

On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle

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Structural models of default calibrated to historical default rates, recovery rates, and Sharpe ratios typically generate Baa–Aaa credit spreads that are significantly below historical values. However, this “credit spread puzzle” can be resolved if one accounts for the fact that default rates and Sharpe ratios strongly covary; both are high during recessions and low during booms. As a specific example, we investigate credit spread implications of the Campbell and Cochrane (1999) pricing kernel calibrated to equity returns and aggregate consumption data. Identifying the historical surplus consumption ratio from aggregate consumption data, we find that the implied level and time variation of spreads match historical levels well. (*JEL* G12, G13)

Standard structural models of default are known to significantly underestimate credit spreads for corporate debt, especially for investment grade bonds of short maturity. Early work on this topic includes that of Jones, Mason, and Rosenfeld (1984), who find that the Merton (1974) model generates spreads that are far below empirical observation for investment grade firms. Although subsequent work (e.g., Eom et al. 2004) documents that various structural models can generate diverse predictions for credit spreads, Huang and Huang (2003) show that once these models are calibrated to be consistent with historical default and recovery rates, they all produce very similar credit spreads that fall well below historical averages. In particular, they find that the theoretical average four-year Baa–Treasury spread is approximately 32 basis points (bp) and relatively stable across all models they consider. This contrasts sharply with their

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reported historical average Baa–Treasury spread of 158bp. Similarly, they find a theoretical average four-year Aaa–Treasury spread of about 1bp, well below their reported historical average of 55bp.

The standard explanation for the discrepancies between observed and theoretically predicted spreads is that these models only capture credit risk and ignore other factors that can drive a wedge between the prices of Treasuries and corporate bonds, such as tax asymmetries, call/put/conversion options, and differences in liquidity between the Treasury and corporate bond markets.¹ Note, though, that if the level of credit spread due to these other factors is of similar magnitude for Aaa and Baa bonds, then the Baa–Aaa spread should be mostly due to credit risk. Yet even in this case, the results of Huang and Huang imply a large disparity between theory and historical observation, as their predicted Baa–Aaa spread of $(32 - 1)$ 31bp falls far short of their reported historical value of $(158 - 55)$ 103bp. Thus, their findings suggest that expected returns on a portfolio that is long Baa bonds and short Aaa bonds are rather large compared with the underlying risks involved. We refer to this finding as the “credit spread puzzle.”

Note that this credit spread puzzle is reminiscent of the so-called equity premium puzzle in that the historical returns on equity also appear to be too high for the risks involved. Given that corporate bonds and equities are contingent claims to the same firm value, they necessarily share many of the same systematic risk sources. As such, it seems natural to ask whether these two puzzles are related. This question is the focus of our paper.

To motivate our analysis of this question, consider a defaultable discount bond with maturity T issued by a firm with (random) default time τ .² If the firm defaults before the maturity date (i.e., if $\tau < T$), the bondholder receives $(1 - L_\tau)$, where L_τ is the loss rate given default. If instead the firm does not default before the maturity date, the bondholder receives \$1. Combining these two possibilities, we can express the cash flows of this risky bond as $X = (1 - \mathbf{1}_{\{\tau \leq T\}} L_\tau)$, where the indicator function $\mathbf{1}_{\{\tau \leq T\}}$ equals one if $(\tau \leq T)$ and zero otherwise. Under some relatively weak no-arbitrage restrictions (see, e.g., Duffie (1996) or Cochrane (2001)), it follows that there exists a pricing kernel Λ such that the bond price satisfies the following relation:

$$\begin{aligned} P &= E[\Lambda(1 - \mathbf{1}_{\{\tau \leq T\}} L_\tau)] \\ &= E[\Lambda] E[1 - \mathbf{1}_{\{\tau \leq T\}} L_\tau] + \text{Cov}[\Lambda, (1 - \mathbf{1}_{\{\tau \leq T\}} L_\tau)] \\ &= \frac{1}{R^f} (1 - E[\mathbf{1}_{\{\tau \leq T\}} L_\tau]) - \text{Cov}[\Lambda, \mathbf{1}_{\{\tau \leq T\}} L_\tau]. \end{aligned} \quad (1)$$

¹ Many papers have attempted to decompose credit spreads into its various components. See, for example, Duffie and Singleton (1997), Duffie (1999), Elton et al. (2001), Leland (2004), Geske and Delianedis (2003), Driessen (2005), Feldhutter and Lando (2008), Ericsson and Renault (2006), Longstaff, Mithal, and Neis (2005), and Chen, Lesmond, and Wei (2007).

² This firm is presumed to have several other debt issuances outstanding, so default may occur before T , after T , or never at all, which corresponds to $\tau = \infty$.

Here, $R^f = \frac{1}{E[\Lambda]}$ is the risk-free rate. By calibrating their models to match historical expected default and recovery rates, Huang and Huang force all models to agree on the expected future cash flows $E[1 - \mathbf{1}_{\{\tau \leq T\}} L_\tau]$ (i.e., the first term on the right-hand side). Thus, Huang and Huang's rather surprising result—that so many different structural models of default produce very similar spreads—implies that the second term of Equation (1) does not vary significantly across the models they investigate. Moreover, Equation (1) implies that a structural model can produce low prices for risky bonds (and thus high spreads) only if it can generate (at least) one of the following two channels:

1. A strong positive covariance between the pricing kernel (Λ_t) and the default time ($\mathbf{1}_{\{\tau \leq T\}}$).
2. A strong positive covariance between the pricing kernel (Λ_t) and the loss rate (L_τ).

Within a structural model of default, channel 1 can be broken down further into two components. This is because structural models typically assume that default is triggered the first time an asset value process $\{V_t\}$ crosses the default boundary $\{B_t\}$.

As such, if we define the default time τ via

$$\tau := \inf\{t : V_t \leq B_t\},$$

then Equation (1) can be rewritten as

$$P = \frac{1}{R^f} (1 - E[\mathbf{1}_{\{\tau \leq T\}} L_\tau]) - \text{Cov}[\Lambda, \mathbf{1}_{\{\inf\{t : V_t \leq B_t\} \leq T\}} L_\tau]. \quad (2)$$

Equation (2) implies that we can refine our analysis above as follows. In order for a structural model to produce large credit spreads (conditional on a given expected historical loss rate), it must generate (at least) one of the following channels:

- 1a. A strong negative covariance between the pricing kernel (Λ_t) and asset prices (V_t).
- 1b. A strong positive covariance between the pricing kernel (Λ_t) and the default boundary (B_t).
2. A strong positive covariance between the pricing kernel (Λ_t) and the loss rate (L_τ).

That is, in order to explain the credit spread puzzle, a model needs to generate low cash flows for risky bonds when marginal utility is high (that is, during recessions). These low cash flows can be generated through either high default rates or low recovery rates. Further, high default rates during recessions can be generated either through firm value dropping toward the default boundary or through the default boundary rising up toward firm value.

Note that it is channel 1a that researchers often pursue when attempting to explain the equity premium puzzle. Motivated by this insight, we investigate whether pricing kernels that have been engineered to explain the equity premium puzzle, such as the habit-formation model of Campbell and Cochrane (1999), can also explain the credit spread puzzle. The Campbell and Cochrane (1999) model is an ideal candidate for this investigation because it is parsimonious and can successfully capture many salient features of historical equity returns, such as high equity premia and strongly time-varying Sharpe ratios. Below, we argue that time-varying Sharpe ratios are also essential for explaining observed Baa–Aaa spreads. We also explore what roles channels 1b and 2 play in capturing the credit spread puzzle.

Our main findings are as follows. First, our model cannot explain either the average level or the time variation of the short-maturity Aaa–Treasury spread. Simply put, the historical default frequencies appear to be too low to be explained from a credit perspective. This result is consistent with interpreting both the level and the time variation of the Aaa–Treasury spread to be mostly unrelated to default.³ Interestingly, because there is a strong correlation between the Aaa–Treasury spread and the Baa–Aaa spread, it appears that liquidity, defined as the nondefault component of spreads, moves with the business cycle.⁴ If so, then during recessions, firms might need to issue bonds at yield spreads that are above “fair compensation” for credit risk, which in turn might generate countercyclical default boundaries.⁵ This argument provides one motivation for investigating the possibility that default boundaries move with the business cycle (i.e., channel 1b).

Second, we find that the Campbell-Cochrane model with a *constant* default boundary generates average levels and time variation for Baa–Aaa spreads that fit historical values better than the benchmark case (Merton 1974), but still fall well short of historical values. Further, this model incorrectly predicts that expected future default probabilities are procyclical. The reason this occurs is that Sharpe ratios are highest during recessions in the Campbell-Cochrane model. Thus, firm value tends to drift away from the default boundary faster during recessions.

Finally, when our model is calibrated to capture the countercyclical nature of defaults, it generates an average level and time variation of Baa–Aaa spreads that agree well with observation. We consider two different mechanisms

³ Several papers have argued that the Treasury rate is not the appropriate measure for the “risk-free rate” due to its extreme liquidity and “benchmark status.” See, for example, Grinblatt (1995), Collin-Dufresne and Solnik (2001), Longstaff (2004), Hull, Predescu, and White (2005), Blanco, Brennan, and Marsh (2005), and Feldhutter and Lando (2008).

⁴ Admittedly, an alternative explanation for the time variation in the Aaa–Treasury spread is a “Peso” problem, where investors account for the possibility of a so-far-unobserved event in which several Aaa firms simultaneously default.

⁵ There is empirical evidence suggesting that financial constraints tighten during recessions (e.g., Gertler and Gilchrist 1994, Kashyap, Stein, and Wilcox 1993). This finding also has an impact on firms, leverage decisions—e.g., Korajczyk and Levy (2003), Hennessy and Levy (2007).

to capture countercyclical default rates: a countercyclical default boundary,⁶ and countercyclical idiosyncratic volatility. Both mechanisms perform well in matching historical level and time variation of spreads.

In sum, we find that the Campbell-Cochrane pricing kernel combined with a model calibrated to match the countercyclical nature of default rates is consistent with historical Baa–Aaa spreads. The intuition for this result is as follows. Because the majority of defaults occur during recessions, the cash flows of a well-diversified portfolio of Baa bonds can be replicated by a portfolio that is long the “risk-free” bond but short those state-contingent claims that pay off during recessions. As such, if one specifies a pricing kernel that imputes high prices for these recessionary state-contingent claims, then the price of the replicating portfolio (and hence the price of the Baa bond portfolio) will be significantly less than the price of the risk-free bond, implying large credit spreads. We choose to investigate the credit spread implications of the Campbell-Cochrane pricing kernel precisely because it generates very high prices for those state-contingent claims that pay off in the worst states of nature. It does so by generating time-varying Sharpe ratios that are high during recessions.

To provide additional support for our claim that time variation in Sharpe ratios is essential for explaining the credit spread puzzle, we also investigate the credit spread implications for the pricing kernel of Bansal and Yaron (2004), which specifies Epstein-Zin (1989) preferences and small but persistent shocks to expected consumption growth. In its simplest form (i.e., their Case I), Bansal and Yaron’s model is able to generate a high equity risk premium, as we show in Appendix C. However, because this model generates little time variation in Sharpe ratios, it cannot explain the credit spread puzzle. In fact, its predictions barely differ from those of the benchmark model.⁷

After demonstrating that the Campbell-Cochrane model (calibrated to match historical Sharpe ratios, default rates, and recovery rates) can capture historical credit spread levels, we then investigate its time series predictions. The model predicts that credit spreads are a function of a single state variable S , which Campbell-Cochrane refer to as the surplus consumption ratio. Using historical consumption data, we identify the time series for this variable, which, in turn, identifies the model-implied time series of spreads.

⁶ It is difficult to assess whether the time-varying default boundary implied by the model is consistent with empirical observation, because default boundaries are not directly observable. (See the discussion surrounding Equation (27) below.) Still, we note that such a boundary is consistent with the empirical findings of Collin-Dufresne, Goldstein, and Martin (2001), Elton et al. (2001), and Schaefer and Strebulaev (2008), who document that market-wide (e.g., Fama-French factors, VIX) factors are economically and statistically significant for predicting changes in credit spreads even after controlling for all factors (e.g., leverage, firm value, volatility, etc.) that standard structural models suggest should be sufficient statistics. These factors might capture the time variation in the default boundary, which is not observable and likely imperfectly measured by leverage.

⁷ As we discuss in the appendix, the more general model considered by Bansal and Yaron (2004), which allows for shocks to both growth rate and stochastic volatility, generates time variation in Sharpe ratios as well. We emphasize that our focus is not on comparing the pricing kernels of Campbell-Cochrane versus Bansal-Yaron models per se, but rather on comparing those pricing kernels that do or do not generate large time variation in market prices of risk (i.e., Sharpe ratios).

We find the consumption-implied Baa–Aaa spreads fit the mean (113bp vs. 120bp) and variation (76bp vs. 70bp) of historical spreads quite well. Their correlation in levels for the whole sample is 72%; in changes the correlation is 49% (46% for the 1919–1945 period and 58% for the 1946–2004 period). Given the well-known failure of consumption-based models to price financial securities (e.g., Hansen and Singleton 1982), the strong link between the surplus consumption ratio and credit spreads is impressive.

Our paper is related to the extant literature in many ways. First, the strong link between observed credit spreads and our model-implied spreads (which are derived from historical consumption data) provides some justification for the common practice of using credit spreads to estimate the equity premium (e.g., Chen, Roll, and Ross 1986, Keim and Stambaugh 1986, Campbell 1987, Fama and French 1989, and 1993, Campbell and Ammer 1993, and Jagannathan and Wang 1996). Second, by showing that it does a good job at capturing the level and time variation in spreads, we provide out-of-sample support for the habit-formation model, which had been engineered to explain the equity premium puzzle. We also find, however, that the historical surplus consumption ratio is much more highly correlated with credit spreads than with the price-dividend (P/D) ratio. The weaker role of the P/D ratio might be related to corporate payout policies.

Related papers by Chen (2007) and Bhamra, Kuehn, and Strebulaev (2007) extend our analysis by attempting to explain historical credit spread levels while endogenizing the capital structure decision of the firm. Rather than using a Campbell–Cochrane framework, both papers instead choose to use a framework similar to that of Bansal and Yaron (2004) combined with a Markov regime switching process as in Hackbarth, Miao, and Morellec (2006). Although our framework differs from theirs, all three assume significant variation in the market prices of risk over the business cycle in order to capture historical spread levels. Separately, Almeida and Philippon (2007) use the insights of this paper to argue that the valuation of financial distress costs are higher than typically estimated because they are more likely to be incurred during times of high marginal utility, leading firms to be more conservative in their capital structure decisions.

Finally, while there is a rapidly growing body of literature that estimates the real and risk-neutral default probability and the implied default risk premium, here we attempt to explain both the equity premium and credit spread simultaneously, thus linking the macroeconomics and credit risk literature.⁸

The rest of the paper is as follows. In Section 1, we report historical data on the level and time variation of credit spreads, leverage, and default probabilities. In Section 2, we use the Merton (1974) model as a benchmark to identify the credit spread puzzle and to explain why it is important to calibrate the models

⁸ See, for example, Fons (1987), Delianedis and Geske (1998), Delianedis, Geske, and Corzo (1998), Bohn (2000), Elton et al. (2001), Cooper and Davydenko (2004), Driessen (2005), Longstaff, Mithal, and Neis (2005), Campello, Chen, and Zhang (2008), Saita (2006), Berndt et al. (2005), and Pan and Singleton (2005).

to historical default rates, recovery rates, and Sharpe ratios. In Section 3, we review the pricing kernel of Campbell-Cochrane and present its implications for credit spreads. In Section 4, we compare historical and consumption-implied credit spreads. We conclude in Section 5. In an appendix, we demonstrate that in spite of its ability to capture the equity premium puzzle, the simplest model of Bansal and Yaron (2004) cannot capture the credit spread puzzle due to its inability to generate sufficient time variation in Sharpe ratios.

1. Historical Data and Summary Statistics

In this section, we report summary statistics related to macroeconomic variables and default risk. In Panel A of Table 1, the P/D ratio is 27.77 for 1919–2001 and 23.40 for 1919–1997.⁹ Using the Moody's 2005 annual report, we find the average four-year future cumulative default rate for Baa-rated bonds to be 1.55% with a standard deviation of 1.04% for 1970–2001. Using data from the Federal Reserve, the mean composite Baa–Aaa spread is 109bp with a standard deviation of 41bp during this period. A longer data set that includes the Depression era provides similar results for the average Baa–Aaa spread but with significantly higher default rates. However, as in Campbell and Cochrane (1999), who calibrate their model to postwar data (when the equity premium was significantly higher than in the longer data set), we attempt to capture the statistics of these shorter data. We do this for two reasons: first, current prices may reflect a belief that there is a better understanding of the economy so that it is unlikely that the United States will ever again experience a depression with such severity. Second, some of the data used to calibrate the model goes back only to 1970.¹⁰

We consider three different proxies for the leverage ratio. The first proxy is book leverage (BLV), calculated as the ratio of book debt (obtained from COMPUSTAT) to (book debt + market equity). The second proxy is market leverage (MLV), defined as the ratio of market debt to (market debt + market equity). In particular, we use the Lehman Brothers fixed-income data set to estimate the market value of debt by first determining the market value of debt *per dollar of face value* for each firm year and then scaling this number by the book debt.¹¹ The third proxy is the inverse distance-to-default (IDD), which is defined as the ratio of $(0.5 \times \text{long term book debt} + \text{short term book debt})$ to (market debt + market equity). This last measure is similar to that used by Moody's KMV for estimating expected default frequencies (EDF). All measures cover the 1974–1998 period due to limitations of the Lehman Brothers fixed-income

⁹ The data used are obtained from Robert Shiller's Web site (<http://www.econ.yale.edu/shiller/data.htm>). Note that the P/D ratio does not consider equity repurchases, and is thus biased upward (see, e.g., Boudoukh et al. 2007).

¹⁰ Summary statistics for a sample with all data available for 1974–1998 are very similar to those in Table 1.

¹¹ We restrict our sample to bonds with nonmatrix prices. We do not exclude the callable bonds because the aggregate Baa over Aaa spread, which we intend to fit, also has this feature.

Table 1
Summary statistics

Panel A: Summary statistics

Variable	Mean	Std.	Min	Max
P/D ratio	27.77	14.54	10.12	85.42
Baa–Aaa spread (%)	1.09	0.41	0.60	2.33
Four-year default probability (%)	1.55	1.04	0.00	3.88
Book leverage of Baa	0.45	0.09	0.27	0.62
Market leverage of Baa	0.44	0.08	0.29	0.59
Inverse of the DD of Baa	0.28	0.07	0.16	0.42

Panel B: Correlation matrix of some benchmark variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
P/D ratio (1)	1.00						
Consumption growth (2)	0.14 0.22	1.00					
Baa–Aaa spread (3)	–0.37 0.00	–0.32 0.00	1.00				
Four-year default probability (4)	0.19 0.30	0.21 0.24	0.34 0.05	1.00			
Book leverage of Baa (5)	–0.70 0.00	–0.26 0.20	0.57 0.00	0.10 0.63	1.00		
Market leverage of Baa (6)	–0.61 0.00	–0.16 0.45	0.49 0.01	0.06 0.79	0.97 0.00	1.00	
Inverse of the DD of Baa (7)	–0.71 0.00	–0.40 0.05	0.60 0.00	0.14 0.52	0.96 0.00	0.87 0.00	1.00

Panel C: Regressions of default probability on Baa–Aaa spread

Dependent variable	Intercept	Baa–Aaa	adj. R-square
Four-year default rate (<i>t</i> -stat)	0.57 (1.01)	0.86 (2.07)	8.80

The statistics of different variables cover different periods in Panel A. The P/D ratio covers the 1919–2001 period. The four-year-ahead cumulative default probability and the Baa–Aaa spread cover the 1970–2001 period. The three leverage measures cover the 1974–1998 period. Among them, book leverage is defined as the ratio of book debt to (book debt + market equity); market leverage is defined as the ratio of market debt to (market debt + market equity); the inverse of the distance-to-default (DD) is defined as the ratio of $(0.5 \times \text{long term book debt} + \text{short term book debt})$ to (market debt + market equity). In Panel B, the first (second) row is the correlation (*p*-value). The correlation statistics use the maximum common sample size between two series. In Panel C, the first row is the OLS regression coefficients. Newey-West *t*-statistics are reported in the second row, where four lags are chosen.

data set. We report only the leverage ratios of Baa-rated bonds. IDD is on average 28%, much lower than BLV (45%) and MLV (44%). We present the correlation matrix in Panel B. The following three patterns can be observed. First, the Baa–Aaa spread is countercyclical: it covaries negatively with both the P/D ratio and the real per capita consumption. Second, the four-year future default rate is significantly positively correlated to Baa–Aaa spread. In Panel C, we regress the four-year forward cumulative default rate on the Baa–Aaa spread, which yields a significant coefficient of 0.86. As we intend to fit both default rates and credit spreads, we will also match this regression coefficient in the model.

Third, the three leverage ratio measures are countercyclical in that they are significantly negatively correlated to the P/D ratio and positively correlated

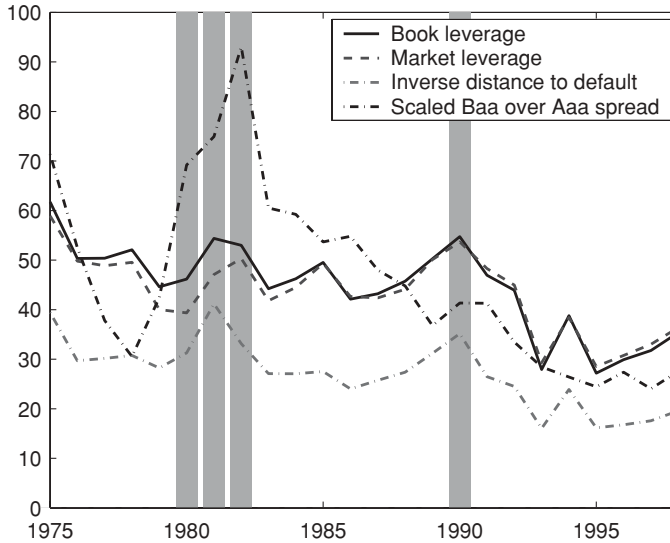


Figure 1
Time series of three different leverage ratios: book leverage, market leverage, inverse distance-to-default for Baa-rated firms for 1975–1998
We also report the time series of Baa–Aaa spreads and NBER recessions.

to the Baa–Aaa spread. We emphasize that this result is not obvious a priori. Rather, it is due to the manner in which rating agencies choose to perform their job. Indeed, had rating agencies decided to base their credit rating on the current level of firm leverage (holding other characteristics, such as volatility, constant), then leverage ratios *within a given rating* would be relatively constant over the business cycle.¹² In practice, however, while rating agencies impose more downgrades than upgrades during recessions (thus performing a “credit refreshment” for the indices), they tend to “rate through the cycle” to some degree. Therefore, even within a given credit rating, leverage ratios are countercyclical. To see this graphically, we plot in Figure 1 the three leverage ratios of Baa-rated bonds as well as Baa–Aaa spread for the 1975–1998 period. We refer to a particular year as a recession year if there are at least five months in that year defined as being in recession by NBER. During the two recession periods, the three leverage ratios go up, reflecting the fact that rating agencies rate through the cycle to some extent.

As we demonstrate below, a key statistic to which we calibrate our models is the average Sharpe ratio for a typical firm. Because equities and bonds should have similar (instantaneous) Sharpe ratios for the same firm, and because equities are both more liquid and associated with more reliable data, we calculate Sharpe ratios using equity returns. Based on the whole universe of the CRSP

¹² If rating agencies followed such a strategy, the fact that aggregate leverage ratios are higher during recessions would be reflected in a larger fraction of firms falling into the lower rating levels during recessions.

monthly tape (1927–2005), the median (mean) firm-level Sharpe ratio is 0.23 (0.17), approximately one-half the value of the Sharpe ratio for the market portfolio (0.43). This reflects the fact that average firm volatility is approximately twice the level of market volatility. As a second estimate, we consider a shorter period (1974–1998) for which we have rating data, and thus can estimate an average Sharpe ratio using only Baa firms. Given that historical returns over a few decades can generate poor proxies for ex ante expected returns (i.e., the numerator of the Sharpe ratio) at the individual firm level, we have adopted the following approach. We first estimate the conditional beta for each firm month (using trailing data in the previous 60 months). We then multiply the excess return of the market portfolio by beta for each month, and regard its mean as the expected risk premium for each firm. Finally, we calculate the volatility of equity return and, subsequently, the Sharpe ratio for each firm. For the 1974–1998 period, the median (mean) Sharpe rate is 0.22 (0.23). Given these rather consistent results over both the short and long data sets, below we calibrate all models to an average Sharpe ratio of 0.22 for the representative Baa firm. This value seems sensible as market betas of Baa-rated firms are close to one, but their return volatilities are about twice as large as market return volatilities.

Below, we attempt to explain the historical Baa–Aaa spread solely in terms of credit risk. As such, we are implicitly assuming that there is little difference between those components of Baa and Aaa bond yields that are unrelated to credit, such as differences due to liquidity or callability. Schultz (2001, Table III) provides some support for this assumption in that he finds little evidence for differences in liquidity between Aaa and Baa bonds.

We also try to assess empirically the component of the composite Baa–Aaa spread due to the call option embedded in many corporate bonds. At first, one might think that it is more valuable for Baa bonds than for Aaa bonds because Baa yields are more volatile than Aaa yields. However, this volatility effect is mitigated by the “interaction effect” between the option to default and the option to call (Kim, Ramaswamy, and Sundaresan 1993, Acharya and Carpenter 2002). Following Duffee (1999) and Huang and Huang (2003), we estimate the Baa–Aaa spread for the set of noncallable corporate bonds in the Lehman Brothers fixed-income data set. We find the average Baa–Aaa spread for noncallable industrial bonds in the maturity range of two to seven years to be 102bp for the 1985–1995 period. If we consider the longer period 1974–1998, the estimated spread is 94bp, but the number of noncallable bonds is relatively low before 1985. We conclude that the component of the composite Baa–Aaa spread reported above due to the call feature is likely relatively small, in the range of 7bp–15bp.

In summary, then the following hold:

- Baa–Aaa spreads are high (all bonds = 109bp, noncallable bonds = 94bp–102bp) and rather volatile (41bp standard deviation).

- Baa default rates are low on average (e.g., the four-year cumulative default probability is 1.55%) and volatile.
- Forward default rates are countercyclical in that the regression coefficient of forward default rates on spreads is 0.86 and statistically significant.
- Leverage ratios are countercyclical (both in terms of P/D ratios and consumption growth) and positively related to credit spreads.
- Average Sharpe ratios for Baa firms (≈ 0.22) are approximately one-half the average Sharpe ratio for the market portfolio (≈ 0.43).

2. A Benchmark Model (Merton 1974)

In this section, we investigate a simple variation of the Merton (1974) model. This model is useful for three reasons. First, it provides a magnitude for the credit spread puzzle. Second, it reveals that three quantities are crucial for the calibration of structural models, namely: the default rate, the recovery rate, and the Sharpe ratio. Finally, this benchmark case allows us to see how the three channels mentioned above can be used to help explain the credit spread puzzle.

We specify firm value dynamics under both the historical (P-) and risk-neutral (Q-) measures as

$$\frac{dV}{V} + \delta dt = \mu dt + \sigma dz \quad (3)$$

$$= r dt + \sigma dz^Q. \quad (4)$$

Here, the dividend yield (δ), expected return (μ), asset volatility (σ), and risk-free rate (r) are all assumed constant. It is convenient to define the asset Sharpe ratio as

$$\theta \equiv \frac{\mu - r}{\sigma}. \quad (5)$$

In the spirit of Merton (1974), we assume that the only liability of the firm is a zero-coupon bond with maturity T . Further, we assume that default can occur only at maturity and only if firm value $V(T)$ falls below an exogenously specified default boundary B .¹³ Finally, we assume that bondholders receive a *constant* recovery rate $(1 - L)$ if default occurs. Thus, L can be interpreted as the loss rate given default.

Under these assumptions, we show in Appendix A that the credit spread $(y - r)$ on a date- T zero-coupon bond is

$$(y - r) = - \left(\frac{1}{T} \right) \log \{ 1 - L N [N^{-1}(\pi^P) + \theta \sqrt{T}] \}. \quad (6)$$

¹³ We note that in the original Merton (1974) framework, the default boundary B equals the face value of debt F . However, given that historical recovery rates are 44.9%, this assumption implies that bankruptcy costs are approximately 55.1%, which is difficult to believe. Moreover, there are several papers (e.g., Leland 2004, Davydenko 2006) that provide some evidence suggesting that the default boundary is probably closer to 70–75% of the face value of debt.

Table 2
Baa–Aaa spreads as a function of Sharpe ratio for the benchmark model

Sharpe	<i>T</i> = 4 years			<i>T</i> = 10 years		
	Baa	Aaa	Baa–Aaa	Baa	Aaa	Baa–Aaa
0.15	44.0	1.6	42.4	67.7	12.0	55.7
0.20	54.9	2.2	52.7	88.1	17.4	70.7
0.25	68.1	3.0	65.1	112.8	24.6	88.2
0.30	83.7	4.1	79.6	141.7	34.2	107.5
0.35	102.0	5.5	96.5	175.1	46.6	128.5
0.40	123.4	7.4	116.0	212.9	62.2	150.7

The four-year Baa (Aaa) default rate is 1.55% (0.04%). The 10-year Baa (Aaa) default rate is 4.89% (0.63%). The recovery rate is 0.449.

Here, the function $N(\cdot)$ is the cumulative normal. The implication of this equation is that, even though the model is specified by seven parameters $\{r, \mu, \sigma, \delta, V_0, B, L\}$, credit spreads depend only on three combinations of these parameters: expected default rate π^P , expected loss rate L , and asset Sharpe ratio θ . We note that Huang and Huang (2003) calibrate their models to historical estimates of $\{\pi^P, L\}$. Given the analysis above, we take this one step further and calibrate all of our models to match historical estimates of $\{\pi^P, L, \theta\}$. As demonstrated in Table 2, credit spreads are very sensitive to Sharpe ratios. Indeed, if the Sharpe ratio is specified to equal 0.35 (similar to the Sharpe ratio of the market portfolio), then the Merton model can explain historical Baa–Aaa spreads. However, for a Sharpe ratio equal to our calibrated number 0.22, Baa–Aaa spreads are about 57bp. This estimate is much smaller than our empirical estimate of 94bp–102bp. We refer to this difference as the credit spread puzzle.¹⁴

Equation (6) identifies the sources of the credit spread puzzle: (i) expected default rates π^P are low, (ii) recovery rates $(1 - L)$ are substantial, and (iii) Sharpe ratios θ of individual firms are low due to a sizable level of idiosyncratic risk. The intuition for this last source is that as idiosyncratic risk increases (i.e., Sharpe ratio decreases), defaults become less systematic, and then the risk-premiums associated with corporate bonds (i.e., the second term in Equation (1)) decrease.

2.1 The “convexity effect”

In the benchmark case above, we assumed that the initial DD $\log(\frac{V_0}{B})$ is a constant across firms and time. Although we cannot directly observe the default boundary, the fact that leverage ratios (see Figure 1) are time dependent strongly suggests that the initial DD is also time varying. Indeed, in a recent paper, David (2007) argues that, because credit spreads are a convex function of the solvency

¹⁴ This benchmark model assumes that firm value dynamics follow a diffusion process, in turn constraining the distribution of future firm values to be log-normal under both measures. To check how limiting this assumption is, we also consider in an appendix a simple jump-diffusion model that permits distributions to be skewed. We find similar results, suggesting that this constraint is not the main source of the credit spread puzzle.

ratio (or inverse leverage ratio), ignoring this heterogeneity causes a large *downward* bias in predicted spreads. In this section, however, we demonstrate that if a model is calibrated to match historical default rates, recovery rates, and Sharpe ratios, then neglecting this heterogeneity in fact generates an error of only a few basis points. Moreover, this bias is actually in the *opposite direction* than that reported in David (2007).¹⁵

Consider the Merton model above with parameters $r = 0.05$, $\mu = 0.10$, $\delta = 0.06$, $\theta = 0.22$ (implying that $\sigma = 0.227$), $T = 4$, $L = 0.551$, and $B = 35.6$. Assume one-half of the firms have an initial value of $V_0 = 120$ and the other half $V_0 = 80$. From Equation (6), the average credit spread is

$$\begin{aligned}\overline{\text{CS}} &= \frac{1}{2} \text{CS}(V_0 = 120) + \frac{1}{2} \text{CS}(V_0 = 80) \\ &= \frac{1}{2}(12.69) + \frac{1}{2}(100.23) \\ &= 56.46\text{bp}.\end{aligned}\tag{7}$$

Next, compare this result to credit spreads obtained when all firm values start at $V_0 = 100$ and the model is calibrated to match historical default rates, recovery rates, and Sharpe ratios. For this example, we find that choosing $B = 0.397$ generates a default rate $\pi^P(V_0 = 100) = 1.55\%$. The resulting spread is 59.9bp, just 3bp above the “true value.” Thus, we see that approximating a heterogeneous initial solvency ratio by a constant provides a very good approximation to the true value if the model is calibrated to match historical default rates, recovery rates, and Sharpe ratios.^{16, 17, 18}

¹⁵ The reason that David (2007) attributes much of the credit spread to this “convexity effect” is that he does not hold Sharpe ratios constant in his attribution (i.e., comparative static) analysis. However, our analysis (and Table 2 above) emphasize that holding Sharpe ratios constant is essential.

¹⁶ To provide some intuition for why ignoring heterogeneity in initial solvency ratios generates a slight upward (instead of a downward) bias in spreads, consider the extreme example where the “actual economy” is composed of two types of firms: the first type has an initial leverage ratio that virtually guarantees default. The proportion of firms of this type is π^P . The second type has an initial leverage ratio that virtually guarantees the firm will not default. The proportion of firms of this type is $(1 - \pi^P)$. Note that by construction, this model will match historical default rate π^P , and therefore will also match expected losses. Further note that neither type of bond will command a risk premium because the cash flows of each are deterministic; the type-2 bonds are guaranteed to receive \$1, and the type-1 bonds are guaranteed to receive the recovery rate. Now, if one were to approximate this “actual” economy with a fictional economy where there is only one type of firm with initial leverage ratio set equal to the average leverage ratio of the actual economy, and if one were to calibrate this economy to match historical default rates, recovery rates, and Sharpe ratios, then both the actual economy and this fictional economy would agree on expected losses (i.e., the first term on the right-hand side of Equation (1)). However, the fictional economy would also have a second component due to risk-premiums (i.e., the second term on the right-hand side of Equation (1)) via covariance between the risky cash flows and the pricing kernel. Thus, by ignoring the heterogeneity of initial leverage ratios, the fictional economy would have higher average spreads than the actual economy. The example given in the text is less extreme than the one given here, but the same intuition holds; heterogeneity implies that those firms that start closer to the default boundary are likely to default due to idiosyncratic risk only. But such risks do not command a risk premium.

¹⁷ We note that for a rating agency to give a high-leverage firm the same rating as a low-leverage firm, the high-leverage firm will typically have a lower asset volatility. This will reduce any bias even further.

¹⁸ David (2007) attributes his good estimates for the Baa–Aaa spread to the heterogeneity in initial leverage ratios, and the “convexity effect” that it creates. Since we have demonstrated that such a convexity effect is small (and

Summarizing this section, we find that our benchmark Merton model, calibrated to historical default rates, recovery rates, and Sharpe ratios, generates a Baa–Aaa spread of 57bp, well below historical values. Moreover, our jump-diffusion example in the appendix and the multiple examples investigated by Huang and Huang (2003) demonstrate that the low implied Baa–Aaa spread is extremely insensitive to many modifications of this benchmark model that have been suggested in the extant literature. That is, calibrating structural models of default to historical default rates, recovery rates, and Sharpe ratios imposes a lot of discipline on these models that makes it difficult for them to capture the credit spread puzzle. Below, we investigate whether models that incorporate channels 1a, 1b, and 2 can fare better.

3. A Model with Time-Varying Sharpe Ratios

Time variation in Sharpe ratios has long been recognized as an important channel for explaining the high risk premium on stocks (the equity premium puzzle). As such, it seems natural to investigate how much it can contribute to our understanding of the observed Baa–Aaa spread. Here we choose to investigate the framework of Campbell and Cochrane (1999) because it has been successful at fitting many salient features of equity returns, including the strong time variation in Sharpe ratios. In a sense, our investigation of credit spreads provides an out-of-sample test of the Campbell-Cochrane model, which was reverse-engineered to match equity data. If the model is a good description of the world, then its pricing kernel should be relevant for pricing both equityholders’ and bondholders’ claims to a firm’s cash flows.

3.1 The Campbell-Cochrane framework

Slightly modifying their notation, Campbell and Cochrane (1999) specify the utility function of the representative agent in an exchange economy as

$$U(C_t, \hat{C}_t, t) = e^{-\alpha t} \frac{(C_t - \hat{C}_t)^{1-\gamma} - 1}{1-\gamma}, \quad (8)$$

where \hat{C} is an exogenous habit. Campbell-Cochrane define the surplus consumption ratio as $S \equiv (\frac{C - \hat{C}}{\hat{C}})$. For convenience, they also define the logarithms of consumption and surplus consumption via

$$c \equiv \log C \quad \text{and} \quad s \equiv \log S.$$

actually moves in the wrong direction), this raises the question why David (2007) can explain high credit spreads. We claim that the high Baa–Aaa spread obtained in David (2007) is due to his counterfactually high calibration for the Sharpe ratio. Indeed, note that David (2007, p. 24) reports an example where the Sharpe ratio is 0.322, the recovery rate is 0.51, and the 10-year default probability is 0.045. His model generates a credit spread of 131bp. We emphasize that the Merton (1974) model (with no heterogeneity in initial leverage ratios!) generates an almost identical value of 130.2bp when the terms in Equation (6) are calibrated to these numbers. That is, the good results found by David (2007) are actually due to the Sharpe ratio he calibrates his model to, rather than to any “convexity effect.”

Because the dividend is perishable and there are no investment opportunities, it follows that in equilibrium consumption equals the dividend payment. Further, the pricing kernel is equal to the marginal utility of the representative agent:

$$\begin{aligned}\Lambda_t &= U_c(C_t, \widehat{C}_t, t) \\ &= e^{-\alpha t} (C - \widehat{C})^{-\gamma} \\ &= e^{-\alpha t} e^{-\gamma s} e^{-\gamma c}.\end{aligned}\tag{9}$$

Campbell-Cochrane (1999) specifies the log-consumption and log-dividend processes as

$$\Delta c = g_c \Delta t + \sigma_c \Delta z_c \tag{10}$$

$$\Delta d = g_d \Delta t + \sigma_d \left(\rho_{cd} \Delta z_c + \sqrt{1 - \rho_{cd}^2} \Delta z_d \right) \tag{11}$$

and the log surplus consumption ratio dynamics as¹⁹

$$\Delta s = \begin{cases} \kappa(\bar{s} - s)\Delta t + \sigma \left[\frac{1}{\bar{s}} \sqrt{1 - 2(s - \bar{s})} - 1 \right] \Delta z & \text{for } s \leq s_{\max} \\ \kappa(\bar{s} - s)\Delta t & \text{for } s > s_{\max}, \end{cases} \tag{12}$$

where

$$\bar{s} \equiv \sigma \sqrt{\frac{\gamma}{\kappa}} \tag{13}$$

$$s_{\max} \equiv \bar{s} + \frac{1}{2}(1 - \bar{s}^2). \tag{14}$$

This specification generates an economy with a constant real risk-free rate (for $s < s_{\max}$):

$$r_f = \alpha + \gamma g_c - \frac{1}{2} \gamma \kappa. \tag{15}$$

The price-consumption ratio for the claim to consumption can be written as

$$\left(\frac{P(t)}{C(t)} \right) = E_t \left[\frac{\Lambda(t+1) C(t+1)}{\Lambda(t) C(t)} \left(1 + \frac{P(t+1)}{C(t+1)} \right) \right] \tag{16}$$

$$= E_t \left[\sum_{j=1}^{\infty} \frac{\Lambda(t+j) C(t+j)}{\Lambda(t) C(t)} \right]. \tag{17}$$

An analogous formula holds for the P/D ratio.

¹⁹ We use the parameter κ instead of $(1 - \phi)$ because κ , which has units of inverse time, can be easily “annualized” if first measured using a different frequency. In contrast, annualizing ϕ is more intricate and approximate.

While their framework does not provide analytic solutions for the price-consumption ratio, Equations (16) and (17) suggest two numerical schemes for estimating this ratio. In particular, Equation (16) can be estimated by using a recursive scheme to obtain a self-consistent solution for $\frac{P}{C}$. Alternatively, Equation (17) can be estimated using Monte Carlo methods. Unfortunately, both methods are vulnerable to certain types of errors. Indeed, both Cosimano, Chen, and Himonas (2004) and Chen, Collin-Dufresne, and Goldstein (2003) report significant discrepancies between their findings and those of Campbell and Cochrane (1999). Below, we use the Chen, Collin-Dufresne, and Goldstein (2003) approach when calibrating the model.²⁰

3.2 Calibration

Following Campbell-Cochrane, we calibrate the consumption dynamics $g_c = 0.0189$ and $\sigma_c = 0.015$ to match their historical averages. Further, the historical average real risk-free rate $r_f = 0.0094$ is used to calibrate $\alpha = 0.133$ via Equation (15). Finally, $\kappa = 0.138$ is chosen to match the serial correlation of the log P/D ratio. We then choose g_d , σ_d , ρ_{cd} , and γ to best match historical data on equity. The higher growth rate on dividends compared with consumption captures the leveraged nature of equity (Gennotte and Marsh 1993, Abel 1999, and 2008, and Chen, Collin-Dufresne, and Goldstein 2003). The results are given in Table 3. We see that the model does a good job at capturing historical levels and volatilities of both the P/D ratio and excess returns, as well as the historical Sharpe ratio of the market portfolio.

Structural models of default (Black and Scholes 1973, Merton 1974) take the firm value process (i.e., the claim to (dividends plus interest)) as the fundamental state variable rather than the equity value process (i.e., the claim to dividends). As such, we specify the log-aggregate output process $\epsilon(t)$ as

$$\Delta\epsilon(t) = g_\epsilon \Delta t + \sigma_\epsilon \left(\rho_{\epsilon c} \Delta z_c + \sqrt{1 - \rho_{\epsilon c}^2} \Delta z_\epsilon \right). \quad (18)$$

With γ determined from the equity data²¹ and $g_c = 0.0189$ chosen to match the consumption growth rate,²² we choose σ_ϵ and $\rho_{\epsilon c}$ to best match historical moments. These results are also given in Table 3. Historical values were estimated assuming historical weighted averages of debt and equity returns, where the weights were determined from historical leverage ratios. For comparison, we also include parameter estimates used in Campbell-Cochrane. As noted in their paper, the empirical correlation between dividends and consumption is very sensitive to data sample they choose. Also note that higher correlation

²⁰ Similar to the approach of “importance sampling,” this approach involves changing the probability measure to simulate “well-behaved” processes.

²¹ We choose γ to best match equity data, because this is the most easily estimated and most studied. Note that the other parameters of the dividend process—namely, g_d , σ_d , and ρ_{cd} —are not used in the analysis below.

²² Here, we are interpreting the claim to output as a nonleveraged security, and hence, should have a growth rate equal to that of consumption.

Table 3
Parameter choices and fitted moments for the Campbell-Cochrane model

Panel A: Parameter inputs

Type of cash flow ($i = d, \epsilon$)	g_i	σ_i	γ	ρ_{ci}
Dividends	0.040	0.080	2.45	0.60
Output	0.0189	0.063	2.45	0.48
CC dividends	0.0189	0.112	2.00	0.2

Panel B: Model outputs

Type of cash flow	$\exp(E[p - d])$	$\sigma(p - d)$	$\frac{E[r - r_f]}{\sigma(r - r_f)}$	$E[r - r_f]$	$\sigma(r - r_f)$
Claim to dividends	24	.21	.44	.073	.17
Historical equity	25	.26	.43	.067	.16
Claim to output	23	.15	.44	.053	.12
Historical (debt + equity)	19	.20	.43	.050	.10

Panel A reports the parameter calibrations for dividend and output (dividends plus interest) processes. Panel B reports the fitted sample moments of claims to dividends versus historical values; and claims to output (dividends plus interest) versus historical values.

estimates may proxy for the fact that consumption and dividends are expected to be cointegrated in the long run, even though the Campbell-Cochrane model does not specify this directly.

With this calibration in place, we now estimate credit spreads. First, we determine the aggregate price-output ratio $I(s_t)$ as a function of the single state variable s_t using the method of Chen, Collin-Dufresne, and Goldstein (2003).

We then determine aggregate firm value $V(\epsilon_t, s_t)$ by noting that price equals output multiplied by the price-output ratio:

$$V(\epsilon_t, s_t) = e^{\epsilon_t} I(s_t). \quad (19)$$

Given the dynamics of log-aggregate output ϵ_t in Equation (18) and the estimated functional form for the price-output ratio $I(s_t)$, it is straightforward to demonstrate that the dynamics of aggregate firm value under both the P and Q measures take the forms

$$\frac{\Delta V(t)}{V(t)} = (\lambda(s_t) + r - \delta(s_t)) \Delta t + \sigma(s_t) \Delta z_V(t) \quad (20)$$

$$= (r - \delta(s_t)) \Delta t + \sigma(s_t) \Delta z_V^Q(t). \quad (21)$$

Here, the risk-premium $\lambda(s_t)$, the dividend yield $\delta(s_t)$ (which equals the inverse price-output ratio ($\frac{1}{I}$)), and volatility $\sigma(s_t)$ are all functions of s_t and independent of ϵ_t . That is, as noted by Campbell and Cochrane (1999), $s(t)$ is the only state variable driving asset return dynamics.

Up to this point, cash flow dynamics have been calibrated to match aggregate dividends, and firm value dynamics have been specified to match the claim to aggregate dividends. In order to study credit spreads on bonds issued by individual firms, we now model firm-level dynamics. In the spirit of, for example,

a CAPM framework, we assume that the return dynamics for a typical firm follow²³

$$\frac{\Delta P(t)}{P(t)} = \frac{\Delta V(t)}{V(t)} + \sigma_{\text{idio}} \Delta z_{\text{idio}}(t). \quad (22)$$

We choose σ_{idio} to match a population average Sharpe ratio of 0.22. We find this value to be $\sigma_{\text{idio}} = 0.208$. Without loss of generality, we set initial firm value to $P(0) = 1$.

3.3 Constant default boundary case

In this section, we assume that the default boundary is constant and calibrate it to match historical default rates. To match the historical Baa (Aaa) four-year default rate of $\pi_{\text{Baa}} = 1.55\%$ ($\pi_{\text{Aaa}} = 0.04\%$), we set the default boundary location to $B_{\text{Baa}} = 0.356$ ($B_{\text{Aaa}} = 0.208$). Hence, firm value must fall to 35.6% of its original value for the Baa-firm to default. We note that, consistent with Leland (2004), Davydenko (2006), and others, the Baa-default boundary is significantly below the average initial leverage ratio of 0.44.

Because the Campbell-Cochrane model is calibrated in real terms while corporate bonds are written in nominal terms, we need to account for inflation. For simplicity, we assume a constant inflation rate of 3%. Hence, the nominal growth rate of output is $g_c = 0.03 + 0.0189 = 0.0489$. The coupon rate for the Treasury bond is set equal to the sum of the real risk-free rate plus inflation.²⁴ The coupon rate on the corporate bond is set equal to the real risk-free rate plus inflation plus 100bp. As such, both bonds are issued near par value.²⁵

Upon default, we assume that the agent immediately receives a recovery rate of 0.449, consistent with the recovery rate of Moody's 2005 report. All future promised coupon payments receive zero recovery. We then estimate Baa–Treasury and Aaa–Treasury spreads as a function of $s(0)$. The results are tabulated in Table 4.

The model generates an average four-year Baa–Aaa spread of 77bp. Although this is a significant improvement over the benchmark Merton model of Baa–Aaa_{Merton} = 57bp, these results fall far short of the historical composite Baa–Aaa spread of 109bp (or the range of 94bp–102bp obtained for noncallable Baa–Aaa bond spreads). Furthermore, the model generates a time variation in spreads of only 12bp, well below the historical value of 41bp. Finally, this model predicts

²³ Equation (22) assumes that idiosyncratic risk is independent across firms and that all firms load on a single “market factor.” A more general model could permit multiple “market factors” and industries to have correlated idiosyncratic risks. We leave these interesting questions for future work.

²⁴ Recall that $r(s)$ is a constant for all values of $s < s_{\text{max}}$, and that, in discrete time, s can actually be greater than s_{max} . Hence, $r(s)$ is stochastic in the Campbell-Cochrane framework. However, in the continuous-time version of the Campbell-Cochrane model discussed in the appendix, interest rates are truly constant because s_{max} constitutes a natural boundary.

²⁵ As expected, we find that credit spreads generated from this model are extremely insensitive to the specification of the coupon rate. Indeed, ignoring coupon payments completely increases spreads by only 2bp on 4Y bonds.

Table 4**Model-generated four-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the constant default boundary case**

$s(0)$	Population Distribution (%)	Baa			Aaa			Baa–Aaa Spread (bp)
		Spread over Treasury (bp)	Q -Default Rate (%)	P -Default Rate (%)	Spread over Treasury (bp)	Q -Default Rate (%)	P -Default Rate (%)	
–2.96	0.06	91.3	6.61	1.22	6.0	0.40	0.03	85.3
–2.86	0.07	90.1	6.50	1.36	5.9	0.39	0.04	84.2
–2.76	0.09	88.0	6.30	1.43	5.8	0.37	0.04	82.2
–2.66	0.11	85.5	6.12	1.54	5.3	0.34	0.04	80.2
–2.56	0.13	80.5	5.77	1.75	5.0	0.32	0.05	75.5
–2.46	0.15	76.0	5.46	1.89	4.8	0.31	0.05	71.2
–2.36	0.14	66.2	4.76	2.08	4.0	0.25	0.06	62.2
–2.27	0.04	52.9	3.82	2.20	3.0	0.17	0.06	49.9
	Average	82.3	5.90	1.55	5.2	0.34	0.04	77.1
	Std. Dev.	12.7	0.90	0.41	1.0	0.07	0.01	11.7

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be independent of $s(0)$ and constant over time ($B_{\text{Baa}} = 0.356$, $B_{\text{Aaa}} = 0.208$). Population averages over the steady-state distribution are then determined.

that four-year forward default rates are strongly procyclical. That is, the four-year forward probability of default *increases* with the initial value s_0 . This occurs because Sharpe ratios ($\frac{\mu-r}{\sigma}$) are high in recessions. Thus, for a given level of volatility σ , expected growth rates μ are high, which tend to push firm values away from the default boundary. To quantify this result, we estimate the theoretical regression coefficient for the four-year future default rates on spreads via

$$\begin{aligned}
\beta_{\text{theory}} &= \frac{\text{cov}_{ss}(\text{def rate}, \text{spread})}{\text{var}_{ss}(\text{spread})} \\
&= \frac{\sum_j \pi_{ss}(s_j)(\text{def rate}_j - E_{ss}[\text{def rate}])(\text{spread}_j - E_{ss}[\text{spread}])}{\sum_j \pi_{ss}(\text{spread}_j - E_{ss}[\text{spread}])^2} \\
&= -3.61.
\end{aligned} \tag{23}$$

This result contrasts significantly with the empirical result of $\beta = +0.86$ reported in the previous section.

In addition to the four-year results, we repeat the same procedure on ten-year spreads in Table 5. We find Baa–Aaa spreads to be 99bp, well short of the 131bp empirical estimate reported by Huang and Huang.

In summary then, compared with historical data, the Campbell-Cochrane model with a constant initial leverage ratio generates a predicted Baa–Aaa spread that is (i) too low on average, (ii) not sufficiently volatile, and (iii) varies negatively with four-year forward default rates, in conflict with observation. We interpret these results as follows: although within the Campbell-Cochrane framework the representative agent is willing to pay a large premium for securities that pay off in bad states, the constant boundary specification suggests that defaults happen too frequently in the future when the current state of nature is good.

Table 5
Model-generated 10-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the constant default boundary case

$s(0)$	Population Distribution (%)	Baa			Aaa			Baa-Aaa Spread (bp)
		Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	
-2.96	0.06	155.0	27.05	3.74	47.3	9.15	0.42	107.7
-2.86	0.07	149.9	26.22	3.86	44.5	8.58	0.47	105.4
-2.76	0.09	144.7	25.33	4.40	42.4	8.17	0.54	102.3
-2.66	0.11	138.5	24.33	4.72	39.4	7.60	0.59	99.1
-2.56	0.13	131.8	23.21	5.20	36.6	7.05	0.66	95.2
-2.46	0.15	124.9	22.08	5.86	34.2	6.59	0.76	90.7
-2.36	0.14	116.4	20.71	6.64	30.8	5.93	0.91	85.6
-2.27	0.04	104.7	18.86	7.36	26.0	5.03	1.05	78.7
	Average	141.8	24.88	4.89	42.5	8.19	0.63	99.4
	Std. Dev.	26.8	4.44	1.39	13.9	2.69	0.22	13.5

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be independent of $s(0)$, and constant over time ($B_{\text{Baa}} = 0.295$, $B_{\text{Aaa}} = 0.171$). Population averages over the steady-state distribution are then determined.

Our results can be understood as follows. In order to capture high spreads, we need a low bond price $P(t) = E[\frac{\Lambda(T)}{\Lambda(t)} X(T)]$. The Campbell-Cochrane pricing kernel determines $\frac{\Lambda(T)}{\Lambda(t)}$, and historical losses pin down $E[X(T)]$. Thus, the only remaining degree of freedom is the covariance between $\frac{\Lambda(T)}{\Lambda(t)}$ and $X(T)$. In practice, default rates are high (i.e., cash flows X are low) and Sharpe ratios are high (i.e., marginal utility is high) during recessions. Unfortunately, the current specification of the model is not capturing the countercyclical nature of the cash flows associated with risky bonds. In the next section, we improve upon this.

3.4 Countercyclical initial DD

In the previous section, we assumed that the initial DD ($\frac{V_0}{B}$) was independent of the initial surplus consumption ratio $S(0)$, and thus constant over the business cycle. Note, however, that the initial DD can be rewritten as the product of two terms:

$$\left(\frac{V_0}{B}\right) = \left(\frac{V_0}{F}\right) \times \left(\frac{F}{B}\right),$$

where F is the face value of outstanding debt. Now, it is standard in structural models to assume that the default boundary is a constant fraction of the face value of debt outstanding, implying that the second term ($\frac{F}{B}$) is constant.²⁶ However, because leverage ratios ($\frac{F}{V_0}$) vary over the business cycle (see

²⁶ For example, in Merton (1974), B is assumed equal to F . For other models (e.g., Leland 1994, Leland and Toft 1996), the ratio ($\frac{F}{B}$) is a constant.

Figure 1), it is reasonable to assume that initial distances-to-default are in fact procyclical.

To investigate how important this time variation in initial DD is for spreads, we first regress the MLV ratio of Baa-rated firms on the exponential surplus consumption ratio²⁷ and obtain the following relation:

$$MLV_{Baa}[S(0)] = 0.52 - .61S(0). \quad (24)$$

As such, in this section, we assume that the location of the default boundary is a constant fraction of the initial leverage:

$$B_{Baa}[S(0)] = \psi_{Baa}[0.52 - .61S(0)] \quad (25)$$

$$B_{Baa}[S(0)] = \psi_{Baa}[0.52 - .61S(0)]. \quad (26)$$

$\psi_{Baa} = 0.750$ and $\psi_{Baa} = 0.435$ are chosen to match the historical default rates. The implication is that, in worse economic times, Baa and Aaa firms begin closer to the default boundary. Once again, we emphasize that this result is due to the fact that credit rating agencies choose to rate-through-the-cycle to some degree. The results are given in Table 6. We find that the average Baa–Aaa spread increases only slightly (from 77bp to 81bp) compared with the constant default boundary case. This result is driven by two partially offsetting effects. First, as we discussed previously, correctly accounting for heterogeneity in initial leverage ratios slightly *reduces* spread levels, because when the model is calibrated to match historical default rates, it is those firms that start closer to the default boundary that tend to default. However, by starting closer to the default boundary, default is more idiosyncratic, implying a lower overall risk premium and in turn a lower credit spread via Equation (1). Hence, from this effect alone, we would expect a credit spread that is a few basis points lower than the 77bp obtained in the constant boundary case.

Second, in this framework, firms start with shorter distances-to-default in recessions. But this is precisely when the market price of risk in the Campbell–Cochrane framework is highest. This will generate higher spreads for the risky debt, *ceteris paribus*. As already mentioned, these two partially offsetting effects generate a spread level of 81bp, only 4bp above the constant boundary case.

Although the average credit spread level is barely affected, the population standard deviation increases considerably (from 11bp to 21bp) compared with the constant boundary case. Still, this too falls far short of the empirical result of 41bp. Furthermore, while forward default rates are not as countercyclical as before, they still remain countercyclical, with a regression coefficient of $\beta_{theory} = -0.71$. Although this is a significant improvement from the value of

²⁷ We construct a time series of the surplus consumption ratio from aggregate consumption data and using our calibration of the process given in Equation (12). More details are given in Section 5.

Table 6
Model-generated four-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the countercyclical initial distance-to-default case

$s(0)$	Population Distribution (%)	Baa			Aaa			Baa–Aaa Spread (bp)
		Spread over Treasury (bp)	Q -Default Rate (%)	P -Default Rate (%)	Spread over Treasury (bp)	Q -Default Rate (%)	P -Default Rate (%)	
–2.96	0.06	103.9	7.41	1.47	6.9	0.46	0.04	97.0
–2.86	0.07	98.5	7.03	1.48	6.5	0.43	0.04	92.0
–2.76	0.09	92.4	6.61	1.57	6.0	0.40	0.04	86.4
–2.66	0.11	87.6	6.27	1.60	5.7	0.38	0.04	81.9
–2.56	0.13	80.1	5.74	1.68	5.1	0.33	0.04	75.0
–2.46	0.15	69.6	4.99	1.72	4.5	0.28	0.04	65.1
–2.36	0.14	58.7	4.22	1.74	3.3	0.19	0.04	55.4
–2.27	0.04	44.6	3.22	1.76	2.4	0.13	0.04	42.2
	Average	86.8	6.20	1.55	5.6	0.37	0.04	81.2
	Std. Dev.	23.2	1.63	0.18	1.7	0.13	0.01	21.5

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be a function of initial leverage via $B(S(0)) = \psi(0.52 - .61S(0))$. ψ is chosen to match historical default rates. Population averages over the steady-state distribution are then determined.

–3.61 obtained when heterogeneity in initial leverage ratios is ignored, it is still far from the historical value of +0.86.

These findings suggest that time-varying initial DD due to leverage alone is not sufficient to generate the appropriate level of credit spread, nor does it induce countercyclical default rates. Previously, we identified three channels that can be used to increase credit spreads while matching historical default rates. Channel 1a has been captured by using the Campbell-Cochrane pricing kernel. We now investigate channel 1b—time-varying default boundaries—to see if this property can capture the historical credit spread data better.

3.5 Countercyclical default boundary case

Although face values of debt and recovery rates are readily observable, it is very difficult to separately identify bankruptcy costs and the location of the default boundary B . To see this, it is convenient to write the recovery rate as a product of two terms:

$$(1 - L) = \left(\frac{(1 - L)F}{B} \right) \left(\frac{B}{F} \right). \quad (27)$$

The first term on the right-hand side reflects how much is recovered by debtholders as a fraction of firm value B at default. Equivalently, the term $[1 - (\frac{(1-L)F}{B})]$ defines bankruptcy costs. The second term captures how far firm value can drop below the face value of debt before default occurs. Although we can readily observe face values of debt F and recovery rates $(1 - L)$, we cannot directly observe the values of either term on the right-hand side. Thus, we can estimate the location of the default boundary B only indirectly.²⁸

²⁸ Recent work has attempted to estimate average values for these numbers. Leland (2004) and Davydenko (2006) estimate $(\frac{B}{F}) \approx 0.73$.

Some models (e.g., Leland 1994) endogenously determine the default boundary to be a constant fraction of the level of debt outstanding. Other models (e.g., Longstaff and Schwartz 1995, Collin-Dufresne and Goldstein 2001) exogenously specify the default boundary and make no connection between the default boundary and outstanding debt level. The dynamics and the location of the default boundary are not observable, so researchers must specify the default boundary based on indirect information.²⁹

As noted previously, Collin-Dufresne, Goldstein, and Martin (2001), Elton et al. (2001), and Schaefer and Strebulaev (2008) all document that market-wide (e.g., VIX, Fama-French) factors have additional predictive power for changes in credit spreads even after controlling for all variables that standard structural models claim are sufficient to determine spreads. One way to capture this empirical fact is to assume that default boundaries are dynamic and are affected by economic conditions. As such, and given that a constant default boundary assumption generates procyclical default rates, in this section, we generalize Equations (25) and (26) so that the default boundary is a function of both the initial $S(0)$ and current $S(t)$ values of the surplus consumption ratio:³⁰

$$B_{\text{Baa}}(S(t), S(0)) = \psi_{\text{Baa}}^* [0.52 - 0.61S(0)][1 - \text{slope} * (S(t) - \bar{S})] \quad (28)$$

$$B_{\text{Baa}}(S(t), S(0)) = \psi_{\text{Baa}}^* [0.52 - 0.61S(0)][1 - \text{slope} * (S(t) - \bar{S})]. \quad (29)$$

Now, recall from Table 1 that we reported two features of the data that would be desirable to match: (i) the estimated historical regression coefficient (and standard error) between four-year future default probabilities and Baa–Aaa spreads is $\beta \sim .86 \pm .42$ and (ii) the standard deviation of the unconditional Baa–Aaa distribution is 41bp. As in the previous section, we choose $\psi_{\text{Baa}} = 0.696$ and $\psi_{\text{Baa}} = 0.394$ to perfectly match historical default rates. This leaves us only a single parameter *slope* to match these two features. We choose *slope* = 4, which generates a standard deviation of 46bp and a regression coefficient of +0.585 (well within one standard error of the point estimate). As reported in Tables 7 and 8, with this calibration, we find the average 4-year Baa–Aaa credit spread to be 107bp and the 10-year Baa–Aaa credit spread to be 124bp, both in excellent agreement with historical findings.

In summary, the Campbell-Cochrane model (calibrated to match many properties associated with equity returns) can successfully capture the level and standard deviation of the Baa–Aaa spread and the correlation between spreads and future default rates if we match the countercyclical nature of default rates by assuming a countercyclical default boundary. This is all accomplished while calibrating the model to match average historical default rates, recovery rates,

²⁹ A literature that studies the difficulty of specifying default boundary includes, among others, Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Davydenko and Strebulaev (2007), Hackbarth, Miao, and Morellec (2006), and Davydenko (2006).

³⁰ We choose the boundary to be linear in S rather than $s = \log S$ because S is bounded by $S \in (0, S_{\max} \approx 0.1)$ whereas s has no minimum.

Table 7**Model-generated four-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the countercyclical default boundary case**

$s(0)$	Population Distribution (%)	Baa			Aaa			Baa–Aaa Spread (bp)
		Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	
–2.96	0.05657	151.1	10.65	1.74	11.4	0.80	0.06	139.7
–2.86	0.07159	140.4	9.92	1.72	10.4	0.73	0.05	130.0
–2.76	0.08983	127.6	9.05	1.65	9.4	0.65	0.05	118.2
–2.66	0.10887	112.4	8.00	1.57	7.9	0.54	0.05	104.5
–2.56	0.12837	94.6	6.77	1.48	6.8	0.46	0.04	87.8
–2.46	0.14735	78.2	5.62	1.34	5.5	0.36	0.02	72.7
–2.36	0.14429	56.1	4.05	1.21	3.9	0.24	0.04	52.2
–2.27	0.03753	33.6	2.43	0.98	2.1	0.11	0.02	31.5
Average		115.6	8.18	1.55	8.5	0.59	0.04	107.1
Std. Dev.		50.3	3.45	0.29	4.0	0.30	0.01	46.3

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be a function of both initial state of the economy and current state of the economy: $B_{\text{Baa}}(S(t), S(0)) = \psi_{\text{Baa}}^* [0.52 - 0.61S(0)][1 - \text{slope} * (S(t) - \bar{S})]$. The parameters $\{\psi_{\text{Baa}}, \psi_{\text{Baa}}^*\}$ are chosen to match historical default rates. *Slope* is chosen to closely capture both the historical variation in spreads and the regression coefficient of spreads on future default rates. Population averages over the steady-state distribution are then determined.

Table 8**Model-generated 10-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the countercyclical default boundary case**

$s(0)$	Population Distribution (%)	Baa			Aaa			Baa–Aaa Spread (bp)
		Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	
–2.96	0.06	211.4	35.26	4.55	68.6	13.01	0.55	142.8
–2.86	0.07	201.4	33.76	4.59	64.0	12.14	0.57	137.4
–2.76	0.09	189.0	31.98	4.73	59.2	11.25	0.59	129.8
–2.66	0.11	177.7	30.31	4.95	53.9	10.28	0.61	123.8
–2.56	0.13	163.0	28.09	5.12	48.7	9.31	0.65	114.3
–2.46	0.15	147.0	25.69	5.33	42.9	8.24	0.69	104.1
–2.36	0.14	130.3	23.13	5.38	36.4	7.02	0.73	93.9
–2.27	0.04	109.2	19.83	5.67	28.7	5.58	0.77	80.5
Average		181.8	30.79	4.87	58.0	11.03	0.61	123.8
Std. Dev.		48.1	7.13	0.56	22.4	4.13	0.10	26.3

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be a function of both initial state of the economy and current state of the economy: $B_{\text{Baa}}(S(t), S(0)) = \psi_{\text{Baa}}^* [0.52 - 0.61S(0)][1 - \text{slope} * (S(t) - \bar{S})]$. The parameters $\{\psi_{\text{Baa}}, \psi_{\text{Baa}}^*\}$ are chosen to match historical default rates. *Slope* is chosen to closely capture both the historical variation in spreads and the regression coefficient of spreads on future default rates. Population averages over the steady-state distribution are then determined.

and Sharpe ratios. We note, however, that our results cannot explain either the average level or the time variation of the Aaa–Treasury spread. Taking at face value the prediction of the model, our results seem to suggest (in line with Huang and Huang 2003, Feldhutter and Lando 2008, and others) that much of the Aaa–Treasury spread is due to factors independent of credit risk.

3.6 Time-varying idiosyncratic risk

In the previous section, we showed that the Campbell-Cochrane pricing kernel generated procyclical default rates if initial distances-to-default (and

idiosyncratic volatility) were specified as constants. Then, we demonstrated that if we calibrate default boundary dynamics to match the countercyclical nature of defaults, the level and time variation in Baa–Aaa spreads can be explained. However, because the locations of default boundaries are not directly observable, we cannot provide direct empirical support for the existence of such countercyclical default boundaries. Still, the countercyclical nature of defaults is observable, so we know that *something* in the original model needs to be changed.

Here, for the purpose of robustness, we show that the model does nearly as well in explaining the credit spread puzzle if, instead of time-varying default boundaries, we specify time-varying idiosyncratic risk to match the countercyclical nature of defaults. In particular, we assume that idiosyncratic risk takes the form

$$\sigma_{\text{idio}}(S) = \sigma_0 + \sigma_1(S - \bar{S}). \quad (30)$$

To check whether there is evidence for time-varying idiosyncratic risks, we regress estimates of idiosyncratic risks on our time-series estimates for S (described in detail below). We find empirically that $\hat{\sigma}_1 \sim -.65 \pm .60$.

If we choose $\sigma_1 = -1.1$ (a number that is within one standard deviation of the point estimate), $\sigma_0 = 0.212$, and the (constant) default boundary $B = 0.342$ to match the historical Sharpe ratio 0.22, default rate 0.0155, and the countercyclical nature of defaults $\beta = +0.91$, this model generates an average spread of 96bp and a standard deviation of 39bp, in good agreement with historical values. If instead we choose $\sigma_1 = -0.65$ (to match the point estimate), $\sigma_0 = 0.211$, and the (constant) default boundary $B = 0.348$ to match the historical Sharpe ratio 0.22 and the default rate 0.0155, the model generates an average spread of 89bp with a standard deviation of 28bp and a β of +0.31. Note that this value of β is somewhat short of its historical value, which we have calibrated previous models to. Thus, for consistency, here we also account for the fact that initial leverage ratios are time varying via Equation (25). We find that this model generates a β of +0.85, an average spread of 91bp, and a standard deviation of 38bp. Therefore, this method of calibration also does an excellent job at capturing historical level and time variation of Baa–Aaa spreads.

The implication of our findings is that so long as the model can capture the countercyclical nature of default rates (regardless of mechanism), the Campbell-Cochrane pricing kernel generates levels and time variation of Baa–Aaa spreads that are broadly consistent with historical data. The two different mechanisms we consider (time-varying default boundary or idiosyncratic risk) have similar implications for spreads. Once again, this can be understood from the relation $P = E[\Lambda X]$: the Campbell-Cochrane pricing kernel combined with any structural model that generates realistic (i.e., countercyclical) cash flows X for corporate bonds will generate a low price (i.e., high yield) for Baa bonds.

3.7 Procyclical recovery rates

So far, we have investigated how channels 1a and 1b discussed in the introduction can be used to help explain the credit spread puzzle. Recently, several papers (e.g., Altman, Resti, and Sironi 2004 and 2005, Acharya, Bharath, and Srinivasan 2007) have noted that recovery rates are procyclical.³¹ In this section, we investigate what effect this channel has on credit spreads.

We specify the recovery rate to be of the form

$$\text{recovery} = t_0 + t_1 S(\tau), \quad (31)$$

where τ is the default date. Unfortunately, the annual recovery rate from Moody's covers only the period 1982–2004, and its relation with surplus consumption ratio is noisy for this short sample period. To overcome this problem, we first regress the recovery rate on the cycle dummy and find that

$$\text{recovery} = 0.47 - 0.15 * \text{cycle}, \quad (32)$$

where the slope coefficient has a t -statistic of 2.48. We then regress the cycle dummy on $S(\tau)$ for the full sample (1919–2004) and find that

$$\text{cycle} = 0.5148 - 3.8029 S(\tau), \quad (33)$$

with t -statistic equal to 2.37 for the slope coefficient. We substitute cycle as a function of $S(\tau)$ from the second equation into the first equation and find that $t_1 = 0.59$. We then choose $t_0 = 0.418$ to match historical recovery rates. This specification increases the four-year Baa–Aaa spread by only a few basis points. However, we note that the parameter t_1 might not be well estimated, and that a larger estimate would increase spreads further.

In summary, historical Baa–Aaa spreads are consistent with a standard asset pricing framework if one chooses a pricing kernel (such as that from Campbell-Cochrane 1999) that generates strongly countercyclical Sharpe ratios and a structural model of default that captures the historical countercyclical nature of defaults. Accounting for countercyclical recovery rates can increase spreads even further.

4. Predicted versus Historical Time Series of Credit Spreads

While most existing studies investigating the credit spread puzzle have focused on the level of spreads, it is also important to look at the time-series properties predicted by the model. The parsimonious Campbell-Cochrane model makes strong predictions about the time-series properties of credit spreads, equity premium, and P/D ratios because all are driven by a single state variable S , the surplus consumption. We investigate these implications below.

³¹ The literature that is related to the recovery rate upon default includes, among others, Asquith, Gertner, and Scharfstein (1994), Franks and Torous (1994), Hotchkiss (1995), Izvorski (1997), Pulvino (1998), Andrade and Kaplan (1998), Thorburn (2000), and Dieckmann, Martin, and Strickland (2006).

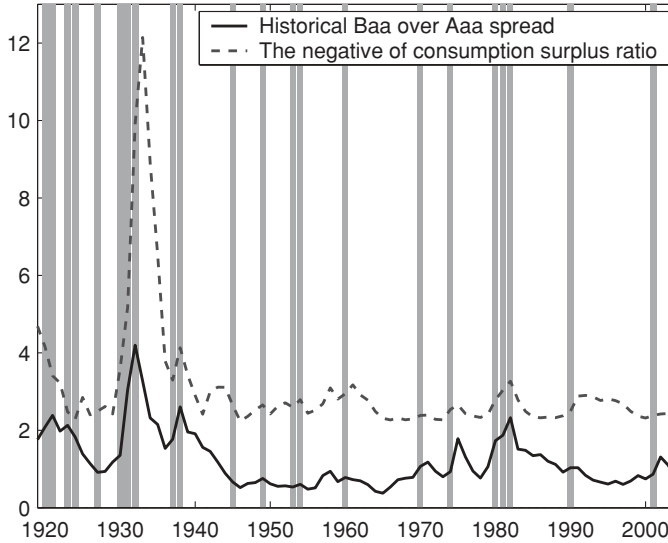


Figure 2
The relation between historical credit spread and the negative of the surplus consumption ratio from the Campbell-Cochrane model
 We plot the negative of the latter for visual convenience.

4.1 Simulated and historical variables

Following Campbell and Cochrane (1999), we obtain the innovation of the historical consumption growth time series using Equation (10). This innovation is then used to construct the time series of the surplus consumption ratio using Equation (12) with parameter values as calibrated in Section 3.2 and assuming its initial value is equal to its long-run average. Given the time series of the surplus consumption ratio and our calibrated model of Section 3, we back out all relevant financial variables and compare them with historical data. We first plot in Figure 2 the negative of the historical surplus consumption ratio and the historical Baa–Aaa spread for the 1919–2004 period. They appear highly correlated. Both rise during recessions and peak in the Great Depression. This suggests that the credit spread is very much driven by aggregate consumption shocks and that the credit spread backed out using consumption data will be able to track its historical counterpart well.

Figure 3 confirms this conjecture. In the upper panel, we plot the simulated credit spread and the equity premium as well as the historical credit spread. As is clear, the simulated and historical credit spreads exhibit similar time-series dynamics throughout the business cycles for the 86 years we examine. In the lower panel, we plot the changes of the simulated and historical credit spreads. Again, these changes fit each other very well.

Table 9 quantifies the implications of the graphs. For the 86-year sample, the simulated credit spread performs well in capturing the historical mean

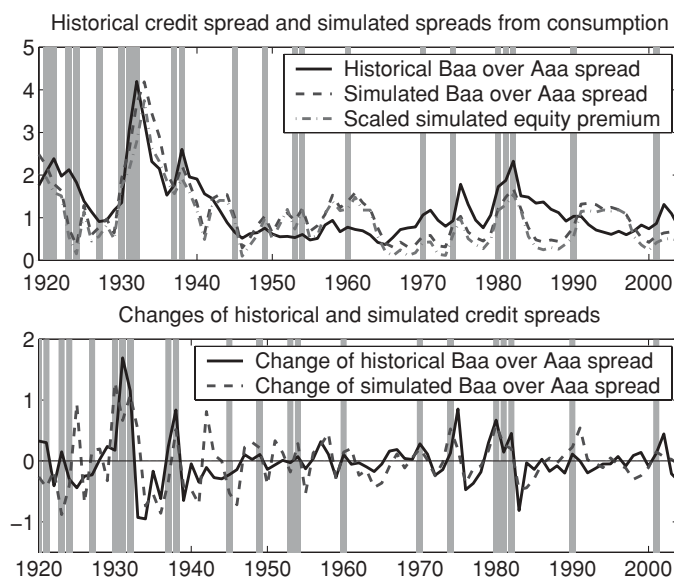


Figure 3
The levels and changes of historical and simulated credit spreads and simulated equity premium implied by the Campbell-Cochrane model

The simulated data are backed out from historical surplus consumption ratio. The scaled simulated equity premium is the simulated equity premium divided by eight.

(113bp versus 120bp), the standard deviation (76bp versus 70bp), the minimum (29bp versus 37bp), and the maximum (419bp versus 420bp). The mean of the simulated and historical credit spread are 84bp and 90bp for the postwar period; the corresponding standard deviations are 38bp and 39bp, respectively.

For the full sample, the simulated credit spread is 72% correlated with the historical credit spread; the corresponding number drops to 23% for the postwar period. A closer look at the data suggests that this drop in the correlation during the postwar period is driven by the last decade of the sample. Nevertheless, the simulated spreads and historical spreads still track each other quite well during this period, as shown in Figure 3. Indeed, the correlation of the changes of the simulated and historical credit spreads is 49% for the full sample, but increases to 58% for the postwar period; the dynamics of the two time series fit better in the latter sample.

Therefore, the consumption-implied credit spread does a very good job at tracking both the levels and the changes of its historical counterpart. Given the well-known failure of consumption-based models to price assets (e.g., Hansen and Singleton 1982), the close link between surplus consumption ratio and credit spread is impressive. The evidence provides further support to the habit formation model and suggests that aggregate credit spreads are driven by aggregate consumption risk. This also provides some support for the widespread use of credit spreads as a proxy for the equity premium (e.g., Jagannathan and

Table 9
Simulated and historical statistics generated from the Campbell-Cochrane model

Panel A: Summary statistics								
Variable	1919–2004				1946–2004			
	Mean	Std.	Min	Max	Mean	Std.	Min	Max
SS	1.13	0.76	0.29	4.19	0.84	0.38	0.29	1.68
SEP	7.55	5.48	0.77	29.99	5.55	3.39	0.77	12.37
Spread	1.20	0.70	0.37	4.20	0.90	0.39	0.37	2.33
Panel B: Cross correlations of levels								
	1919–2004				1946–2004			
	SUR	SS	SEP	PD	SUR	SS	SEP	PD
PD	0.21	–0.31	–0.31		0.22	–0.22	–0.22	
Spread	–0.72	0.72	0.68	–0.31	–0.23	0.19	0.18	–0.13
Panel C: Cross correlations of the changes								
	1919–2004				1946–2004			
	SUR	SS	SEP	PD	SUR	SS	SEP	PD
PD	0.16	–0.26	–0.29		0.35	–0.38	–0.36	
Spread	–0.49	0.45	0.38	–0.20	–0.58	0.56	0.54	–0.28

SUR is surplus consumption ratio. *SS* is simulated Baa–Aaa spread; *SEP* is simulated equity premium; *Spread* is the actual Baa–Aaa spread; *PD* is the actual P/D ratio. Panel A reports the summary statistics of the variables. Panel B reports the correlation of the levels of the variables, and Panel C the changes of the variables. Most correlation coefficients are significant at the 1% level.

Wang 1996). Figure 3 shows that the model-implied equity premium is closely related to spreads.³²

4.2 Credit spread versus P/D ratio

The historical surplus consumption ratio correlates much better with the historical credit spread (72%) than with the P/D ratio (21%). To elaborate, we plot in Figure 4 the demeaned historical surplus consumption ratio and demeaned historical P/D ratio. The dividend-price ratio does not capture the Great Depression well: there are at least three periods (1920s, 1940s, and 1950s) during which the P/D ratio is lower than in the Great Depression. The P/D ratio is lowest in the 1950s even though the surplus consumption ratio is above average; the P/D ratio explodes in the 1990s while the surplus consumption ratio appears to be quite stationary. This relatively weak relation between the two ratios can also be seen in Campbell and Cochrane (1999, Figure 9).

We can also back out the time series of implied credit spread using the historical P/D ratio. In that case, however, as shown in Figure 5, we do not see a dramatic spike of the implied credit spread during the Great Depression. Rather, there are three periods during which the implied credit spread is higher than in the Great Depression. The implied credit spread approaches zero in the 1990s while the historical spread appears stationary. Therefore, the credit spread implied from the P/D ratio does a much poorer job in tracking the historical spread than the credit spread implied from the surplus consumption

³² One caveat is that the CC model is a one-state variable model. Therefore, in time series, all variables of interest will be highly correlated within the model.

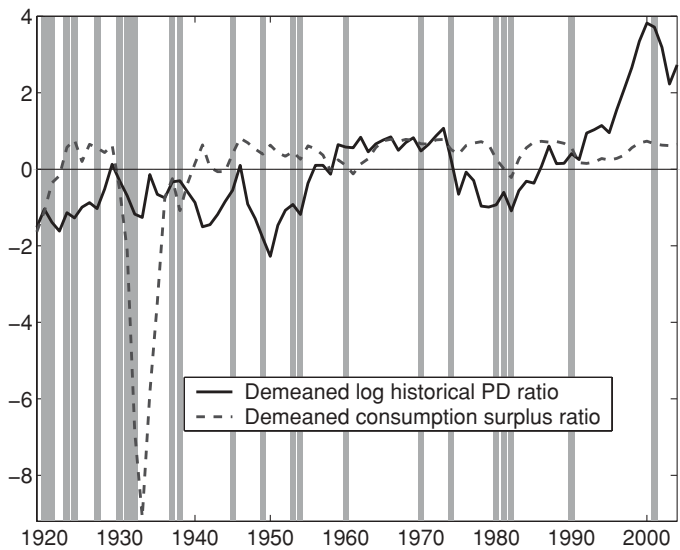


Figure 4
The relation between demeaned log historical P/D ratio and demeaned surplus consumption ratio
The demeaned log historical P/D ratio is further scaled to make the variation clear in the graph.

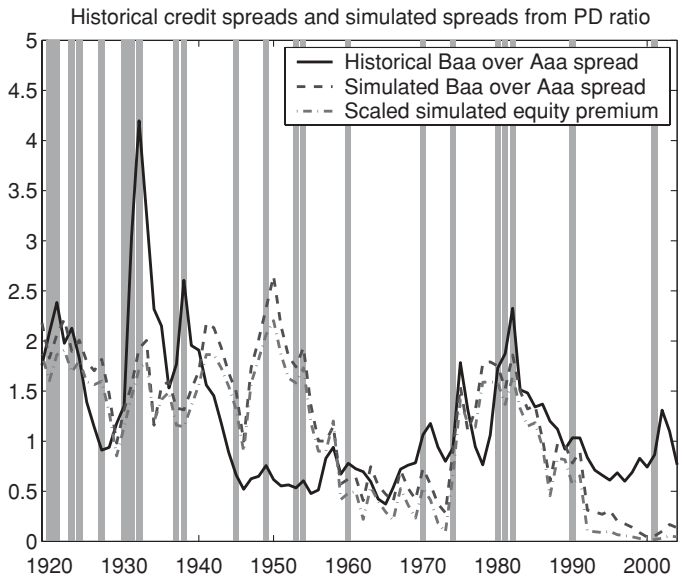


Figure 5
The levels of historical and simulated credit spreads and simulated equity premium
The simulated data are backed out from historical P/D ratio. The scaled simulated equity premium is the simulated equity premium divided by eight.

ratio. Put differently, aggregate credit spread has much tighter relation with consumption and appears to be more systematic than the P/D ratio, which has long been regarded as a main state variable in equilibrium models. One possible reason is that the P/D ratio is affected by corporate payout policies that are not necessarily related to aggregate consumption risk.

5. Conclusion

Assuming that the components of credit spreads unrelated to default (e.g., liquidity, taxes, callability, etc.) are similar across ratings levels, it follows that the Baa–Aaa spread should be explainable from a purely credit perspective. Expanding upon Huang and Huang (2003), we demonstrate that the benchmark Merton (1974) model implies that, when comparing across models, it is important to calibrate all models to historical default rates, recovery rates, and Sharpe ratios. Once this is done, the benchmark Merton model predicts that the historical four-year Baa–Aaa spread should be 57bp, which falls far short of the historical value of 94bp to 102bp. We refer to this difference as the credit spread puzzle and investigate whether it can be resolved.

We identify three channels that can potentially increase predicted spreads. One of these channels, time-varying market prices of risk, is the same channel often used to explain the equity premium puzzle. Intuitively, if a structural model can capture the empirical fact that most defaults occur during recessions, and if we specify a pricing kernel with time-varying market prices of risk that generates high Arrow/Debreu prices in recessions, it then follows from the no-arbitrage relation $P(t) = E[\frac{\Lambda(T)}{\Lambda(t)} X(T)]$ that we should be able to explain low prices (i.e., large yields) for Baa corporate debt. As a specific example, we show that the Campbell-Cochrane (1999) pricing kernel combined with some mechanism to match the countercyclical nature of defaults (either countercyclical default boundaries or countercyclical idiosyncratic volatility) does an excellent job of capturing the level and time variation of Baa–Aaa spreads. We also show (in the appendix) that a pricing kernel that explains the equity premium with a constant Sharpe ratio (such as the long-run risk model I of Bansal and Yaron 2004) cannot explain the credit spread puzzle.

Finally, we use historical consumption data to back out model predictions for the time series of credit spreads. We find that the model fits the dynamics of its historical counterpart well.

Appendix A: Proof of Equation (6)

We assume that the zero-coupon risky bond can default only at maturity T , and it does so only if the firm value at maturity falls below the default boundary B . Moreover, if default occurs, bondholders receive a *constant* recovery rate of $(1 - L)$. Hence, the date- T cash flows of this risky bond are

$$\begin{aligned} \$1 & \quad \text{if } V(T) > B \\ \$ (1 - L) & \quad \text{if } V(T) < B. \end{aligned} \tag{A.1}$$

It is convenient to define

$$v(t) \equiv \log \left(\frac{V(t)}{B} \right). \quad (\text{A.2})$$

From Itô's lemma, this implies

$$dv(t) = \left(\mu - \delta - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (\text{A.3})$$

$$= \left(r - \delta - \frac{\sigma^2}{2} \right) dt + \sigma dz^Q. \quad (\text{A.4})$$

Thus, $v(T)$ is normally distributed under both measures

$$v(T) | v(0) \sim N^P \left(v(0) + \left(\mu - \delta - \frac{\sigma^2}{2} \right) T, \sigma^2 T \right) \quad (\text{A.5})$$

$$\sim N^Q \left(v(0) + \left(r - \delta - \frac{\sigma^2}{2} \right) T, \sigma^2 T \right), \quad (\text{A.6})$$

where $N[\cdot]$ is the cumulative normal distribution. Now, note that default occurs if $v(T) < 0$. Hence, the P -measure probability (π^P) that the firm defaults is

$$\begin{aligned} \pi^P &= \frac{1}{\sqrt{2\pi\sigma^2 T}} \int_{-\infty}^0 dv \exp \left[- \left(\frac{1}{2\sigma^2 T} \right) \left(v - v(0) - \left(\mu - \delta - \frac{\sigma^2}{2} \right) T \right)^2 \right] \\ &= N \left[- \left(\frac{1}{\sqrt{\sigma^2 T}} \right) \left(\log \left(\frac{V_0}{B} \right) + \left(\mu - \delta - \frac{\sigma^2}{2} \right) T \right) \right]. \end{aligned} \quad (\text{A.7})$$

Similarly, the risk-neutral probability of default (π^Q) is

$$\pi^Q = N \left[- \left(\frac{1}{\sqrt{\sigma^2 T}} \right) \left(\log \left(\frac{V_0}{B} \right) + \left(r - \delta - \frac{\sigma^2}{2} \right) T \right) \right]. \quad (\text{A.8})$$

Combining Equations (5), (A.7), and (A.8), we see that

$$\pi^Q = N[N^{-1}(\pi^P) + \theta\sqrt{T}]. \quad (\text{A.9})$$

As is well known, the price of a zero-coupon risky bond can be determined via

$$\begin{aligned} B^T(0) &= e^{-rT} \mathbb{E}_0^Q [\mathbf{1}_{\{v(T)>0\}} + (1-L) \mathbf{1}_{\{v(T)<0\}}] \\ &= e^{-rT} [1 - L\pi^Q]. \end{aligned} \quad (\text{A.10})$$

If we write the bond price in terms of its yield to maturity $B = e^{-yT}$, then the credit spread, $(y - r)$, can be written as

$$(y - r) = - \left(\frac{1}{T} \right) \log [1 - L\pi^Q]. \quad (\text{A.11})$$

Combining Equations (A.9) and (A.11), we find Equation (6):

$$(y - r) = - \left(\frac{1}{T} \right) \log \{ 1 - L N[N^{-1}(\pi^P) + \theta\sqrt{T}] \}. \quad (\text{A.12})$$

Appendix B: A Simple Benchmark Case with Jumps

We specify firm value dynamics under both the historical (P -) and risk-neutral (Q -) measures as

$$\frac{dV}{V} + \delta dt = \mu dt + \sigma dz - (dN(t) - \lambda dt) \quad (\text{B.1})$$

$$= r dt + \sigma dz^Q - (dN(t) - \lambda^Q dt). \quad (\text{B.2})$$

Here, the dividend yield (δ), expected return (μ), asset volatility (σ), and risk-free rate (r) are all assumed constant. Further, we assume that $N(t)$ is a counting process with intensity λ under the physical measure and λ^Q under the risk-neutral measure.

It is convenient to define the Brownian motion market price of risk as

$$\theta \equiv \frac{\mu - r + \lambda - \lambda^Q}{\sigma}. \quad (\text{B.3})$$

The contribution to the expected return in the asset value process in excess of the risk-free rate due to jump to default risk is measured by the difference $\lambda^Q - \lambda$.

In the spirit of Merton (1974), we assume that default of a zero-coupon bond can occur only at maturity T , and this occurs if firm value $V(T)$ falls below the default boundary B . Finally, if default occurs, we assume that bondholders receive a *constant* recovery rate $(1 - L)$.

Under these assumptions, we find that the credit spread $(y - r)$ on a date- T zero-coupon bond is

$$(y - r) = - \left(\frac{1}{T} \right) \log[1 - L Q(T)], \quad (\text{B.4})$$

where the risk-neutral default probability is given by

$$Q(T) = 1 - e^{-\lambda^Q T} (1 - N[N^{-1}(e^{\lambda T} (\pi^P - 1) + 1) + \theta \sqrt{T}]).$$

The implication of this equation is that, even though the model is specified by nine parameters $\{r, \mu, \sigma, \delta, V_0, B, L, \lambda, \lambda^Q\}$, credit spreads depend only on five combinations of these parameters: expected default rate π^P , expected recovery rate $(1 - L)$, Brownian motion market price of risk θ , the physical measure jump intensity λ , and the jump-risk premium often measured by the ratio λ^Q/λ .

Simple numerical experiments show that for reasonable jump-risk premiums ($\lambda^Q/\lambda \in (1, 5)$) consistent with some of the empirical evidence (e.g., Driessen 2005, Berndt et al. 2005), introducing jumps to default has only a marginal impact on predicted spreads, once the model is calibrated to fit the historical default rate, total Sharpe ratio, and recovery rates.³³

Appendix C: Long-Run Cash-Flow Risk and the Credit Spread Puzzle

In this section, we investigate the credit spread implications of the model of Bansal and Yaron (2004). In contrast to the Campbell-Cochrane model, which captures the historical equity premium with iid consumption but time-varying risk aversion generated by the habit process, the Bansal-Yaron model has a standard (Epstein-Zin 1991 type) utility function but modifies the consumption process. In particular, it allows both its growth rate and volatility to follow highly persistent mean-reverting stochastic processes. Bansal-Yaron argue that in finite samples, their consumption process cannot be distinguished statistically from the iid consumption process assumed

³³ Results are available upon request. We considered various values of historical default probability λ so that the four-year historical default probability due to jumps explains up to one-half the observed default rate, and jump-risk premium so that $\lambda^Q/\lambda \in (1, 5)$.

in Campbell-Cochrane. However, it explains the equity premium puzzle using a very different mechanism than Campbell-Cochrane—namely, consumption/cash-flow risk as opposed to the risk-premium/discount rate risk in Campbell-Cochrane.³⁴ Studying the implications of long-run cash-flow risk for spreads is interesting for two reasons. First, it provides a potential alternative explanation for credit spread level and variation. The results of the calibration can help us sort out which components are more important for spreads. Second, looking at the implications of this model for credit spreads provides an out-of-sample test of the two explanations of the equity premium: cash-flow risk versus risk aversion. It seems natural to expect that a model that can explain many features of equity prices should also be able to explain corporate bond prices accurately. A failure along that dimension might indicate that the model is misspecified or that bond and equity markets are segmented. Admittedly, both models are highly stylized and may illustrate two different mechanisms, both of which may be at work in the data.

As explained above, Bansal-Yaron (2004) proposes two models: one where only the growth rate of consumption is time varying (Case I) and one where both the growth rate and volatility of consumption are stochastic (Case II). Here we investigate Case I only, as their Case II considers stochastic cash-flow volatility, which leads to time-varying risk-premiums in equilibrium. The consequence is that the pricing kernel for Case II becomes similar to the pricing kernel in the Campbell-Cochrane model in that they both display countercyclical risk-premiums. In contrast, Bansal-Yaron's Case I model has essentially constant risk-premiums (as we show below) and is nevertheless able to explain the high equity premium due solely to long-run cash-flow risk. It is thus interesting to compare its predictions for spreads to those of the Campbell-Cochrane model as it isolates the contribution of cash-flow risk versus time-varying risk-premiums.

We first describe the continuous-time version of the Bansal-Yaron (2004) model we implement, then the calibration and the results.

C.1: The long-run cash-flow risk model

We consider the following model for aggregate consumption growth:

$$d \log C_t = (\mu_c + x_t)dt + \sigma_c dZ_c(t) \quad (\text{C.1})$$

$$dx_t = -\kappa_x x_t dt + \sigma_x dZ_x(t), \quad (\text{C.2})$$

where C_t is the aggregate consumption process.

The representative agent has recursive utility (of the Kreps-Porteus-Epstein-Zin type) given by

$$J(t) = E_t \left[\int_t^\infty f(c_s, J_s) ds \right],$$

where the normalized aggregator function (with $\theta = \frac{1-\gamma}{1-\rho}$) is given by³⁵

$$f(c, J) = \frac{\beta u_\rho(c)}{((1-\gamma)J)^{1/\theta-1}} - \beta \theta J,$$

and $u_\rho(c) = \frac{c^{1-\rho}}{1-\rho}$ is the standard CES utility function (to which the preferences above reduce in the case $\gamma = \rho$, i.e., $\theta = 1$).

In this affine economy, the value function can be solved explicitly. It is equal to

$$J(t) = I(x_t)^\theta u_\gamma(c_t),$$

³⁴ The intuition often used to explain the difference between these two frameworks stems from the Gordon Growth model, in which the P/D ratio equals one divided by the difference between the discount rate and the dividend growth rate: $V/D = 1/(k - g)$. The Campbell-Cochrane model emphasizes time-varying discount rates k whereas Bansal and Yaron (2004) emphasize time-varying growth rate risk g for asset prices.

³⁵ The special cases $\rho = 1$ and $\gamma = 1$ can be treated similarly. The results are available upon request.

where the function $I(x)$ solves the ODE³⁶

$$\frac{\sigma_x^2}{2} I_{xx} - \kappa_x x I_x + I \left((1 - \rho)(\mu_c + x) + (1 - \gamma)(1 - \rho) \frac{\sigma_c^2}{2} - \beta \right) + (\theta - 1) \frac{(I_x)^2}{I} \frac{\sigma_x^2}{2} + 1 = 0. \quad (C.3)$$

Further, it is well known (e.g., Duffie 1996) that the pricing kernel in this economy is given by

$$\Lambda(t) = e^{\int_0^t f_J(c_s, J_s) ds} f_c(c_t, J_t) = e^{-\beta \theta t - \int_0^t \frac{1-\theta}{I(x_u)} du} c_t^{-\gamma} I(x_t)^{\theta-1}. \quad (C.4)$$

It can be shown that the claim to aggregate consumption, the risk premium, and the risk-free rate are equal to

$$S^c(t) = I(x_t) c_t \quad (C.5)$$

$$\left(E \left[\frac{dS_t}{S_t} \right] + \frac{1}{I(x_t)} - r_t dt \right) = \gamma \sigma_c^2 + (1 - \theta) \frac{I_x^2}{I} \sigma_x^2 \quad (C.6)$$

$$r(t) = \beta + \mu_c + \frac{\sigma_c^2}{2} - \gamma(1 + \rho) \frac{\sigma_c^2}{2} - (1 - \theta) \frac{I_x^2}{I} \sigma_x^2. \quad (C.7)$$

While the ODE satisfied by the price-consumption ratio $I(x_t)$ does not have, to our knowledge, a closed-form solution it can be approximated quite accurately as an exponential affine solution. We follow the approach of Benzoni et al. (2005), which consists in solving the following problem:

$$\min_g E[(g(x) - 1)^2]$$

s.t.

$$\frac{\sigma_x^2}{2} \tilde{I}_{xx} - \kappa_x x \tilde{I}_x + \tilde{I} \left((1 - \rho)(\mu_c + x) + (1 - \gamma)(1 - \rho) \frac{\sigma_c^2}{2} - \beta \right) + (\theta - 1) \frac{(\tilde{I}_x)^2}{\tilde{I}} \frac{\sigma_x^2}{2} + g(x) = 0.$$

Essentially, the idea is to approximate as closely as possible the constant 1 appearing in the ODE by a function $g(x)$ chosen such that the resulting ODE has an explicit solution. If the approximation is good in the sense that $g(x) \approx 1$ on the entire domain of x , then we expect $\tilde{I}(x) \approx I(x)$. The approximation can be made more and more accurate as we choose the function $g(x)$ to better approximate 1 over the range of x . We choose a family of functions $g_N(x) = e^{A+Bx+Cx^2} \sum_{i=0}^N n_i x^i$ so that $\tilde{I}_N(x) = e^{A+Bx+Cx^2} \sum_{i=0}^M m_i x^i$. We find that for the problem at hand, the simple first-order approximation $N = 1$ is very accurate (in that the solution hardly changes by going to higher orders) and that the coefficient C can be set to zero without significantly affecting the results. This is consistent with Bansal-Yaron (2004), who find that in this case, the equilibrium pricing kernel displays constant risk-premiums. Indeed, in that case, the solution is

$$I(x_t) = e^{A+Bx_t},$$

which delivers a pricing kernel with the following dynamics:

$$\frac{d\Lambda(t)}{\Lambda(t)} = -r(t)dt - \gamma \sigma_c dZ_c(t) - (1 - \theta) B \sigma_x dZ_x(t)$$

with

$$r(t) = r_0 + \rho x_t$$

³⁶ From the definition of the utility function, one can use the fact that $E[dJ(t) + f(c_t, J_t)dt] = 0$ to verify the guessed functional form for $J(t)$.

Table 10
Parameter choices for the Bansal-Yaron (2004) one-channel model

Panel A: Parameter inputs

Aggregate consumption	μ_c	σ_c	κ_x	σ_x
	0.0189	0.015	0.25	0.015
Preference	γ	ρ	β	$\theta = \frac{1-\gamma}{1-\rho}$
	8.5	0.5	0.02	-15

with

$$r_0 = \beta + \rho \left(\mu_c + \frac{\sigma_c^2}{2} \right) - \gamma(1 + \rho) \frac{\sigma_c^2}{2} - (1 - \theta) B^2 \sigma_x^2.$$

We present the results for that choice of approximation to the pricing kernel. The parameter calibration is tabulated in Table 10. The parameters for aggregate consumption, output, and dividends are chosen equal to those used for the Campbell-Cochrane economy. The parameters for preferences are chosen to be consistent with those used by Bansal-Yaron and such that we fit the same equity risk premium, P/D ratio, and Sharpe ratio on equity as in the Campbell-Cochrane economy presented above.

Using these numbers, we obtain the following approximate price-consumption ratio, pricing kernel dynamics, and interest rate:

$$I(x) = \exp(4.08143 + 1.87375 x_t) \quad (\text{C.8})$$

$$\frac{d\Lambda(t)}{\Lambda(t)} = -r(x_t) dt - 0.1275 dZ_c(t) - 0.4497 dZ_x(t) \quad (\text{C.9})$$

$$r(x_t) = 0.02175 + 0.5 x_t. \quad (\text{C.10})$$

We note that because the approximate pricing kernel is affine, one way to check the consistency of the approximation is that our approximate solution to the consumption claim $S^c(t) = I(x_t)c_t$ should be very close to the present value of the consumption stream evaluated at the pricing kernel ($E[\int_t^\infty \frac{\Lambda(u)}{\Lambda(t)} c_u du]$). In other words, we should have

$$I(x) \approx \int_t^\infty E \left[\frac{\Lambda_s}{\Lambda_t} \frac{c_s}{c_t} ds \right] =: I_{\text{aff}}(x), \quad (\text{C.11})$$

where the left-hand side is given in Equation (C.8) and the right-hand side can be computed explicitly using the dynamics for Λ given in (C.9) above (as an integral of terms similar to those found in Vasicek 1977). Figure 6 demonstrates that the two solutions are virtually indistinguishable.

With the model thus calibrated, we can move on to value the claim to aggregate dividends and output. We follow Bansal-Yaron (2004) (and our Campbell-Cochrane economy) and model dynamics of aggregate dividends D_t and output O_t as follows:

$$d \log D_t = (\mu_d + \phi_d x_t) dt + \sigma_d \left(\rho_d dZ_c(t) + \sqrt{1 - \rho_d^2} dZ_d(t) \right) \quad (\text{C.12})$$

$$d \log O_t = (\mu_o + \phi_o x_t) dt + \sigma_o \left(\rho_o dZ_c(t) + \sqrt{1 - \rho_o^2} dZ_o(t) \right). \quad (\text{C.13})$$

We fix the parameters of these processes to be equal to those set for the Campbell-Cochrane economy. The new parameters (ϕ_o, ϕ_d) are calibrated to match the predictions of the Campbell-Cochrane economy for the P/D ratio, Sharpe ratio, risk premium for the claim to output, and claim to dividends. In both cases, the claim to dividend and output can be valued in a closed form using the affine pricing kernel and the affine dynamics as (we write the equations for the claim to output

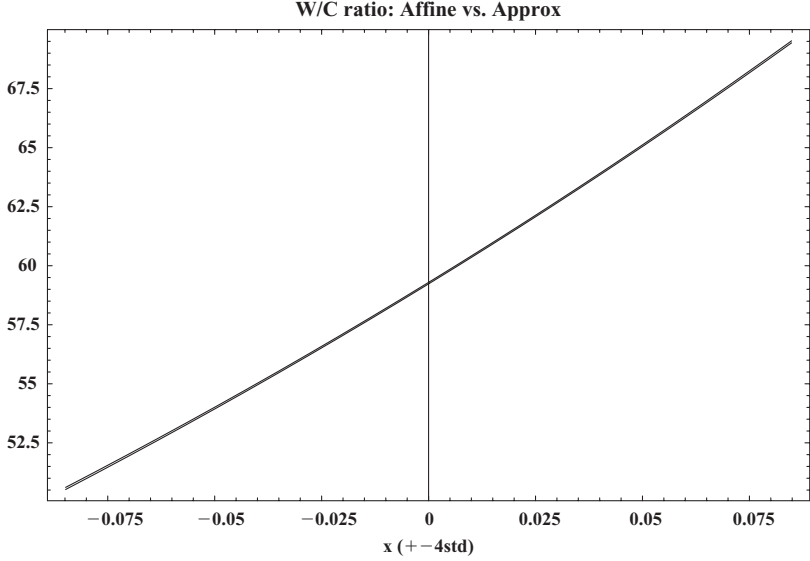


Figure 6
First-order approximation to wealth-consumption ratio given by Equation (C.8) versus the closed-form solution computed with approximate affine pricing kernel as in Equation (C.11)

only—the dividend case can be solved similarly)

$$S^o(t) = I^o(x_t) O_t, \quad (\text{C.14})$$

where the price-output ratio is given by

$$I^o(x_t) = \int_t^\infty \mathbb{E} \left[\frac{\Lambda_s}{\Lambda_t} \frac{O_s}{O_t} ds \right]. \quad (\text{C.15})$$

Equation (C.15) can be expressed as an integral of closed-form zero-coupon bond prices, which is easily computed using Mathematica. Again, we can also approximate this integral of an exponential affine solution as a simple exponential formula, following the approach outlined above. For example, for the P/D ratio, we find that

$$I^o(x_t) \approx e^{3.03567 + 7.42131 x_t}. \quad (\text{C.16})$$

Figure 7 plots the exact and approximate solutions.

Given how good the approximation is, we use the approximate solution for the price-output ratio for our simulations to determine credit spreads. Table 11 gives the parameters chosen for the Bansal-Yaron economy and resulting price dividend/output ratios for this long-run risk economy. These numbers can be compared with Table 4 in the Campbell-Cochrane case. They are all very similar (except for the volatility of risk-premiums, which is virtually zero in this economy; again, the idea is to compare the credit spread implications of two different frameworks, both of which can generate a large equity premium).

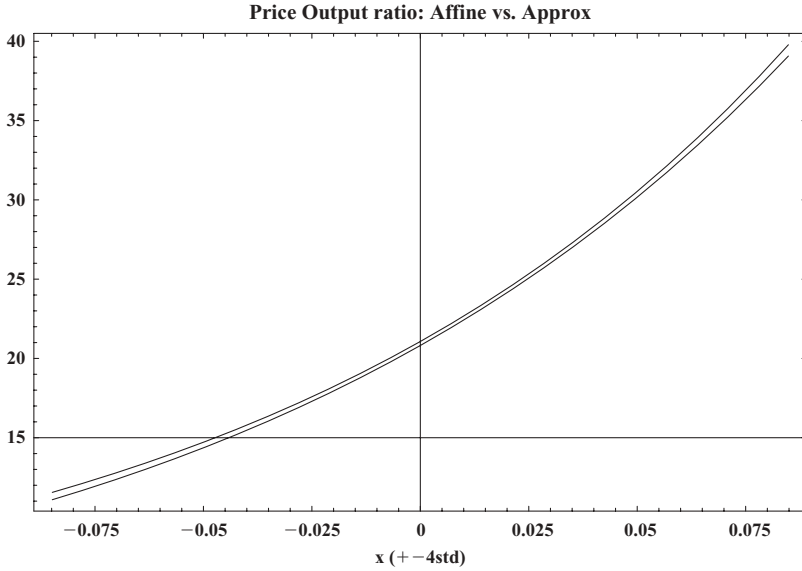


Figure 7
First-order approximation to price-output ratio given by Equation (C.16) versus the closed-form solution computed with affine pricing kernel as in Equation (C.15)

Table 11
Parameter choices and fitted moments for the Bansal-Yaron (2004) model

Panel A: Parameter inputs

Type of cash flow	μ_i	ϕ_i	σ_i	ρ_i
Dividends ($i = d$)	.040	3.5	.080	.60
Output ($i = o$)	.0189	2.7	.063	.48

Panel B: Model outputs

Type of cash flow	$\exp(E[p - d])$	$\sigma(p - d)$	Sharpe	$E[r - r_f]$	$\sigma(r - r_f)$	$\sigma(\mu - r_f)$
Claim to dividends	24.6	.215	.43	.074	.17	0.0011
Historical equity	25	.26	.43	.067	.16	n.a.
Claim to output	21.1	.155	.42	.053	.12	0.0007
Historical (debt + equity)	19	.20	.43	.050	.10	n.a.

Panel A reports the parameter calibrations for dividend and output (dividends plus interest) processes for the Bansal-Yaron model. Panel B reports the sample moments of claims to dividends versus historical values; and claims to output (dividends plus interest) versus historical values.

C.2: Credit spreads in the long-run cash-flow risk model

As a result of our calibration, we obtain the dynamics of the claim to aggregate output endogenously (i.e., with dynamics consistent with the claim to aggregate dividends):

$$\frac{dS^o(t)}{S^o(t)} = (r(x_t) + \frac{1}{I^o(x_t)} + \theta(x_t))dt + \sigma_o(\rho_o dZ_c(t) + \sqrt{1 - \rho_o^2} dZ_o(t)) + \frac{I_x^o(x_t)}{I^o(x_t)} \sigma_x dZ_x(t).$$

We note that this is similar to the Campbell-Cochrane model except that the state-variable driving drift and diffusion of the value process is the growth rate of aggregate consumption and not the surplus consumption ratio. Qualitatively, however, the models are very different. In the Campbell-Cochrane model, most of the action comes from the time variation in the (countercyclical)

Table 12
Implied spreads for the Bansal-Yaron (2004) Case 1 model

	Baa ($T = 4$ years)	Aaa ($T = 4$ years)
Simulation number	100000	200000
dt	0.001	0.001
σ_{idio}	0.209	0.209
V_0/B	0.3246	0.1883
P -default rate	0.0155 (± 0.0003)	0.0005 (± 0.00005)
Std. dev. of P -default rate	0.0040	0.00017
Q -default rate	0.0437 (± 0.0006)	0.0022 (± 0.0001)
Std. dev. of Q -default rate	0.0085	0.0007
Credit spread	0.0055	0.0003
Std. dev. of spread	0.0011	0.00007
Regression coefficient	3.68	2.56

risk-premium θ , whereas in the long-run risk model, the risk premium is essentially constant (in fact, using the exponential affine approximation as in Equation (C.16), it is exactly constant) and most of the action comes from time variation in the payout ratio $1/I^o(x)$ and the risk-free rate $r(x)$.

To determine the impact on credit spreads, we use the same approach as in the Campbell-Cochrane model. Namely, we consider that the typical Baa firm has a firm value process of the form

$$\frac{dV(t)}{V(t)} = \frac{dS^o(t)}{S^o(t)} + \sigma_{\text{idio}} dZ_{\text{idio}}(t).$$

The idiosyncratic volatility is set to match the historically measured ratio of idiosyncratic to total volatility for typical Baa firms ($\sigma_{\text{idio}} = 0.2090$). We assume that the firm defaults the first time that $V(t)$ hits a constant boundary B . In the event of default, the bond recovers a fixed fraction of principal. We set the default boundary and recovery rates to match the historical default frequency and recovery rates. Predicted spreads are reported in Table 12.

We see that the resulting Baa–Aaa spread of 52bp is barely different from the base-case Black-Scholes-Merton solution and much smaller than the spread explained in the constant default boundary Campbell-Cochrane model (77bp). In other words, time-varying payout ratio has only a marginal effect on spreads. This suggests that a mechanism that relies on long-run cash-flow risk but does not generate any time variation in risk-premiums cannot successfully explain the size of observed credit spreads. Time variation in risk-premiums appears crucial for the latter. We note that, unlike in the Campbell-Cochrane model, allowing the default boundary to be time varying will not improve the predictions of the long-run risk model. Indeed, the default probability is already countercyclical in the long-run risk model with a constant default boundary (the regression coefficient of default probabilities on spreads is positive). Thus, it is not possible to shift default probability mass from good states to bad states via a countercyclical default boundary (which is necessary to keep average P -probability unchanged and raise the Q -measure probability) without making observed default rates too countercyclical. So, introducing a default boundary with the right properties might improve the predictions with respect to average spread levels, but will lead to too much predictability in default rates as a function of spreads, which further suggests that time variation in risk-premiums is an important ingredient for explaining asset pricing puzzles.

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