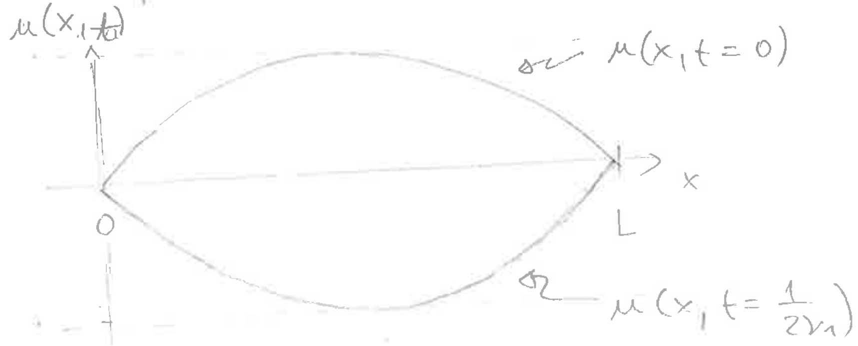
• Diskretne (lastne) frekvence ν_n, odvisno od L in c

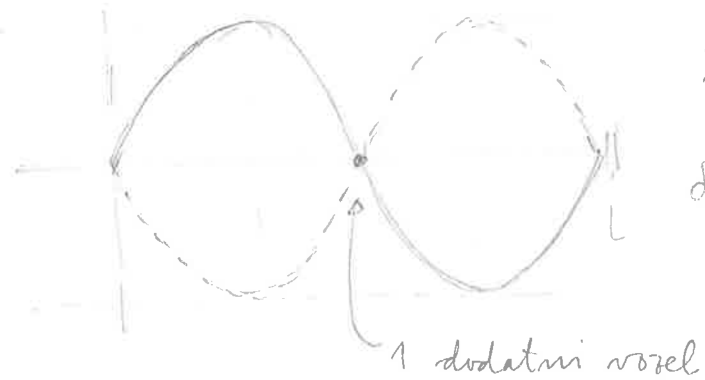
<u>n = 1</u>: $\lambda_1 = 2L$, $k_1 = \frac{\pi}{L}$, $\nu_1 = \frac{c}{2L}$

$$\delta = 0;\ t_1 = 0:\ u(x, t=0) = 2u_0 \sin\left(\frac{\pi}{L} x\right)$$

$$t_2 = \frac{1}{2\nu_1}:\ u\left(x, t = \frac{1}{2\nu_1}\right) = -2u_0 \sin\left(\frac{\pi}{L} x\right)$$



<u>n = 2</u>



$$\lambda_2 = L \Rightarrow k_2 = \frac{2\pi}{L},\ \nu_2 = \frac{c}{\lambda_2} = \frac{c}{L}$$

$$\delta = 0;\ t_1 = 0:\ u(x,t) = 2u_0 \sin\left(\frac{2\pi}{L} x\right)$$

$$t_2 = \frac{1}{2\nu_2}:\ u(x,t) = -2u_0 \sin\left(\frac{2\pi}{L} x\right)$$

<u>n = 3</u>: Domača naloga!

(10) Odboj valovanja

• Valovanje se na meji med dvema sredstvoma ($c_1 \neq c_2$) vedno (vsaj delno) odbije

• Račun za splošen val (motnjo), ki se širi po vrvi (1D): dve različni vrvi ($c_1 \neq c_2$), staknjeni pri $x = 0$; val se iz leve ($x < 0$) širi proti desni, $f_1(x - c_1 t)$.

Del vala se na meji odbije, $g_1(x+c_1t)$, del pa gre preko meje naprej, $f_2(x-c_2t)$:

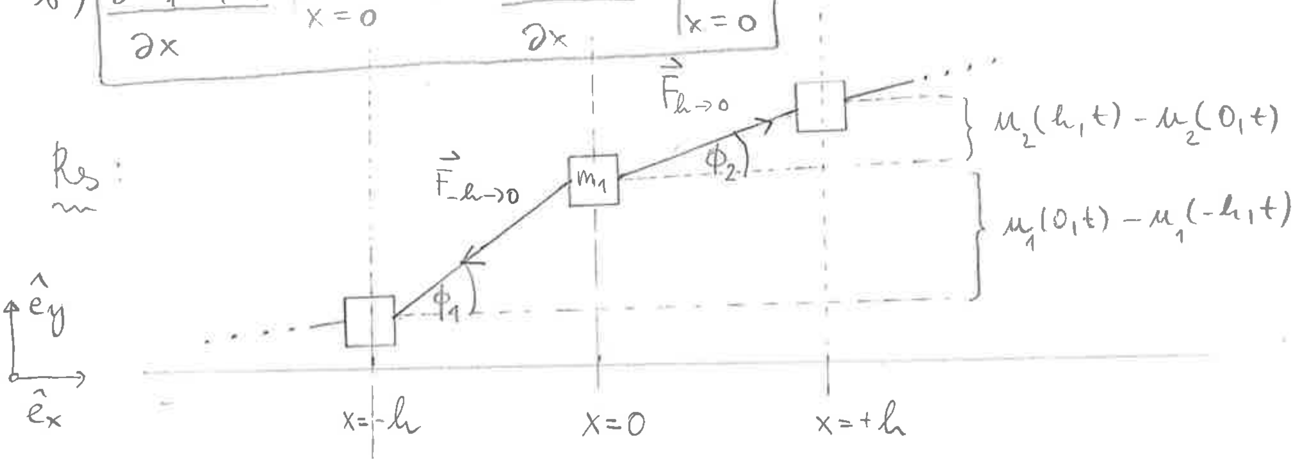
$$\boxed{\begin{aligned} x\leq 0:\ & u_1(x,t) = f_1(x-c_1t) + g_1(x+c_1t) \\ x\geq 0:\ & u_2(x,t) = f_2(x-c_2t) \end{aligned}}$$

Robna pogoja: pri $x=0$ (na meji)

a) $\boxed{u_1(x=0,t) = u_2(x=0,t)}$ (vrv se ne strga)

b) $\boxed{\left.\frac{\partial u_1(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial u_2(x,t)}{\partial x}\right|_{x=0}}$

Rs:



$$\mathrm{tg}\,\phi_1 = \frac{u_1(0,t)-u_1(-h,t)}{h} \xrightarrow[h\to 0]{} \left.\frac{\partial u_1(x,t)}{\partial x}\right|_{x=0}$$

$$\mathrm{tg}\,\phi_2 = \frac{u_2(h,t)-u_2(0,t)}{h} \xrightarrow[h\to 0]{} \left.\frac{\partial u_2(x,t)}{\partial x}\right|_{x=0}$$

$$\left.\begin{aligned} F_{h\to 0,x} &= F_0 \\ F_{-h\to 0,x} &= -F_0 \end{aligned}\right\}\ \text{sila, s katero je napeta vrv}$$

$$\Rightarrow \vec{F}_{-h\to 0} = \begin{bmatrix} -F_0 \\ -F_0\,\mathrm{tg}\,\phi_1 \\ 0 \end{bmatrix}, \quad \vec{F}_{h\to 0} = \begin{bmatrix} F_0 \\ F_0\,\mathrm{tg}\,\phi_2 \\ 0 \end{bmatrix}$$

$$\Rightarrow \vec{F} = \vec{F}_{-h\to 0} + \vec{F}_{+h\to 0} = \begin{bmatrix} 0 \\ F_0(\mathrm{tg}\,\phi_2 - \mathrm{tg}\,\phi_1) \\ 0 \end{bmatrix}$$

$$m_1 \ddot{u}_1(0,t) = F_0(\operatorname{tg}\phi_2 - \operatorname{tg}\phi_1)$$

$$m_L \cdot \frac{h}{l} \ddot{u}_1(0,t) = F_0(\operatorname{tg}\phi_2 - \operatorname{tg}\phi_1) \quad ; m_L = \text{masa leve vrvi}$$

$$h \to 0: \quad 0 = F_0\left(\frac{\partial u_2(x,t)}{\partial x}\bigg|_{x=0} - \frac{\partial u_1(x,t)}{\partial x}\bigg|_{x=0} \right)$$

$$\Rightarrow \frac{\partial u_1(x,t)}{\partial x}\bigg|_{x=0} = \frac{\partial u_2}{\partial x}(x,t)\bigg|_{x=0} \checkmark$$

$$\boxed{f_1(x-c_1t)} = f_1\left[c_1\left(\frac{x}{c_1}-t\right)\right] = f_1\left[-c_1\left(t-\frac{x}{c_1}\right)\right] \boxed{\equiv \tilde{f}_1(\chi_1); \ \chi_1 \equiv t - \frac{x}{c_1}} \text{; vpadni val}$$

$$\boxed{g_1(x+c_1t)} = \quad \boxed{\equiv \tilde{g}_1(\xi_1); \ \xi_1 \equiv t + \frac{x}{c_1}} \text{; odbiti val}$$

$$\boxed{f_2(x-c_2t)} = \quad \boxed{\equiv \tilde{f}_2(\chi_2); \ \chi_2 \equiv t - \frac{x}{c_2}} \text{; prepuščeni val}$$

$$\Rightarrow \frac{\partial \tilde{f}_1(\chi_1)}{\partial x}\bigg|_{x=0} = \frac{\partial \tilde{f}_1(\chi_1)}{\partial \chi_1} \cdot \frac{\partial \chi_1}{\partial x}\bigg|_{x=0} = -\frac{1}{c_1} \frac{\partial \tilde{f}_1(\chi_1)}{\partial \chi_1}\bigg|_{x=0} = -\frac{1}{c_1}\frac{\partial \tilde{f}_1(t)}{\partial t}$$

$$\Rightarrow \frac{\partial \tilde{g}_1(\xi_1)}{\partial x}\bigg|_{x=0} = +\frac{1}{c_1}\frac{\partial \tilde{g}_1(t)}{\partial t}$$

$$\Rightarrow \frac{\partial \tilde{f}_2(\chi_2)}{\partial x}\bigg|_{x=0} = -\frac{1}{c_2}\frac{\partial \tilde{f}_2(t)}{\partial t}$$

$$u_1(x=0,t) = \tilde{f}_1(t) + \tilde{g}_1(t)$$

$$u_2(x=0,t) = \tilde{f}_2(t)$$

$$\frac{\partial u_1(x,t)}{\partial x}\bigg|_{x=0} = -\frac{1}{c_1}\frac{\partial \tilde{f}_1(t)}{\partial t} + \frac{1}{c_1}\frac{\partial \tilde{g}_1(t)}{\partial t}$$

$$\frac{\partial u_2(x,t)}{\partial x}\bigg|_{x=0} = -\frac{1}{c_2}\frac{\partial \tilde{f}_2(t)}{\partial t}$$

Robna pogoja:

a) $$\boxed{\tilde{f}_1(t) + \tilde{g}_1(t) = \tilde{f}_2(t)}$$

b) $$\frac{1}{C_1}\frac{\partial \tilde{f}_1(t')}{\partial t'} - \frac{1}{C_1}\frac{\partial \tilde{g}_1(t')}{\partial t'} = \frac{1}{C_2}\frac{\partial \tilde{f}_2(t')}{\partial t'}$$

$$\Rightarrow \int_{t_0}^{t}\left[\frac{1}{C_1}\frac{\partial \tilde{f}_1(t')}{\partial t'} - \frac{1}{C_1}\frac{\partial \tilde{g}_1(t')}{\partial t'}\right]dt' = \int_{t_0}^{t}\frac{1}{C_2}\frac{\partial \tilde{f}_2(t')}{\partial t'}\,dt'$$

$$\Rightarrow \frac{1}{C_1}\tilde{f}_1(t) - \frac{1}{C_1}\tilde{g}_1(t) = \frac{1}{C_2}\tilde{f}_2(t) + K \quad ; \; K = \frac{1}{C_1}\tilde{f}_1(t_0) - \frac{1}{C_1}\tilde{g}_1(t_0) - \frac{1}{C_2}\tilde{f}_2(t_0)$$

$$t_0 \to -\infty : \; \tilde{f}_1(t_0) = \tilde{g}_1(t_0) = \tilde{f}_2(t_0) = 0 \quad \Rightarrow K = 0$$

$$\Rightarrow \boxed{\frac{1}{C_1}\tilde{f}_1(t) - \frac{1}{C_1}\tilde{g}_1(t) = \frac{1}{C_2}\tilde{f}_2(t)}$$

$$\frac{2}{C_1}\tilde{f}_1(t) = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\tilde{f}_2(t)$$

$$\Rightarrow \frac{2}{C_1}\tilde{f}_1(t) = \frac{C_1 + C_2}{C_1 C_2}\tilde{f}_2(t)$$

$$\Rightarrow \tilde{f}_2(t) = \tilde{f}_1(t)\frac{2C_2}{C_1 + C_2} \quad \Rightarrow \boxed{\left|\frac{\tilde{f}_2(t)}{\tilde{f}_1(t)}\right| = \frac{2C_2}{C_1 + C_2}}$$ prepustnost

$$\frac{2}{C_1}\tilde{g}_1(t) = \left(\frac{1}{C_1} - \frac{1}{C_2}\right)\tilde{f}_2(t)$$

$$\Rightarrow \frac{2}{C_1}\tilde{g}_1(t) = \frac{C_2 - C_1}{C_1 C_2}\tilde{f}_2(t)$$

$$\Rightarrow \boxed{\tilde{g}_1(t)} = \frac{C_2 - C_1}{2C_2}\tilde{f}_2(t) = \frac{C_2 - C_1}{2C_2}\,\frac{2C_2}{C_2 + C_1}\tilde{f}_1(t) = \boxed{\frac{C_2 - C_1}{C_2 + C_1}\tilde{f}_1(t)}$$

$$\boxed{\left|\frac{\tilde{g}_1(t)}{\tilde{f}_1(t)}\right| = \frac{|C_2 - C_1|}{C_1 + C_2}}$$ odbojnost

(11) Energija

- potovanje motnje → potovanje energije

Valovanje po vijačni vzmeti

- diskretni model; utež, ki je v ravnovesju v legi x, in vzmet med x in x+h

$$\boxed{W_k = \frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 \left|\frac{\partial u(x,t)}{\partial t}\right|^2 = \frac{1}{2}\frac{mh}{l}\left|\frac{\partial u(x,t)}{\partial t}\right|^2}$$

$$\Rightarrow \boxed{\frac{W_k}{h} = \frac{1}{2}\frac{m}{l}\left|\frac{\partial u(x,t)}{\partial t}\right|^2}$$

$$W_{pr} = \frac{1}{2} k_1 (\Delta h)^2 = \frac{1}{2} k_1 \left[u(x+h,t) - u(x,t)\right]^2$$

$$= \frac{1}{2}\frac{kl}{h}\left[u(x+h,t) - u(x,t)\right]^2$$

$$= \frac{1}{2}\frac{klh}{h^2}\left[u(x+h,t) - u(x,t)\right]^2$$

$$= \frac{1}{2} klh \left[\frac{u(x+h,t) - u(x,t)}{h}\right]^2$$

$$\Rightarrow \frac{W_{pr}}{h} = \frac{1}{2} kl \left[\frac{u(x+h,t) - u(x,t)}{h}\right]^2$$

$$\Rightarrow h \to 0: \quad \boxed{\frac{W_{pr}}{h} = \frac{1}{2} kl \left|\frac{\partial u(x,t)}{\partial x}\right|^2}$$

$$= \frac{1}{2} kl \frac{l}{m}\frac{m}{l}\left|\frac{\partial u(x,t)}{\partial x}\right|^2$$

$$\boxed{= \frac{1}{2} c^2 \frac{m}{l}\left|\frac{\partial u(x,t)}{\partial x}\right|^2} \quad ; \quad c^2 = \frac{kl^2}{m}$$

Prožna palica (longitudinalno valovanje (zvok), $\lambda \gg 2r$)

$$\left.\begin{aligned} m &= \rho V = \rho S l \\ k &= \frac{ES}{l} \end{aligned}\right\} \Rightarrow \frac{W_k(x,t)}{h} = \frac{1}{2}\rho S \left|\frac{\partial u(x,t)}{\partial t}\right|^2$$

$$\Rightarrow \boxed{w_k(x,t) \equiv \frac{W_k(x,t)}{\Delta V} = \frac{W_k(x,t)}{Sh} = \frac{1}{2}\rho \left|\frac{\partial u(x,t)}{\partial t}\right|^2}$$

$$\Rightarrow \frac{W_{pr}(x,t)}{h} = \frac{1}{2} c^2 \frac{m}{l} \left|\frac{\partial u(x,t)}{\partial x}\right|^2$$

$$= \frac{1}{2} c^2 \rho S \left|\frac{\partial u(x,t)}{\partial x}\right|^2$$

$$\Rightarrow \boxed{w_{pr}(x,t) \equiv \frac{W_{pr}(x,t)}{\Delta V} = \frac{W_{pr}(x,t)}{Sh} = \frac{1}{2} c^2 \rho \left|\frac{\partial u(x,t)}{\partial x}\right|^2} ; \quad c^2 = \frac{E}{\rho}$$

Energija sinusnega valovanja

valovni vektor (ne koeficient vzmeti)

$$u(x,t) = u_0 \sin(kx - \omega t + \delta) ; \quad k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega}{c} \text{ ali } c = \frac{\omega}{k} \text{ ali } \omega = ck$$

$$\Rightarrow \frac{\partial u(x,t)}{\partial t} = -u_0 \omega \cos(kx - \omega t + \delta)$$

$$\frac{\partial u(x,t)}{\partial x} = u_0 k \cos(kx - \omega t + \delta)$$

$$\Rightarrow w_k(x,t) = \frac{1}{2} \rho u_0^2 \omega^2 \cos^2(kx - \omega t + \delta)$$

$$w_{pr}(x,t) = \frac{1}{2} \overbrace{c^2 \rho u_0^2 k^2}^{\omega^2} \cos^2(kx - \omega t + \delta) = \frac{1}{2} \rho u_0^2 \omega^2 \cos^2(kx - \omega t + \delta) = w_k(x,t)$$

$$\Rightarrow \boxed{w(x,t) = w_k(x,t) + w_{pr}(x,t) = 2w_k(x,t) = \rho\omega^2 u_0^2 \cos^2(kx-\omega t+\delta)}$$

Povprečna energija v intervalu $[0,t_0]$:

$$\boxed{\bar{w}(x,[0,t_0]) = \frac{1}{t_0}\int_0^{t_0} w(x,t)\,dt}$$

$$= \frac{\rho\omega^2 u_0^2}{t_0}\int_0^{t_0} \cos^2(kx-\omega t+\delta)\,dt$$

$z = kx-\omega t+\delta$

$\Rightarrow dt = -\frac{dz}{\omega}$

$\Rightarrow z_1 = kx+\delta$

$\Rightarrow z_2 = kx+\delta-\omega t_0$

$= kx+\delta-\frac{2\pi}{t_0}t_0$

$= z_1-2\pi$

$$= -\frac{\rho\omega^2 u_0^2}{2\pi}\int_{z_1}^{z_1-2\pi} \cos^2 z\,dz$$

$$= \frac{\rho\omega^2 u_0^2}{2\pi}\underbrace{\int_{z_1-2\pi}^{z_1} \cos^2 z\,dz}_{\frac{2\pi}{2}}$$

$$= \boxed{\frac{1}{2}\rho\omega^2 u_0^2}$$

$\bar{W} = \bar{w}\cdot\Delta V = \bar{w}\,S\cdot ct$

$\boxed{\perp \text{ na } c}$

Energijski tok valovanja:

$P = \frac{\bar{W}}{t}$; $[P] = W$

Gostota energijskega toka

$\boxed{j_W = \frac{P}{S} = \bar{w}c}$; $[j_W] = \frac{W}{m^2}$

Primeri

- $j_W = 1{,}4\ \mathrm{kW/m^2}$ (sončna svetloba na vrhu atmosfere)
- $j_W = 10^{-12}\ \mathrm{W/m^2}$ (meja slišnosti zvoka s frekvenco $\sim 1\,\mathrm{kHz}$)
- $j_W = 1\ \mathrm{W/m^2}$ (meja bolečine)

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(12) Zvok

- longitudinalno valovanje v snovi

a) V tanki dolgi elastični palici $(2r \ll \lambda,\ l \geq \lambda)$ ✓

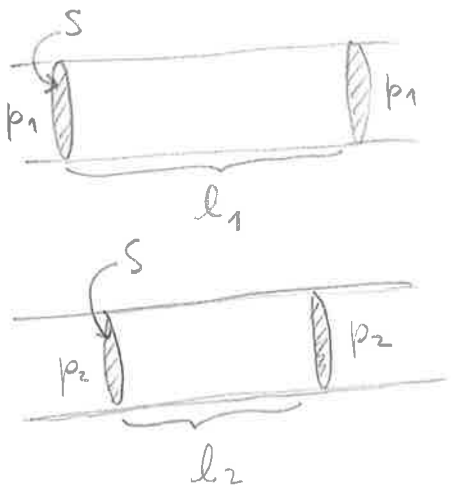
$$c^2 = \frac{E}{\rho}$$

---

b) V tekočini, zaprti v tanki dolgi cevi

- viskoznost zanemarljiva (zanemarljive strižne sile med tekočino in steno cevi)
- toga cev (S = konst.)

Stiskanje tekočine v cevi:



$$\left.\begin{array}{l} V_1 = S l_1 \\ \\ V_2 = S l_2 \end{array}\right\}$$

$\Delta V \equiv V_2 - V_1 = S\Delta l\ ;\ \Delta l \equiv l_2 - l_1$

$$\Rightarrow \boxed{\frac{\Delta V}{V_1}} = \frac{S\Delta l}{S l_1} = \boxed{\frac{\Delta l}{l_1}}$$

$\Delta p \equiv p_2 - p_1$

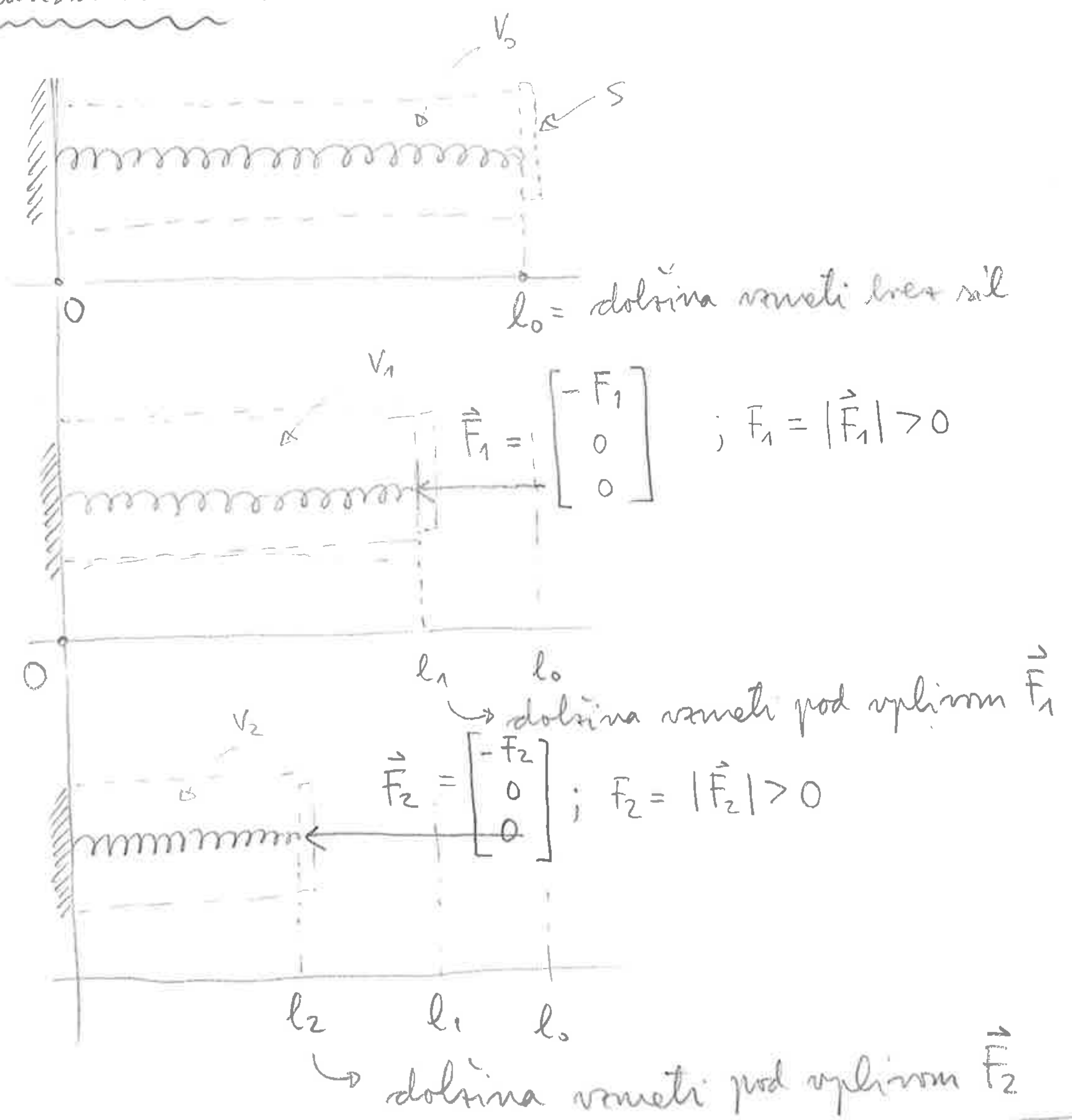
· Stisljivost: $\boxed{\chi \equiv -\frac{1}{V_1}\frac{\Delta V}{\Delta p}} \Rightarrow \Delta p = -\frac{1}{\chi}\frac{\Delta V}{V_1} \Rightarrow \boxed{\Delta p = -\frac{1}{\chi}\frac{\Delta l}{l_1}}$

$$\Rightarrow \Delta p = p_2 - p_1 = \frac{F_2}{S} - \frac{F_1}{S} = \frac{\Delta F}{S} \; ; \; \Delta F \equiv F_2 - F_1$$

$$\Rightarrow \boxed{\Delta F = -\frac{S}{\chi l_1}\Delta l}$$

Hookov zakon, glej nadaljevanje!

· Mehanski model (zanemarimo Brownovo gibanje molekul tekočine)



$\Delta l_1 \equiv l_1 - l_0 \; (<0)$, $\Delta l_2 \equiv l_2 - l_0 \; (<0)$; $\boxed{\Delta l \equiv l_2 - l_1 = \Delta l_2 - \Delta l_1}$ (<0)

Hookov zakon: $-F_1 = +k_0 \Delta l_1 \Rightarrow \boxed{F_1 = -k_0 \Delta l_1}$ (predznak $-$ smer $\vec{v}$)

$$-F_2 = +k_0 \Delta l_2 \Rightarrow \boxed{F_2 = -k_0 \Delta l_2}$$

$$\Rightarrow \boxed{F_{x-h\to x} = -k_1(h'-h_0)} \;; \qquad h' < h_0 \Rightarrow F_{x-h\to x} > 0 \; (\rightarrow) \checkmark$$

$$= -k_1(h' - h + h - h_0)$$

$$\boxed{= -k_1[u(x,t) - u(x-h,t)] - k_1(h-h_0)}$$

$$\Rightarrow \boxed{F_{x+h\to x} = +k_1[u(x+h,t) - u(x,t)] + \underset{\substack{\wedge \\ 0}}{k_1(h-h_0)}} < 0 \; (\leftarrow)$$

$$\Rightarrow F_x = F_{x+h\to x} + F_{x-h\to x}$$ (ni strižnih sil sten na tekočino)

$$= k_1[u(x+h,t) - 2u(x,t) + u(x-h,t)]$$

$$= \frac{k l_1}{h}[u(x+h,t) - 2u(x,t) + u(x-h,t)]$$

$$\Rightarrow m_1 \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{k l_1}{h}[\ldots]$$ (2. Newtonov zakon za maso, ki je v ravnovesju na mestu x)

$$\Rightarrow \boxed{m \frac{h}{l_1} \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{k l_1}{h}[\ldots]}$$

$$\Rightarrow \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{k l_1^2}{m}\left[\frac{u(x+h,t) - 2u(x,t) - u(x-h,t)}{h^2}\right]$$

$$\Rightarrow h \to 0 : \boxed{\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{k l_1^2}{m} \frac{\partial^2 u(x,t)}{\partial x^2}}$$ (zvezna limita)

je prostornina tekočine v cevi

$$k \simeq k_0 = \frac{S}{\chi l_1} \Rightarrow \boxed{c^2 = \frac{k l_1^2}{m} = \frac{S l_1^2}{\chi \cancel{l_1} m} = \frac{V}{\chi m} = \frac{1}{\chi \rho}}$$

$$\boxed{\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}}$$

$$\Delta F \equiv F_2 - F_1 \Rightarrow F_2 = F_1 + \Delta F$$

$$\Delta l_2 = \Delta l_1 + \Delta l$$

$$\Rightarrow \cancel{F_1} + \Delta F = -k_0(\Delta l_1 + \Delta l) = \cancel{-k\Delta l_1} - k_0 \Delta l$$

$$\Rightarrow \boxed{\Delta F = -k_0 \Delta l}$$

$\Rightarrow$ iz primerjave $\boxed{k_0 = \dfrac{S}{\chi l_1}}$

$$\Delta V_1 = V_1 - V_0 = S(l_1 - l_0) = S\Delta l_1$$

$$\frac{\Delta V}{V_0} = \frac{\cancel{S}\Delta l_1}{\cancel{S} l_0} = -\chi p_1$$

- Diskretni model: stisnjena masivna vzmet

l_0: prosta dolžina (nestisnjene vzmeti); k_0: koeficient vzmeti

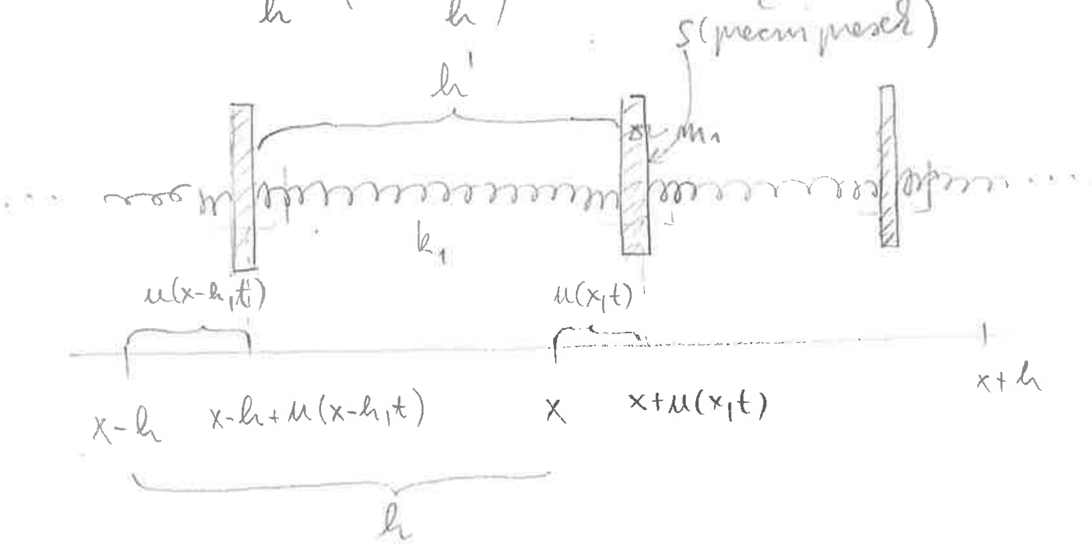
h_0: razdalja med dvema masama

$\vec{F}_1$: $l_0 \to l_1$; $k_0 \to k$

$h_0 \to h$

$$k = k_0 \frac{l_0}{l_1} = k_0 \frac{l_0}{l_1 - l_0 + l_0} = k_0 \frac{l_0}{l_0 + \Delta l_1} = k_0 \frac{\cancel{l_0}}{\cancel{l_0}\left(1 + \frac{\Delta l_1}{l_0}\right)} \overset{\frac{\Delta l_1}{l_0} \ll 1}{\approx} k_0\left(1 - \frac{\Delta l_1}{l_0}\right) \approx k_0$$

$$k_1 = \frac{k l_1}{h} \quad \left(= \frac{k_0 l_1}{h}\right)$$



$$u(x-h,t) + h' = h + u(x,t) \Rightarrow h' - h = u(x,t) - u(x-h,t)$$

• Še povezava med p_1 (tlakom v tekočini brez motnje) $p(x,t)$ in odmikom $u(x,t)$:

2. Newtonov zakon: $$m_1 \frac{\partial^2 u(x,t)}{\partial t^2} = F_x$$

$$\frac{mh}{l_1} \frac{\partial^2 u(x,t)}{\partial t^2} = F_x$$

zvezna limita $h \to 0$: $$\frac{mh}{l_1} \frac{\partial^2 u(x,t)}{\partial t^2} = F_x \to 0$$

$$\Rightarrow F_{x+h\to x} + F_{x-h\to x} = 0$$

$$\Rightarrow |F_{x+h\to x}| = |F_{x-h,x}| \equiv p(x,t)\,S$$

$$\Rightarrow p(x,t) = \frac{1}{S}|F_{x+h\to x}|$$

$$= \frac{1}{S}\left| k_1[u(x+h,t)-u(x,t)] + k_1(h-h_0)\right| \quad < 0$$

$$= \frac{1}{S}\left\{ -k_1[u(x+h,t)-u(x,t)] - k_1(h-h_0)\right\}$$

$$= -\frac{k l_1}{S}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] - \frac{k l_1}{S}\left(\frac{h-h_0}{h}\right)$$

$$= -\frac{k l_1}{S}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] - \frac{k l_1}{S}\left(\frac{mh-mh_0}{mh}\right)$$

$$= -\frac{k l_1}{S}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] - \frac{k l_1}{S}\frac{\overbrace{(l_1-l_0)}^{\Delta l_1}}{l_1} \qquad \left(k = \frac{S}{\chi l_1}\right)$$

$$= -\frac{1}{\chi}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] - \frac{S}{\chi l_1}\frac{1}{S}\Delta l_1$$

$$= -\frac{1}{\chi}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] - \frac{1}{\chi}\frac{\Delta l_1}{l_1}$$

$$\left(\frac{\Delta l_1}{l_1} = \frac{\Delta l_1}{l_0 + l_1 - l_0} = \frac{\Delta l_1}{l_0+\Delta l_1} = \frac{\Delta l_1}{l_0}\frac{1}{\left(1+\frac{\Delta l_1}{l_0}\right)} \simeq \frac{\Delta l_1}{l_0}\left(1-\frac{\Delta l_1}{l_0}\right) \simeq \frac{\Delta l_1}{l_0} = \frac{\Delta V_1}{V_0}\right)$$

$$= -\frac{1}{\chi}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] - \frac{1}{\chi}\frac{\Delta V_1}{V_0}$$

$$= -\frac{1}{\chi}\left[\frac{u(x+h,t)-u(x,t)}{h}\right] + p_1$$

$$\Rightarrow h \to 0: \boxed{p(x,t) = p_1 - \frac{1}{\chi}\frac{\partial u(x,t)}{\partial x}}$$

$u = u_0 \sin(kx - \omega t + \delta)$ (sinusni val)

$$\Rightarrow \frac{\partial u(x,t)}{\partial x} = k u_0 \cos(kx-\omega t+\delta) = \frac{\omega u_0}{c}\cos(kx-\omega t+\delta)$$

$$\Rightarrow p = p_1 - \frac{\omega u_0}{\chi c}\cos(kx-\omega t+\delta)$$

amplituda, s katero niha tlak okoli p_1

Stisljivost idealnega plina

a) Izotermna

$$\boxed{pV = p_0 V_0 = K} \Rightarrow V = \frac{K}{p} \Rightarrow \boxed{\frac{dV}{dp} = -\frac{K}{p^2} = -\frac{K}{p}\cdot\frac{1}{p} = -\frac{V}{p}}$$

$$\Rightarrow -\frac{1}{V}\frac{dV}{dp} = \boxed{\frac{1}{p} = \chi}$$

b) Adiabatna (hitro stiskanje in razpenjanje, skoraj nič toplote se ne izmenja med posameznimi deli plina)

$$\boxed{pV^{\varkappa} = p_0 V_0^{\varkappa} = K'}$$

$$\varkappa = \frac{c_p}{c_v} = \begin{cases} \frac{5}{3} \simeq 1{,}67 & \text{za 1-atomne pline} \\ \frac{7}{5} = 1{,}4 & \text{za 2-atomne pline} \\ \frac{8}{6} \simeq 1{,}3 & \text{za 3-atomne pline} \end{cases}$$

$$\Rightarrow p^{\frac{1}{\varkappa}} V = p_0^{1/\varkappa} V_0 = K'^{1/\varkappa} = K$$

$$\Rightarrow V = K p^{-1/\varkappa}$$

$$\Rightarrow \frac{dV}{dp} = -\frac{1}{\varkappa} K p^{-1/\varkappa - 1} = -\frac{1}{\varkappa p} K p^{-1/\varkappa} = -\frac{1}{\varkappa p} V$$

$$\Rightarrow -\frac{1}{V}\frac{dV}{dp} = \boxed{\frac{1}{\varkappa p} = \chi}$$ ✓

Primerjava c v plinih in kapljevinah

$c = \sqrt{\frac{1}{\chi\rho}}$; $\rho(\text{kapljevine}) > \rho(\text{plini})$, $\chi(\text{kapljevine}) \ll \chi(\text{plini})$ $\Rightarrow \chi\rho(\text{kapljevine}) < \chi\rho(\text{plini})$

$\Rightarrow c(\text{kapljevine}) > c(\text{plini})$

Domače naloge:

(1) Izračunaj hitrost zvoka v zraku pri $T = 0°C$ ($= 273\,K$) in $p = 10^5\,Pa$!

$M_z = 29\,\frac{kg}{}$, $\varkappa = 1{,}4$

(2) Meja slišnosti zvoka frekvence $1000\,Hz$ je pri jakosti $\approx 10^{-12}\,W/m^2s$.

a) Kolikšna je amplituda u_0 odmikov atomov zraka od ravnovesne lege pri tej jakosti?

b) Kolikšni tlačni amplitudi ustreza?