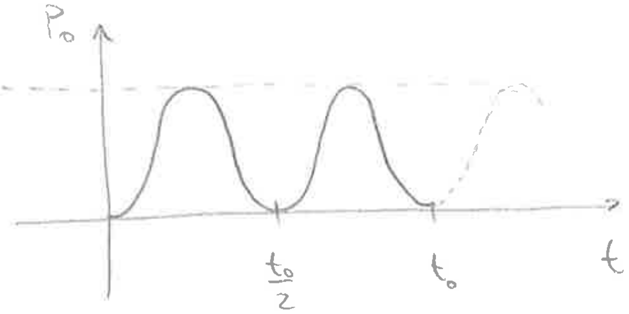
$$\left.\begin{aligned} U &= U_0 \sin(\omega t) \\ I &= I_0 \sin(\omega t) \end{aligned}\right\} \Rightarrow P = \underbrace{U_0 I_0}_{P_0} \sin^2(\omega t)$$



Povprečna moč $\bar{P}$ na intervalu $[0, t_0]$:

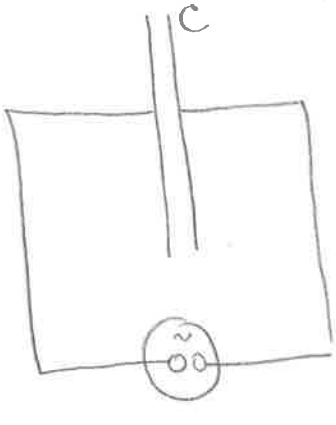
$$\bar{P} = \frac{1}{t_0}\int_0^{t_0} P\,dt = U_0 I_0 \underbrace{\frac{1}{t_0}\int_0^{t_0} \sin^2(\omega t)\,dt}_{\parallel} = \frac{U_0 I_0}{2}$$

$1/2$ (povprečna vrednost $\sin^2(\omega t)$ na $[0, t_0]$; D.N.)

Def: $U_{ef} \equiv \frac{U_0}{\sqrt{2}}$, $I_{ef} \equiv \frac{I_0}{\sqrt{2}}$ $\Rightarrow$ $\boxed{\bar{P} = U_{ef} I_{ef}}$

Primer: $U_0 = 310\ \mathrm{V} \Rightarrow U_{ef} = \frac{U_0}{\sqrt{2}} = 220\ \mathrm{V}$

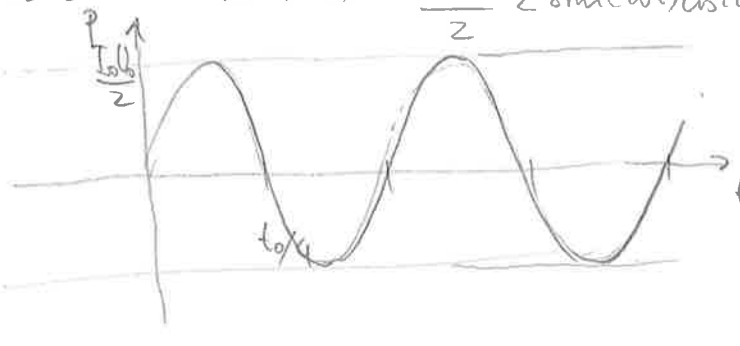
Primer:



$$U = U_0 \sin(\omega t)$$

$$I = I_0 \sin(\omega t - \underbrace{\delta_C}_{= -\frac{\pi}{2}}) = I_0 \sin(\omega t + \tfrac{\pi}{2}) = I_0 \cos(\omega t)$$

$$\Rightarrow P_g = I_0 U_0 \sin(\omega t)\cos(\omega t) = \frac{I_0 U_0}{2}\, 2\sin(\omega t)\cos(\omega t) = \frac{I_0 U_0}{2}\sin(2\omega t)$$



$$\boxed{\bar{P} = 0}$$

Splošni primer: $z = |z| e^{i\delta}$

$$U_g = U_0 \sin(\omega t)$$

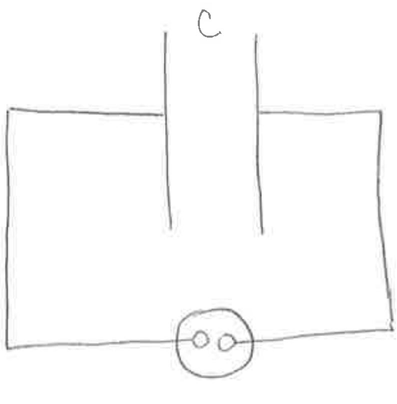
$$I = I_0 \sin(\omega t - \delta) \qquad \left(I_0 = \frac{U_0}{|z|} \right)$$

$$\Rightarrow P_g = UI = U_0 I_0 \sin(\omega t) \sin(\omega t - \delta)$$

$$\Rightarrow \boxed{\overline{P_g} = \frac{U_0 I_0}{2} \cos \delta}$$ (Domača naloga)

---

7 Energija kondenzatorja, električnega polja



$$U_g : U_g(t_1) \longrightarrow U_g(t_2)$$

$$\boxed{U_g + U_c' + \cancel{U_z'} = 0}$$

$$\boxed{U_c'(L \to D) = -\frac{q_c}{C}}$$ napetost na levi plošči

$$\boxed{I = \dot{q}_c}$$

$$\Rightarrow P_g = U_g I = -U_c' I = \frac{q_c}{C} \cdot \dot{q}_c = \left(\frac{q_c^2}{2C} \right)^{\cdot}$$

$$\Rightarrow dA_{el} = P_g \, dt$$

$$\Rightarrow A_{el} = \int_{t_1}^{t_2} P_g \, dt = \frac{q_c^2}{2C} \Big|_{t_1}^{t_2} = \frac{q_c^2(t_2)}{2C} - \frac{q_c^2(t_1)}{2C}$$

$$= \frac{C U_c'^2(t_2)}{2} - \frac{C U_c'^2(t_1)}{2}$$

$$\boxed{W_c \equiv \frac{1}{2} C U_c'^2} \Rightarrow \boxed{A_{el} = \Delta W_c}$$

$W_C = \frac{1}{2} C U_C'^2$

$= \frac{1}{2} \frac{\varepsilon_0 S}{l} E^2 l^2$

$= \frac{1}{2} S l \varepsilon_0 E^2$

$= \boxed{\frac{1}{2} V \varepsilon_0 E^2 = W_{el.\,polja}}$

prostornina med ploščama kondenzatorja

~ prostornina el. polja

$$\boxed{w_{el.\,polja} = \frac{W_{el.\,polja}}{V} = \frac{1}{2} \varepsilon_0 E^2}$$

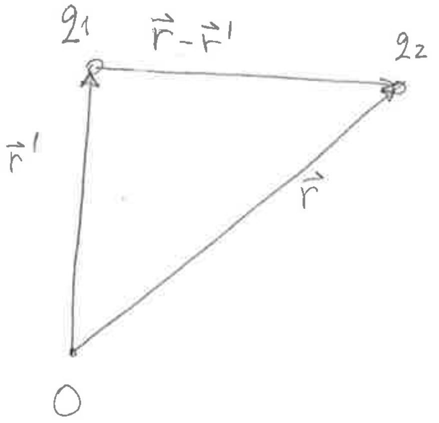
- Poleg energije ima polje tudi gibalno količino (nekaj malega o tem v nadaljevanju).

IV. MAGNETNO POLJE

- magnetostatika

① Magnetna sila, gostota magnetnega polja

Spomnimo se iz elektrostatike:



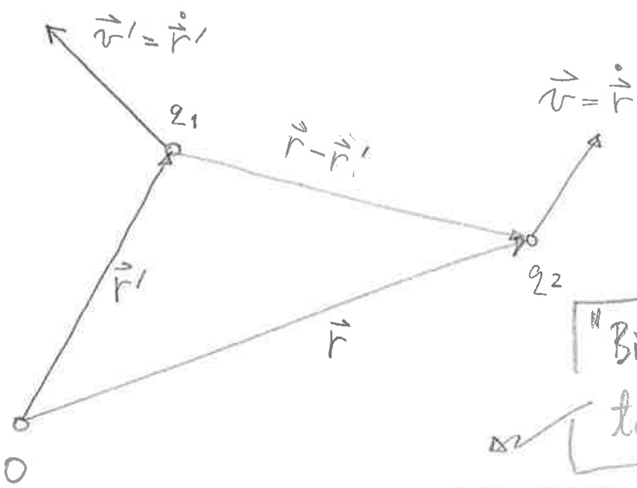
$\vec{F}_{el} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ (Coulombov zakon)

el. sila med točkastima mirujočima nabojema.

$\vec{E}_{\vec{r}'}(\vec{r}) = \frac{q_1(\vec{r}')}{4\pi\varepsilon_0} \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

$\Rightarrow \vec{F}_{el} = q_2(\vec{r}) \vec{E}_{\vec{r}'}(\vec{r})$

Dodatna (= magnetna) sila med gibajočimi naboji !?



"Biot-Savartov" zakon za točkaste naboje (zakon narave)

$$\boxed{\vec{F}_{mag}(q_1 \rightarrow q_2) = q_2 \vec{v} \times \left(\frac{\mu_0 q_1}{4\pi} \vec{v}\,' \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right)}$$

; $\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am$ indukcijska konstanta

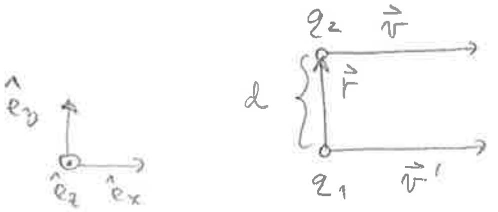
Vpeljemo: $\boxed{\vec{B}_{\vec{r}\,'}(\vec{r}) = \frac{\mu_0 q_1}{4\pi} \vec{v}\,' \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}}$ gostota magnetnega polja v točki $\vec{r}$ zaradi gibajočega se naboja q_1 v točki $\vec{r}\,'$

$$\boxed{[\vec{B}_{\vec{r}\,'}(\vec{r})] = \frac{Vs}{Am} \cdot As \cdot \frac{m}{s} \cdot \frac{1}{m^2} = \frac{Vs}{m^2} = T} \text{ (tesla)}$$

$$\Rightarrow [\vec{F}_{mag}(q_1 \rightarrow q_2)] = As\,\frac{m}{s}\,\frac{Vs}{m^2} = \frac{VAs}{m} = \frac{J}{m} = N \checkmark$$

$\Rightarrow \boxed{\vec{F}_{mag} = q_2 \vec{v} \times \vec{B}}$ magnetna sila na naboj q_2 s hitrostjo $\vec{v}$ v polju $\vec{B}$ naboja q_1

Primer: v sistemu S: $\vec{v} = \vec{v}\,' = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$, $\vec{r}\,' = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\vec{r} = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} = \vec{r} - \vec{r}\,'$



$$\Rightarrow \vec{v}\,' \times (\vec{r} - \vec{r}\,') = \begin{bmatrix} 0 \\ 0 \\ vd \end{bmatrix} \Rightarrow \vec{v} \times [\vec{v}\,' \times (\vec{r} - \vec{r}\,')] = \begin{bmatrix} 0 \\ -v^2 d \\ 0 \end{bmatrix}$$

$$\boxed{\vec F_{mag}(q_1\to q_2) = -\frac{\mu_0 q_1 q_2}{4\pi d^2} v^2 \hat e_y} \Rightarrow F_{mag}(q_1\to q_2) = |\vec F_{mag}(q_1\to q_2)| = \frac{\mu_0 |q_1 q_2|}{4\pi d^2} v^2$$

Za primerjavo:

$$F_{el}(q_1\to q_2) = \frac{|q_1 q_2|}{4\pi\varepsilon_0 d^2}$$

$$\Rightarrow \boxed{\frac{F_{mag}}{F_{el}} = \varepsilon_0\mu_0 v^2 = \frac{8{,}85\cdot10^{-12}\,As}{Vm}\cdot\frac{4\pi\cdot10^{-7}\,Vs}{Am}\cdot v^2 = 8{,}85\cdot4\pi\cdot10^{-19}\,\frac{s^2}{m^2}\,v^2 = \frac{v^2}{\underbrace{9\cdot10^{16}\,m^2/s^2}_{c_0^2}}}$$

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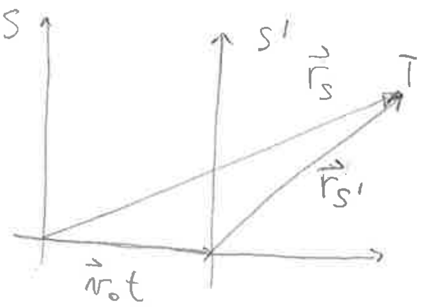
1. težava: sistem S', ki se s hitrostjo $\vec v_0 = \vec v$ giblje glede na S



$\vec r_{S'}$ točke T v S'

$\vec r_S = \vec v_0 t + \vec r_{S'} \Rightarrow \vec r_{S'} = \vec r_S - \vec v_0 t$

$t = t'$ (čas v obeh sistemih teče enako hitro)

Galilejeve transformacije

$\Rightarrow \vec v_{S'}$: hitrost točke T v sistemu S'

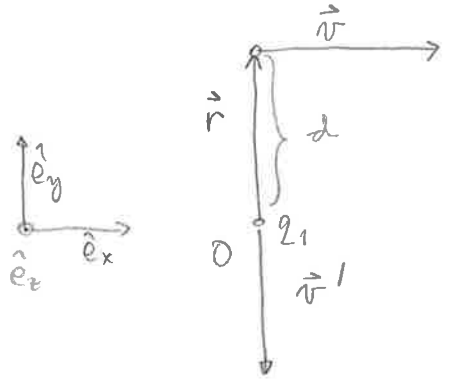
$$\vec v_{S'} = \frac{d\vec r_{S'}}{dt'} = \frac{d\vec r_{S'}}{dt} = (\vec r_S - \vec v_0 t)^{\cdot} = \dot{\vec r} - \vec v_0 = \vec v - \vec v = 0$$

$\Rightarrow$ v sistemu S' hitrost obeh nabojev 0 $\Rightarrow \boxed{\vec F'_{mag}(q_1\to q_2) = 0}$

$\Rightarrow$ (magnetna) sila med telesoma odvisna od tega, iz katerega sistema telesi opazujemo (?!?)

Rešitev: posebna teorija relativnosti; Lorentzove transformacije (Moderna fizika?)

Primer: $\vec r\,' = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $\vec r = \begin{bmatrix}0\\d\\0\end{bmatrix}$, $\vec v = \begin{bmatrix}v\\0\\0\end{bmatrix}$, $\vec v\,' = \begin{bmatrix}0\\-v\\0\end{bmatrix}$; $\vec r - \vec r\,' = \begin{bmatrix}0\\d\\0\end{bmatrix} = -(\vec r\,' - \vec r)$



a) $\vec B_{\vec r\,'}(\vec r) = \frac{\mu_0}{4\pi} q_1 \, \vec v\,' \times \frac{(\vec r - \vec r\,')}{|\vec r - \vec r\,'|^3} = 0 \Rightarrow \vec F_m(q_1 \to q_2) = 0$

b) $B_{\vec r}(\vec r\,') = \frac{\mu_0}{4\pi} q_2 \, \vec v \times \frac{(\vec r\,' - \vec r)}{|\vec r\,' - \vec r|^3} = -\frac{\mu_0}{4\pi} \frac{q_2 v}{d^2} \hat e_z$

$\vec v \times (\vec r - \vec r\,') = \begin{bmatrix}0\\0\\-vd\end{bmatrix}$

$\Rightarrow \vec F_m(q_2 \to q_1) = q_1 \vec v\,' \times B_{\vec r}(\vec r\,') =$

$= +\frac{\mu_0}{4\pi} \frac{q_1 q_2 v^2}{d^2} \hat e_x \quad (\neq 0)$

Težava:

$\vec F_m(q_1 \to q_2) \neq -F_m(q_2 \to q_1)$ (!?) (Ne velja 3. Newtonov zakon!?!)

$\int \vec F_m(q_1 \to q_2)\,dt = \Delta \vec G_2 = 0 = \vec G_2' - \vec G_2$

$\int \vec F_m(q_2 \to q_1)\,dt = \Delta \vec G_1 = \vec G_1' - \vec G_1 \neq 0$

$\Rightarrow \Delta \vec G = \Delta \vec G_1 + \Delta \vec G_2 = \vec G_1' + \vec G_2' - (\vec G_1 + \vec G_2) = \vec G' - \vec G \neq 0$

$\Rightarrow \boxed{\vec G' \neq \vec G}$

$\vec G$ izoliranega sistema se ne ohranja (!?)

Rešitev: $\vec G$ izoliranega sistema se ohranja, če prištejemo še gibalno količino EM polja (FTP, Moderna fizika)

Sestavljena gibajoča se nabita telesa: superpozicija "točkastih" teles:

$dq = \rho_e' dV'$; $dq\,\vec v' = \rho_e' \vec v' dV = \vec j_e' dV'$; dV' okolica točke $\vec r'$

$$d\vec B = \frac{\mu_0}{4\pi} \vec j_e' \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} dV'$$

← Načelo superpozicije (podobno kot pri elektrostatiki)

$$\Rightarrow \boxed{\vec B = \int_{V'} \frac{\mu_0}{4\pi} \vec j_e' \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} dV'}$$

(posplošeni Biot-Savartov zakon)*

Poseben primer: tok po tanki žici

$\vec j_e' = j_e' \hat e_j$

$dV' = dS' ds'$; $dS' \perp \vec j_e' \Rightarrow \int j_e' dS' = I$

$$\Rightarrow \vec B = \frac{\mu_0}{4\pi} \int \left\{ \int j_e' \hat e_j \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} dS' \right\} ds'$$

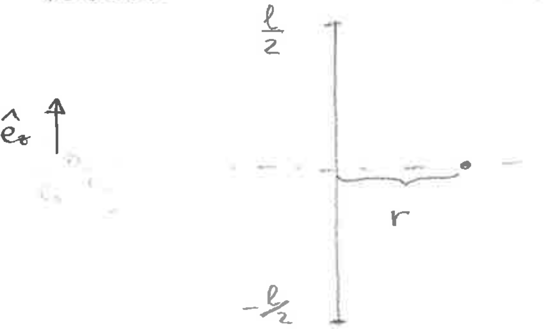
$$\simeq \frac{\mu_0}{4\pi} \int \left\{ \int j_e' dS' \right\} \hat e_j \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} ds' \quad \text{(tanka žica)}$$

$$= \boxed{\frac{\mu_0 I}{4\pi} \int d\vec s' \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} \; ; \; d\vec s' \equiv ds\,\hat e_j}$$

(Biot-Savartov zakon)*

* oba posledici (izreka) zakona o magnetnem polju v okolici gibajočega se točkastega naboja in principa superpozicije.

Primer: $\vec B$ na simetrali dolgega ravnega vodnika s tokom



Cilindrični sistem: $\vec r = r\hat e_\rho$, $\vec r' = z'\hat e_z$ $\Rightarrow \vec r - \vec r' = r\hat e_\rho - z'\hat e_z$

$d\vec s' = dz'\hat e_z$

$$\Rightarrow |\vec{r}-\vec{r}'|^2 = r^2+z'^2$$

$$\Rightarrow |\vec{r}-\vec{r}'|^3 = (r^2+z'^2)^{3/2}$$

$$\hat{e}_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \hat{e}_\varrho = \begin{bmatrix} \cos\phi \\ \sin\phi \\ 0 \end{bmatrix}, \quad \hat{e}_\phi = \begin{bmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{bmatrix} \quad ; \quad \hat{e}_z \times \hat{e}_\varrho = \hat{e}_\phi$$

$$\Rightarrow \quad \hat{e}_z \times (\vec{r}-\vec{r}') = \hat{e}_z \times (r\hat{e}_\varrho - z'\hat{e}_z) = r\hat{e}_\phi$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I r}{4\pi}\hat{e}_\phi \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{dz'}{(r^2+z'^2)^{3/2}} = \frac{\mu_0 I r}{2\pi}\hat{e}_\phi \int_0^{l/2} \frac{dz'}{(r^2+z'^2)^{3/2}}$$

$$\int \frac{dz'}{(r^2+z'^2)^{3/2}} = \frac{1}{r^2}\frac{z'}{(r^2+z'^2)^{1/2}} + C \quad (\text{preveri})$$

$$\Rightarrow \int_0^{l/2} \frac{dz'}{(r^2+z'^2)^{3/2}} = \frac{l}{2r^2}\frac{1}{(r^2+l^2/4)^{1/2}} \overset{l \gg r}{\approx} \frac{1}{r^2}$$

$$\Rightarrow \boxed{\vec{B} \simeq \frac{\mu_0 I}{2\pi r}\hat{e}_\phi}$$

Za dani primer izračunamo integral

$$\oint_{\partial S} \vec{B}\cdot d\vec{s}$$

po poljubni zaključeni zanki ∂S (robu ploskve S) v ravnini $z=0$:

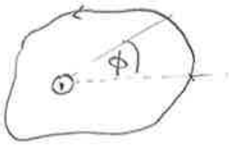
$$\oint_{\partial S} \vec{B}\cdot d\vec{s} = \int_{t_1}^{t_2} \vec{B}\cdot\dot{\vec{r}}\, dt$$

$$\vec{r} = r\hat{e}_\varrho \Rightarrow \dot{\vec{r}} = \dot{r}\hat{e}_\varrho + r\dot{\phi}\hat{e}_\phi$$

$$\Rightarrow \vec{B}\cdot\dot{\vec{r}} = \frac{\mu_0 I}{2\pi r}\hat{e}_\phi\cdot(\dot{r}\hat{e}_\varrho + r\dot{\phi}\hat{e}_\phi) = \frac{\mu_0 I}{2\pi}\dot{\phi}$$

$$\Rightarrow \oint_{\partial S} \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi}\int_{t_1}^{t_2} \dot{\phi}\, dt = \frac{\mu_0 I}{2\pi}\left[\phi(t_2) - \phi(t_1)\right]$$

a)



$$\phi(t_2)-\phi(t_1)=2\pi \Rightarrow \boxed{\oint_{\partial S}\vec{B}\cdot d\vec{s}=\mu_0 I}$$

b)

$$\phi(t_2)-\phi(t_1)=0 \Rightarrow \boxed{\oint_{\partial S}\vec{B}\cdot d\vec{s}=0}$$

zaobjeti tok

$$\Rightarrow \boxed{\oint_{\partial S}\vec{B}\cdot d\vec{s}=\mu_0 I}$$

(Poseben primer izreka o magnetni napetosti (Ampèrovega izreka))

Posplošitev: $I=\int_S \vec{j}_e\, dS$ (zaobjeti tok)

$$\Rightarrow \boxed{\oint_{\partial S}\vec{B}\cdot d\vec{s}=\mu_0\int_S \vec{j}_e\cdot d\vec{S}}$$

Izrek o magnetni napetosti v integralni obliki.

Stokesov izrek: $\oint_{\partial S}\vec{B}\cdot d\vec{s}=\int_S(\nabla\times\vec{B})\cdot d\vec{S}$

$$\Rightarrow \int_S(\nabla\times\vec{B})\cdot d\vec{S}=\mu_0\int_S \vec{j}_e\cdot d\vec{S}\ ;\ \forall S$$

$$\Rightarrow \boxed{\nabla\times\vec{B}=\mu_0\vec{j}_e}$$

Izrek o magnetni napetosti v diferencialni obliki.