

# 10 "Indukcija" (navidezna indukcija) - Lorentzova sila (Hallov pojav)

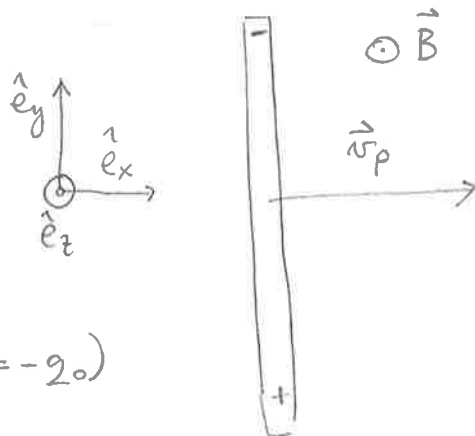
Primer: premikanje vodnika po statičnem zmanjšanju  $\vec{B}$  (gugalka)

- $\vec{B} = B \hat{e}_z ; B > 0$

- $\vec{v}_p = v_p \hat{e}_x ; v_p > 0$

- $l$  = dolžina prečke

- $e^-$ : prosti nosilci ( $q = -q_0$ )



$$\vec{F}_{mag} = q \vec{v} \times \vec{B}$$

$$= -q_0 \vec{v}_p \times \vec{B}$$

$$= -q_0 v_p B (\hat{e}_x \times \hat{e}_z)$$

$$= q_0 v_p B \hat{e}_y \quad ; \quad \hat{e}_x \times \hat{e}_z = -\hat{e}_y$$

Steče kratkotrajni tok, presežek negativnih nabojev na zgornjem delu prečke ( $y \rightarrow \frac{l}{2}$ ), presežek pozitivnega naboja na spodnjem delu prečke ( $y \rightarrow -\frac{l}{2}$ )  $\Rightarrow$  ustvari se električno polje

- $\vec{E} = E \hat{e}_y \Rightarrow \vec{E}$  (polje),  $\vec{v}_p$ ,  $\vec{B}$ : parna pravokotni vektorji.

$\Rightarrow$  na proste nosilce naboja deluje tudi električna sila

$$\vec{F}_el = q \vec{E}$$

$$= -q_0 E \hat{e}_y$$

$$= q_0 E (-\hat{e}_y)$$

$\Rightarrow$  V stacionarnih razmerah, ko toka ni več, ravnovesje obeh sil!

$$\vec{F}_{\text{mag}} + \vec{F}_{\text{el}} = 0$$

$$q_0 (v_p B - E) \hat{e}_y = 0$$

$$\Rightarrow E = v_p B \quad (= q_{\text{ind}})$$

$$\Rightarrow \vec{E} = v_p B \hat{e}_y$$

Napetost med koncema prečke:

$$\vec{r}(t_1) = \begin{bmatrix} 0 \\ -\frac{l}{2} \\ 0 \end{bmatrix}, \vec{r}(t_2) = \begin{bmatrix} 0 \\ \frac{l}{2} \\ 0 \end{bmatrix} \quad (\text{v sistemu, v katerem prečka miruje})$$

C: krivulja z točkama v  $\vec{r}(t_1)$  in  $\vec{r}(t_2)$

$$U(\vec{r}_1 \rightarrow \vec{r}_2) = - \int_C \vec{E} \cdot d\vec{s} = - \int_{t_1}^{t_2} \vec{E} \cdot \dot{\vec{r}} dt = - \int_{t_1}^{t_2} E \dot{y} dt = - E \int_{t_1}^{t_2} \dot{y} dt = - E y \Big|_{t_1}^{t_2} = - E l = - v_p B l$$

$\dot{\vec{r}} = \dot{x} \hat{e}_x + \dot{y} \hat{e}_y + \dot{z} \hat{e}_z$

Če se prečka premika v dolgo nasprotno smer:

$$\cdot \vec{v}_p = v_p (-\hat{e}_x); v_p > 0$$

$$\cdot U(\vec{r}_1 \rightarrow \vec{r}_2) = +v_p B l$$

Napetost lahko izrazimo z mešanim produktom:

$$U(\vec{r}_1 \rightarrow \vec{r}_2) = \vec{v}_p \cdot (\vec{B} \times \vec{\ell}); \quad \vec{\ell} = l \hat{e}_\ell; \quad \hat{e}_\ell = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \quad (\text{mešani produkt})$$

$$\cdot \text{v našem primeru: } \vec{r}_2 - \vec{r}_1 = l \hat{e}_y \Rightarrow \hat{e}_\ell = \hat{e}_y$$

$$\begin{aligned} \Rightarrow U(\vec{r}_1 \rightarrow \vec{r}_2) &= v_p \hat{e}_x \cdot (B l \hat{e}_z \times \hat{e}_y) \\ &= v_p B l \hat{e}_x \cdot (\hat{e}_z \times \hat{e}_y) \\ &= v_p B l \hat{e}_x \cdot (-\hat{e}_x) \\ &= -v_p B l \quad \checkmark \end{aligned}$$

$$\begin{aligned}
 \Rightarrow U(\vec{r}_1 \rightarrow \vec{r}_2) &= v_p (-\hat{e}_x) \cdot (\vec{B} \times \vec{\ell}) \\
 &= -v_p B \ell \hat{e}_x \cdot (\hat{e}_z \times \hat{e}_y) \\
 &= -v_p B \ell \hat{e}_x \cdot (-\hat{e}_x) \\
 &= v_p B \ell \quad \checkmark
 \end{aligned}$$

Enačba

$$U(\vec{r}_1 \rightarrow \vec{r}_2) = \vec{v}_p \cdot (\vec{B} \times \vec{\ell})$$

velja tudi, da  $\vec{v}_p, B, \vec{\ell}$  niso paroma pravokotni.

Preploščev enačbe, ko  $\vec{B}$  ni konstantno in/ali prečja (volum.)  
ni sama in/ali  $\vec{v}_p$  ni konstanta:

$$U(\vec{r}_1 \rightarrow \vec{r}_2) = \int_C \vec{v}_p \cdot (\vec{B} \times d\vec{\ell}), \text{ kjer integriramo vzdolž vodnika (prečje)}$$

Primer: palico v zunanjem  $\vec{B}$  ntimo okrog nje in med krajšč;

$$\vec{B} = B \hat{e}_z, B > 0$$

$$\omega : \text{kotna hitrost } (\dot{\phi})$$

$$\vec{r} = r \hat{e}_s \text{ (cilindrični koordinatni sistem); } r = \text{zmest.}$$

$$\vec{v}_p = \dot{r} \hat{e}_s + r \dot{\phi} \hat{e}_\phi = r \dot{\phi} \hat{e}_\phi = r \omega \hat{e}_\phi$$

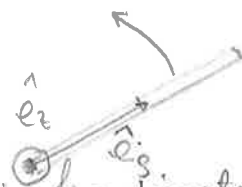
$$d\vec{\ell} = \hat{e}_s dr$$

$$\Rightarrow dU_i = \vec{v}_p \cdot (\vec{B} \times d\vec{\ell})$$

$$= \omega r \hat{e}_\phi \cdot (B dr \hat{e}_z \times \hat{e}_s)$$

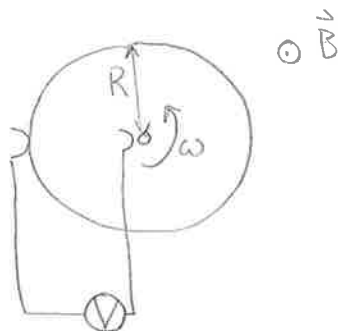
$$= \omega B r dr \hat{e}_\phi \cdot (\underbrace{\hat{e}_z \times \hat{e}_s}_{\hat{e}_\phi})$$

$$= \omega B r dr$$

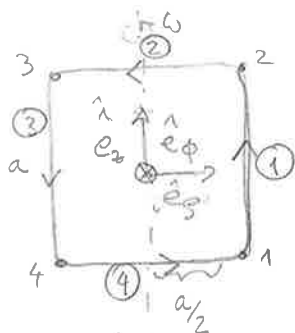


$$\Rightarrow \boxed{U_i = \int_0^R \omega B r dr = \frac{1}{2} \omega B R^2}$$

Primer: diska, ki se vrti v zunanjem  $\vec{B}$



Primer: kvadratna zanka se vrti v zunanjem  $\vec{B}$



$$\vec{B} = B \hat{e}_y; B > 0$$

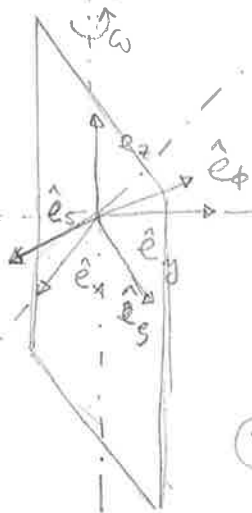
$$\vec{S} = S \hat{e}_s = a^2 (-\hat{e}_\phi) \quad (\hat{e}_s = -\hat{e}_\phi)$$

$$\textcircled{1} \vec{v}_p = \frac{a}{2} \omega \hat{e}_\phi = \frac{a}{2} \omega (-\sin \phi \hat{e}_x + \cos \phi \hat{e}_y)$$

$$\vec{l} = a \hat{e}_z$$

$$\Rightarrow \vec{B} \times \vec{l} = a B (\hat{e}_y \times \hat{e}_z) = a B \hat{e}_x$$

$$\Rightarrow \boxed{U_1 = \vec{v}_p \cdot (\vec{B} \times \vec{l}) = \frac{a^2}{2} B \omega [-\sin \phi \hat{e}_x + \cos \phi \hat{e}_y] \cdot \hat{e}_x} \\ = \boxed{-\frac{a^2}{2} B \omega \sin \phi}$$



$$\textcircled{3} \vec{v}_p = -\frac{a}{2} \omega \hat{e}_\phi$$

$$\vec{l} = -a \hat{e}_z$$

$$\Rightarrow \boxed{U_3 = U_1 = -\frac{a^2}{2} B \omega \sin \phi}$$

$$\textcircled{2} d\vec{l} = dr (-\hat{e}_s) = -dr (\cos \phi \hat{e}_x + \sin \phi \hat{e}_y), \vec{v}_p = r\omega (-\sin \phi \hat{e}_x + \cos \phi \hat{e}_y)$$

$$\Rightarrow \vec{B} \times d\vec{l} = -B dr [\hat{e}_y \times (\cos \phi \hat{e}_x + \sin \phi \hat{e}_y)] = -B dr [\cos \phi (\hat{e}_y \times \hat{e}_x) + \sin \phi (\hat{e}_y \times \hat{e}_y)] \\ = B \omega \phi dr \hat{e}_z$$

$$\Rightarrow dU_2 = \vec{v}_p \cdot (\vec{B} \times d\vec{l})$$

$$= B \omega \phi r dr (-\sin \phi \hat{e}_x + \cos \phi \hat{e}_y) \cdot \hat{e}_z$$

$$= 0$$

$$\Rightarrow \boxed{U_2 = 0}$$

$$\textcircled{4} \quad U_4 = 0$$

$$\Rightarrow \boxed{U = U_1 + U_2 + U_3 + U_4 = -a^2 B \omega \sin \phi} ; \text{generator izmenljive napetosti!}$$

$$\Phi_B = \vec{S} \cdot \vec{B} = S B \hat{e}_S \cdot \hat{e}_B$$

$$= a^2 B (-\hat{e}_\phi) \cdot \hat{e}_y$$

$$= a^2 B (\sin \phi \hat{e}_x - \cos \phi \hat{e}_y) \cdot \hat{e}_y$$

$$= -a^2 B \cos \phi$$

$$\Rightarrow \frac{d\Phi_B}{dt} = a^2 B \omega \sin \phi \quad (\omega = \dot{\phi})$$

$$\Rightarrow \boxed{U = - \frac{d\Phi_B}{dt}} \quad (1)$$

- $\Phi_B = \Phi_B(t)$  zaradi  $\vec{S} = \vec{S}(t)$ : ( $\vec{S} = a^2 \hat{e}_S = a^2 (-\hat{e}_\phi) = a^2 (\sin \phi \hat{e}_x - \cos \phi \hat{e}_y)$ ;  $\phi = \phi(t)$ )
- Enačba (1) odpre npr. pri intenziji diska v ravnanju  $\vec{B}$  (ni splošna!)

Primerjaj z:

$$\oint_{\partial S} \vec{E} \cdot d\vec{s} = -U_i = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$\vec{E} = \vec{S}$  in smer  $\vec{B}$  konstantna;

$$\Rightarrow \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = \frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} = \frac{\partial \Phi_B}{\partial t}$$

$$\Rightarrow \boxed{U_i = \frac{\partial \Phi_B}{\partial t}} \quad (2)$$

- $\Phi_B = \Phi_B(t)$  zaradi  $B = B(t)$

- Enačba (2) velja splošno (zakon o elektrom. napetosti)

## 11 EM valovanje

Maxwellove enačbe v diferencialni obliki:

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 \quad (= \rho / \epsilon \epsilon_0)$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \quad (= \mu \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t})$$

pri čemer

$$c_0 \equiv \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 299792458 \text{ m/s}$$

$$c = \frac{c_0}{\sqrt{\mu \epsilon}} \stackrel{\mu=1}{\approx} \frac{c_0}{\sqrt{\epsilon}} = \frac{c_0}{n} ; n = \sqrt{\epsilon} \geq 1 \quad (\Rightarrow c \leq c_0)$$

$\rho = 0, \vec{j} = 0$ :

M1:  $\nabla \cdot \vec{B} = 0$

M2:  $\nabla \cdot \vec{E} = 0$

M3:  $\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$

M4:  $\nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \quad (= \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t})$

$\downarrow$                        $\downarrow$   
 v vakuumu      v snovi

Mat! •  $\vec{E}$  poljubno vektorsko polje (lahko jakost el. polja, lahko  $\vec{B}$ , ali pa katerokoli drugo vektorsko polje)

$$\boxed{\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}}$$

pri čemer:

$$\left. \begin{aligned} \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \\ \vec{E} &= \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \end{aligned} \right\} \Rightarrow \nabla^2 \vec{E} = \begin{bmatrix} \nabla^2 E_x \\ \nabla^2 E_y \\ \nabla^2 E_z \end{bmatrix}$$

M3:  $\nabla \times (\nabla \times \vec{E}) = \nabla \times \left( -\frac{\partial \vec{B}}{\partial t} \right)$  ( $\vec{E}$  jakost el. polja  
 $\vec{B}$  gostota mag. polja)

$$\underbrace{\nabla(\nabla \cdot \vec{E})}_{\substack{\text{II (M2)} \\ 0}} - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} \underbrace{(\nabla \times \vec{B})}_{\substack{\text{II (M4)} \\ \frac{1}{c_0^2} \frac{\partial \vec{E}}{\partial t}}}$$

$$\Rightarrow -\nabla^2 \vec{E} = -\frac{1}{c_0^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} = \frac{1}{c_0^2} \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Valovna enačba za  $\vec{E}$  v prostoru  
 (v suhi:  $c_0^2 \rightarrow c^2$ )

Podobno!

M4:  $\nabla \times (\nabla \times \vec{B}) = \frac{1}{c_0^2} \nabla \times \left( \frac{\partial \vec{E}}{\partial t} \right)$

$$\underbrace{\nabla(\nabla \cdot \vec{B})}_{\substack{\text{II (M1)} \\ 0}} - \nabla^2 \vec{B} = \frac{1}{c_0^2} \frac{\partial}{\partial t} \underbrace{(\nabla \times \vec{E})}_{\substack{\text{II (M3)} \\ -\frac{\partial \vec{B}}{\partial t}}}$$

$$\Rightarrow -\nabla^2 \vec{B} = -\frac{1}{c_0^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{B} = \frac{1}{c_0^2} \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Valovna enačba za  $\vec{B}$  v praznem prostoru  
(v zvezi:  $c_0^2 \rightarrow c^2$ )

### Komentarji:

- EM valovanje (motnja v  $\vec{E}$  in  $\vec{B}$ ) se lahko širi skozi prazen prostor
- Hitrost  $c_0$  EM valovanja v praznem prostoru:  $c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \text{konst.}$   
(enaka v vseh inercialnih koordinatnih sistemih (!?!)) ;

### Michelson-Morley

- EM valovanje se lahko širi tudi skozi snov:  $c_0 \rightarrow c = \frac{c_0}{n} \leq c_0$

### Primer rešitve:

$$\vec{E} = \begin{bmatrix} E_{0,x} \sin(kx - \omega t + \delta_{E_x}) \\ E_{0,y} \sin(kx - \omega t) \\ E_{0,z} \sin(kx - \omega t + \delta_{E_z}) \end{bmatrix}; \quad \vec{B} = \begin{bmatrix} B_{0,x} \sin(kx - \omega t + \delta_{B_x}) \\ B_{0,y} \sin(kx - \omega t + \delta_{B_y}) \\ B_{0,z} \sin(kx - \omega t + \delta_{B_z}) \end{bmatrix}; \quad \vec{c}_0 = c_0 \hat{e}_x$$

$$\text{Res rešitev: } \left. \begin{aligned} \nabla^2 \vec{E} &= -k^2 \vec{E} \\ \frac{\partial^2 \vec{E}}{\partial t^2} &= -\omega^2 \vec{E} \end{aligned} \right\} \Rightarrow -k^2 \vec{E} = -\frac{\omega^2}{c_0^2} \vec{E} \quad (k = \frac{\omega}{c}) \checkmark$$

Podobno za  $\vec{B}$ .

### Dodatni pogoji

$$\bullet \text{ M2: } \nabla \cdot \vec{E} = \frac{\partial}{\partial x} E_x = k E_{0,x} \cos(kx - \omega t + \delta_{E_x}) = 0 \quad \forall x, t$$

$$\Rightarrow E_{0,x} = 0 \quad (\vec{E} \perp \vec{c}_0)$$

$$\bullet \text{ M1: } \Rightarrow B_{0,x} = 0 \quad (\vec{B} \perp \vec{c}_0) \quad \left. \begin{aligned} & \Rightarrow E_{0,x} = 0 \quad (\vec{E} \perp \vec{c}_0) \\ & \Rightarrow B_{0,x} = 0 \quad (\vec{B} \perp \vec{c}_0) \end{aligned} \right\} \text{ EM valovanje transverzalno}$$



$$\Rightarrow \vec{E} = \begin{bmatrix} 0 \\ E_{0y} \sin(kx - \omega t) \\ E_{0z} \sin(kx - \omega t + \delta_{Ez}) \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} 0 \\ B_{0y} \sin(kx - \omega t + \delta_{By}) \\ B_{0z} \sin(kx - \omega t + \delta_{Bz}) \end{bmatrix}$$

$$M4: \nabla \times \vec{B} = \frac{1}{c_0^2} \frac{\partial \vec{E}}{\partial t}$$

$$\frac{1}{c_0^2} \frac{\partial \vec{E}}{\partial t} = \begin{bmatrix} 0 \\ -\frac{\omega}{c_0^2} E_{0y} \cos(kx - \omega t) \\ -\frac{\omega}{c_0^2} E_{0z} \cos(kx - \omega t + \delta_{Ez}) \end{bmatrix}$$

$$\begin{aligned} \nabla \times \vec{B} &= \begin{bmatrix} \frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y \\ \frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z \\ \frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\partial}{\partial x} B_z \\ \frac{\partial}{\partial x} B_y \end{bmatrix} = \begin{bmatrix} 0 \\ -k B_{0z} \cos(kx - \omega t + \delta_{Bz}) \\ k B_{0y} \cos(kx - \omega t + \delta_{By}) \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -\frac{\omega}{c_0} B_{0z} [\cos(kx - \omega t) \cos \delta_{Bz} - \sin(kx - \omega t) \sin \delta_{Bz}] \\ \frac{\omega}{c_0} B_{0y} [\cos(kx - \omega t + \delta_{Ez}) \cos(\delta_{By} - \delta_{Ez}) - \sin(kx - \omega t + \delta_{Ez}) \sin(\delta_{By} - \delta_{Ez})] \end{bmatrix} \end{aligned}$$

$$\Rightarrow \frac{1}{c_0} E_{0y} \cos(kx - \omega t) = B_{0z} [\cos(kx - \omega t) \cos \delta_{Bz} - \sin(kx - \omega t) \sin \delta_{Bz}] \quad (1)$$

$$\frac{1}{c_0} E_{0z} \cos(kx - \omega t + \delta_{Ez}) = -B_{0y} [\cos(kx - \omega t + \delta_{Ez}) \cos(\delta_{By} - \delta_{Ez}) - \sin(kx - \omega t + \delta_{Ez}) \sin(\delta_{By} - \delta_{Ez})] \quad (2)$$

$\forall x, t$

$$(1) \underline{kx - \omega t = \frac{\pi}{2}} : -B_{0z} \sin \delta_{Bz} = 0 \Rightarrow \delta_{Bz} = 0, \pi$$

$$\underline{kx - \omega t = 0} : \frac{E_{0y}}{c_0} = B_{0y} \cos \delta_{Bz}$$

$$\bullet \delta_{Bz} = \pi \Rightarrow B_{0y} = -\frac{E_{0y}}{c_0} < 0 //$$

$$\bullet \boxed{\delta_{Bz} = 0} \Rightarrow \boxed{B_{0y} = \frac{E_{0y}}{c_0}} (> 0 \vee)$$

$$(2) \underline{kx - \omega t + \delta_{Ez} = \frac{\pi}{2}} : B_{0y} \sin(\delta_{By} - \delta_{Ez}) = 0 \Rightarrow \delta_{By} - \delta_{Ez} = 0, \pi$$

$$\underline{kx - \omega t + \delta_{Ez} = \delta} : \frac{E_{0z}}{c_0} = -B_{0y} \cos(\delta_{By} - \delta_{Ez})$$

$$\bullet \delta_{By} - \delta_{Ez} = 0 \Rightarrow B_{0y} = -\frac{E_{0z}}{c_0} < 0 //$$

$$\bullet \delta_{By} - \delta_{Ez} = \pi \Rightarrow B_{0y} = \frac{E_{0z}}{c_0} > 0 \checkmark$$

$\Downarrow$

$$\delta_{By} = \delta_{Ez} + \pi$$

$$\Rightarrow \sin(kx - \omega t + \delta_{Bz}) = \sin(kx - \omega t + \delta_{Ez} + \pi) \\ = -\sin(kx - \omega t + \delta_{Ez})$$

$$\Rightarrow \vec{B} = \begin{bmatrix} 0 \\ -\frac{E_{0z}}{c_0} \sin(kx - \omega t + \delta_{Ez}) \\ \frac{E_{0y}}{c_0} \sin(kx - \omega t) \end{bmatrix}$$

$$\bullet \Rightarrow \vec{B} \cdot \vec{E} = 0 \quad (\vec{B} \perp \vec{E})$$

$$\Rightarrow \boxed{\vec{B} \perp \vec{E} \perp \vec{c}_0}$$

$$\Rightarrow \boxed{B^2 = \frac{E_{0z}^2}{c_0^2} \sin^2(kx - \omega t + \delta_{Ez}) + \frac{E_{0y}^2}{c_0^2} \sin^2(kx - \omega t) = \frac{E^2}{c_0^2}}$$

$$\Rightarrow \boxed{\vec{B} \times \vec{c}_0 = \begin{bmatrix} 0 \\ c_0 B_z \\ -c_0 B_y \end{bmatrix} = \begin{bmatrix} 0 \\ E_{0y} \sin(kx - \omega t) \\ E_{0z} \sin(kx - \omega t + \delta_{Ez}) \end{bmatrix} = \vec{E}}$$

## Energijshi tok EM valovanja

$$w_E = \frac{1}{2} \epsilon_0 E^2$$

$$w_B = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{1}{2} \frac{E^2}{\frac{\mu_0}{\epsilon_0 c^2}} = \frac{1}{2} \frac{\epsilon_0 \mu_0}{\mu_0} E^2 = \frac{1}{2} \epsilon_0 E^2 = w_E$$

$$\Rightarrow w = w_E + w_B = \epsilon_0 E^2$$

$$\Rightarrow \boxed{\vec{j}_{EM} = w \vec{c} = \epsilon_0 E^2 c \hat{e}_x = \epsilon_0 c^2 \frac{E^2}{c} \hat{e}_x = \frac{\epsilon_0}{\epsilon_0 \mu_0 c} \frac{E^2}{\mu_0 c} \hat{e}_x = \frac{1}{\mu_0 c} \frac{E^2}{c} \hat{e}_x}$$

Hluti velfa:

$$\vec{E} \times \vec{B} = \begin{bmatrix} E_y B_z - E_z B_y \\ E_z B_x - E_x B_z \\ E_x B_y - E_y B_x \end{bmatrix} = \begin{bmatrix} E_y B_z - E_z B_y \\ 0 \\ 0 \end{bmatrix}$$

$$= \left[ \frac{E_{0y}^2}{c_0} \sin^2(kx - \omega t) + \frac{E_{0z}^2}{c_0} \sin^2(kx - \omega t + \delta_{E,z}) \right] \hat{e}_x$$

$$= \frac{E^2}{c_0} \hat{e}_x$$

$$\Rightarrow \boxed{\vec{S} \equiv \vec{E} \times \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0 c_0} \frac{E^2}{c_0} \hat{e}_x = \vec{j}_{EM}} \quad (\text{Poytingov vektor})$$