Modeling and Control of Mechatronic Systems

## Lab III: Position Controller Design Guide

This document describes the design of a position control system for the position control laboratory experiment carried out at the position control rig in L204 of the School of Mechanical and Manufacturing Engineering. The control software is already operational on the control computer. The students are expected to bring along the controller coefficients to be entered into the program. The software is written by Mr. Jim Sanderson and conforms to the block diagram shown in Fig. 1(a). This block diagram can be simplified to obtain that shown in Fig. 1(b). The control design follows the Direct Analytical Design method to design the internal velocity feedback loop and then root-locus method to design the external position feedback loop. The design specifications are:

- ullet A speed control time constant of au ms.
- ullet A sampling frequency of  $f_s$  Hz.
- Zero steady state errors in both velocity and position loops.
- Critically damped position controller response.

## 1 Speed Controller Design

The system transfer function is obtained by finding a first order approximation to the open loop data. This is shown in Fig. 2. The equation of the fitted curve is,

$$y(t) = xxxxxx(1 - e^{-t/0.xxx})$$

Note that the response corresponds to an applied voltage of 24 volts. Hence the transfer function that relates the applied voltage to the input shaft position is,

$$G_p'(s) = rac{xxxxxx/24.0}{s(1+0.xxxs)}$$

Hence, the transfer function relating the input voltage to the input shaft speed in rad is,

$$G_p(s) = rac{xxxxxx/xx}{s(1+0.xxxs)} rac{2\pi}{8192}$$

Hence,

$$G_p(s) = rac{x.xxxx}{s(1+0.xxxs)}$$

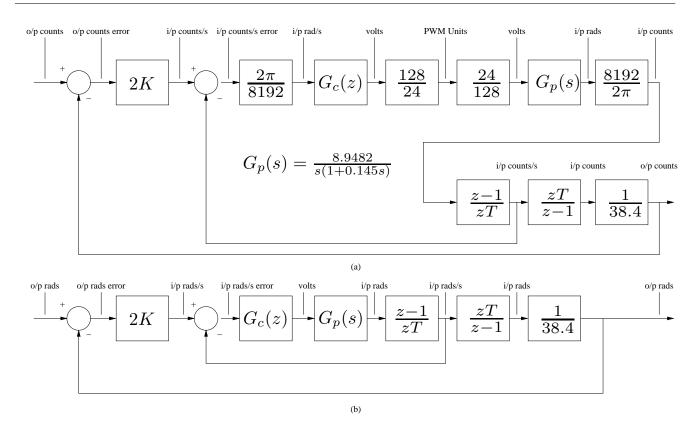


Figure 1: (a) Program implementation, (b) Simplified block diagram

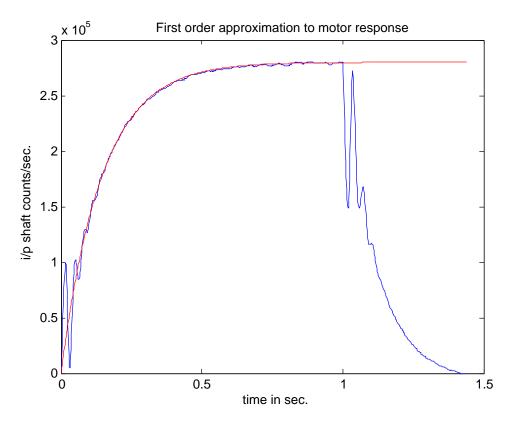


Figure 2: First order curve fitted to motor speed data

The discrete version of  $G_p(s)$  is,

$$G_p'(z) = xxxxxxxxxxrac{(z+0.xxxxx)}{(z-1)(z-0.xxxxx)}$$

Including the numerical differentiator,

$$G_p(z) = 0.xxxxxrac{(z+0.xxxxx)}{z(z-0.xxxxx)}$$

The desired pole location is given by,

$$z = e^{sT} = e^{rac{-0.xxx}{0.xxx}} = 0.xxxx$$

Hence,

$$F(z) = rac{b_0(z+0.xxxxx)}{z(z-0.xxxxx)}$$

A pole at origin is added to F(z) to fulfil causality constraint. For zero steady state error,

$$\left. F(z) = rac{b_0(z+0.xxxxx)}{z(z-0.xxxxxx)} 
ight|_{z=1} = 1$$

Therefore,

$$F(z) = rac{0.xxxxx(z+0.xxxxx)}{z(z-0.xxxxx)}$$

Using

$$G_c(z) = rac{1}{G_p(z)}rac{F}{1-F} \ G_c(z) = rac{0.xxxxx}{z^2-0.xxxxx}rac{z}{z-0.xxxxx}$$

Therefore, the difference equation to be implemented is

$$\mathsf{m}(\mathsf{k}) = 0.89004 \mathsf{m}(\mathsf{k}\text{-}1) + 0.10996 \mathsf{m}(\mathsf{k}\text{-}2) + 0.72935 \mathsf{e}(\mathsf{k}) - 0.70463 \mathsf{e}(\mathsf{k}\text{-}1)$$

## 2 Position Controller Design

Combining the entire velocity loop, the integrator, the 2K gain, and the gear ratio of 1/38.4, the forward path transfer function applicable to the position control loop is,

$$G(z) = rac{0.xxxxxxxK(z+0.xxxxx)}{(z-1)(z-0.xxxxx)}$$

Given that this transfer function has an integrator (as evidenced by the pole at z=1), a pure constant can give zero steady state response. Critically damped response can be obtained by choosing the break away point on the root locus. This gives a gain value of,

$$K = 225.04$$

and a break away point of

$$z = 0.88612$$

that corresponds to a time constant of 41.3 ms.