearbox:**
   Pros: Smoother operation due to gradual tooth engagement, handles higher loads, potential to transfer power and motion between either right or parallel-angle shafts.
   Cons: More complex design, higher cost, potential for axial thrust.
   Applications: Automotive transmission, rolling mills, conveyors.

In drones, generally gearbox is not used in drones because drones should be light weight to maximize flight effectiveness. Adding gear box can cause increase weight. Drones have battery where gearbox can introduce energy loss due to frction also maintenance also can be costly. Along with that, adding gear box can make design complex for drone. But sometimes for retractable landing of drones, gearbox can be used.

Q7.

Ans:

Q.7.

For SCARA, $n=3$ $(i=1,2,3)$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \left[\begin{array}{c|c|c} R_0^0 \begin{bmatrix} 0\\0\\1 \end{bmatrix} \times (O_3 - O_0) & R_0^1 \begin{bmatrix} 0\\0\\1 \end{bmatrix} \times (O_3 - O_1) & R_0^2 \begin{bmatrix} 0\\0\\1 \end{bmatrix} \\ \hline R_0^0 \begin{bmatrix} 0\\0\\1 \end{bmatrix} & R_0^1 \begin{bmatrix} 0\\0\\1 \end{bmatrix} & \begin{bmatrix} 0\\0\\0 \end{bmatrix} \end{array}\right]$$

$$O_0 = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1 C_1 \\ S_1 & C_1 & 0 & l_1 S_1 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \rightarrow O_1$$

$$H_0^2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 + l_2 C_{12} \\ S_{12} & C_{12} & 0 & l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \quad \rightarrow O_2$$

$$H_0^3 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 + l_2 C_{12} \\ S_{12} & C_{12} & 0 & l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & h_1 - l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = H_0^2 \cdot H_2^3 \quad \rightarrow O_3$$

$$z_0 = R_0^0 \begin{bmatrix} 0\\0\\1 \end{bmatrix} = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$$

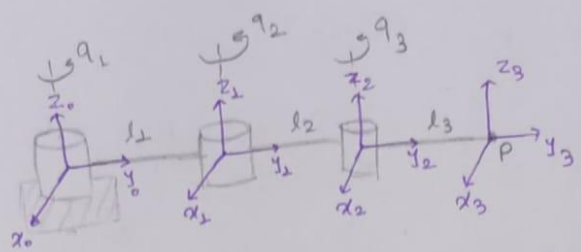
$$z_1 = R_0^1 \begin{bmatrix} 0\\0\\1 \end{bmatrix} = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$$

$$z_2 = R_0^2 \begin{bmatrix} 0\\0\\1 \end{bmatrix} = R_0^1 \cdot R_1^2 \begin{bmatrix} 0\\0\\1 \end{bmatrix} = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix} = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} & 0 \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q9.
Ans:

Q.9 →



$$R_0^1 = R_{z,q_1} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} -l_1 \sin q_1 \\ l_1 \cos q_1 \\ 0 \end{bmatrix}$$

$$H_0^1 = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{R_0^1} & O_1 \\ C_1 & -S_1 & 0 & -l_1 S_1 \\ S_1 & C_1 & 0 & l_1 C_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_1^2 = R_{z,q_2} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} -l_2 \sin q_2 \\ l_2 \cos q_2 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & -l_2 S_2 \\ S_2 & C_2 & 0 & l_2 C_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = H_0^1 \cdot H_1^2 = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{R_0^2} & O_2 \\ C_{12} & -S_{12} & 0 & -l_1 S_1 - l_2 S_{12} \\ S_{12} & C_{12} & 0 & l_1 C_1 + l_2 C_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_2^3 = R_{z,q_3} = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 \\ \sin q_3 & \cos q_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} -l_3 \sin q_3 \\ l_3 \cos q_3 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} C_3 & -S_3 & 0 & -l_3 S_3 \\ S_3 & C_3 & 0 & l_3 C_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
H_0^3 = H_0^2 \cdot H_2^3 = \begin{bmatrix} \boxed{\begin{matrix} C_{123} & -S_{123} & 0 \\ S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{matrix}}^{R_0^3} & \boxed{\begin{matrix} -l_1S_1 - l_2S_{12} - l_3S_{123} \\ l_1C_1 + l_2C_{12} + l_3C_{123} \\ 0 \end{matrix}} \rightarrow O_3 \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}
$$

$$
J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}
$$

$$
= \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}
$$

~~where~~

$$
J = \begin{bmatrix} -l_1C_1 - l_2C_{12} - l_3C_{123} & -l_2C_{12} - l_3C_{123} & -l_3C_{123} \\ -l_1S_1 - l_2S_{12} - l_3S_{123} & -l_2S_{12} - l_3S_{123} & -l_3S_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}
$$