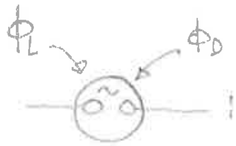
5 Izmenična napetost in izmenični tok

• ponavadi sinusna napetost, lahko tudi stopničasta, žagasta, ...

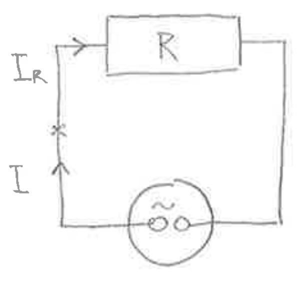


: generator sinusne napetosti

$U_g = U_0 \sin(\omega t)$; $\omega = 2\pi\nu = \frac{2\pi}{t_0}$; $U_0 > 0$ (amplituda)

$\boxed{U_g \equiv \phi_L - \phi_D}$ (razlika potencialov levega in desnega priključka)

Primer: izmenični (sinusni) tok skozi upornik



$U_g = U_0 \sin(\omega t)$

Predpostavke:

• $R_ž \ll R$ ($\Rightarrow U_ž' \ll U_R'$)

• 1. in 2. Kirchhoffov izrek še vedno veljata (kljub temu, da napetosti in el. polja niso več konstantni)

• velja Ohmov zakon

2. Kirchhoffov izrek: $U_g + U_R' + \cancel{U_ž'} = 0 \Rightarrow \boxed{U_g + U_R' = 0}$

Ohmov zakon: $\boxed{U_R' = -I_R R}$

$\boxed{U_g = I_R R}$

1. Kirchhoffov izrek: $\boxed{I = I_R}$

$\boxed{U_g = IR}$

$\Rightarrow \boxed{I = \frac{U_g}{R} = \frac{U_0}{R}\sin(\omega t)}$: tudi tok sinusen, enaka ν kot U

• mikroskopska slika (model): (sinusno) nihanje nabojev okoli ravnovesne lege

I bi radi zapisali v naslednji obliki:

$\boxed{I = I_0 \sin(\omega t - \delta_R)}$; $I_0 > 0$ (amplituda toka)

δ_R : fazni zamik med napetostjo in tokom

Iz primerjave

$$\frac{U_0}{R}\sin(\omega t) = I_0 \sin(\omega t - \delta_R)$$

sledi:

$$I_0 = \frac{U_0}{R}$$ in $\delta_R = 0$ (I in U nihata istočasno, brez zamika)

Def: impendanca upornika R: $\boxed{z_R \equiv |z_R| e^{i\delta_R}}$,

pri čemer

$$\boxed{|z_R| \equiv \frac{U_0}{I_0}} = R$$

Impendanca v splošnem (po definiciji) kompleksno število, vendar

$$z_R = R \in \mathbb{R}$$

Smisel vpeljave kompleksnih števil v nadaljevanju.

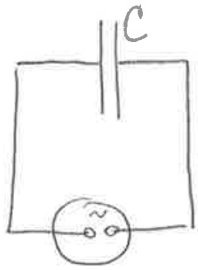
Povzetek primera:

$$U_R' = -U_g = -U_0 \sin(\omega t)$$

$$I_R = I = I_0 \sin(\omega t - \delta_R)$$

$$\delta_R = 0, \quad \frac{U_0}{I_0} = |z_R| = R \quad ; \quad z_R = |z_R| e^{i\delta_R} = R$$

Primer: kondenzator v tokokrogu z generatorjem izmenične (sinusne) napetosti



$U_g = U_0 \sin(\omega t)$

Predpostavke:

- $U_z' \ll U_C'$
- Kirchhoffova zakona še vedno veljata

$\underset{\sim\sim}{2.K.i.}$: $U_g + U_c' + U_Z' \simeq \boxed{U_g + U_c' = 0}$

- Spomnimo se: $\boxed{U_c' = -\frac{q_c}{C}}$; q_c = naboj na levi plošči kondenzatorja

$\Rightarrow \boxed{q_c = C U_g = C U_0 \sin(\omega t)}$

Spomnimo se (prehodni pojavi s kondenzatorjem):

$$I = \dot{q}_c$$

$$\Rightarrow \boxed{I = \omega C U_0 \cos(\omega t)}$$

Po zgledu iz prejšnjega primera:

$$\boxed{I = I_0 \sin(\omega t - \delta_c)}$$

$$\Rightarrow \omega C U_0 \cos(\omega t) = I_0 \sin(\omega t - \delta_c)$$

$$= I_0 [\sin(\omega t)\cos\delta_c - \cos(\omega t)\sin\delta_c] \; ; \; \forall t$$

$t_1 = 0$: $\boxed{\omega C U_0 = -I_0 \sin\delta_c}$

$t_2 = \frac{t_0}{4}$ $(\omega t_2 = \frac{\pi}{2})$: $I_0 \cos\delta_c = 0 \Rightarrow \boxed{\delta_c = \pm\frac{\pi}{2}}$

$\delta_c = +\frac{\pi}{2}$: $\omega C U_0 = -I_0 \Rightarrow I_0 = -\omega C U_0 < 0$ //

$\boxed{\delta_c = -\frac{\pi}{2}}$: $\boxed{\omega C U_0 = I_0} > 0$ ✓

$\underset{\sim}{\text{Def}}$: $\boxed{|Z_c| = \frac{U_0}{I_0} = \frac{1}{\omega C}}$; $\boxed{Z_c = |Z_c| e^{i\delta_c} = \frac{1}{\omega C} e^{-i\frac{\pi}{2}} = -\frac{i}{\omega C}}$

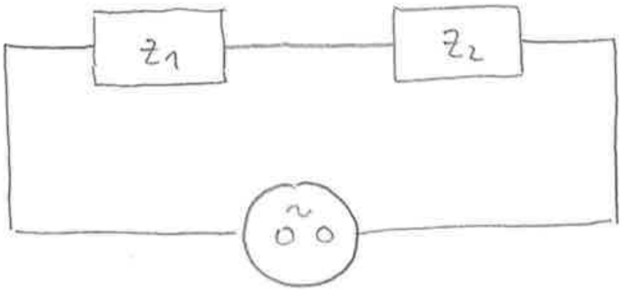
odvisnost od ν

Povzetek primera: $U_C' = -U_g = -U_0 \sin(\omega t)$

$$I_C = I = I_0 \sin(\omega t - \delta_C)$$

$$\delta_C = -\frac{\pi}{2} \; ; \; \frac{U_0}{I_0} = |Z_C| = \frac{1}{\omega C} \; ; \; Z_C = |Z_C| e^{i\delta_C} = -\frac{i}{\omega C}$$

Primer: zaporedno vezana porabnika z impedancama Z_1 in Z_2



$$Z_1 = |Z_1| e^{i\delta_1}$$

$$Z_2 = |Z_2| e^{i\delta_2}$$

$$U = U_0 \sin(\omega t)$$

$$I = I_0 \sin(\omega t - \delta)$$

Nadomestna impedanca (impedanca tokokroga):

$$Z = |Z| e^{i\delta} \; ; \; |Z| = \frac{U_0}{I_0}$$

1. K.i.: $\boxed{I = I_1 = I_2 = I_0 \sin(\omega t - \delta)}$

1. porabnik: če $I_1 = I_0 \sin(\omega t - \delta_1) \Rightarrow U_1' = -U_{1,0} \sin(\omega t)$

$\Rightarrow$ če $I_1 = I_0 \sin(\omega t) \Rightarrow U_1' = -U_{1,0} \sin(\omega t + \delta_1)$

$\Rightarrow$ če $I_1 = I_0 \sin(\omega t - \delta) \Rightarrow U_1' = -U_{1,0} \sin(\omega t + \delta_1 - \delta)$,

pri čemer je $U_{1,0} = I_0 |Z_1|$

$\Rightarrow$ če $\boxed{I_1 = I_0 \sin(\omega t - \delta) \Rightarrow U_1' = -|Z_1| I_0 \sin(\omega t + \delta_1 - \delta)}$

2. porabnik: če $\boxed{I_2 = I_0 \sin(\omega t - \delta) \Rightarrow U_2' = -|Z_2| I_0 \sin(\omega t + \delta_2 - \delta)}$

2.K.i.: $U_g + U_1' + U_2' = 0$

$$\Rightarrow |Z| I_0 \sin(\omega t) - |Z_1| I_0 \sin(\omega t + \delta_1 - \delta) - |Z_2| I_0 \sin(\omega t + \delta_2 - \delta) = 0$$

$$\boxed{\begin{aligned} \Rightarrow |Z| \sin(\omega t) = {} & |Z_1| \left[\sin(\omega t)\cos(\delta_1 - \delta) + \cos(\omega t)\sin(\delta_1 - \delta)\right] \\ & + |Z_2| \left[\sin(\omega t)\cos(\delta_2 - \delta) + \cos(\omega t)\sin(\delta_2 - \delta)\right] \end{aligned}} \quad \forall t$$

$t_1 = 0$: $\quad 0 = |Z_1| \sin(\delta_1 - \delta) + |Z_2| \sin(\delta_2 - \delta) \quad \} \times i$

$t_2 = \frac{t_0}{4}$: $\quad |Z| = |Z_1| \cos(\delta_1 - \delta) + |Z_2| \cos(\delta_2 - \delta)$

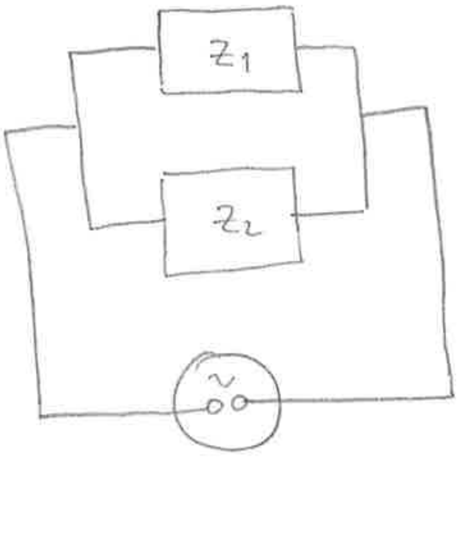
Σ : $\quad |Z| = |Z_1| \left[\cos(\delta_1 - \delta) + i \sin(\delta_1 - \delta)\right] + |Z_2| \left[\cos(\delta_2 - \delta) + i \sin(\delta_2 - \delta)\right]$

$$|Z| = |Z_1| e^{+i(\delta_1 - \delta)} + |Z_2| e^{+i(\delta_2 - \delta)}$$

$$\Rightarrow \quad |Z| e^{i\delta} = |Z_1| e^{i\delta_1} + |Z_2| e^{i\delta_2}$$

$$\Rightarrow \quad \boxed{Z = Z_1 + Z_2}$$ (posplošitev enačbe $R = R_1 + R_2$)

Primer: Nadomestna impedanca dveh vzporedno vezanih porabnikov



$$\boxed{\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}}$$, pri čemer je

$$\frac{1}{Z} = \frac{1}{|Z|} e^{-i\delta}$$

(posplošitev enačbe $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$)

Domača naloga: nadomestna impendanca za R in C (zaporedna in vzporedna vezava)

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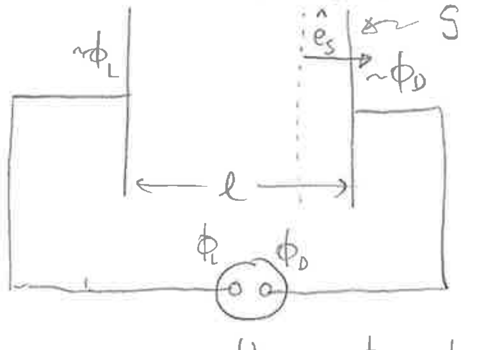
⑥ Električno delo in električna moč



- $\hat{e}_s = \hat{e}_x$ (izberemo)
- zanemarljiva upornost žic

$$U_g \equiv \phi_L - \phi_D$$

$$U_g > 0 \Rightarrow \phi_L > \phi_D \Rightarrow \vec{E}_g = E_g \hat{e}_g ;\ \hat{e}_g = \hat{e}_x \qquad ;\ E_g = \frac{|U_g|}{l}$$

$$U_g < 0 \Rightarrow \phi_L < \phi_D \Rightarrow \qquad \hat{e}_g = -\hat{e}_x$$

$$\vec{j} = q_1 n \vec{v} = \underbrace{|q_1| n v}_{\substack{\parallel \\ j_e > 0}} \underbrace{\frac{q_1}{|q|} \hat{e}_v}_{\hat{e}_j} = j_e \hat{e}_j$$

$$I = \vec{j}_e \cdot \vec{S} = j_e S\, \hat{e}_j \cdot \hat{e}_s = |I|\, \hat{e}_j \cdot \hat{e}_x$$

$$= \begin{cases} |I| > 0 ;\ \hat{e}_j = \hat{e}_x \ (\text{od } L \to D) \\ -|I| < 0 ;\ \hat{e}_j = -\hat{e}_x \ (\text{od } D \to L) \end{cases}$$

- Celotni gibljivi naboj: $q = q_1 n V = q_1 n S l$
- Električna sila v polju $\vec{E}_g$ generatorja: $\vec{F}_{el,g} = q \vec{E}_g$
- Delo dA_g sile $\vec{F}_{el,g}$, ko se težišče naboja premakne za $\vec{v}\,dt$:

$$dA_g = \vec{F}_{el,g} \cdot \vec{v}\, dt$$

$$\Rightarrow \quad P_g \equiv \frac{\partial A_g}{\partial t} = \vec{F}_{el,g} \cdot \vec{v} = q_1 n \vec{v} S l \vec{E}_g = j_e S l \vec{E}_g \cdot \hat{e}_j = \ldots$$

$$\ldots = \underbrace{j S}\,\underbrace{l E_g}\, \hat{e}_g \cdot \hat{e}_j$$

$$= |I|\,|U_g|\, \hat{e}_g \cdot \hat{e}_j$$

a) $U_g > 0,\ I > 0 \Rightarrow \hat{e}_j = \hat{e}_x$ in $\hat{e}_j = \hat{e}_x \Rightarrow \hat{e}_g \cdot \hat{e}_j = +1 \Rightarrow |I||U_g| = U_g I$

b) $U_g < 0,\ I < 0 \Rightarrow \hat{e}_j = -\hat{e}_x$ in $\hat{e}_j = -\hat{e}_x \Rightarrow \hat{e}_g \cdot \hat{e}_j = +1 \Rightarrow |I||U_g| = U_g I$

c) $U_g > 0,\ I < 0 \Rightarrow \hat{e}_j = \hat{e}_x$ in $\hat{e}_j = -\hat{e}_x \Rightarrow \hat{e}_g \cdot \hat{e}_j = -1 \Rightarrow -|I||U_g| = U_g I$

d) $U_g < 0,\ I > 0 \Rightarrow \hat{e}_j = -\hat{e}_x$ in $\hat{e}_j = \hat{e}_x \Rightarrow \hat{e}_g \cdot \hat{e}_j = -1 \Rightarrow -|I||U_g| = U_g I$

$$\Rightarrow \boxed{P_g = U_g I}$$

- Pozor: j je v smeri celotnega $\vec{E}$, ne pa nujno v smeri $\vec{E}_g$ (posameznega) generatorja (lahko tudi drugi generatorji, kondenzatorji, ...)

V zgornjem primeru: $\vec{E} = \vec{E}_g$; $\hat{e}_j = \hat{e}_g \Rightarrow P_g = |I||U_g| > 0$

Sila (električnega) upora: $\vec{v} = \text{konst.} \Rightarrow \vec{F}_R = -\vec{F}_{el,g}$

$\Rightarrow A_R = -A_g$

$\Rightarrow P_R = -P_g$

$$\Rightarrow P_R = -|I||U_g| = -|I||U_R'|$$

Ohmov zakon: $|U_R'| = R|I_R| = R|I|$

$$\Rightarrow \boxed{P_R = -R|I_R|^2 = -R I_R^2 = -\frac{|U_R|^2}{R} = -\frac{U_R^2}{R}}$$

Primer: izmenična napetost