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Laboratory Experiment 3 – Design and Implementation of a Position Controller

MTRN3020 - Modelling and Control of Mechatronic Systems

I verify that the contents of this report are my own work

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1. Introduction

This laboratory experiment aims to design and implement a position controller in an experimental control system to achieve a specified position. The controller will be designed using the direct analytical method for the internal velocity loop and the root locus method for the external position feedback loop [1]. This method will give the controller's transfer function and gain value which will be put into a difference equation form to obtain the coefficients required for the laboratory experiment. Part A of the experiment will verify the designed position controller is correct. This will be performed using a Simulink model using the designed controller and checking that the steady-state error is 0 and the time constant matches the desired time constant found in Table 1. Part B of the experiment will analyse the effect of modelling uncertainties with the controller. This will be performed using the previously created Simulink model and altering the arm length of the control system. The deviation in the response of the system should be explainable by the change that occurred. Both simulated results from Parts A and B will be compared against the experimental data to determine the validity of the simulations and designed controller and draw parallels to real-world applications.

Table 1. Design Parameters

Design Parameter	**Value / Data File**
Desired Time Constant (s)	0.048
Sampling Time (s)	0.009
Design File	ROT335
Test System File	ROT445

2. Aim

This experiment aims to design and model a controller using the direct analytical design method for the internal velocity loop and the root locus method for the feedback loop to control the position of an experimental control system. This controller will aim to generate a signal that will attempt to attain the specified position. The implementation of the experiment and the simulation of the model through MATLAB’s Simulink will produce experimental and simulation results to analyse and discuss.

3. Experimental Procedure

Using the direct analytical method, calculate the transfer function for the internal velocity feedback loop of the controller. Convert the transfer function into a difference equation form to obtain the coefficients of the controller. Using the root locus method, find the forward path transfer function related to the position control loop and its gain value for a zero steady-state response. Using the calculated coefficients and gain value, run the experimental simulation for Part A with the length of the arm adjusted to the designed parameter and save the results. Once complete, change the arm length to the specified length for the test system file and run the experimental simulation for Part B and save the results.

Model the experimental setup in MATLABs Simulink using the provided design parameters and designed controller. Simulate Part A and B of the experiment using the designed model and respective plant transfer functions for the respective arm lengths. Save the simulation results for both parts and superimpose the experimental and simulation results for Parts A and B on separate graphs and compare the compatibility of the results.

4. Controller Design Calculation

The design of the position controller was based on the block diagram seen below in Figure 1.

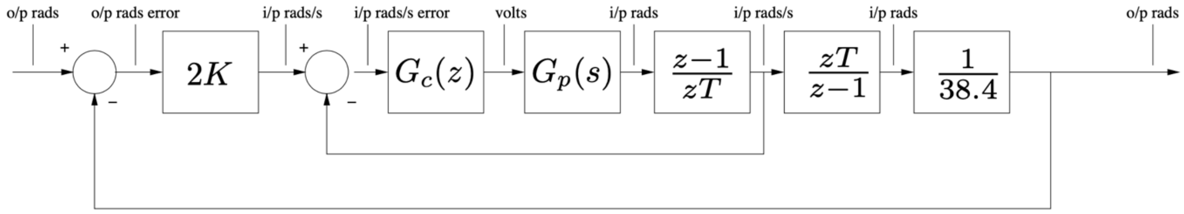

Figure 1. Position Control Block Diagram [1]

The internal velocity feedback loop of the controller was designed using the Direct Analytical Design Method and the external position feedback loop was designed using the Root-Locus Method. The desired time constant and sampling time design specifications are stated in Table 1. Additionally, the system is designed to have zero steady-state errors in both the velocity and position loops and have a critically damped position controller response [1].

To obtain the system transfer function, a first-order response fitting to the motor response when an input voltage of 24V was performed on the open-loop data as seen in Figure 2. The equation for the fitted curve is seen below in (1),

$$y(t) = 280400\left(1 - e^{\frac{-t}{0.103}}\right) \tag{1}$$

This corresponds to the transfer function seen in (3), which has been normalised to an input voltage of 1V and relates the input voltage to the input shaft speed in rad/s.

$$G_p(s) = \frac{\left(\frac{280400}{24}\right)}{s(1+0.103s)}\frac{2\pi}{8192} \tag{2}$$

$$G_p(s) = \frac{8.96100}{s(1+0.103s)} \tag{3}$$

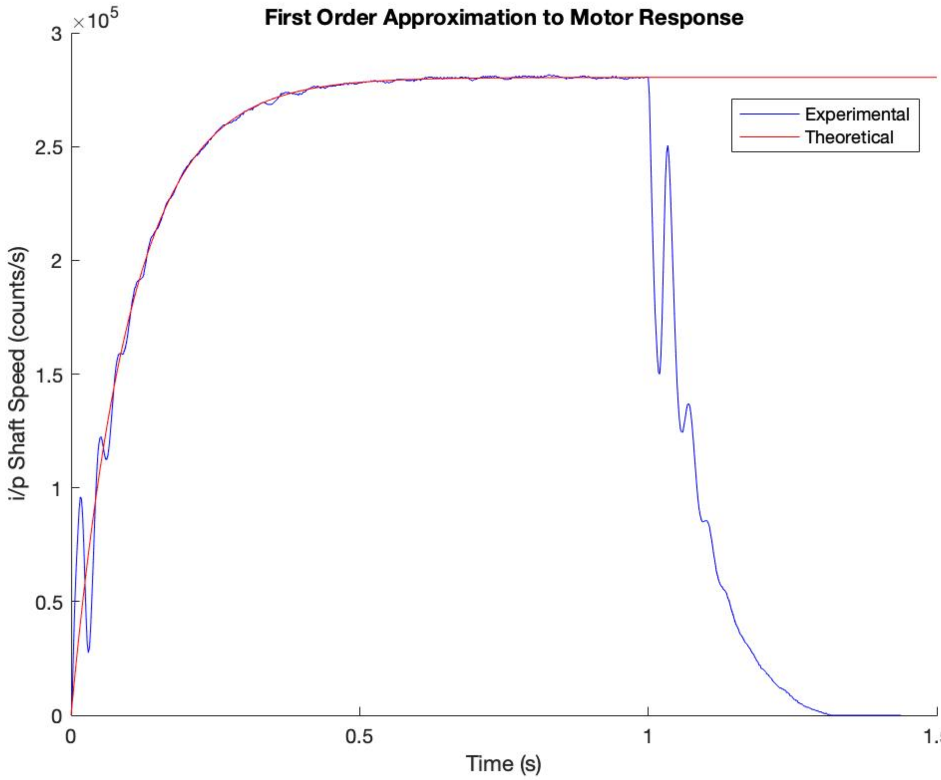

Figure 2. First Order Curve Fitted to Motor Speed Data for an Applied Voltage of 24V

To determine the discrete version of $G_p(s)$ including the numerical differentiator, MATLAB's $\mathrm{c2dm}$ function was used with a sampling time of 0.009 seconds as required in Table 1. Hence,

$$G_p(z) = \frac{0.38034(z + 0.97130)}{z(z - 0.91633)} \tag{4}$$

The desired time constant is 0.048 seconds, and the sampling time is 0.009 seconds as seen in Table 1. This can be used to calculate the desired pole in the z-plane.

$$z_p = e^{sT} = e^{\frac{-0.009}{0.048}} = 0.82903 \tag{5}$$

$F(z)$ is derived below in (6). The ringing from the pole in the obtained plant, $G_p(z)$, has been eliminated and the causality constraint has been adhered to.

$$F(z) = \frac{b_0(z + 0.97130)}{z(z - 0.82903)} \tag{6}$$

Applying the zero steady-state design requirement, $F(1) = 1$, b_0 is found, and $F(z)$ is now,

$$F(1) = \frac{b_0(1 + 0.97130)}{1(1 - 0.82903)} = 1 \tag{7}$$

$$b_0 = 0.08673 \tag{8}$$

$$F(z) = \frac{0.08673(z + 0.97130)}{z(z - 0.82903)} \tag{9}$$

The transfer function of the controller, $G_c(z)$, is obtained using the expression,

$$G_c(z) = \frac{1}{G_p(z)} \frac{F(z)}{\left(1 - F(z)\right)} \tag{10}$$

Hence, substituting (4) and (9) into (10), $G_c(z)$ is obtained as,

$$G_c(z) = \frac{0.22803z^2 - 0.20895z}{z^2 - 0.91576z - 0.08424} \tag{11}$$

The difference equation of the velocity controller is,

$$\frac{M(z)}{E(z)} = \frac{0.22803 - 0.20895z^{-1}}{1 - 0.91576z^{-1} - 0.08424z^{-2}} \tag{12}$$

$$m(k) = 0.91576m(k-1) + 0.08424m(k-2) + 0.22803e(k) - 0.20895e(k-1) \tag{13}$$

By combining the entire velocity loop, the $2K$ gain and the gear ratio of the 1/38.4, the forward path transfer function applicable to the position control loop is calculated as,

$$G(z) = \frac{0.00004486K(z + 0.97130)}{(z - 1)(z - 0.82903)} \tag{14}$$

The K can be found by plotting the root locus as seen in Figure 3 and getting the gain value at the breakaway point. This was chosen as the transfer function in (14) has a zero steady-state response due to the integrator. Hence at the breakaway point of $z = 0.91$,

$$K = 86.3 \tag{15}$$

Which corresponds to a time constant of 95.4ms.

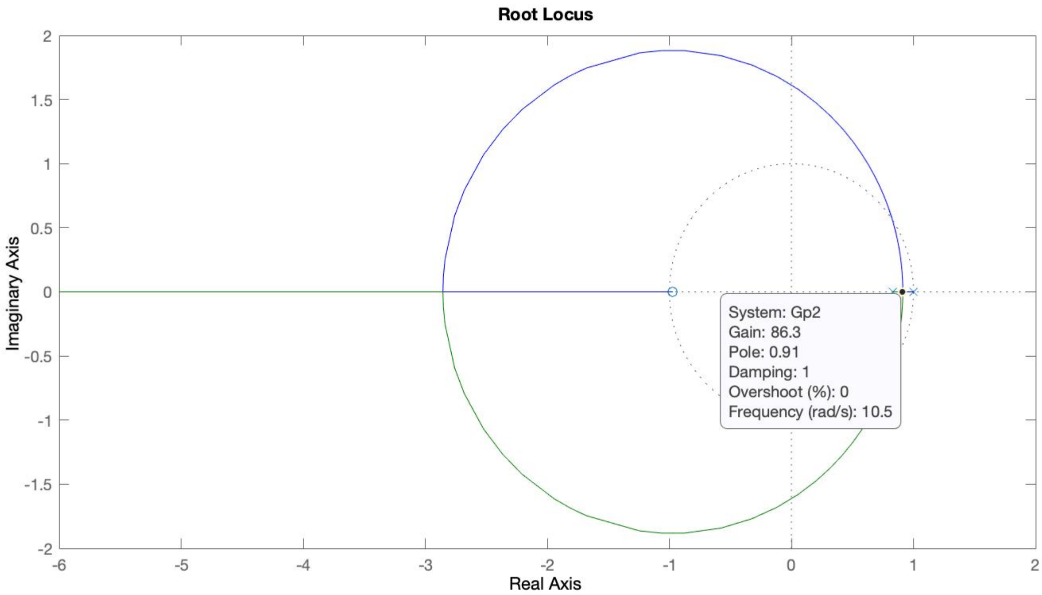

Figure 3. Root Locus Plot of the Control System

Hence the final transfer function for the control is calculated as,

$$G(z) = \frac{0.00387146(z + 0.97130)}{(z - 1)(z - 0.82903)} \tag{16}$$

5. Simulink Block Diagram

The system used during the experiment is modelling using MATLAB's Simulink. The block diagram is shown below in Figure 4.

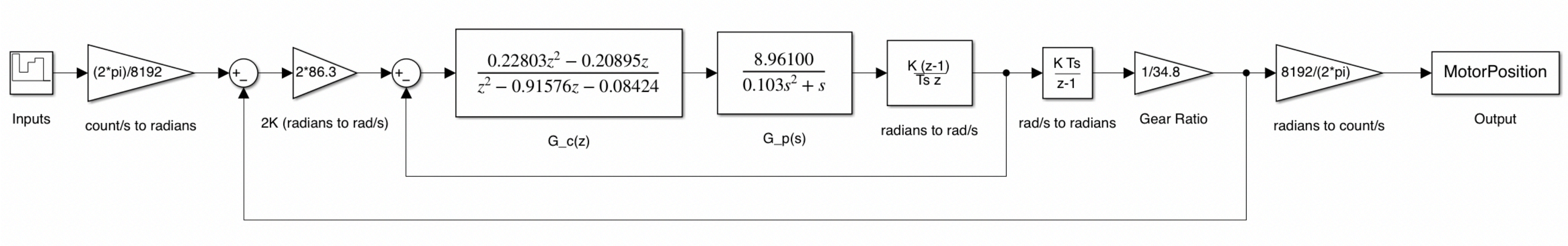

Figure 4. MATLAB Simulink Block Diagram for Position Controller Experiment

The model takes in the desired motor positions in counts/s as an input. It is converted from counts/s to radians before the error is calculated at the summation node through the input and a feedback loop. This value is multiplied by the gain, $2K$, giving the input velocity in rad/s. The value is passed through a summation node used to calculate the error in the velocity before it is passed into the designed controller, outputting the voltage and then into the plant of the respective dataset. This value is then derived to get the velocity in rad/s and is passed back into the second summation node mentioned prior and is also integrated to get the position in radians. This is multiple by the gear ratio and passed back into the first summation node to calculate the error in position. It is also converted to counts/s giving the desired output of the position of the arm.

6. Part A – Design Verification Results and Discussion

The superposition of the experimental and Simulink plot for Part A is seen below in Figure 5.

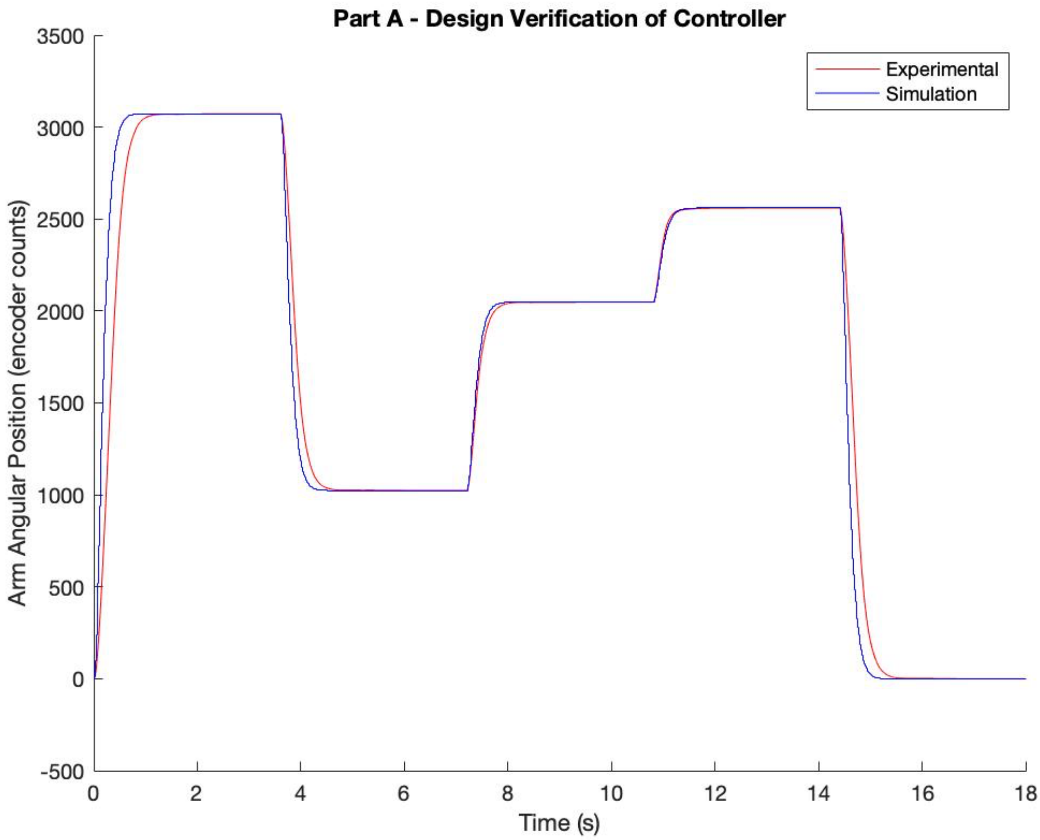

Figure 5. MATLAB Plot of Experimental and Simulated Arm Angular Positions for Designed Controller with an Arm Length of 335mm

By visual inspection, the experimental and Simulink model data are seen to follow a similar plot. It is however observed that the desired 0.048 second time constant is not achieved when the position is changed. The simulated model provides a much faster time constant compared to the experimental results obtained. The other design specification is to attain zero steady error, which the simulated model has achieved implying it was correctly designed in that aspect.

There are several possible reasons for this deviation in the desired time constant between the simulated and experimental plots. Firstly, there may be a backlash in the gears of the arm, where the delay from the movement of the first gear to the interaction with the second gear may lead to inaccurate data on the position of the arm as well as lead to delayed responses to errors in the position of the arm. Secondly, the encoder used in the experimental setup may be prone to a measurement error. Based on the plot obtained, the encoder may be providing overestimated position values which would result in the slower desired time constant seen in the experimental data. Finally, the time constant deviation may be accounted for by the accuracy level of the coefficients used by the controller as well as the gain value obtained during the design phase. During the design stage, the plant was designed by a first-order response fitting to the motor response. The motor response was heavily impacted by noise, hence calculated plant may have been inaccurate to a certain degree. In addition, the simulated controller used coefficients accurate to 5 decimal places whilst the experimental setup used coefficients accurate to 6-7 decimal places. The difference in accuracy levels introduces a small difference between the calculated signals for the experimental and simulated which may account for the varying time constants.

7. Part B – Disturbance Rejection Results and Discussion

The superposition of the experimental and Simulink plot for Part B is seen below in Figure 6 with a close up of the plot in Figure 7.

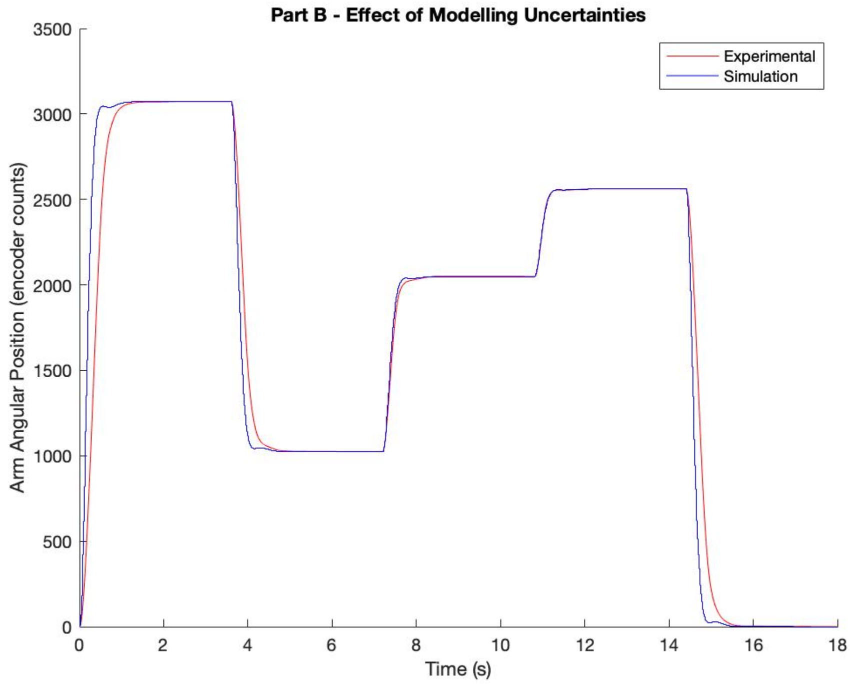

Figure 6. MATLAB Plot of Experimental and Simulated Arm Angular Positions for Designed Controller with an Arm Length of 445mm

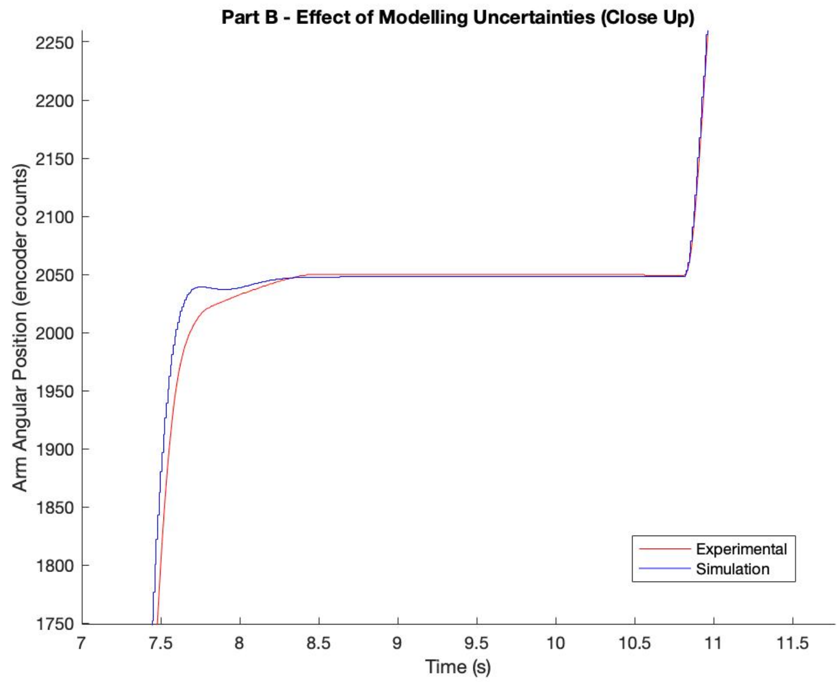

Figure 7. MATLAB Plot of Experimental and Simulated Arm Angular Positions for Designed Controller with an Arm Length of 445mm (Close Up)

In Part B, the experimental setup was adjusted to have an arm length of 445mm compared to the 335mm arm length the controller was designed for. Hence, the plant's transfer function in the simulation was changed to the following transfer function,

$$G_p(s) = \frac{8.92905}{s(1 + 0.145s)} \tag{17}$$

By visual inspection, the effect of modelling uncertainties with the designed controller produced similar plots for both the experimental and Simulink models. However, as mentioned in Part A, the desired time constant was not achieved. This is most evident when large position changes occur and less so in smaller position changes. As discussed in Part A this behaviour is due to either backlash between gears, encoder error or the level of accuracy of the controller design or a combinate of the three. Additionally, the increase in arm length has increased the time constant of the experimental data as the motor experiences a heavier arm than what it was designed for.

In Figure 7, it is evident that the introduction of the uncertainty has introduced some steady-state error. Although, it is seen that this error is very minimal, approximately $\pm\, 0 - 3$ encoder counts and could be considered negligible.

Figure 7 also reveals there is a undershoot in the simulated response. This can be explained by the change in arm length modelling uncertainty in the system. As mentioned, the controller designed for an arm length of 335mm was used with the plant transfer function for the 445mm arm length. As the arm length was changed from 335mm to 445mm, the motor experiences a heavier load due to the redistribution of weight along the system. Hence, this increased weight with the old controller results in an undershoot of the position.

8. Conclusion

In conclusion, the direct analytical method and the root locus method were used to design a position controller cable of setting an experimental arm to a specified position. The controller was found to be capable of achieving the desired position with an acceptable level of accuracy but failed to satisfy the desired time constant. It was seen that when the controller was used with uncertainties, it performed to an acceptable level, achieving the specified positions with minimal error, and failing to meet the desired time constant as before. Extrapolating this experiment to a real-world setting, certain vehicles such as a crane car may use a controller similar to the one produced to position the crane arm. In this example, the length of the crane arm is likely to change during operation and as such the controller will need to be able to position the crane within a reasonable tolerance as researched in this lab.

References

[1] J. Katupitiya, *Lab III : Position Controller Design Guide,* 2022.