⇒ naboja q_p in q_e po absolutni vrednosti enaka, po predznaku pa nasprotna

$$\boxed{\begin{aligned} q_p &= +q_0 \doteq 1{,}602176634 \cdot 10^{-19}\ \text{As} \\ q_{e^-} &= -q_0 = -1{,}602\ldots \cdot 10^{-19}\ \text{As} \end{aligned}}$$

As : amper-sekunda (coulomb, C)

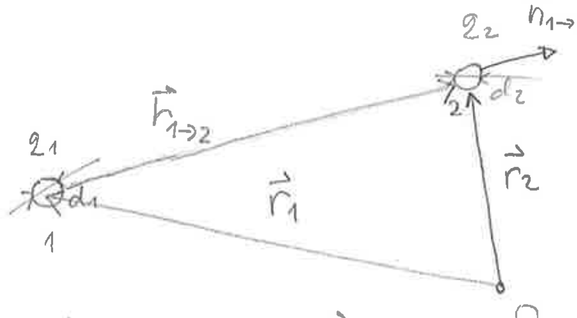
q_0 : osnovni naboj

$q = \pm N q_0 \quad (N \in \mathbb{N}_0)$; $N \to \infty \Rightarrow q$ je lahko obravnavamo kot zvezne.

⇒ telesa z nabojem enakega predznaka se odbijajo, različnega predznaka pa privlačijo (za primerjavo: gravitacija samo privlačna)

- $F_{el} \propto \frac{1}{r^2}$ (C. de Coulomb, 18. stol.)

Dve nabiti telesi:



$\vec{r}_{1\to2} = \vec{r}_2 - \vec{r}_1$

– telesi drobni: $d_1, d_2 \ll |\vec{r}_{1,2}| = r_{1\to2}$

– smerni vektor: $\vec{n}_{1\to2} = \frac{\vec{r}_{1\to2}}{r_{1\to2}}$

$$\boxed{\vec{F}_{el,1\to2} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{1\to2}^2}\,\vec{n}_{1\to2} = \frac{q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{4\pi\varepsilon_0 |\vec{r}_2 - \vec{r}_1|^3}}$$

Coulombov zakon

$$\boxed{\varepsilon_0 \doteq 8{,}85 \cdot 10^{-12} \frac{(\text{As})^2}{\text{Nm}^2}}$$

influenčna konstanta

$$\frac{Nm}{As} \equiv V \text{ (volt)} \quad \Rightarrow \quad \boxed{\varepsilon_0 \simeq 8{,}9 \cdot 10^{-12} \, \frac{As}{Vm}}$$

← Newtonov gravitacijski zakon

(Primerjava z $\vec{F}_g$: $\vec{F}_{g,1\to 2} = -\dfrac{\varkappa m_1 m_2}{r_{1\to 2}^2}\, \vec{n}_{1\to 2}$)

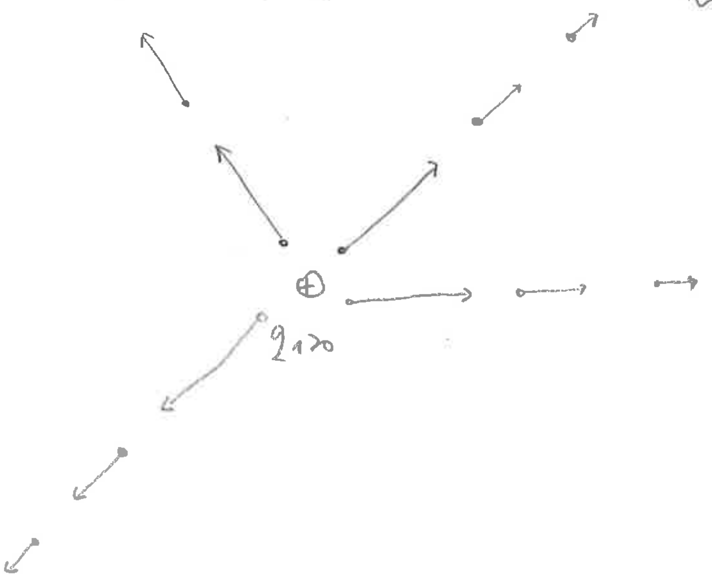
Pojem polja: q_1, ki ga postavimo v prazen prostor, spremeni lastnosti prostora v svoji okolici. Prostor s tako spremenjenimi lastnostmi imenujemo električno polje. Lastnosti prostora spremenjene: če vanj postavimo drug naboj, q_2, nanj deluje (električna) sila $\vec{F}_{el,1\to 2}$.

$\vec{E}_1(\vec{r}_2)_{1\to 2}$: jakost el. polja, ki ga ustvari q_1 (na mestu $\vec{r}_1$) na mestu $\vec{r}_2$

$$\boxed{\vec{E}_1(\vec{r}_2) \equiv \frac{\vec{F}_{el,1\to 2}}{q_2}} \qquad [\vec{E}_1] = \frac{N}{As} = \frac{Nm}{As\,m} = \frac{V}{m}$$

$$\Rightarrow \boxed{\vec{E}_1(\vec{r}_2) = \frac{q_1 \, \hat{e}_{1\to 2}}{4\pi\varepsilon_0 r_{1\to 2}^2} = \frac{q_1 (\vec{r}_2 - \vec{r}_1)}{4\pi\varepsilon_0 |\vec{r}_2 - \vec{r}_1|^3}} \quad ; \; q_1 \text{ izvor polja}$$

- Vektorsko polje, ponazorimo ga s silnicami



Zakon (načelo, predpostavka, ...) superpozicije

a) Diskretno porazdeljeni (točkasti) naboji

$\vec{E}_1(\vec{r})$: polje q_1 (na mestu $\vec{r}_1$) v točki T s krajevnim vektorjem $\vec{r}$

$\vec{E}_2(\vec{r})$: q_2 $\vec{r}_2$

⋮

$\vec{E}_i(\vec{r})$ q_i $\vec{r}_i$

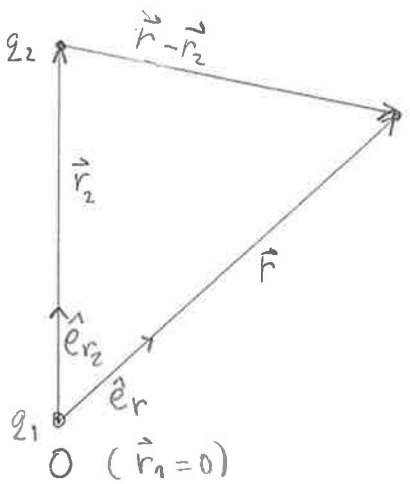
$\vec{E}(\vec{r})$: skupno el. polje (jakost el. polja) na mestu $\vec{r}$

Zakon: $\boxed{\vec{E}(\vec{r}) = \sum_i \vec{E}_i(\vec{r})}$

Primer: el. polje v okolici el. dipola

$q_1 = q > 0$

$q_2 = -q (< 0)$



$\hat{e}_r = \frac{\vec{r}}{r}$, $\hat{e}_{r_2} = \frac{\vec{r}_2}{r_2}$; $r_2 = |\vec{r}_2| = r_d$

$\vec{r}_d \equiv -\vec{r}_2 = -r_d \hat{e}_{r_2}$

$\Rightarrow \vec{r}_2 = -\vec{r}_d = r_d \hat{e}_{r_2}$

$\Rightarrow \boxed{\vec{r}-\vec{r}_2} = \vec{r} + \vec{r}_d$

$= r\hat{e}_r - r_d \hat{e}_{r_2}$

$= r(\hat{e}_r - \varepsilon \hat{e}_{r_2})$; $\varepsilon \equiv \frac{r_d}{r}$

Splošno: $\vec{E}_1(\vec{r}) = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{e}_r = \frac{q\,\hat{e}_r}{4\pi\varepsilon_0 r^2} = \frac{q\,\vec{r}}{4\pi\varepsilon_0 r^3}$

$\vec{E}_2(\vec{r}) = \frac{q_2}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3} = \frac{-q(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_2|^3}$

$\Rightarrow \boxed{\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r})}$

V primeru, ko je T daleč stran od dipola (ko je $\varepsilon = \frac{r_d}{r} \ll 1$), se splošni izraz za $\vec{E}(\vec{r})$ poenostavi. Uporabimo linearni Taylorjev razvoj naslednjih

Funkcij:

$$(1)\quad |\vec r-\vec r_2|^2 = r^2(\hat e_r - \varepsilon \hat e_{r_2})(\hat e_r - \varepsilon\hat e_{r_2}) \overset{\varepsilon\ll 1}{\simeq} r^2(1-2\varepsilon \hat e_r \cdot \hat e_{r_2})$$

$$(2)\quad |\vec r-\vec r_2| = \sqrt{|\vec r-\vec r_2|^2} \simeq r\sqrt{1-2\varepsilon \hat e_r \cdot \hat e_{r_{12}}} \simeq r(1-\varepsilon \hat e_r\cdot \hat e_{r_2})$$

$$(3)\quad |\vec r-\vec r_2|^3 \simeq \{r(1-\varepsilon \hat e_r\cdot\hat e_{r_2})\}^3 \simeq r^3(1-3\varepsilon \hat e_r\cdot \hat e_{r_2})$$

$$(4)\quad \frac{1}{1-3\varepsilon\hat e_r \hat e_{r_2}} \simeq 1+3\hat e_r\hat e_{r_2}$$

$$(5)\quad (\hat e_r-\varepsilon\hat e_{r_2})(1+3\varepsilon\hat e_r\cdot\hat e_{r_2}) \simeq \hat e_r - \varepsilon\hat e_{r_2} + 3\varepsilon(\hat e_r\cdot\hat e_{r_2})\hat e_r$$

$$\Rightarrow\quad E_2(\vec r) = -\frac{q(\vec r-\vec r_2)}{4\pi\varepsilon_0|\vec r-\vec r_2|^3}$$

$$\simeq \frac{-qr(\hat e_r-\varepsilon\hat e_{r_2})}{4\pi\varepsilon_0 r^3(1-3\varepsilon\hat e_r\cdot\hat e_{r_2})}$$

$$\simeq -\frac{q}{4\pi\varepsilon_0 r^2}(\hat e_r-\varepsilon\hat e_{r_2})(1+3\varepsilon\hat e_r\hat e_{r_2})$$

$$\simeq -\frac{q}{4\pi\varepsilon_0 r^2}(\hat e_r-\varepsilon\hat e_{r_2}+3\varepsilon(\hat e_r\cdot\hat e_{r_2})\hat e_r)$$

$$= \frac{-q\hat e_r}{4\pi\varepsilon_0 r^2} + \frac{q\varepsilon}{4\pi\varepsilon_0 r^2}\{\hat e_{r_2} - 3(\hat e_r\cdot\hat e_{r_2})\hat e_r\} \qquad \varepsilon = \frac{r_d}{r}$$

$$= -\vec E_1 + \frac{1}{4\pi\varepsilon_0 r^3}\{q r_d\hat e_{r_2} - 3(\hat e_r\cdot q r_d\hat e_{r_2})\hat e_r\} \qquad r_d\hat e_{r_2} = -\vec r_d$$

$$= -\vec E_1 + \frac{1}{4\pi\varepsilon_0 r^3}\{-q\vec r_d + 3(\hat e_r\cdot q\vec r_d)\hat e_r\}$$

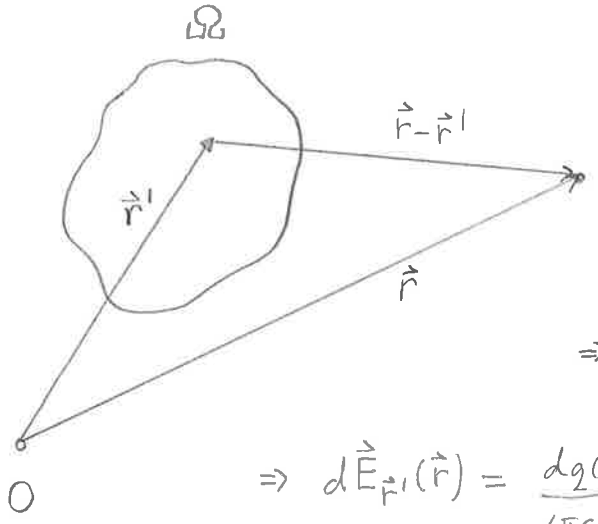
$$= -\vec E_1 + \frac{1}{4\pi\varepsilon_0 r^3}\{3(\vec p_e\cdot\hat e_r)\hat e_r - \vec p_e\},$$

pri čemer smo vpeljali električni dipolni moment

$$\boxed{\vec p_e \equiv q\vec r_d} \quad ; \quad [p_e] = \mathrm{Asm}$$

$$\Rightarrow \boxed{\vec E = \vec E_1+\vec E_2 \simeq \frac{1}{4\pi\varepsilon_0 r^3}\{3(\vec p_e\cdot\hat e_r)\hat e_r - \vec p_e\}} \quad ; \quad \boxed{E \propto \frac{1}{r^3}}$$

b) zvezno porazdeljen naboj



$$\rho(\vec{r}\,') = \frac{dq}{dV'_\Omega}(\vec{r}\,')$$; (prostorska) gostota naboja

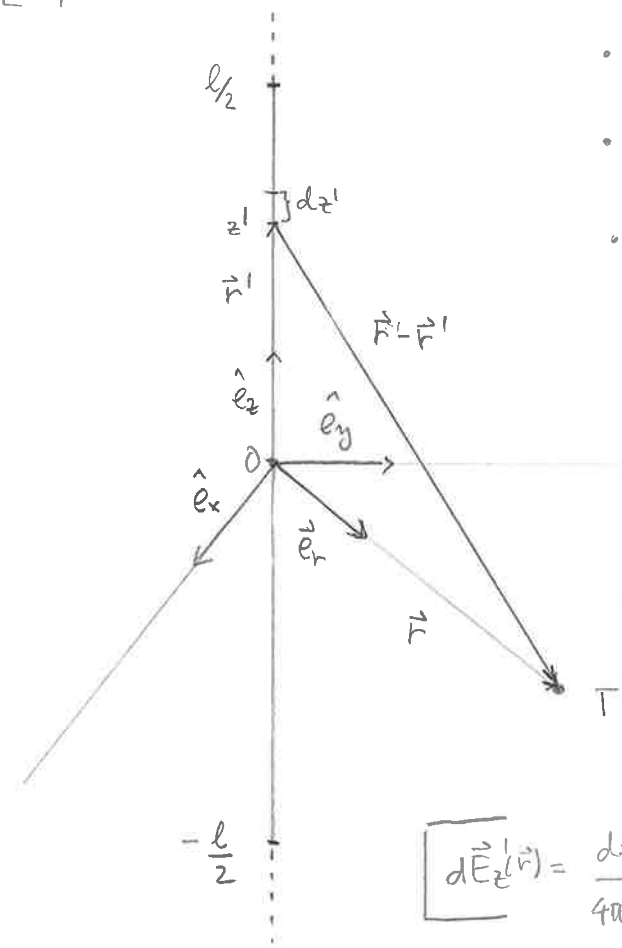
$$q = \int_\Omega \rho(\vec{r}\,')\, dV'_\Omega \qquad [\rho(\vec{r})] = \frac{As}{m^3}$$

$$\Rightarrow dq(\vec{r}\,') = \rho(\vec{r}\,')\cdot dV'$$

$$\Rightarrow d\vec{E}_{\vec{r}\,'}(\vec{r}) = \frac{dq(\vec{r}\,')\,(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}\,'|^3} = \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\,dV'_\Omega}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}\,'|^3}$$

$\Rightarrow$ Zakon: $$\boxed{\vec{E}(\vec{r}) = \int d\vec{E}_{\vec{r}\,'}(\vec{r}) = \int_\Omega \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\,dV'_\Omega}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}\,'|^3}}$$

Primer: $\vec{E}$ na simetrali enakomerno nabite žice



- T na simetrali žice (v ravnini xy)
- $\rho_l(z) = \frac{dq}{dz}(z)$: dolžinska gostota naboja; $[\rho_l] = \frac{As}{m}$
- žica enakomerno nabita: $\rho_l \neq \rho_l(z)$

$\rho_l = \frac{q}{l}$ $\to$ celotni naboj na žici; $l \to$ dolžina žice

$\Rightarrow dq = dz'\cdot\frac{q}{l}$: naboj koščka žice dolžine dz'

- r = oddaljenost T od žice

$$\left.\begin{aligned}\vec{r}\,' &= z'\hat{e}_z\\ \vec{r} &= r\hat{e}_r\end{aligned}\right\} \Rightarrow \vec{r}-\vec{r}\,' = r\hat{e}_r - z'\hat{e}_z$$

$$\Rightarrow |\vec{r}-\vec{r}\,'| = \sqrt{r^2+z'^2} \quad (\hat{e}_r \perp \hat{e}_z)$$

$$\boxed{d\vec{E}_{z'}(\vec{r}) = \frac{dz'\rho_l\,|\vec{r}-\vec{r}\,'|}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}\,'|^3}}$$

$$\boxed{= \frac{\rho_l\,(r\hat{e}_r - z'\hat{e}_z)\,dz'}{4\pi\varepsilon_0\,(r^2+z'^2)^{3/2}}}$$

$$\Rightarrow \vec{E}(\vec{r}) = \int d\vec{E}_z'(\vec{r}) = \int_{-\frac{\ell}{2}}^{\ell/2} \frac{\rho_e}{4\pi\varepsilon_0} \frac{(r\hat{e}_r - z'\hat{e}_z)}{(r^2+z'^2)^{3/2}}\, dz'$$

$$= \frac{\rho_e}{4\pi\varepsilon_0}\left[r\hat{e}_r I_1 - \hat{e}_z I_2 \right], \quad \text{kjer} \quad I_1 = \int_{-\frac{\ell}{2}}^{\ell/2} \frac{dz'}{(r^2+z'^2)^{3/2}}$$

$$I_2 = \int_{-\ell/2}^{\ell/2} \frac{z'\, dz'}{(r^2+z'^2)^{3/2}}$$

$I_2 = 0$ (integral lihe funkcije na simetričnem intervalu $[-\ell/2, \ell/2]$)

I_1: $\displaystyle\int \frac{dz'}{(r^2+z'^2)^{3/2}} = \frac{1}{r^2}\frac{z'}{(r^2+z'^2)^{1/2}} + C$ (za D.N.: preveri z odvajanjem!)

$$\Rightarrow I_1 = \int_{-\ell/2}^{\ell/2} \frac{dz'}{(r^2+z'^2)^{3/2}} = 2\int_0^{\ell/2} \frac{dz'}{(r^2+z'^2)^{3/2}} = \frac{2}{r^2}\frac{z'}{(r^2+z'^2)^{1/2}}\Big|_0^{\ell/2} = \frac{\ell}{r^2}\frac{1}{(r^2+\ell^2/4)^{1/2}}$$

$$\Rightarrow \boxed{\vec{E}(\vec{r})} = \frac{\rho_e}{4\pi\varepsilon_0}\, r\, \frac{\ell}{r^2}\frac{1}{(r^2+\ell^2/4)^{1/2}}\,\hat{e}_r$$

$$= \frac{q}{\ell}\frac{1}{4\pi\varepsilon_0}\frac{\ell}{r}\frac{1}{(r^2+\ell^2/4)^{1/2}}\,\hat{e}_r$$

$$\boxed{= \frac{q}{4\pi\varepsilon_0 r}\frac{1}{(r^2+\ell^2/4)^{1/2}}\,\hat{e}_r}$$

V primeru dolge žice ($\ell \gg r$): $\frac{\ell^2}{4} \gg r^2 \Rightarrow \frac{\ell^2}{4} + r^2 \simeq \frac{\ell^2}{4}$

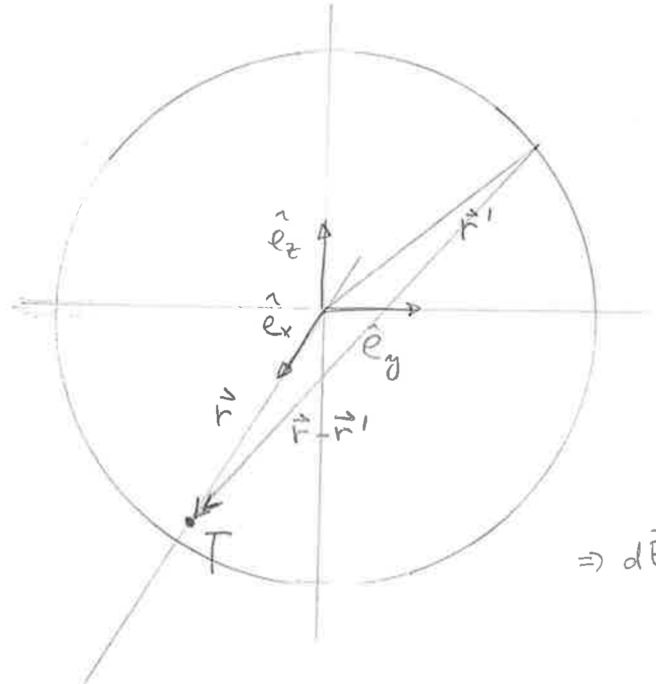
$$\Rightarrow \sqrt{\frac{\ell^2}{4}+r^2} \simeq \frac{\ell}{2}$$

$$\Rightarrow \boxed{\vec{E}(\vec{r})} \simeq \frac{q}{\ell}\frac{1}{2\pi\varepsilon_0 r}\,\hat{e}_r$$

$$\boxed{= \frac{\rho_e}{2\pi\varepsilon_0 r}\,\hat{e}_r} \qquad \left(E(\vec{r}) \propto \frac{1}{r}\right)$$

$$\boxed{= \frac{\rho_e}{2\pi\varepsilon_0}\frac{\vec{r}}{|\vec{r}|^2}}$$

Primer: $\vec{E}$ na simetrali enakomerno nabite zanke



$$\vec{r} = r\hat{e}_x$$

$$\vec{r}\,' = r'\cos\phi\,\hat{e}_y + r'\sin\phi\,\hat{e}_z$$

$$\Rightarrow \vec{r}-\vec{r}\,' = r\hat{e}_x - r'\cos\phi\,\hat{e}_y - r'\sin\phi\,\hat{e}_z$$

$$\Rightarrow |\vec{r}-\vec{r}\,'|^3 = (r^2+r'^2)^{3/2}$$

$$\rho_e = \frac{q}{2\pi r'} = \frac{dq}{dl'} = \frac{dq}{r'd\phi}$$

$$\Rightarrow dq = \rho_e \cdot r'd\phi$$

$$\Rightarrow d\vec{E}_\phi(\vec{r}) = \frac{dq\,(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}\,'|^3}$$

$$= \frac{\rho_e r'(r\hat{e}_x - r'\cos\phi\,\hat{e}_y - r'\sin\phi\,\hat{e}_z)}{4\pi\varepsilon_0 (r^2+r'^2)^{3/2}}$$

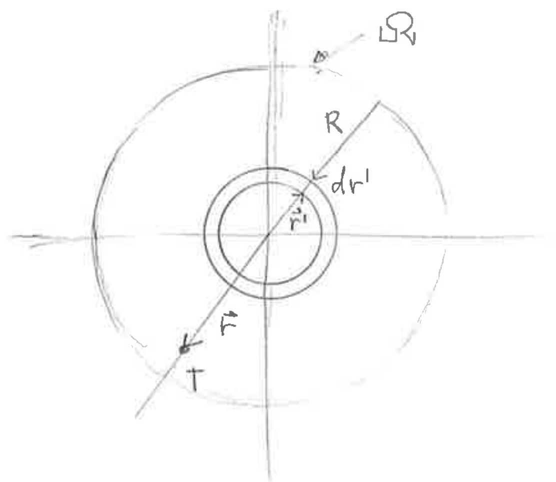
$$\Rightarrow \vec{E}(\vec{r}) = \int d\vec{E}_\phi(\vec{r})$$

$$= \frac{\rho_e r'}{4\pi\varepsilon_0 (r^2+r'^2)^{3/2}}\left[r\hat{e}_x I_1 - r'\hat{e}_y I_2 - r'\hat{e}_z I_3\right]$$

$$I_1 = \int_0^{2\pi} d\phi = 2\pi\,,\quad I_2 = \int_0^{2\pi}\cos\phi\,d\phi = 0\,,\quad I_3 = \int_0^{2\pi}\sin\phi\,d\phi = 0$$

$$\Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{\rho_e}{2\varepsilon_0}\frac{r'r}{(r^2+r'^2)^{3/2}}\hat{e}_x} = \frac{q}{2\pi r'}\frac{1}{2\varepsilon_0}\frac{r'r}{(r^2+r'^2)^{3/2}}\hat{e}_x = \underline{\underline{\frac{q}{4\pi\varepsilon_0}\frac{r}{(r^2+r'^2)^{3/2}}\hat{e}_x}}$$

Primer: Uporabimo zgornji izraz in izračunajmo $\vec{E}(\vec{r})$ na simetrali enakomerno nabite okrogle plošče



$$S' = \pi R^2,\quad \rho_{S'} = \frac{q}{\pi R^2},\quad dS' = 2\pi r'dr$$

$$\rho_{S'} = \frac{dq}{dS'} \Rightarrow dq = \rho_{S'}\cdot dS' = \frac{q}{\pi R^2}\cdot 2\pi r'dr' = \frac{2q}{R^2}r'dr'$$

$$\Rightarrow d\vec{E}_{r'}(\vec{r}) = \frac{dq}{4\pi\varepsilon_0}\frac{r}{(r^2+r'^2)^{3/2}}\hat{e}_x$$

$$= \frac{qr}{2\pi\varepsilon_0 R^2}\frac{r'dr'}{(r^2+r'^2)^{3/2}}\hat{e}_x$$

$$\Rightarrow E(\vec{r}) = \frac{q r}{2\pi\varepsilon_0 R^2}\,\hat{e}_x\, I \;; \quad I = \int_0^{R} \frac{r'dr'}{(r^2+r'^2)^{3/2}} = \frac{1}{2}\int_{u_1}^{u_2} u^{-3/2}\,du = -u^{-1/2}\Big|_{u_1}^{u_2} = \frac{1}{r}\left[1 - \frac{1}{\sqrt{1+R^2/r^2}}\right]$$

$$u = r'^2 + r^2 \Rightarrow u_1 = r^2,\ u_2 = r^2 + R^2,\ r'dr' = \frac{du}{2}$$

$$\Rightarrow \boxed{E(\vec{r}) = \underbrace{\frac{q}{\pi R^2}}_{\rho_s'}\frac{1}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1+R^2/r^2}}\right]\hat{e}_x}$$

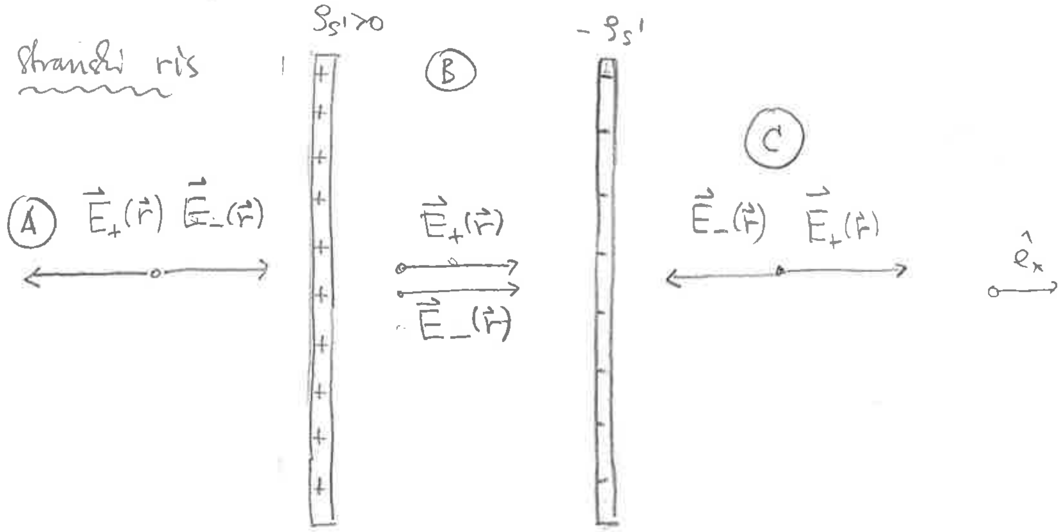
Velika plošča: $R/r \gg 1$

$\Rightarrow \boxed{\vec{E}(\vec{r}) \simeq \frac{\rho_s'}{2\varepsilon_0}\hat{e}_x}$ $\quad \hat{e}_x$: polje pravokotno na ploščo, iz plošče ven za $q>0$

: ne pada z razdaljo

Primer: Dve enaki, nasprotno enakomerno nabiti, vzporedni, veliki plošči na razdalji l ($l \ll a, b$); ploščati kondenzator

Stranski ris



(A) $\vec{E}(\vec{r}) = \vec{E}_+(\vec{r}) + \vec{E}_-(\vec{r}) = 0$

(B) $\boxed{E(\vec{r})} = 2\vec{E}_+(\vec{r}) = \frac{\rho_s'}{\varepsilon_0}\hat{e}_x = \boxed{\frac{q}{S'\varepsilon_0}\hat{e}_x}$

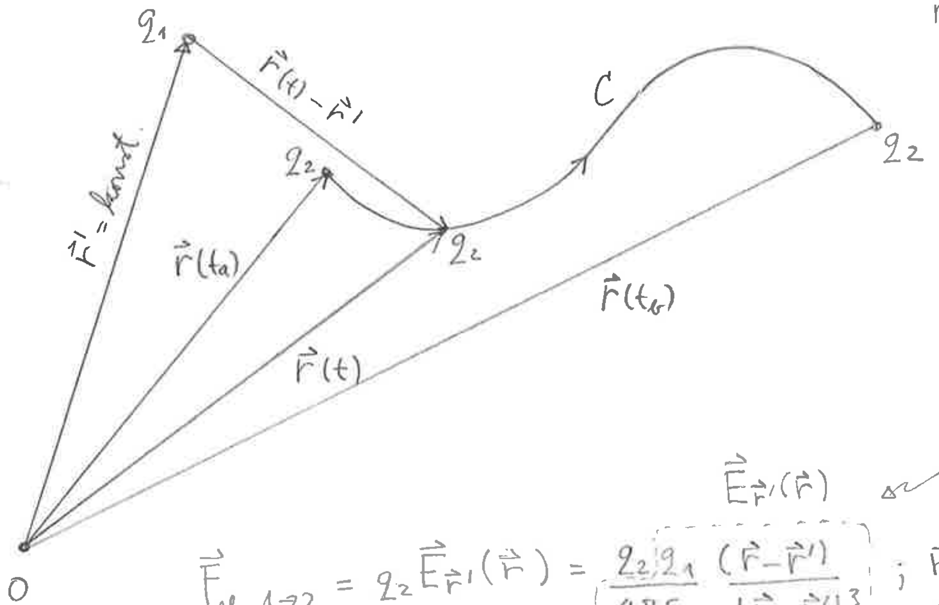
$S' = ab$ = površina (ene strani) vsake izmed plošč

(C) $\vec{E}(\vec{r}) = 0$

$l \ll a, b$: polje med ploščama $\approx$ homogeno ($\vec{E}$ = konst.)

(2) Delo električne sile; el. napetost in potencial

a) $\vec{E}_{\vec{r}'}(\vec{r})$ v okolici točkastega naboja



$$\vec{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}, \quad \vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Coulombov zakon

$$\vec{F}_{el,1\to 2} = q_2 \vec{E}_{\vec{r}'}(\vec{r}) = \underbrace{\frac{q_2 q_1}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}_{\vec{E}_{\vec{r}'}(\vec{r})} \quad ; \quad \vec{r} = \vec{r}(t), \quad \vec{r}' = \text{konst.}$$

A_{el}: delo električne sile 1. telesa na 2. telo pri premikanju 2. telesa po krivulji C

$$A_{el} \equiv \int_C \vec{F}_{el,1\to 2} \cdot d\vec{s} \qquad \text{(definicija dela sile)}$$

$$\equiv q_2 \int_C \vec{E}_{\vec{r}'}(\vec{r}) \cdot d\vec{s} \quad (*) \qquad \text{(definicija } \vec{E}_{\vec{r}'}(\vec{r})\text{)}$$

$$\equiv q_2 \int_{t_a}^{t_b} \vec{E}_{\vec{r}'}[\vec{r}(t)] \cdot \dot{\vec{r}}\, dt \qquad \text{(definicija krivuljnega integrala)}$$

$$= q_2 \int_{t_a}^{t_b} \frac{q_1}{4\pi\varepsilon_0} \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \dot{\vec{r}}\, dt \qquad \text{(Coulombov zakon)}$$

$$= q_2 \int_{t_a}^{t_b} \frac{q_1}{4\pi\varepsilon_0} \frac{(\vec{r}-\vec{r}')\cdot(\dot{\vec{r}}-\dot{\vec{r}}')}{|\vec{r}-\vec{r}'|^3}\, dt \qquad (\dot{\vec{r}}'=0)$$

Trdim: $\dfrac{(\vec r-\vec r\,')(\dot{\vec r}-\dot{\vec r}\,')}{|\vec r-\vec r\,'|^3} = \left\{-\dfrac{\dot 1}{|\vec r-\vec r\,'|}\right\}$

Res: $\left\{-\dfrac{\dot 1}{|\vec r-\vec r\,'|}\right\} = \left\{-\left[(\vec r-\vec r\,')^2\right]^{-1/2}\right\}^{\dot{}} = -\left(-\frac{1}{2}\right)\left[(\vec r-\vec r\,')^2\right]^{-3/2}\cdot 2(\vec r-\vec r\,')\cdot(\dot{\vec r}-\dot{\vec r}\,')$

$$= \frac{(\vec r-\vec r\,')(\dot{\vec r}-\dot{\vec r}\,')}{\left[(\vec r-\vec r\,')^2\right]^{3/2}} = \frac{(\vec r-\vec r\,')(\dot{\vec r}-\dot{\vec r}\,')}{|\vec r-\vec r\,'|^3} \checkmark$$

$$\Rightarrow \boxed{A_{el}} = q_2\int_{t_a}^{t_b}\left\{-\frac{q_1}{4\pi\varepsilon_0|\vec r-\vec r\,'|}\right\}^{\dot{}} dt$$

$$= q_2\left\{-\frac{q_1}{4\pi\varepsilon_0|\vec r-\vec r\,'|}\right\}_{\vec r=\vec r(t_b)} - q_2\left\{-\frac{q_1}{4\pi\varepsilon_0|\vec r-\vec r\,'|}\right\}_{\vec r=\vec r(t_a)}$$

$$\boxed{= -q_2\left\{\frac{q_1}{4\pi\varepsilon_0|\vec r(t_b)-\vec r\,'|} - \frac{q_1}{4\pi\varepsilon_0|\vec r(t_a)-\vec r\,'|}\right\}}$$ $\vec F_{el,1\to 2}$ konservativna!

Def: $\boxed{W_{ep} \equiv \dfrac{q_2 q_1}{4\pi\varepsilon_0|\vec r-\vec r\,'|}}$; električna potencialna energija q_2 v polju q_1

$[W_{ep}] = \dfrac{A^2s^2Vm}{As\ m} = VAs = \dfrac{Nm}{As}As = Nm = J \checkmark$

$$\Rightarrow \boxed{A_{el} = -\Delta W_{ep}} = -\left(W_{ep}(t_b) - W_{ep}(t_a)\right)$$

Def: $\boxed{\phi(\vec r) \equiv \dfrac{q_1}{4\pi\varepsilon_0|\vec r-\vec r\,'|}}$ potencial $\vec E_{\vec r\,'}(\vec r)$ v točki $\vec r$

$[\phi] = \dfrac{As}{As}\dfrac{Vm}{m} = V$

$$\Rightarrow \boxed{W_{ep} = q_2\phi(\vec r)}$$

$$\Rightarrow A_{el} = -q_2\left\{\phi[\vec r(t_b)] - \phi[\vec r(t_a)]\right\} = -q_2\Delta\phi$$

Def: $\boxed{U[\vec r(t_b),\vec r(t_a)] \equiv \phi[\vec r(t_b)] - \phi[\vec r(t_a)] = \Delta\phi}$ električna napetost med točkama $\vec r(t_b)$ in $\vec r(t_a)$ v polju $\vec E_{\vec r\,'}(\vec r)$

$$\Rightarrow \boxed{A_{el} = -q_2 U[\vec r(t_b),\vec r(t_a)]}^{(**)}$$

Iz primerjave enačb (*) in (**) sledi:

$$\boxed{U[\vec{r}(t_b)-\vec{r}(t_a)] = -\int_C \vec{E}_{\vec{r}'}(\vec{r})\cdot d\vec{s}}$$; $\vec{r}(t_b)$ in $\vec{r}(t_a)$ končna in začetna točka C

• Komentar: v elektrostatiki (statično el. polje) je napetost med dvema točkama neodvisna od poti (dobro definirana); glej nadaljevanje spodaj.

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b) $\vec{E}(\vec{r})$ v okolici zvezno porazdeljenega naboja

• glavne ideje enake kot v primeru a)

$$\vec{E}(\vec{r}) = \int_\Omega \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3} dV'_\Omega$$; el. polje v točki $\vec{r}$ (na mestu q_2)

$\vec{F}_{el} = q_2 \vec{E}(\vec{r})$; sila na q_2 v polju $E(\vec{r})$

$$A_{el} \equiv \int_C \vec{F}_{el}\cdot d\vec{s}$$

$$\equiv \int_C q_2 \vec{E}(\vec{r})\cdot d\vec{s}$$

$$= q_2 \int_C \vec{E}(\vec{r})\, d\vec{s}$$

$$= q_2 \int_{t_a}^{t_b} \vec{E}[\vec{r}(t)]\cdot\dot{\vec{r}}\, dt$$

$$= q_2 \int_{t_a}^{t_b} \left\{ \int_\Omega \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3} dV'_\Omega \right\}\cdot\dot{\vec{r}}\, dt$$

$$= q_2 \int_\Omega \frac{\rho(\vec{r}')}{4\pi\varepsilon_0} \left\{ \int_{t_a}^{t_b} \frac{(\vec{r}-\vec{r}')\cdot\dot{\vec{r}}}{|\vec{r}-\vec{r}'|^3} dt \right\} dV'_\Omega$$ (dovolj gladke funkcije pod integralom)

$$= q_2 \int_\Omega \frac{\rho(\vec{r}')}{4\pi\varepsilon_0} \left\{ \int_{t_a}^{t_b} \frac{(\vec{r}-\vec{r}')\cdot(\dot{\vec{r}-\vec{r}'})}{|\vec{r}-\vec{r}'|^3} dt \right\} dV'_\Omega$$ ($\dot{\vec{r}}'=0$)

⋮

$\vdots$

$$= q_2 \int_{\Omega} \frac{\rho(\vec r')}{4\pi\varepsilon_0} \left\{ \int_{t_a}^{t_b} \overset{\cdot}{\left[-\frac{1}{|\vec r-\vec r'|}\right]} dt \right\} dV'_{\Omega}$$

$$= q_2 \int_{\Omega} \frac{\rho(\vec r')}{4\pi\varepsilon_0} \left\{ -\frac{1}{|\vec r(t_b)-\vec r'|} + \frac{1}{|\vec r(t_a)-\vec r'|} \right\} dV'_{\Omega}$$

$$= q_2 \left\{ -\int_{\Omega} \frac{\rho(\vec r')\, dV'_{\Omega}}{4\pi\varepsilon_0 |\vec r(t_b)-\vec r'|} + \int_{\Omega} \frac{\rho(\vec r')\, dV'_{\Omega}}{4\pi\varepsilon_0 |\vec r(t_a)-\vec r'|} \right\};$$ tudi tukaj el. sila konservativna!

Def: $\boxed{\phi(\vec r) \equiv \int_{\Omega} \frac{\rho(\vec r')\, dV'_{\Omega}}{4\pi\varepsilon_0 |\vec r-\vec r'|}}$

$$\boxed{W_{ep} \equiv q_2 \phi(\vec r)}$$

$$\boxed{U[\vec r(t_b), \vec r(t_a)] \equiv \phi[\vec r(t_b)] - \phi[\vec r(t_a)]} \equiv \Delta\phi$$

$$\Rightarrow \boxed{\begin{aligned} A_{el} &= -\Delta W_{ep} \\ &= -q_2 \Delta\phi \\ &= -q_2 U[\vec r(t_b), \vec r(t_a)] \end{aligned}}$$

Prav tako velja:

$$\boxed{U[\vec r(t_b), \vec r(t_a)] = -\int_C \vec E \cdot d\vec s}\;;$$ $\vec r(t_b), \vec r(t_a)$: končna in začetna točka krivulje.

Izrek o električni napetosti

Spomnimo se:

$$\vec E(\vec r) = \int_{\Omega} \frac{\rho(\vec r')}{4\pi\varepsilon_0} \frac{(\vec r-\vec r')}{|\vec r-\vec r'|^3} dV'_{\Omega} = \begin{bmatrix} E_x(\vec r) \\ E_y(\vec r) \\ E_z(\vec r) \end{bmatrix}; \quad \vec r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \vec r' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix};$$

$$|\vec r-\vec r'| = \left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{1/2}$$

$$\Rightarrow E_x(\vec r) = \int_{\Omega} \frac{\rho(\vec r')}{4\pi\varepsilon_0} \frac{(x-x')}{|\vec r-\vec r'|^3} dV_{\Omega}; \quad E_y(\vec r) = \ldots, \quad E_z(\vec r) = \ldots$$

Mat: $\mathrm{grad}\,\phi \equiv \nabla\phi \equiv \begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \frac{\partial\phi}{\partial y} \\ \frac{\partial\phi}{\partial z} \end{bmatrix}$

$$\phi(\vec{r}) = \int_{\Omega} \frac{\rho(\vec{r}\,')}{4\pi\varepsilon_0} \frac{dV'_{\Omega}}{|\vec{r}-\vec{r}\,'|}$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\left\{ \int_{\Omega} \frac{\rho(\vec{r}\,')}{4\pi\varepsilon_0} \frac{1}{|\vec{r}-\vec{r}\,'|} dV'_{\Omega} \right\}$$

$$= \int_{\Omega} \frac{\rho(\vec{r}\,')}{4\pi\varepsilon_0} \frac{\partial}{\partial x}\left[\frac{1}{|\vec{r}-\vec{r}\,'|}\right] dV'_{\Omega} \qquad \text{(dovolj gladke funkcije)}$$

$$\frac{\partial}{\partial x}\left[\frac{1}{|\vec{r}-\vec{r}\,'|}\right] = \frac{\partial}{\partial x}\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-1/2}$$

$$= -\frac{1}{2}\left[\ldots\right]^{-3/2} \cdot 2(x-x')$$

$$= -\frac{(x-x')}{|\vec{r}-\vec{r}\,'|^3}$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = -\int_{\Omega} \frac{\rho(\vec{r}\,')}{4\pi\varepsilon_0} \frac{(x-x')}{|\vec{r}-\vec{r}\,'|^3} dV'_{\Omega} = -E_x$$

$$\Rightarrow \frac{\partial\phi}{\partial y} = \ldots = -E_y$$

$$\Rightarrow \frac{\partial\phi}{\partial z} = \ldots = -E_z$$

$\boxed{\vec{E} = -\nabla\phi}$

$\vec{E}$: potencialno polje

Mat: $\mathrm{rot}\,\vec{E} \equiv \nabla\times\vec{E} \equiv \begin{bmatrix} \partial_y E_z - \partial_z E_y \\ \partial_z E_x - \partial_x E_z \\ \partial_x E_y - \partial_y E_x \end{bmatrix}$

Izrek: $\boxed{\nabla\times\vec{E} = 0}$ Izrek o el. napetosti v diferencialni obliki.

Rez: $\nabla\times\vec{E} = -\nabla\times(\nabla\phi) = -\begin{bmatrix} \partial_y\partial_z\phi - \partial_z\partial_y\phi \\ \partial_z\partial_x\phi - \partial_x\partial_z\phi \\ \partial_x\partial_y\phi - \partial_y\partial_x\phi \end{bmatrix} = \vec{0}$

Mat: Stokesov izrek: $\oint_{\partial S} \vec{E}\cdot d\vec{s} = \int_{S} (\nabla\times\vec{E})\cdot d\vec{S}$

$$\oint_{\partial S} \vec{E}\cdot d\vec{s} = -U[\vec{r}(t_b)=\vec{r}(t_a), \vec{r}(t_a)] =$$ - napetost po (poljubni) zaključeni poti

$\nabla\times\vec{E} = 0 \Rightarrow \boxed{\oint_{\partial S} \vec{E}\cdot d\vec{s} = 0}$ Izrek o el. napetosti v integralni obliki: napetost po poljubni zaključeni poti = 0.

Komentar: omejena veljavnost izreka o el. napetosti

- mirujoči naboji
- brez (časovno spreminjajočega se) magnetnega polja

Dopolnitev: (Faradayev) zakon o el. napetosti (indukcijski zakon)