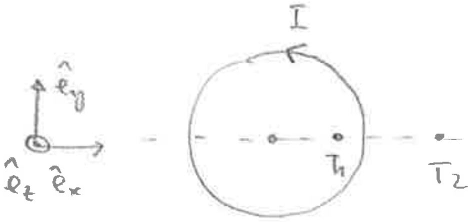
Primer: $\vec{B}$ v okolici dolge tuljave ($l >> R$)

a) $\vec{B}$ v okolici zanke

- zanka v ravnini, vzporedni z ravnino x-y ($\hat{e}_n = \hat{e}_z$); z'; R = polmer zanke
- točka T v ravnini x-y, na osi x, za D oddaljena od O ($D > R$: izven zanke; $D < R$: v zanki)



$$\vec{r}\,' = R\hat{e}_\rho + z'\hat{e}_z$$

$$\vec{r} = D\hat{e}_x = D\cos\phi\,\hat{e}_\rho - D\sin\phi\,\hat{e}_\phi$$

$$\Rightarrow \boxed{\vec{r} - \vec{r}\,' = (D\cos\phi - R)\hat{e}_\rho - z'\hat{e}_z - D\sin\phi\,\hat{e}_\phi}$$

$$\Rightarrow |\vec{r} - \vec{r}\,'|^2 = D^2\cos^2\phi + R^2 - 2DR\cos\phi + z'^2 + D^2\sin^2\phi$$

$$= D^2 + R^2 - 2DR\cos\phi + z'^2$$

$$\Rightarrow \boxed{|\vec{r} - \vec{r}\,'|^3 = \left[D^2 + R^2 - 2DR\cos\phi + z'^2\right]^{3/2}}$$

$$\boxed{d\vec{s} = R\,d\phi\,\hat{e}_\phi} \Rightarrow \boxed{d\vec{s} \times (\vec{r} - \vec{r}\,') = \{R(D\cos\phi - R)\underbrace{\hat{e}_\phi \times \hat{e}_\rho}_{-\hat{e}_z} - Rz'\underbrace{\hat{e}_\phi \times \hat{e}_z}_{\hat{e}_\rho}\}d\phi}$$

$$\boxed{= \{R(R - D\cos\phi)\hat{e}_z - Rz'\hat{e}_\rho\}d\phi}$$

$$\Rightarrow \boxed{d\vec{B} = \frac{\mu_0 I}{4\pi}\, d\vec{s} \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|}}$$

$$\boxed{= \frac{\mu_0 I}{4\pi}\, \frac{\{R(R - D\cos\phi)\hat{e}_z - Rz'\hat{e}_\rho\}\,d\phi}{\left[D^2 + R^2 - 2DR\cos\phi + z'^2\right]^{3/2}}}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{\{R(R - D\cos\phi)\hat{e}_z - Rz'\hat{e}_\rho\}\,d\phi}{[\ \dots\ ]^{3/2}}}$$

---

b) Tuljava: N zank, ena nad drugo

- gosto navita tuljava $\Rightarrow$ zvezna aproksimacija, $\boxed{dI = I\,\frac{N}{l}\,dz'}$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi} \overbrace{I \frac{N}{l}\, dz'}^{dI} \int_0^{2\pi} \frac{\{\ldots\}}{[\ldots]^{3/2}}\, d\phi$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \frac{N}{l} \int_{-l/2}^{l/2} \left(\int_0^{2\pi} \frac{\{\ldots\}}{[\ldots]^{3/2}}\, d\phi \right) dz'$$

$$= \frac{\mu_0 I}{4\pi} \frac{N}{l} \int_0^{2\pi} \left(\int_{-l/2}^{l/2} \frac{\{\ldots\}}{[\ldots]^{3/2}}\, dz' \right) d\phi$$ (zamenjamo vrstni red integriranja)

$$\int_{-l/2}^{l/2} \frac{\{\ldots\}}{[\ldots]^{3/2}}\, dz' = R(R - D\cos\phi)\, \hat{e}_z \int_{-l/2}^{l/2} \frac{dz'}{[D^2+R^2-2DR\cos\phi+z'^2]^{3/2}} - R\hat{e}_\varrho \int_{-l/2}^{l/2} \frac{z'\, dz'}{[D^2+R^2-2DR\cos\phi+z'^2]^{3/2}}$$

Drugi integral je 0 (integral lihe funkcije na simetričnem intervalu), prvi integral pa je integral sode funkcije na simetričnem intervalu

$$\Rightarrow \int_{-l/2}^{l/2} \frac{dz'}{[D^2+R^2-2DR\cos\phi+z'^2]^{3/2}} = 2 \int_0^{l/2} \frac{dz'}{[D^2+R^2-2DR\cos\phi+z'^2]^{3/2}}$$

$$= \frac{2}{D^2+R^2-2DR\cos\phi} \cdot \left. \frac{z'}{[D^2+R^2-2DR\cos\phi+z'^2]^{1/2}} \right|_0^{l/2}$$

$$= \frac{2}{D^2+R^2-2DR\cos\phi} \cdot \frac{l}{2} \frac{1}{[D^2+R^2-2DR\cos\phi+l^2/4]^{1/2}}$$

$l \gg R, D \Rightarrow [D^2+R^2-2DR\cos\phi+l^2/4]^{1/2} \simeq l/2$

$$\Rightarrow \boxed{\int_{-l/2}^{l/2} \frac{dz'}{[D^2+R^2-2DR\cos\phi+z'^2]^{3/2}} \simeq \frac{2}{D^2+R^2-2DR\cos\phi}}$$

$$\Rightarrow \boxed{\vec{B} \simeq \frac{\mu_0 I}{2\pi} \frac{N}{l} \hat{e}_z \int_0^{2\pi} \frac{R(R-D\cos\phi)\, d\phi}{D^2+R^2-2DR\cos\phi} = \frac{\mu_0 I}{2\pi} \frac{N}{l} \hat{e}_z \int_0^{2\pi} \frac{1-a\cos\phi}{a^2+1-2a\cos\phi}\, d\phi \;;\; a = \frac{D}{R}}$$

$$\boxed{a = \begin{cases} >1 \;;\; D>R \\ <1 \;;\; D<R \end{cases}}$$

$\cos\phi$ periodična funkcija na intervalu $[0, 2\pi]$ $\Rightarrow$ integrand tudi periodična funkcija na intervalu $[0, 2\pi]$ $\Rightarrow$ meje integriranja lahko premaknemo iz $[0, 2\pi]$ na $[-\pi, \pi]$, ne da bi se integral spremenil (še vedno integriramo celotno periodo).

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi}\frac{N}{l}\hat{e}_z \int_{-\pi}^{\pi} \frac{1 - a\cos\phi}{a^2 + 1 - 2a\cos\phi} d\phi}$$

Nova spremenljivka: $t = tg(\frac{\phi}{2})$ (Razlog za translacijo $[0, 2\pi] \Rightarrow [-\pi, \pi]$: t je na $[-\pi, \pi]$ zvezna)

$$\Rightarrow \boxed{\cos\phi = \frac{1 - t^2}{1 + t^2}}, \quad \boxed{d\phi = \frac{2\,dt}{1 + t^2}}$$

$$\Rightarrow 1 - a\cos\phi = 1 - a\frac{1 - t^2}{1 + t^2}, \quad 1 + a^2 - 2a\cos\phi = 1 + a^2 - 2a\frac{1 - t^2}{1 + t^2}$$

$$\Rightarrow \boxed{\frac{1 - a\cos\phi}{1 + a^2 - 2a\cos\phi}} = \frac{1 - a\frac{1 - t^2}{1 + t^2}}{1 + a^2 - 2a\frac{1 - t^2}{1 + t^2}} = \frac{1 + t^2 - a(1 - t^2)}{(1 + a^2)(1 + t^2) - 2a(1 - t^2)}$$

$$= \frac{(1 - a) + (1 + a)t^2}{1 + a^2 - 2a + (1 + a^2 + 2a)t^2} = \frac{(1 - a) + (1 + a)t^2}{(1 - a)^2 + (1 + a)^2 t^2} = \boxed{\frac{1}{1 - a}\frac{1 + bt^2}{1 + b^2 t^2}}$$

$$\boxed{b = \frac{1 + a}{1 - a} = \begin{cases} < 0; & a > 1 \ (D > R) \\ > 0; & a < 1 \ (D < R) \end{cases}}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{\pi}\frac{N}{l}\hat{e}_z \frac{1}{1 - a}\int_{-\infty}^{\infty} \frac{1 + bt^2}{1 + b^2 t^2}\frac{dt}{(1 + t^2)}}$$

(preveri za d.n.)

$$\boxed{= \frac{\mu_0 I}{\pi}\frac{N}{l}\hat{e}_z \frac{1}{1 - a}\left[\frac{atan(bt) + atan(t)}{1 + b}\right]_{-\infty}^{\infty}}$$

$$(1 - a)(1 + b) = (1 - a)\left(1 + \frac{(1 + a)}{1 - a}\right) = (1 - a)\frac{(1 - a + 1 + a)}{(1 - a)} = 2$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi}\frac{N}{l}\hat{e}_z \Big[\operatorname{atan}(bt) + \operatorname{atan}(t)\Big]_{-\infty}^{\infty}}$$

• $b > 0$ $(D < R)$: $\quad b\infty = \infty$, $b(-\infty) = -\infty$

$$\Rightarrow \boxed{\vec{B}(\vec{r})} = \frac{\mu_0 I}{2\pi}\frac{N}{l}\hat{e}_z \underbrace{\Big[2\overbrace{\operatorname{atan}(\infty)}^{\pi/2} - 2\overbrace{\operatorname{atan}(-\infty)}^{(-\pi/2)}\Big]}_{2\pi}$$

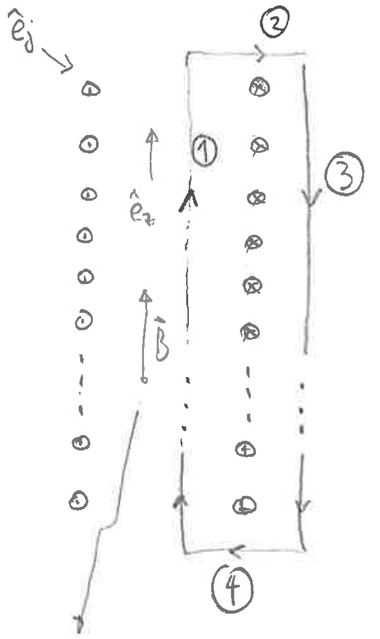
$$\boxed{= \mu_0 I \frac{N}{l}\hat{e}_z}$$ (= homogeno $\vec{B}$ znotraj tuljave)

• $b < 0$ $(D > R)$: $\quad b\infty = -\infty$, $b(-\infty) = \infty$

$$\Rightarrow \boxed{\vec{B}(\vec{r})} = \frac{\mu_0 I}{2\pi}\frac{N}{l}\hat{e}_z \Big[\overbrace{\operatorname{atan}(-\infty) + \operatorname{atan}(\infty)}^{=0} - \big\{\overbrace{\operatorname{atan}(\infty) + \operatorname{atan}(-\infty)}^{=0}\big\}\Big]$$

$$\boxed{= 0}$$ (zanemarljivo $\vec{B}$ zunaj dolge tuljave)

Opomba: Ampère-ov izrek za zgornji primer;



gostotnice (pravilo desnega vijaka)

$$\oint_C \vec{B}\cdot d\vec{s} = \int_{C_1}\vec{B}_1\cdot d\vec{s} + \int_{C_2}\vec{B}_2\cdot d\vec{s} + \int_{C_3}\vec{B}_3\cdot d\vec{s} + \int_{C_4}\vec{B}_4\cdot d\vec{s}$$

$$\vec{B}_3 = 0 \Rightarrow \int_{C_3}\vec{B}_3\cdot d\vec{s} = 0$$

C_2 in C_4 zelo kratki (v primerjavi z l)

$$\Rightarrow \int_{C_2}\vec{B}_2\cdot d\vec{s} \text{, } \int_{C_4}\vec{B}_4\cdot d\vec{s} \simeq 0$$

$$\int_{C_1}\vec{B}_1\cdot d\vec{s} \equiv \int_{t_1}^{t_2}\vec{B}_1\cdot\vec{v}_1\, dt = \int_{t_1}^{t_2}\vec{B}_1\cdot \dot{z}\hat{e}_z\, dt =$$

$$= \mu_0 I \frac{N}{l}\int_{t_1}^{t_2}\dot{z}\,\underbrace{\hat{e}_z\cdot\hat{e}_z}_{1}\, dt = \ldots$$

$$\ldots = \mu_0 I \frac{N}{l} \left(\underbrace{z(t_2)}_{l/2} - \underbrace{z(t_1)}_{-l/2} \right) = \mu_0 I$$

$$\Rightarrow \boxed{\oint_C \vec{B} \cdot d\vec{s} = \int_{C_1} \vec{B}_1 \cdot d\vec{s} = \mu_0 I} \checkmark$$

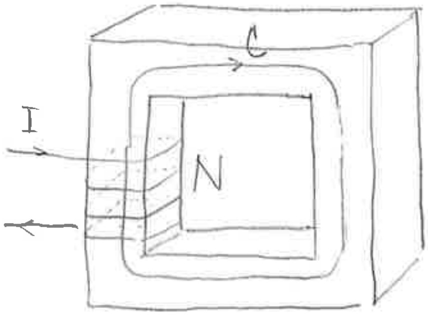
Jedro (železno) v tuljavi:

$\vec{B} \longrightarrow \boxed{\vec{B}' = \mu \vec{B}}$; μ = permeabilnost (železa)

$[\mu] = /$

$\mu_{Fe} \simeq 3000$ (feromagnet)

Navitje na posebnem jedru (transformator, glej tudi nadaljevanje)



- železno jedro dolžine l
- Ampère-ov izrek:

$$\oint_C \vec{B} \cdot d\vec{s} = \mu \mu_0 N I$$

- $B \simeq$ konst., $\vec{B} \parallel d\vec{s} \Rightarrow \oint_C \vec{B} \cdot d\vec{s} = \int_{t_1}^{t_2} \vec{B} \cdot \vec{v}\, dt$; $\vec{v} = \frac{d\vec{s}}{dt}$

$$= \int_{t_1}^{t_2} B v\, dt$$

$$= B \int_{t_1}^{t_2} \frac{ds}{dt} dt$$

$$= B \left(\underbrace{s(t_2)}_{l} - \underbrace{s(t_1)}_{0} \right)$$

$$= B l$$

$$\Rightarrow \boxed{B = \mu \mu_0 I \frac{N}{l}}$$

dolžina jedra

• Poskus: katodna cev, $\vec{F}_{el}$ in $\vec{F}_{mag}$ na nabite delce (e^-) v zunanjem $\vec{E}$ in $\vec{B}$

Magnetna sila na vodnik s tokom

• $\vec{B}$: zunanje magnetno polje

$$\vec{F} = q\vec{v}\times\vec{B} \rightarrow d^3\vec{F} = d^3q\,\vec{v}\times\vec{B} \quad ; \quad d^3q = \rho_e\,dV = q_1 n\,dV$$

$$\Rightarrow \boxed{d^3\vec{F}} = q_1 n\vec{v}\times\vec{B}\,dV$$

$$= \vec{j}_e\times\vec{B}\,dV$$

$$= \boxed{j_e\,\hat{e}_j\times\vec{B}\,dV}$$

$$dV = dS\,ds \quad ; \quad dS \perp \hat{e}_j \;,\; ds \parallel \hat{e}_j$$

$$\Rightarrow \vec{F} = \int_C \left\{ \int_S j_e\,\hat{e}_j\times\vec{B}\,dS \right\} ds$$

• tanka žica : $\vec{B} \sim$ konst. na celotnem preseku žice

$$\Rightarrow \boxed{\vec{F}} = \int_C \overbrace{\left\{ \int_S j_e\,dS \right\}}^{I} \hat{e}_j\times\vec{B}\,ds$$

$$= \int_C I\,d\vec{s}\times\vec{B} \qquad ; \quad d\vec{s} = ds\,\hat{e}_j$$

$$= \boxed{I\int_C d\vec{s}\times\vec{B}} \qquad ; \quad I = \text{konst. (1. Kirchoffov izrek)}$$

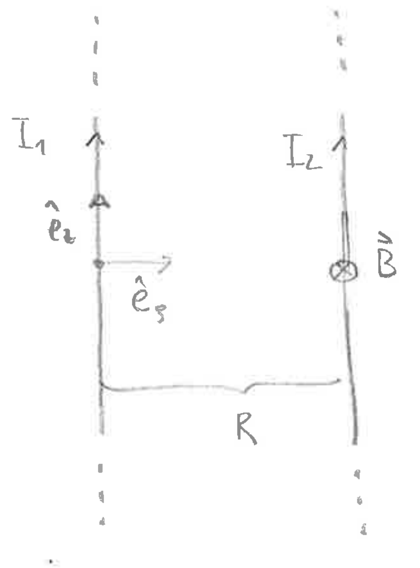
Primer: $\vec{B}$ = konst., C = ravna črta

$$\Rightarrow \boxed{\vec{F}} = I\int_C d\vec{s}\times\vec{B} = I\left\{\int_C d\vec{s}\right\}\times\vec{B} = \boxed{I\vec{l}\times\vec{B}} \quad ; \quad |\vec{l}| = l = \text{dolžina vodnika v } \vec{B}$$

$$\vec{l} \parallel \hat{e}_j$$

Primer: dva dolga vzporedna ravna vodnika na razdalji $R = 1\,m$, po katerih tečeta tokova $I_1 = I_2 = 1\,A$

• naj bosta tokova v isto smer



$\vec{B}$: magnetno polje 1. vodnika na mestu 2. vodnika

$$\vec{B} = \frac{\mu_0 I_1}{2\pi R}\hat{e}_\phi = \frac{\mu_0 I}{2\pi R}\hat{e}_\phi$$

$$\vec{l} = l\,\hat{e}_{j_2} = l\,\hat{e}_z$$

$$\Rightarrow \boxed{\vec{F}_{2\to 1}} = I_2 \vec{l} \times \vec{B}$$

$$= I\,l\,\hat{e}_z \times \frac{\mu_0 I}{2\pi R}\hat{e}_\phi$$

$$= \frac{\mu_0 I^2}{2\pi R}\, l\, \underbrace{\hat{e}_z \times \hat{e}_\phi}$$

$$= \boxed{\frac{\mu_0 I^2}{2\pi}\frac{l}{R}(-\hat{e}_\varrho)}$$ ⇐ privlačna sila

$$\boxed{F_{2\to 1}} = \frac{\mu_0 I^2}{2\pi}\frac{l}{R}$$

$$= \frac{4\pi\cdot 10^{-7}\,Vs \cdot 1A^2 \cdot 1m}{Am \cdot 2\pi \cdot 1m} \quad ; \; l = 1m$$

$$= 2\cdot 10^{-7}\frac{VAs}{m} = \boxed{2\cdot 10^{-7}\,N}$$

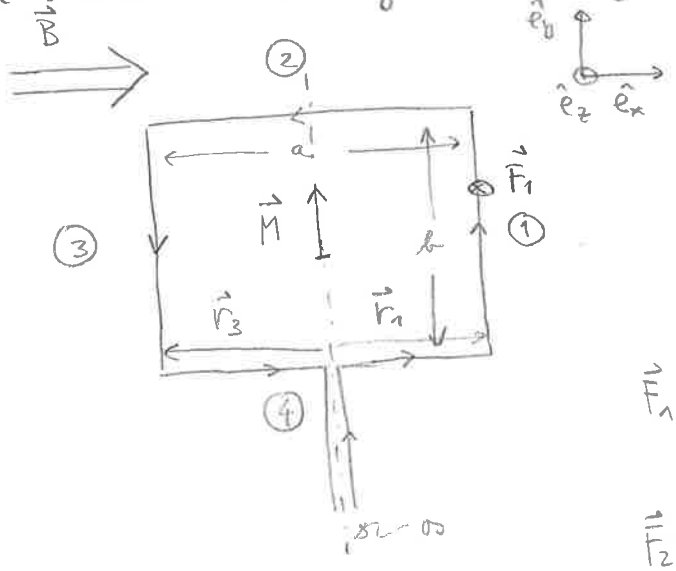
• če tokova v nasprotno smer ⇒ sila odbojna

Definicija enote A (amper): dva zelo dolga vzporedna ravna vodnika na razdalji $R = 1\,m$, po katerih tečeta enaka tokova $I_1 = I_2 = I$. Če je sila 1. vodnika na $l = 1\,m$ drugega vodnika $2\cdot 10^{-7}\,N \Rightarrow I = 1\,A$.

• Poskus

② Navor magnetnih sil, magnetni dipolni moment

• Zanka s tokom v (homogenem) zunanjem magnetnem polju:



$$\vec{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} = B\hat{e}_x$$

$$\vec{l}_1 = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \vec{l}_2 = \begin{bmatrix} -a \\ 0 \\ 0 \end{bmatrix}, \vec{l}_3 = \begin{bmatrix} 0 \\ -b \\ 0 \end{bmatrix}, \vec{l}_4 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{F}_1 = I\vec{l}_1 \times \vec{B} = \begin{bmatrix} 0 \\ 0 \\ -IbB \end{bmatrix}, \quad \vec{F}_3 = I\vec{l}_3 \times \vec{B} = \begin{bmatrix} 0 \\ 0 \\ IbB \end{bmatrix}$$

$$\vec{F}_2 = \vec{F}_4 = \vec{0}$$

$$\vec{r}_1 = \begin{bmatrix} \frac{a}{2} \\ 0 \\ 0 \end{bmatrix}, \vec{r}_3 = -\vec{r}_1 = \begin{bmatrix} -\frac{a}{2} \\ 0 \\ 0 \end{bmatrix}$$

$\vec{F}_1, \vec{F}_3$: dvojica sil ($\vec{F}_3 = -\vec{F}_1$)

$$\boxed{\vec{M}} = \vec{M}_1 + \vec{M}_3$$
$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_3 \times \vec{F}_3$$
$$= \vec{r}_1 \times \vec{F}_1 + (-\vec{r}_1) \times (-\vec{F}_1)$$
$$= 2\vec{r}_1 \times \vec{F}_1$$
$$= 2\begin{bmatrix} 0 \\ \frac{a}{2}IbB \\ \end{bmatrix} = \begin{bmatrix} 0 \\ IabB \\ 0 \end{bmatrix} = \boxed{ISB\hat{e}_y}$$

• 1 ovoj → N ovojev: $\boxed{\vec{M} = NISB\hat{e}_y}$

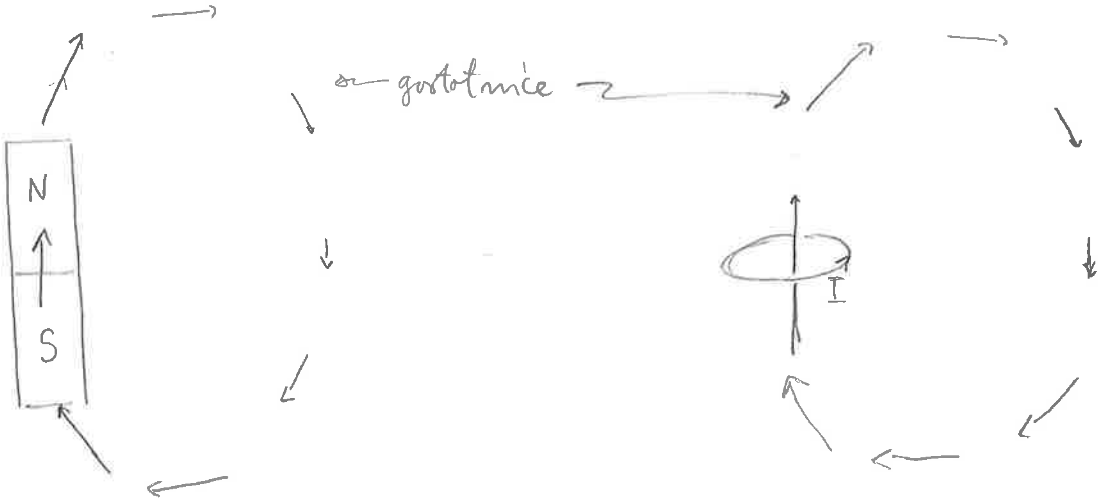
Def: $\vec{p}_m$ = magnetni dipolni moment zanke

$\boxed{\vec{p}_m = \underbrace{NIS}_{p_m}\hat{e}_s}$ $\quad \hat{e}_s$: ⊥ na ravnino zanke, tok in pravilo desnega vijaka

⇒ v našem primeru: $\boxed{\hat{e}_s = \hat{e}_z}$; $\boxed{[\vec{p}_m] = Am^2}$

$$\Rightarrow \boxed{\vec{p}_m \times \vec{B} = p_m B\,\hat{e}_s \times \hat{e}_x = p_m B\,\underbrace{\hat{e}_z \times \hat{e}_x}_{\hat{e}_y} = NISB\hat{e}_y = \vec{M}}$$

- Tudi trajni magnetni dipoli: atomi, elektroni, p, n
- Zemlja tudi magnetni dipol: severni zemljski pol (trenutno) približno sovpada z južnim magnetnim polom, južni zemljski pol pa približno sovpada s severnim magnetnim polom



Delo magnetnih sil

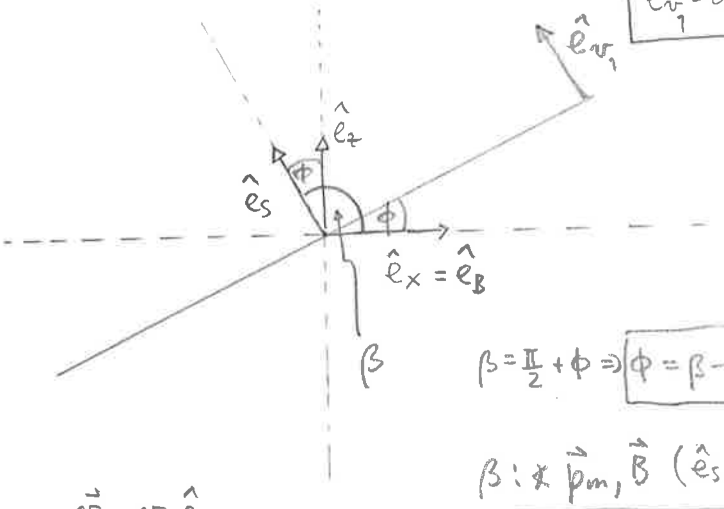
- Nazaj na prejšnji primer (N-ovojev)

$$\vec{F}_1 = NI\,\vec{l}_1 \times \vec{B} = NI\,b\,B\,\hat{e}_1 \times \hat{e}_B = NI\,b\,B\,\hat{e}_y \times \hat{e}_x = NI\,b\,B(-\hat{e}_z)$$

$$\vec{F}_2 = NI\,b\,B\,\hat{e}_z$$

stranski pogled

$$\boxed{\hat{e}_{v_1} = \cos\phi\,\hat{e}_z - \sin\phi\,\hat{e}_x}$$



$$\beta = \frac{\pi}{2} + \phi \Rightarrow \boxed{\phi = \beta - \frac{\pi}{2}}$$

$\beta : \sphericalangle\, \vec{p}_m, \vec{B}$ $(\hat{e}_s, \hat{e}_B)$

$$\Rightarrow \boxed{\vec{p}_m \cdot \vec{B} = p_m B \cos\beta}$$

$$\vec{v}_1 = v_1\,\hat{e}_{v_1} = \frac{a}{2}\dot{\phi}\,\hat{e}_{v_1}$$

$$\boxed{A_1} = \int_C \vec{F}_1 \cdot d\vec{s}$$

$$= \int_{t_1}^{t_2} \vec{F}_1 \cdot \vec{v}_1\,dt$$

$$= \int_{t_1}^{t_2} NI\,b\,B \cdot \frac{a}{2}\dot{\phi}(-\hat{e}_z) \cdot \hat{e}_v\,dt$$

$$= -\frac{1}{2} p_m B \int_{t_1}^{t_2} \dot{\phi}\cos\phi\,dt$$

$$= -\frac{1}{2} p_m B \int_{t_1}^{t_2} \dot{(\sin\phi)}\,dt$$

$$= -\frac{1}{2} p_m B (\sin\phi_2 - \sin\phi_1) \quad ; \quad \boxed{\phi_2 = \phi(t_2),\ \phi_1 = \phi(t_1)}$$

$A_2 = A_1$

$$\Rightarrow \boxed{A = A_1 + A_2 = -p_m B (\sin\phi_2 - \sin\phi_1)}$$

$$\boxed{\sin\phi = \sin\left(\beta - \frac{\pi}{2}\right) = \sin\beta\cos\left(\frac{\pi}{2}\right) - \cos\beta\sin\left(\frac{\pi}{2}\right) = -\cos\beta}$$

$$\Rightarrow \boxed{A = p_m B\cos\beta_2 - p_m B\cos\beta_1}$$ so $\vec{F}_1, \vec{F}_2$ konservativni sili

W_p = energija magnetnega dipola v homogenem zunanjem $\vec{B}$

$$\boxed{W_p \equiv -\vec{p}_m \cdot \vec{B} = -p_m B\cos\beta}$$

$$\Rightarrow \boxed{A = -W_p(2) - W_p(1) = -\Delta W_p}$$

Enako lahko definiramo energijo trajnega magnetnega dipola v zunanjem magnetnem polju.

Poskus: model preprostega elektromotorja (spreminjanje smeri toka)

---

③ Magnetni potencial

- Spomnimo se: el. potencial v okolici nabitega telesa Ω:

$$\phi(\vec{r}) = \int_\Omega \frac{\rho_e(\vec{r}\,')}{4\pi\varepsilon_0} \frac{1}{|\vec{r} - \vec{r}\,'|}\, dV'_\Omega \Rightarrow \vec{E}(\vec{r}) = -\nabla\phi, \ \ldots$$

- Podobno vpeljemo magnetni (vektorski) potencial $\vec{A}$:

$$\boxed{\vec{A}(\vec{r}) \equiv \frac{\mu_0}{4\pi} \int_\Omega \frac{\vec{j}_e(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\, dV'} \ ; \ \boxed{[\vec{A}] = \frac{Vs}{Am} \cdot \frac{A}{m^2} \frac{1}{m} m^3 = \frac{Vs}{m}}$$

Velja (preveri za domačo nalogo):

$$\nabla \times \frac{\vec{j}_e(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = -\vec{j}_e \times \nabla\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right),$$

in (glej poglavje o elektrostatiki):

$$\nabla\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = -\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$$\Rightarrow \boxed{\nabla \times \vec{A}} = \nabla \times \frac{\mu_0}{4\pi}\int_{\Omega_1} \frac{\vec{j}_e(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dV'$$

$$= \frac{\mu_0}{4\pi}\int_{\Omega_1} \nabla \times \frac{\vec{j}_e(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dV'$$

$$= -\frac{\mu_0}{4\pi}\int_{\Omega_1} \vec{j}_e(\vec{r}\,') \times \nabla\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) dV'$$

$$= \frac{\mu_0}{4\pi}\int_{\Omega_1} \vec{j}_e(\vec{r}\,') \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^{3/2}}\,dV'$$

$$\boxed{= \vec{B}(\vec{r})} \qquad \left([\nabla \times \vec{A}] = \frac{1}{m}\,\frac{Vs}{m} = \frac{Vs}{m^2} = T\,\checkmark \right)$$

---

④ Magnetni pretok in izrek o magnetnem pretoku

- Spomnimo se: $\Phi_E = \varepsilon_0 \int_S \vec{E}\cdot d\vec{S}$ in $\oint_{\partial V} \varepsilon_0 \vec{E}\cdot d\vec{S} = \int_V \rho_e \cdot dV$

Podobno definiramo magnetni pretok:

$$\boxed{\Phi_B = \int_S \vec{B}\cdot d\vec{S}} \; ; \; \boxed{[\Phi_B] = \frac{Vs}{m^2}\cdot m^2 = Vs}$$

Trdim: $\boxed{\nabla\cdot\vec{B}=0}$ (izrek o Φ_B v diferencialni obliki)

Rez: $\nabla\cdot\vec{B}=\nabla\cdot(\nabla\times\vec{A})=0$ (div rot $\vec{A}=0$ za $\forall\vec{A}$; preveri za d.n.)

Gaussov izrek

$$\Rightarrow \boxed{\oint_{\partial V}\vec{B}\cdot d\vec{S} = \int_V \nabla\cdot\vec{B}\,dV = 0}$$ (izrek o Φ_B v integralni obliki)

izrek o Φ_B v diferencialni obliki

Φ_B skozi zaključeno ploskev ∂V

Primerjava: V naravi ni magnetnih nabojev (monopolov)

$$\oint_{\partial V}\varepsilon_0\vec{E}\cdot d\vec{S} = q\ (\text{zaobjet})$$

$$\oint_{\partial V}\vec{B}\cdot d\vec{S} = 0$$

---

(5) Povzetek ((in primerjava) elektrostatike in magnetostatike