

Sequence Learning with Neural Networks

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1 Motivation

2 Sequence Models

- RNNs
- Training Challenges
- LSTM and GRU

3 Machine Translation

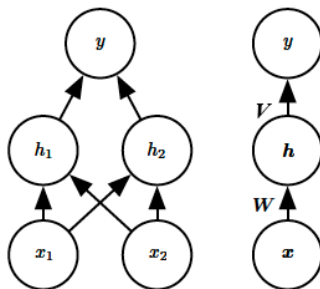
- Attention
- Transformer Model

Examples of problems involving sequence data

- Speech recognition
- Music generation
- Time series forecasting
- Machine translation
- Conversation agents
- Image captioning

Limitations of feedforward networks

Recall the structure of a basic feedforward network:



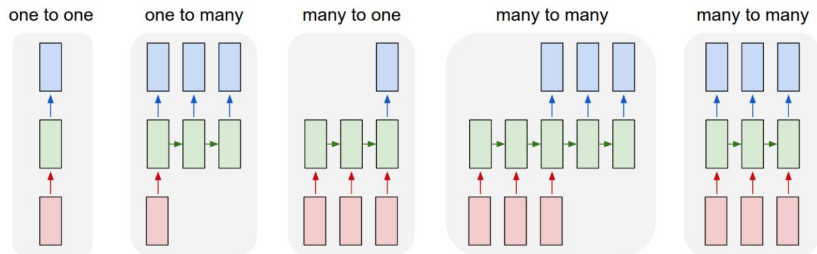
$$h = \sigma_1(b + Wx)$$

$$o = c + Vh$$

$$\hat{y} = \text{softmax}(o)$$

Problem: How to learn from a sequence of inputs x_1, x_2, \dots, x_T ?

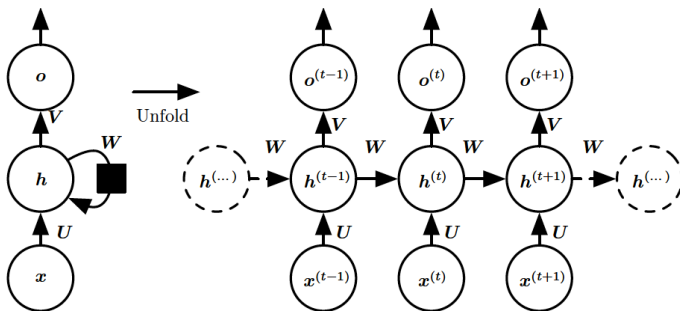
Sequence Models



We will focus on the many-to-many cases. Requirements:

- 1 Output \hat{y}_t should depend on the sequence so far, x_1, \dots, x_t .
- 2 We may not know the length of a particular input sequence ahead of time.

A Simple Recurrent Neural Network



$$h_t = \sigma_1(b + Wh_{t-1} + Ux_t)$$

$$o_t = c + Vh_t$$

$$\hat{y}_t = \text{softmax}(o_t)$$

A Simple Recurrent Neural Network

RNNs allow us to learn a single model, rather than a separate one for each time step.

- 1 The model is specified in terms of *transitions* from one state h_t to the next.
- 2 Parameters are shared across time.

Loss function is a sum of losses at each time step t :

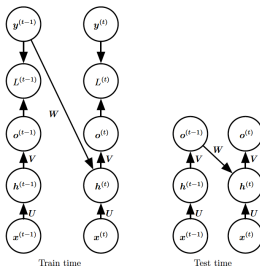
$$L(\hat{y}, y) = \sum_{t=1}^T \ell(\hat{y}_t, y_t)$$

Training RNNs

Training proceeds as before using **backpropagation through time** on the unrolled computation graph.

We cannot easily parallelize training since each step is dependent on the one before it.

Teacher forcing can be used when there are output-to-hidden recurrent connections. Ground truth is fed to the model instead of its own output.



Modeling Joint Probability Distributions

When we use a negative log-likelihood training objective,

$$\ell(\hat{y}_t, y_t) = -\log \Pr(y_t \mid x_1, \dots, x_t)$$

we train the RNN to estimate the conditional distribution of the next sequence element y_t given the past inputs.

We typically use the softmax function as the output layer to obtain normalized probabilities for each class.

$$\text{softmax}(o)_i = \frac{e^{o_i}}{\sum_{j=1}^{\tau} e^{o_j}}$$

Modeling Joint Probability Distributions

RNNs can model arbitrary probability distributions of some sequence y over another sequence x .

$$\Pr(y_1, \dots, y_\tau \mid x_1, \dots, x_\tau) = \prod_{t=1}^{\tau} \Pr(y_t \mid x_1, \dots, x_t)$$

To remove the conditional independence assumption, we can add output-to-hidden connections.

$$\Pr(y_1, \dots, y_\tau \mid x_1, \dots, x_\tau) = \prod_{t=1}^{\tau} \Pr(y_t \mid y_1, \dots, y_{t-1}, x_1, \dots, x_t)$$

Modeling Joint Probability Distributions

Some challenges:

- 1 What if we want a given output y_t to depend on the entire sequence x_1, \dots, x_T ?
- 2 How do we map a sequence x to a sequence y when they can differ in length from each other?

We will see the solutions later in the context of machine translation.

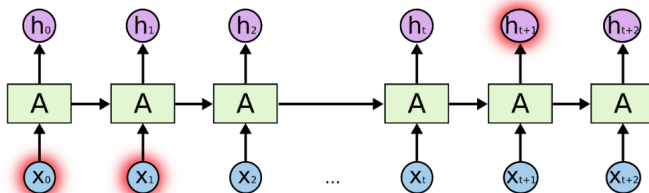
First, a fundamental problem:

RNNs have trouble learning long-term dependencies.

Learning Long-Term Dependencies

Sentence 1: “Jane walked into the room. John walked in too. Jane said hi to...”

Sentence 2: “Jane walked into the room. John walked in too. It was late in the day, and everyone was walking home after a long day at work. Jane said hi to...”



RNNs have trouble learning dependencies from inputs with a large time difference from the predicted output.

Vanishing or Exploding Gradients

Suppose we have an RNN with state h_t , input x_t , and cost \mathcal{E} .¹

$$h_t = W\sigma(h_{t-1}) + Ux_t + b$$

$$\mathcal{E} = \sum_{1 \leq t \leq \tau} \mathcal{E}_t, \quad \mathcal{E}_t = \mathcal{L}(h_t)$$

Let's calculate the gradient over one input sequence.

$$\frac{\partial \mathcal{E}}{\partial W} = \sum_{1 \leq t \leq \tau} \frac{\partial \mathcal{E}_t}{\partial W} \quad (1)$$

$$\frac{\partial \mathcal{E}_t}{\partial W} = \sum_{1 \leq k \leq t} \left(\frac{\partial \mathcal{E}_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial^+ h_k}{\partial W} \right) \quad (2)$$

$$\frac{\partial h_t}{\partial h_k} = \prod_{t \geq i > k} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{t \geq i > k} W^\top \text{diag}(\sigma'(h_{i-1})) \quad (3)$$

¹The original paper uses this formulation instead of $h_t = \sigma(Wh_{t-1} + Ux_t + b)$ and says they are equivalent.

Vanishing or Exploding Gradients

The cause is the Jacobian matrix J . We have a product of $t - k$ Jacobians.

$$J = \frac{\partial h_{k+1}}{\partial h_k} = W^\top \text{diag}(\sigma'(h_k))$$
$$\|J\| = \left\| \frac{\partial h_{k+1}}{\partial h_k} \right\| \leq \underbrace{\|W^\top\|}_{\lambda_1} \underbrace{\|\text{diag}(\sigma'(h_k))\|}_{\gamma} = \lambda_1 \gamma$$

λ_1 is the largest singular value of W . $\gamma = 1$ for tanh and $\gamma = \frac{1}{4}$ for sigmoid.

$$\left\| \prod_{i=k}^{t-1} \frac{\partial h_{i+1}}{\partial h_i} \right\| \leq (\lambda_1 \gamma)^{t-k}$$

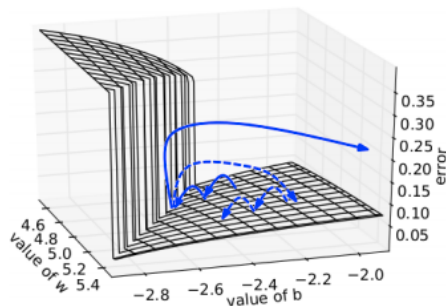
When $t \gg k$:

- If $\lambda_1 < \frac{1}{\gamma}$, the gradients *will* vanish.
- If $\lambda_1 > \frac{1}{\gamma}$, the gradients *may* explode.

How to deal with gradient problems?

- 1 Modify the training algorithm: **gradient clipping**.
 - Commonly used when training all variants of RNNs.
- 2 Use a different activation function: **ReLU**s.
 - Primarily used for deep neural nets (e.g. computer vision).
- 3 Use a more complex neural architecture: **LSTMs and GRUs**.

Gradient Clipping



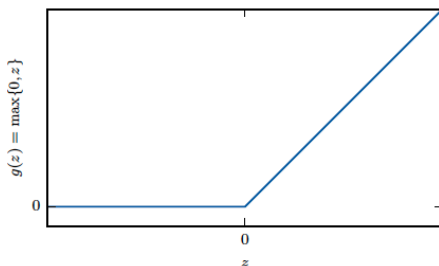
Addresses the problem of exploding gradients by preventing parameter updates from being too large.

```

$$g \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$$
if  $\|g\| \geq \text{threshold}$  then  
     $g \leftarrow \frac{\text{threshold}}{\|g\|} g$ end if
```

Does gradient clipping affect convergence?

ReLU



Since the derivative is 1 when $z > 0$, gradients flow more easily compared to sigmoid or tanh units. Also computationally efficient.

Can lead to “dead neurons” during training. Is this a problem?

Empirically, ReLU has been shown to be very effective.

New Recurrent Architectures

We will introduce the Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU) architectures.

- Address the problem of vanishing gradients.
- Considered the state of the art for many deep learning tasks.
 - Image captioning, parsing, speech recognition, machine translation, reinforcement learning

LSTM and Backpropagating Gradients

Recall the equation to update the cell state:

$$c_t = i_t \odot a_t + f_t \odot c_{t-1}$$

The original LSTM did not have a forget gate, allowing error to flow unchanged from c_t to c_{t-1} .

- Referred to as the constant error carousel.

With a forget gate, if $f_t \approx 1$, we achieve the same effect.

- Can initialize the forget gate bias to 1 before training.

Gated Recurrent Unit (Cho et. al 2014)

$$u_t = \sigma(W_u x_t + U_u h_{t-1} + b_u)$$

$$r_t = \sigma(W_r x_t + U_r h_{t-1} + b_r)$$

$$a_t = \sigma_t(W_i x_t + r_t \odot U_i h_{t-1} + b_i)$$

$$h_t = u_t \odot h_{t-1} + (1 - u_t) \odot a_t$$

Hidden units that learn to capture...

- short-term dependencies will tend to have reset gates that are frequently active.
- longer-term dependencies will have update gates that are mostly active.

Takeaway: Similar performance to LSTM, but fewer parameters.

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Neural Machine Translation in 2016

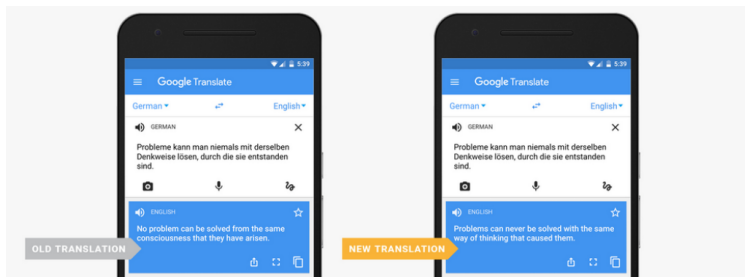
TRANSLATE

Found in translation: More accurate, fluent sentences in Google Translate

Barak Turovsky
Product Lead, Google
Translate

Published Nov 15, 2016

In 10 years, Google Translate has gone from supporting just a few languages to 103, [connecting](#) strangers, [reaching across](#) language barriers and even helping people find [love](#). At the start, we pioneered large-scale statistical machine translation, which uses [statistical models to translate text](#). Today, we're introducing the next step in making Google Translate even better: Neural Machine Translation.



A bilingual translation task:

“The cat sat on the mat.” → “Le chat s’est assis sur le tapis.”

* * *

2003: Neural language model introduced by Bengio et al. This was incorporated into existing phrase-based statistical machine translation (SMT) systems.

2014: Encoder-decoder RNN introduced by Cho et. al for use in SMT.

2015: Attention model proposed by Bahdanu et. al for end-to-end neural machine translation (NMT).

2017: Transformer model proposed by Vaswani et. al. This is the current state-of-the-art.

Word Representations

How should the model encode a word token in a sequence?

Attempt 1: One-hot vectors. Represent every word as an $\mathbb{R}^{|V| \times 1}$ vector with all zero's except for a single one depending on the index of the word in the vocabulary V .

- $e(\text{cat}) = [0 \ 0 \ 0 \ \dots \ 1 \ \dots \ 0]^\top$
- Does not capture semantic similarity between words!
- Scales poorly with size of vocabulary.

Word Representations

Attempt 2: Word embeddings. Model each word in a low-dimensional continuous space with dimension M .

- $e(\text{cat}) = [0.1 \ 0.33 \ 0.72 \ \dots \ 0.59]^\top$
- Intuition: “A word is defined by the company it keeps.”

Has revolutionized natural language processing (NLP) tasks since 2010.

- Popular word embeddings: word2vec, GloVe, ELMo

Encoder-Decoder Model

How to map an input sequence to an output sequence where the lengths are not necessarily the same?

Encoder RNN processes the input sequence and emits a context vector c , which is the model's final hidden state, h_{τ_x} .

$$h_t = f(h_{t-1}, e(x_t))$$

Decoder RNN generates the output sequence based on c and the previous output.

$$s_t = f(s_{t-1}, e(y_{t-1}), c)$$

$$\Pr(y_t) = g(s_t, e(y_{t-1}), c)$$

Jointly trained to maximize $\log \Pr_{\theta}(y_1, \dots, y_{\tau_y} \mid x_1, \dots, x_{\tau_x})$.

Encoder-Decoder Model

Task: English-to-French translation.

Training data: 348M sentences from Europarl, news commentary, etc.

Test data: 3000 sentences from a standard newstest dataset.

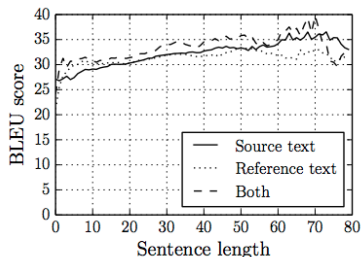
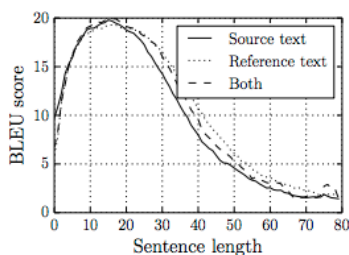


Figure: Translation quality vs. sentence length. RNN left, SMT right.

Encoder-Decoder Model

Source	She explained her new position of foreign affairs and security policy representative as a reply to a question: "Who is the European Union? Which phone number should I call?"; i.e. as an important step to unification and better clarity of Union's policy towards countries such as China or India.
Reference	Elle a expliqué le nouveau poste de la Haute représentante pour les affaires étrangères et la politique de défense dans le cadre d'une réponse à la question: "Qui est qui à l'Union européenne?" "A quel numéro de téléphone dois-je appeler?", donc comme un pas important vers l'unicité et une plus grande lisibilité de la politique de l'Union face aux états, comme est la Chine ou bien l'Inde.
RNNEnc	Elle a décrit sa position en matière de politique étrangère et de sécurité ainsi que la politique de l'Union européenne en matière de gouvernance et de démocratie .
grConv	Elle a expliqué sa nouvelle politique étrangère et de sécurité en réponse à un certain nombre de questions : "Qu'est-ce que l'Union européenne ? " .
Moses	Elle a expliqué son nouveau poste des affaires étrangères et la politique de sécurité représentant en réponse à une question: "Qui est l'Union européenne? Quel numéro de téléphone dois-je appeler?"; c'est comme une étape importante de l'unification et une meilleure lisibilité de la politique de l'Union à des pays comme la Chine ou l'Inde .

Figure: Sample output of two RNN models compared to the SMT system, Moses.

Problem: the neural network must compress all information in the source sentence into a single, fixed-length vector.

Encoder-Decoder Model with Attention

Decoder RNN is similar to before, but each context vector c_t is distinct for each target word y_t :

$$s_t = f(s_{t-1}, e(y_{t-1}), c_t)$$
$$\Pr(y_t) = g(s_t, e(y_{t-1}), c_t)$$

c_t is a weighted sum of *annotations* h_1, \dots, h_{τ_x} to which the encoder maps the input sentence.

$$c_t = \sum_{j=1}^{\tau_x} \alpha_{tj} h_j$$

How are α and h computed?

Encoder-Decoder Model with Attention

The encoder is a “**bidirectional RNN**.”

Forward RNN reads the input as ordered and computes a sequence of forward hidden states $\vec{h}_1, \dots, \vec{h}_{\tau_x}$.

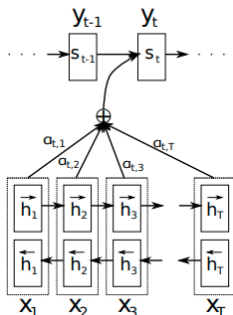
Backward RNN reads the input in reverse and computes a sequence of backward hidden states $\overleftarrow{h}_1, \dots, \overleftarrow{h}_{\tau_x}$.

An annotation for a word x_j is the concatenation of the two hidden states.

$$h_j = \left[\vec{h}_j, \overleftarrow{h}_j \right]$$

Due to the tendency of RNNs to better represent recent inputs, the annotation h_j will be focused on the words around x_j .

Encoder-Decoder Model with Attention



To compute the weight α_{ij} of each annotation h_j , we use an **alignment model**. This is just a single-layer feedforward NN.

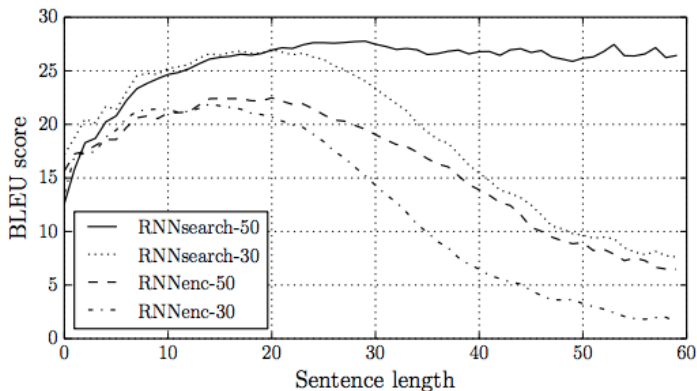
$$e_{ij} = a(s_{i-1}, h_j)$$

$$\alpha_{ij} = \text{softmax}(e_{ij})$$

The alignment model scores how well the inputs around position j and the output at position i match.

Encoder-Decoder Model with Attention

Same training and test data as before (English-to-French).



The encoder-decoder model with attention does much better on longer sentences.

Encoder-Decoder Model with Attention

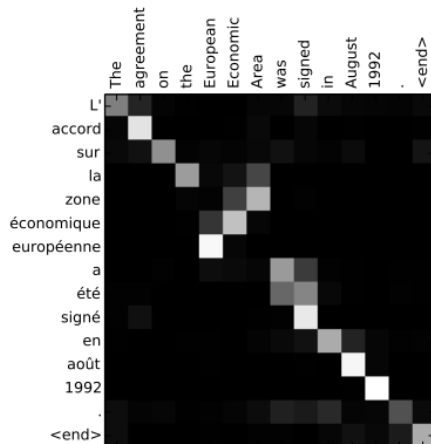


Figure: A sample alignment found by RNNsearch-50.

“Attention is All You Need”

* * *

The transformer model utilizes an encoder/decoder architecture with attention, but no recurrent connections!

Recall that training RNNs is not parallelizable and requires more memory for each example due to the recurrence.

Also achieves state-of-the-art performance on translation tasks.

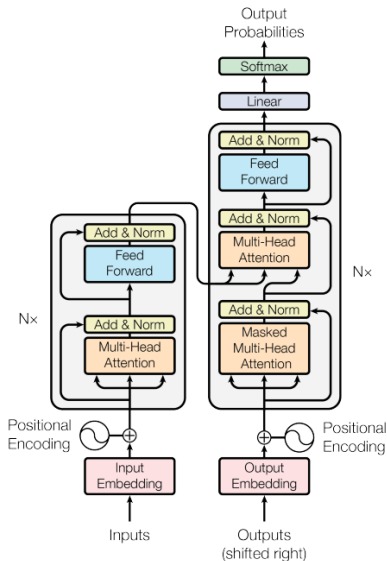
NMT Model Comparison (December 2017)

Layer Type	Complexity per Layer	Sequential Operations	Maximum Path Length
Self-Attention	$O(n^2 \cdot d)$	$O(1)$	$O(1)$
Recurrent	$O(n \cdot d^2)$	$O(n)$	$O(n)$
Convolutional	$O(k \cdot n \cdot d^2)$	$O(1)$	$O(\log_k(n))$
Self-Attention (restricted)	$O(r \cdot n \cdot d)$	$O(1)$	$O(n/r)$

Figure: n is sequence length, d is representation dimension, k is kernel size of convolutions, r is the size of the neighborhood in restricted self-attention.

Self-attention allows the Transformer to be trained in a more parallel fashion on GPU hardware. Typically, $d > n$.

Transformer Model



Transformer Model

No recurrent connections: the entire input sequence/output sequence so far is sent into the model at once (demo).

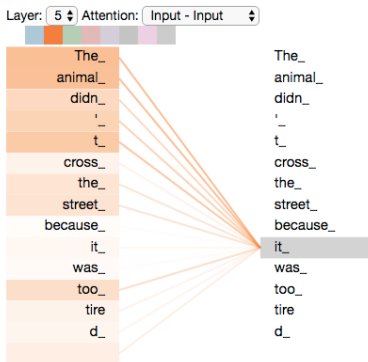
- ① Input words get transformed into “positional embeddings.”
- ② For each encoder layer:
 - ① Apply multi-headed self-attention.
 - ② Send outputs through feedforward network.
- ③ For each output word:
 - ① Transform all previous output words into positional embeddings.
 - ① Apply masked multi-headed self-attention.
 - ② Apply attention using the encoder outputs.
 - ③ Send output through feedforward network.
 - ② Transform output vector into the next word using linear and softmax layers.

Self-Attention

Intuition: When the model processes a word, self-attention allows it to look at other positions in the input sequence to help it determine the optimal encoding for the word.

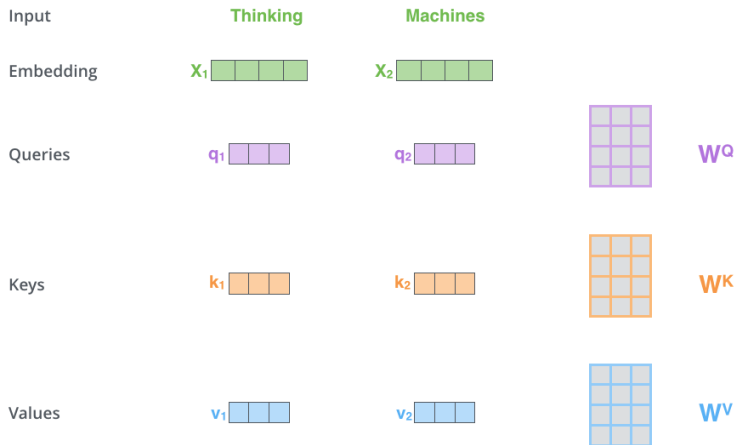
Example: “The animal didn’t cross the street because **it** was too tired.”

- Visualization



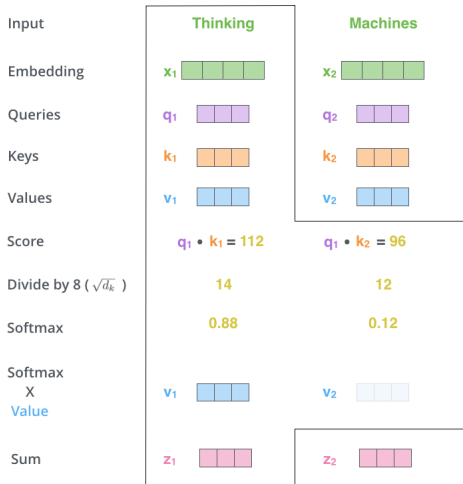
Self-Attention

Step 1: Generate query, key, and value vectors for each embedding using learned matrices W^Q , W^K , W^V .



Self-Attention

Step 2: Compute a score for each key-value pair. Normalize these weights to sum to 1, and compute the output as the weighted sum of the values.



Self-Attention

Multi-headed attention means learning M attention layers in parallel (with different weight matrices).

Step 3 is combining the results using another learned matrix W^O .

1) Concatenate all the attention heads



2) Multiply with a weight matrix W^O that was trained jointly with the model

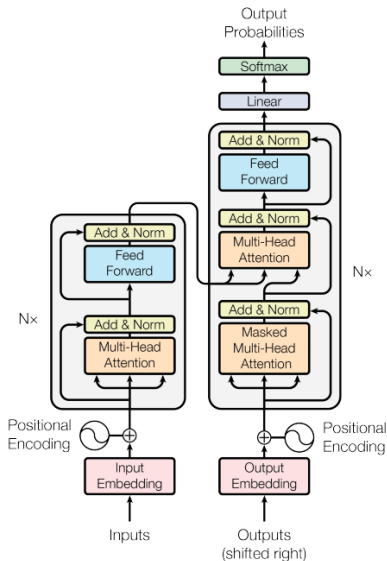
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3) The result would be the Z matrix that captures information from all the attention heads. We can send this forward to the FFNN



Transformer Model



NMT Model Comparison (December 2017)

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	41.29	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$
Transformer (base model)	27.3	38.1	$3.3 \cdot 10^{18}$	
Transformer (big)	28.4	41.8	$2.3 \cdot 10^{19}$	

Figure: The Transformer outperforms other state-of-the-art models at a fraction of the training cost. FLOPS is floating point operations.

Future Directions for NMT

Explore attention and CNNs as an alternative to RNNs.

Many active research areas in NMT, including:

- ① Sub-word or character-level models.
- ② Rare word problem.
- ③ Low-resource languages.
- ④ Efficient decoding.

References I



Goodfellow, Bengio, Courville

Deep Learning

The MIT Press, 2017



Pascanu, Mikolov, Bengio (2013)

On the difficulty of training recurrent neural networks

arXiv preprint [arXiv:1211.5063](https://arxiv.org/abs/1211.5063)



Cho, Bahdanau, Bougares, Schwenk, Bengio (2014)

Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation

arXiv preprint [arXiv:1406.1078](https://arxiv.org/abs/1406.1078)



Bahdanau, Cho, Bengio (2015)

Neural Machine Translation by Jointly Learning to Align and Translate

arXiv preprint [arXiv:1409.0473](https://arxiv.org/abs/1409.0473)



Vaswani, Shazeer, Parmar, Uszkoreit, Jones, Gomez, Kaiser, Polosukhin (2017)

Attention Is All You Need

arXiv preprint [arXiv:1706.03762](https://arxiv.org/abs/1706.03762)

Thank you!