Trditev: $\boxed{\nabla\cdot\vec{B}=0}$ (izrek o Φ_B v diferencialni obliki)

Rez: $\nabla\cdot\vec{B}=\nabla\cdot(\nabla\times\vec{A})=0$ (div rot $\vec{A}=0$ za $\forall\vec{A}$; preveri za d.n.)

$$\Rightarrow \boxed{\oint_{\partial V}\vec{B}\cdot d\vec{S} = \int_V \nabla\cdot\vec{B}\,dV = 0}$$

Gaussov izrek

(izrek o Φ_B v integralni obliki)

izrek o Φ_B v diferencialni obliki

Φ_B skozi zaključeno ploskev ∂V

Primerjava: V naravi ni magnetnih nabojev (monopolov)

$$\oint_{\partial V}\varepsilon_0\vec{E}\cdot d\vec{S} = q \text{ (zaobjet)} \qquad \oint_{\partial V}\vec{B}\cdot d\vec{S}=0$$

---

(5) Povzetek ((in primerjava) elektrostatike in magnetostatike

(1) Izrek o Φ_E: $\boxed{\oint_{\partial V}\varepsilon_0\vec{E}\cdot d\vec{S} = \int_V \rho_e\cdot dV}$ ali $\boxed{\mathrm{div}\,\vec{E} = \rho_e/\varepsilon_0}$

(2) Izrek o el. napetosti: $\boxed{\oint_{\partial S}\vec{E}\cdot d\vec{s}=0}$ ali $\boxed{\nabla\times\vec{E}=0}$

(3) Izrek o Φ_B: $\boxed{\oint_{\partial V}\vec{B}\cdot d\vec{S}=0}$ ali $\boxed{\nabla\cdot\vec{B}=0}$

(4) Izrek o mag. napetosti (Ampère-ov izrek):

$$\boxed{\oint_{\partial S}\vec{B}\cdot d\vec{s} = \int_S \mu_0\vec{j}_e\cdot d\vec{S}} \text{ ali } \boxed{\nabla\times\vec{B}=\mu_0\vec{j}_e}$$

Vsi štirje samo "izreki", ker so neposredne posledice Coulombovega zakona in Biot-Savartovega zakona za točkaste naboje, ter načela superpozicije.

- Veljavnost izrekov je pogojena z veljavnostjo zakonov, na katerih temeljijo: (1) in (2) za mirujoče naboje ter (3) in (4) za stacionarne tokove (glej tudi splošen dokaz Ampère-ovega izreka).

V. ELEKTRODINAMIKA

1) Zakona o električnem in magnetnem pretoku

Številni poskusi kažejo, da izreka o električnem in magnetnem pretoku veljata splošno, tudi v nestacionarnih razmerah $\Rightarrow$ zakona o električnem in magnetnem pretoku (prvi dve Maxwellovi enačbi)

(1) $$\boxed{\oint_{\partial V} \varepsilon_0 \vec{E} \cdot d\vec{S} = \int_V \rho_e \, dV} \quad \text{ali} \quad \boxed{\nabla \cdot \vec{E} = \rho_e / \varepsilon_0}$$

(2) $$\boxed{\oint_{\partial V} \vec{B} \cdot d\vec{S} = 0} \quad \text{ali} \quad \boxed{\nabla \cdot \vec{B} = 0}$$

2) Zakon o magnetni napetosti – Ampère-ov zakon

- Izrek o magnetni napetosti (Ampère-ov izrek):

$$\nabla \times \vec{B} = \mu_0 \vec{j}_e \Rightarrow \mu_0 \nabla \cdot \vec{j}_e = \nabla \cdot (\nabla \times \vec{B}) = 0 \Rightarrow \nabla \cdot \vec{j}_e = 0$$

(?!)

- Po drugi strani (kontinuitetna enačba):

$$\nabla \cdot \vec{j}_e = -\frac{\partial \rho_e}{\partial t}$$

- Iz zakona o el. pretoku

$$\frac{\partial \rho_e}{\partial t} = \frac{\partial}{\partial t} \nabla \cdot (\varepsilon_0 \vec{E}) = \nabla \cdot \left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow \nabla \cdot \vec{j}_e = -\nabla \cdot \left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \neq 0$$

v splošnem, ko $\frac{\partial \vec{E}}{\partial t} \neq 0$

Maxwellov popravek Ampère-ovega izreka:

$$\nabla \times \vec{B} = \mu_0 \vec{j} \longrightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{j} + \vec{G}}$$

$$\Rightarrow \mu_0 \nabla \vec{j} + \mu_0 \nabla \cdot \vec{G} = \nabla \cdot (\nabla \times \vec{B}) = 0$$

$$\Rightarrow -\mu_0 \varepsilon_0 \nabla \cdot \left(\frac{\partial \vec{E}}{\partial t}\right) + \nabla \cdot \vec{G}$$

$$\Rightarrow \boxed{\vec{G} = \frac{1}{c_0^2} \frac{\partial \vec{E}}{\partial t}} \; ; \; \boxed{c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \cdot 10^8 \, m/s}$$

$$(3) \quad \boxed{\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c_0^2} \frac{\partial \vec{E}}{\partial t}} \quad \text{ali}^{*} \quad \boxed{\int_{\partial S} \vec{B} \cdot d\vec{s} = \mu_0 \overbrace{\int_S \vec{j} \cdot d\vec{S}}^{\text{tok}} + \mu_0 \overbrace{\int_S \frac{\partial}{\partial t}(\varepsilon_0 \vec{E}) \cdot d\vec{S}}^{\text{premikalni tok}}},$$

kar je 3. Maxwellova enačba (Ampère-ov zakon): spreminjajoče se električno polje se obda z magnetnim poljem.

* običajen prehod iz diferencialne v integralno obliko:

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c_0^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{j} + \mu_0 \frac{\partial (\varepsilon_0 \vec{E})}{\partial t}$$

$$\Rightarrow \int_S (\nabla \times \vec{B}) \cdot d\vec{S} = \mu_0 \int_S \vec{j}_e \cdot d\vec{S} + \mu_0 \int_S \frac{\partial (\varepsilon_0 \vec{E})}{\partial t} \cdot d\vec{S} \; ; \; \forall S$$

$$\Rightarrow \int_{\partial S} \vec{B} \cdot d\vec{s} = \mu_0 \int_S \vec{j}_e \cdot d\vec{S} + \mu_0 \int_S \frac{\partial (\varepsilon_0 \vec{E})}{\partial t} \cdot d\vec{S} \quad \text{(Stokesov izrek)}$$

③ Indukcijski (Faradayev) zakon

• Izrek o električni napetosti: $\nabla \times \vec{E} = 0$

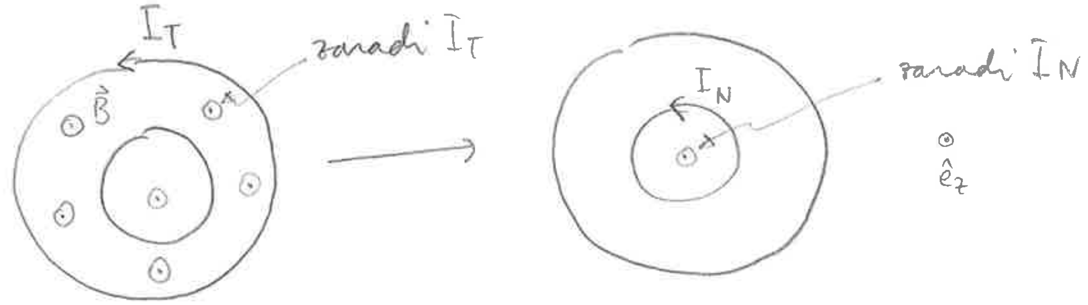
Poskus: navitje (tuljavica) v tuljavi s tokom

- ko v tuljavi s tokom prekinemo tok, po navitju steče kratkotrajen tok
- ko v tuljavi tok ponovno priključimo, po navitju spet steče kratkotrajen tok, tokrat v drugo smer

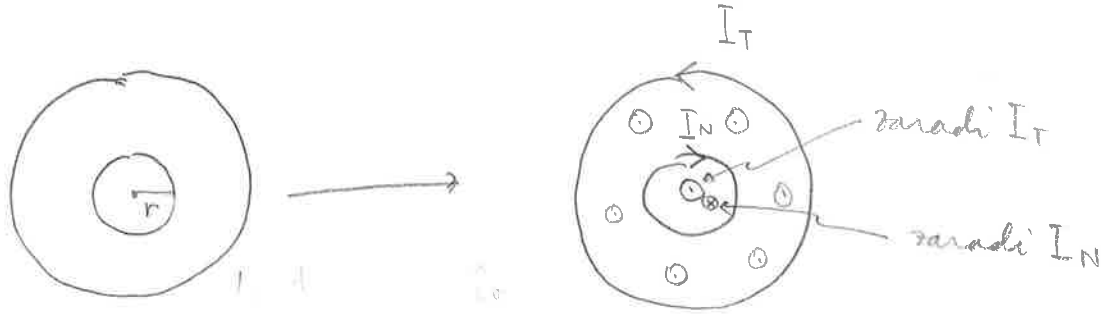
Razlaga: ko spreminjamo (izključujemo ali vključujemo) tok po tuljavi, se v tuljavi spreminja $\vec{B}$, $\frac{\partial \vec{B}}{\partial t} \neq 0$, in spreminjajoče se magnetno polje se obda z električnim poljem, ki požene proste naboje po žicah navitja

Tloris:

a)

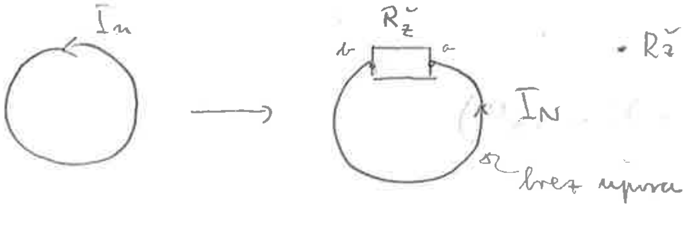


b)



Poskus v nasprotju z izrekom o el. napetosti (in Ohmovim zakonom):

• notranja zanka



• $R_ž$: upornost žice notranje tuljave

$$\nabla \times \vec{E} = 0$$

$$\Rightarrow \int_{S_1} (\nabla \times \vec{E}) \cdot d\vec{S} = 0$$; S_1 = površina enega ovoja notranje tuljave

$$\Rightarrow \oint_{\partial S_1} \vec{E} \cdot d\vec{s} = 0$$; ∂S_1 = en ovoj notranje tuljave

$$\Rightarrow N \oint_{\partial S_1} \vec{E} \cdot d\vec{s} = \oint_{\partial S} \vec{E} \cdot d\vec{s} = 0$$; ∂S = N ovojev notranje tuljave

V nadomestni shemi žica notranje tuljave brez upora $\Rightarrow$ $\vec{E}$ v žicah $= 0$

$$\Rightarrow \oint_{\partial S} \vec{E} \cdot d\vec{s} = \int_{\partial S(a \to b)} \vec{E} \cdot d\vec{s} = -U'_{R\check{z}} = 0$$

$$\Rightarrow \boxed{U'_{R\check{z}} = 0}$$

očitno nasprotje (!?)

Medtem Ohmov zakon narekuje: $\boxed{U'_{R\check{z}} = -I_N R_{\check{z}} \neq 0}$

$\Rightarrow$ Popravek izreka o el. napetosti: zakon o el. napetosti (indukcijski zakon)

(4) $\boxed{\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}}$ ali $\boxed{\oint_{\partial S} \vec{E} \cdot d\vec{s} = -\int_S \dfrac{\partial \vec{B}}{\partial t} \cdot d\vec{S}}$

Lenzovo pravilo

Spreminjajoče se $\vec{B}$ se "obda" z $\vec{E}$.

Spet običajen prehod iz diferencialne v integralno obliko:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \int_S (\nabla \times \vec{E}) \cdot d\vec{S} = \int_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\Rightarrow \int_{\partial S} \vec{E} \cdot d\vec{s} = \int_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$ (Stokesov izrek)

V našem primeru

a) $\vec{B}(t_1) = B\hat{e}_z$, $\vec{B}(t_2) = 0 \Rightarrow \Delta\vec{B} = \vec{B}(t_2) - \vec{B}(t_1) = B(-\hat{e}_z) \Rightarrow \frac{\partial \vec{B}}{\partial t} \propto -\hat{e}_z \Rightarrow -\frac{\partial \vec{B}}{\partial t} \propto \hat{e}_z$

$$\overset{?}{\Rightarrow} \boxed{\vec{E} \propto \hat{e}_\phi} = -\sin\phi\,\hat{e}_x + \cos\phi\,\hat{e}_y$$

(el. polje v žicah motanega navitja v smeri urinega kazalca)

Reš: $\sin\phi = \frac{y}{r}$, $\cos\phi = \frac{x}{r}$; $r = $ konst. (polmer motanega navitja)

$$\Rightarrow \frac{\partial}{\partial x}\sin\phi = -\frac{xy}{r^3} \; ; \; \frac{\partial}{\partial y}\sin\phi = \frac{1}{r} - \frac{y^2}{r^3} \; ; \; \frac{\partial}{\partial z}\sin\phi = 0$$

$$\Rightarrow \frac{\partial}{\partial x}\cos\phi = \frac{1}{r} - \frac{x^2}{r^3} \; ; \; \frac{\partial}{\partial y}\cos\phi = -\frac{xy}{r^3} \; ; \; \frac{\partial}{\partial z}\cos\phi = 0$$

$$\Rightarrow \nabla\times\vec{E} = \nabla\times(-\sin\phi\,\hat{e}_x + \cos\phi\,\hat{e}_y + 0\,\hat{e}_z)$$

$$= \hat{e}_x\left(\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}\cos\phi\right) + \hat{e}_y\left(\frac{\partial}{\partial z}(-\sin\phi) - \frac{\partial}{\partial x}0\right) + \hat{e}_z\left(\frac{\partial}{\partial x}\cos\phi - \frac{\partial}{\partial y}(-\sin\phi)\right)$$

$$= \hat{e}_z\left(\frac{1}{r^2} - \frac{x^2}{r^3} + \frac{1}{r} - \frac{y^2}{r^3}\right)$$

$$= \hat{e}_z\left(\frac{2}{r} - \frac{r^2}{r^3}\right)$$

$$= \hat{e}_z \cdot \frac{1}{r}$$

$$\propto \hat{e}_z \propto -\frac{\partial \vec{B}}{\partial t}$$

$\Rightarrow \hat{e}_j \propto \hat{e}_\phi$: (inducirani) tok I_N steče v smeri $\vec{E}$

$\Rightarrow \vec{B}$ zaradi $I_N \propto \hat{e}_z$ (pravilo desnega vijaka):

$\Rightarrow$ tok steče v smeri, ki nasprotuje spremembi magnetnega pretoka skozi motano tuljavo (Lenzovo pravilo)

b) izračunaj za domačo nalogo $\vec{E} \propto \hat{e}_\phi$ in komentiraj (Lenzovo pravilo)

④ Magnetni pretok skozi tuljavo, po kateri teče tok I

- tok I po tuljavi

$\hat{e}_j = \hat{e}_\phi \Rightarrow \vec{B} = \mu_0 I \frac{N}{l} \hat{e}_z = B\hat{e}_B \ ; \ B = \mu_0 I \frac{N}{l} \ , \ \hat{e}_B = \hat{e}_z$

$\hat{e}_j = -\hat{e}_\phi \Rightarrow \vec{B} = \mu_0 I \frac{N}{l}(-\hat{e}_z) = B\hat{e}_B \ ; \ B = \mu_0 I \frac{N}{l} \ ; \ \hat{e}_B = -\hat{e}_z$

$\vec{S}_1 = S_1 \hat{e}_S \ ; \ \hat{e}_S \equiv \hat{e}_B$ (pravilo desnega vijaka glede na $\hat{e}_j$)

$\Rightarrow \Phi_{B,1} = \vec{B} \cdot \vec{S}_1 = B S_1 \hat{e}_B \cdot \hat{e}_S = B S_1 \ ; \ S_1 =$ "površina" enega ovoja (zanke)

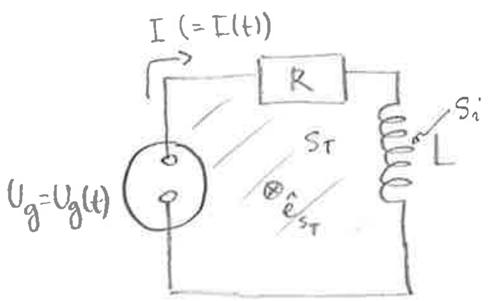
$$\Rightarrow \boxed{\Phi_B = N \Phi_{B,1} = N B S_1 = \frac{\mu_0 N^2 S_1}{l} I}$$

- jedro v tuljavi:

$$\boxed{\Phi_B = \frac{\mu \mu_0 N^2 S_1}{l} I = L I} \ ; \ \boxed{L \equiv \frac{\mu \mu_0 N^2 S_1}{l}} \text{ - induktivnost tuljave}$$

$$\boxed{[L] = \frac{Vs}{Am} \frac{m^2}{m} = \frac{Vs}{A} = H \text{ (henri)}}$$

⑤ Padec napetosti na tuljavi s tokom



- $R_L = 0$ (supraprevodna žica tuljave)
- I_L: tok po tuljavi

  $I_L = I_R = I$ (1. Kirchhoffov izrek)

- dolga, gosto navita tuljava (B zunaj tuljave $\simeq 0$) ; $\vec{B} = \vec{B}_L + \vec{B}_T$, $\vec{B}_T = 0$ (ali vsaj konst.)

• spomnimo se: $|U_R'| = |I_R| R$; padec napetosti na žici z upornostjo R

Košček žice dolžine dl : $|dU_R'| = |I_R| dR$

$$dR = \zeta \frac{dl}{S}, |I_R| = j_e \cdot S ; \quad |dU_R'| = |-\vec{E}\cdot d\vec{s}|$$

$$= |\vec{E}\cdot d\vec{s}|$$

$$= |E\, dl| \quad (\vec{E} \parallel d\vec{s})$$

$$= E\, dl \quad ; \quad E \equiv |\vec{E}|$$

$$\Rightarrow E\, dl = j_e S \zeta \frac{dl}{S}$$

$$\Rightarrow \boxed{E = j_e \zeta}$$

• E_L : absolutna vrednost jakosti el. polja v žici tuljave

• ζ_L : specifična upornost žice tuljave

$$\Rightarrow \boxed{E_L = j_e \zeta_L}$$

$$\Rightarrow \boxed{\zeta_L \to 0 \Rightarrow E_L = 0 \Rightarrow \vec{E}_L = 0 \Rightarrow \int_{C_L} \vec{E}_L \cdot d\vec{s} = 0}$$

Tudi ostale žice iz superprevodnega materiala (materiala z zanemarljivo specifično upornostjo) $\Rightarrow \boxed{\int_{C_ž} \vec{E}_ž \cdot d\vec{s} = 0}$

Zakon o el. napetosti (4. Maxwellova enačba):

$$\oint_{\partial S} \vec{E}\cdot d\vec{s} = -\int_{S} \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$\vec{S} = \vec{S}_T + \sum_i \vec{S}_i$; $\vec{S}_T$ = površina tokokroga (brez tuljave)

$\vec{S}_i$ = površina i-tega tokokroga

$\partial\vec{S}$: celotna pot okrog in okrog skozi generator, upornik in po žicah, vključno z žicami tuljave

- $\vec{E}_g$: el. polje v generatorju ; C_g : krivulja skozi generator
- $\vec{E}_R$: el. polje v uporniku ; C_R : krivulja skozi upornik
- $\vec{E}_{\check{z}}$: el. polje v žicah (brez tuljave) ; $C_{\check{z}}$: krivulja po žicah
- $\vec{E}_L$: el. polje v žici tuljave ; C_L : krivulja po žici tuljave

$$\Rightarrow \boxed{\oint_{\partial S} \vec{E}\cdot d\vec{s}} = \int_{C_g} \vec{E}_g\cdot d\vec{s} + \int_{C_{\check{z}}} \vec{E}_{\check{z}}\cdot d\vec{s} + \int_{C_R} \vec{E}_R\cdot d\vec{s} + \int_{C_L} \vec{E}_L\cdot d\vec{s}$$

$$= \int_{C_g} \vec{E}_g\cdot d\vec{s} + \int_{C_R} \vec{E}_R\cdot d\vec{s} \qquad \left(\int_{C_{\check{z}}} \vec{E}_{\check{z}}\cdot d\vec{s}, \int_{C_L} \vec{E}_L\cdot d\vec{s} \simeq 0 \right)$$

$$= \boxed{-U_g - U_R}$$

Po drugi strani:

$$\boxed{\int_{\vec{S}} \frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}} = \int_{\vec{S}_T} \frac{\partial\vec{B}_T}{\partial t}\cdot d\vec{S} + \sum_{i=1}^{N} \int_{S_i} \frac{\partial\vec{B}_L}{\partial t}\cdot d\vec{S}$$

$$= \sum_{i=1}^{N} \int_{S_i} \frac{\partial\vec{B}_L}{\partial t}\cdot d\vec{S} \qquad \left(\frac{\partial\vec{B}_T}{\partial t} = 0 \right)$$

$$= N \int_{S_1} \frac{\partial\vec{B}_L}{\partial t}\cdot d\vec{S} \qquad (\vec{B}_L \simeq \text{homogeno v celotni tuljavi})$$

$$= N \frac{\partial}{\partial t} \int_{S_1} \vec{B}_L\cdot d\vec{S} \qquad (S_1 \neq S_1(t))$$

...

$$\ldots = N \frac{\partial}{\partial t} \int_{S_1} B_L \, dS \qquad (\vec{B}_L = B_L \hat{e}_B ;\ d\vec{S} = \hat{e}_S \cdot dS ;\ \hat{e}_S = \hat{e}_B)$$

$$= N \frac{\partial}{\partial t} \left(B_L \int_{S_1} dS \right) \qquad (B_L = \text{homogeno po celotnem preseku } S_1)$$

$$= N \frac{\partial}{\partial t} (B_L S_1)$$

$$= N \frac{\partial}{\partial t} \Phi_{B,1}$$

$$= \frac{\partial}{\partial t} (N \Phi_{B,1})$$

$$= \boxed{\frac{\partial}{\partial t} \Phi_{B,L}}$$

$$= \frac{\partial}{\partial t} (LI) \qquad (\Phi_{B,L} = LI)$$

$$\boxed{= L \dot{I}} \qquad (L = \text{konst.})$$

$$\Rightarrow -U_g - U_R' = -L\dot{I}$$

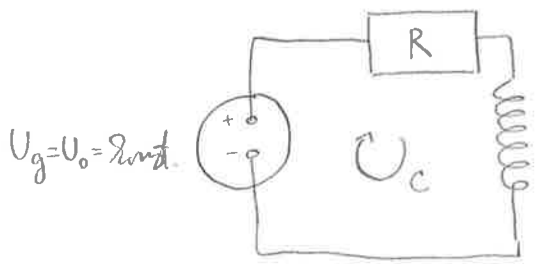
$$\Rightarrow \boxed{U_g + U_R' - L\dot{I} = 0}$$

Def: $\boxed{U_L' \equiv -L\dot{I} \left(= -\frac{\partial}{\partial t} \Phi_{B,L} \right)}$

$$\Rightarrow \boxed{U_g + U_R' + U_L' = 0} \qquad (\text{2. Kirchhoffov izrek za tokokrog s tuljavo})$$

Lahko ga razširimo tudi na tokokroge z dodatnimi elementi (upornik, kondenzatorji, tuljavami, generatorji ...)

Primer: prehodni pojavi s tuljavo



Začetni pogoj: $I(t=0) = I_0$

2. Kirchhoffov izrek: $U_g + U_R' + U_L' = 0$

$$\Rightarrow U_0 - I_R R - \dot{I}_L \cdot L = 0$$

1. Kirchhoffov izrek: $I_R = I_L = I$

$$\Rightarrow U_0 - IR - \dot{I}L = 0$$

$$\Rightarrow \dot{I} + \frac{R}{L} I - \frac{U_0}{L} = 0$$

$$\Rightarrow \dot{I} + \frac{R}{L}\left(I - \frac{U_0}{R}\right) = 0$$

$$\Rightarrow \boxed{\dot{I} + (I - I_\infty)/\tau = 0};$$

$$\boxed{\tau = \frac{L}{R}, \quad I_\infty = \frac{U_0}{R}}$$

$$I' \equiv I - I_\infty \Rightarrow \dot{I}' = \dot{I}$$

$$\Rightarrow \dot{I}' + I'/\tau = 0$$

$$\Rightarrow \frac{\dot{I}'}{I'} = -\frac{1}{\tau}$$

$$\Rightarrow \frac{d}{dt}(\ln I') = -\frac{1}{\tau}$$

$$\Rightarrow \int \frac{d}{dt}(\ln I')\,dt = -\int \frac{1}{\tau}\,dt$$

$$\Rightarrow \ln I' = -\frac{t}{\tau} + \ln K \quad \text{(nedoločeni integral)}$$

$$\Rightarrow \ln I' - \ln K = -t/\tau$$

$$\Rightarrow \ln(I'/K) = -t/\tau$$

$$\Rightarrow I' = K \exp\{-t/\tau\}$$

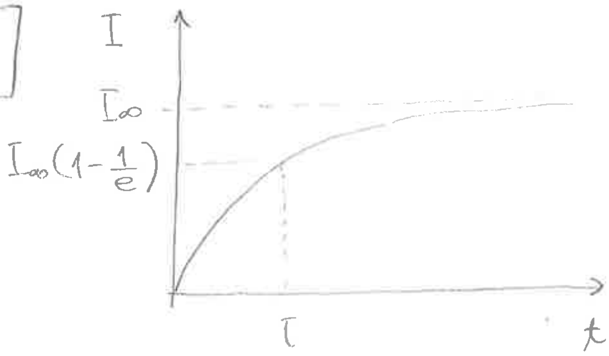
$$\Rightarrow \boxed{I = I_\infty + K \exp\{-t/\tau\}}$$

a) $I_0 = 0$ in $U_0 > 0$

$$\Rightarrow I_\infty = \frac{U_0}{R} > 0$$

$$\Rightarrow I(t=0) = 0 = I_\infty + K \Rightarrow K = -I_\infty$$

$$\Rightarrow \boxed{I = I_\infty \left(1 - \exp\{-t/\tau\}\right)}$$

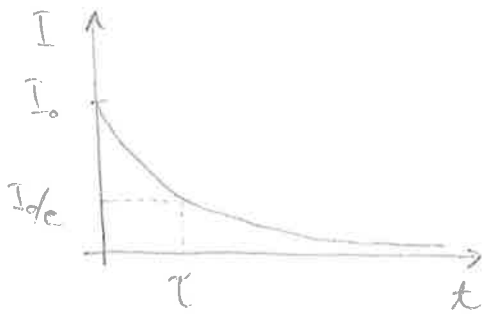


b) $I_0 > 0$ in $U_0 = 0$

$$\Rightarrow I_\infty = \frac{U_0}{R} = 0$$

$$\Rightarrow I(t=0) = I_0 = K$$

$$\Rightarrow \boxed{I = I_0 \exp\{-t/\tau\}}$$



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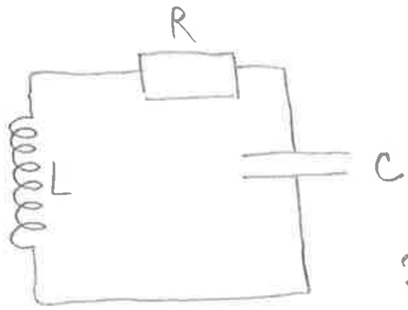
(6) Električni nihajni krog

Primer:



$$\left.\begin{aligned} q_c(t=0) &= q_0 \\ \dot{q}_c(t=0) &= I_0 \end{aligned}\right\} \text{začetna pogoja}$$

2. K. z.: $U_L' + U_R' + U_C' = 0$

$$-L\dot{I} - IR - \frac{q_c}{C} = 0$$

Spomnimo se: $\dot{q}_c = I$ (prehodni pojavi s kondenzatorjem)

$$\Rightarrow L\ddot{q}_c + \dot{q}_c R + \frac{q_c}{C} = 0$$

$q_c \equiv q$ (krajša oznaka) $\Rightarrow \boxed{\ddot{q} + \beta\dot{q} + \omega_0^2 q = 0}$ $\boxed{\beta = \frac{R}{L}\,;\ \omega_0^2 = \frac{1}{LC}}$

enačba (dušenega) nihanja $\Rightarrow$ velja vse, kar smo povedali za dušeno mehansko nihanje (glej poglavje I.)

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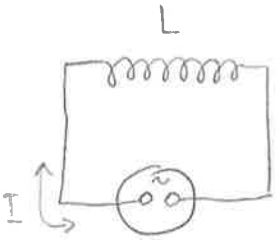
⑦ Izmenični tok skozi tuljavo

Primer:



$U_g = U_0 \sin(\omega t)$

Nastavek: $I = I_0 \sin(\omega t - \delta_L)\,;\ I_0 > 0$

Po drugi strani:

2. K. z.: $U_g + U_L' = 0$

$$\Rightarrow U_g - \dot{I}_L L = 0$$

1. K. z.: $I_L = I$

$$\Rightarrow U_g - \dot{I} L = 0$$

$$\Rightarrow \boxed{U_0 \sin(\omega t)} = I_0 L\omega \cos(\omega t - \delta_L)$$

$$\boxed{= I_0 L\omega\left[\cos(\omega t)\cos\delta_L + \sin(\omega t)\sin\delta_L\right]\,;\ \forall t}$$

$t_1 = 0$: $\sin(\omega t_1) = 0$ in $\cos(\omega t_1) = 1$

$$\Rightarrow 0 = I_0 L\omega \cos\delta_L \Rightarrow \boxed{\delta_L = \pm\frac{\pi}{2}}$$

$t_2 = t_0/4$: $\omega t_2 = \frac{2\pi}{t_0}\frac{t_0}{4} = \frac{\pi}{2} \Rightarrow \sin(\omega t_2) = 1$ in $\cos(\omega t_2) = 0$

$$\Rightarrow U_0 = I_0 L\omega \sin\delta_L$$

a) $\delta_L = -\frac{\pi}{2} \Rightarrow \sin\delta_L = -1 \Rightarrow I_0 = -\frac{U_0}{L\omega} < 0$ //

b) $\boxed{\delta_L = +\frac{\pi}{2}} \Rightarrow \sin\delta_L = 1 \Rightarrow I_0 = \frac{U_0}{L\omega} > 0$ ✓

$$\Rightarrow \boxed{|z_L| = \frac{U_0}{I_0} = L\omega}$$

$$\Rightarrow \boxed{z_L = |z_L| e^{i\delta_L} = L\omega e^{i\frac{\pi}{2}} = i\omega L}$$

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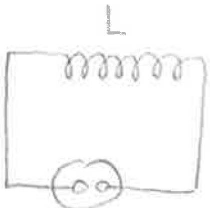
(8) Energija tuljave - magnetnega polja

Primer:



$U_g = U_g(t) \Rightarrow I = I(t)$

2. Kirchhoffov izrek: $U_g + U_L' = 0$

$$\Rightarrow U_g - L\dot{I}_L = 0$$

1. Kirchhoffov izrek: $I_L = I$

$$\Rightarrow U_g = L\dot{I}$$

$$\Rightarrow P_{el} = U_g I = LI\dot{I} = \left(\frac{LI^2}{2}\right)^{\bullet}$$

$$\Rightarrow \boxed{A_{el} = \int_{t_1}^{t_2} P_{el}\, dt = \int_{t_1}^{t_2} \left(\frac{LI^2}{2}\right)^{\bullet} dt = \frac{LI^2}{2}\Big|_{t_1}^{t_2} = \frac{LI_2^2}{2} - \frac{LI_1^2}{2}} ; \quad I_1 = I(t_1), \; I_2 = I(t_2)$$

Delo odvisno le od začetnega in končnega toka $\Rightarrow$ smiselno vpeljati energijo tuljave $\boxed{W_L = \frac{1}{2} L I^2}$

$$\Rightarrow \boxed{A_{el} = \frac{1}{2} L I_2^2 - \frac{1}{2} L I_1^2}$$

$$= W_{L,2} - W_{L,1}$$

$$= \boxed{\Delta W_L}$$

Zapišemo W_L nekoliko drugače:

$$B = \mu_0 \frac{N}{l} I = \frac{L}{NS} I \Rightarrow I = \frac{NS}{L} B \Rightarrow I^2 = \frac{N^2 S^2}{L^2} B^2$$

$$\Rightarrow \boxed{W_L = \frac{1}{2} L I^2}$$

$$= \frac{1}{2} \frac{L \, N^2 S^2 B^2}{L^2}$$

$$= \frac{1}{2} \frac{N^2 S^2 B^2}{L}$$

$$= \frac{1}{2} \frac{N^2 S^2 B^2 l}{\mu_0 N^2 S}$$

$$= \frac{1}{2\mu_0} S l B^2$$

$$= \boxed{V_L \frac{B^2}{2\mu_0}}$$

Idealna tuljava: $V_L = V_B$ (zunaj tuljave ni polja)

$$\Rightarrow W_L = \boxed{V_B \frac{B^2}{2\mu_0} = W_B} \text{ (energija magnetnega polja)}$$

V tuljavi jedro s permeabilnostjo μ:

$$\Rightarrow \boxed{W_B = V_B \frac{B^2}{2\mu\mu_0}}$$

Def: $\boxed{w_B \equiv \frac{W_B}{V_B} = \frac{B^2}{2\mu\mu_0}}$ (gostota energije magnetnega polja).

9 Transformatorji

Primer: eden izmed razlogov za transformacijo toka in napetosti

- manjše naselje, ki troši povprečno el. moč $\bar{P} = 20\,kW$ pri $U_{ef} = 220\,V$ (sinusna napetost)

$\bar{P} = U_{ef} I_{ef} \Rightarrow I_{ef} = \bar{P}/U_{ef} = 91\,A$

- elektrarno in naselje povezuje 1 km bakrenih žic ($\xi = 0{,}017\,\Omega\,mm^2/m$) s prečnim presekom $S = 20\,mm^2$

$\Rightarrow R_ž = \xi \frac{l}{S} \simeq 0{,}85\,\Omega$

$\Rightarrow$ povprečna moč $\bar{P}_R$, ki jo trošijo žice:

$\bar{P}_R = -RI^2 = -7\,kW$ (več kot tretina moči se porabi za gretje žic!?)

Rešitev: tik za elektrarno zvišamo (transformiramo) napetost za (recimo) 10-krat

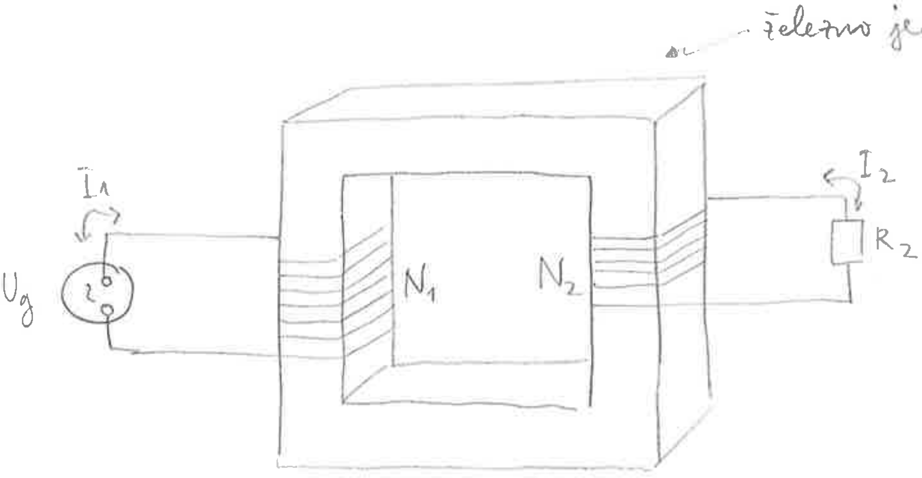
$U_{ef}' = 10\, U_{ef}$

$\Rightarrow I_{ef}' = \bar{P}/U_{ef}' = I_{ef}/10$

$\Rightarrow \bar{P}_R' = -RI_{ef}'^2 = \bar{P}_R/100 = 70\,W$ (~ ena žarnica) ✓

(tik pred naseljem moramo napetost spet znižati (transformirati) na $U_{ef} = 220\,V$).

Princip delovanja transformatorja:



- S: prečni presek jedra
- l: dolžina (obseg) jedra
- $U_g = U_0 \sin(\omega t)$

  $I_1 = I_{1,0} \sin(\omega t - \delta_1)$; $I_{1,0} > 0$

  $I_2 = I_{2,0} \sin(\omega t - \delta_2)$; $I_{2,0} > 0$

- B_1: magn. polje (velikost) v jedru zaradi toka I_1 po primarni tuljavi:

$$B_1 = \mu\mu_0 \frac{N_1}{l} I_1 = \frac{L_1}{N_1 S} I_1 \quad ; \quad L_1 = \frac{\mu\mu_0 N_1^2 S}{l}$$

- B_2: magnetno polje (velikost) v jedru zaradi I_2 po sekundarni tuljavi:

$$B_2 = \mu\mu_0 \frac{N_2}{l} I_2 = \frac{L_2}{N_2 S} I_2 \quad ; \quad L_2 = \frac{\mu\mu_0 N_2^2 S}{l}$$

$$\boxed{\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}}$$

$\Rightarrow$ $\boxed{B = B_1 + B_2 = \frac{L_1}{N_1 S} I_1 + \frac{L_2}{N_2 S} I_2}$: skupno magnetno polje (velikost) v jedru

$$\Rightarrow \boxed{\begin{aligned} \Phi_{B,1} &= N_1 S B = L_1 I_1 + \frac{N_1}{N_2} L_2 I_2 \\ \Phi_{B,2} &= N_2 S B = \frac{N_2}{N_1} L_1 I_1 + L_2 I_2 \end{aligned}}$$

(magnetni pretok skozi obe navitji)

$$\boxed{\begin{aligned} U'_{L,1} &= -\frac{\partial}{\partial t} \Phi_{B,1} = -L_1 \dot{I}_1 - \frac{N_1}{N_2} L_2 \dot{I}_2 \\ U'_{L,2} &= -\frac{\partial}{\partial t} \Phi_{B,2} = -\frac{N_2}{N_1} L_1 \dot{I}_1 - L_2 \dot{I}_2 \end{aligned}}$$

(inducirani napetosti na navitjih)

Očitno: $\boxed{U'_{L,2} = \frac{N_2}{N_1} U'_{L,1}}$ oz. $\boxed{\frac{U'_{L,1}}{U'_{L,2}} = \frac{N_1}{N_2}}$

$\Rightarrow$ $\frac{N_2}{N_1} \gtrless 1$: višanje (nižanje) napetosti $\left(\frac{U'_{L,2}}{U'_{L,1}} \gtrless 1\right)$

- Kako pa je z razmerjem (amplitud) tokov I_2 in I_1 ?

2. Kirchhoffov izrek za primarni tokokrog:

$$\boxed{U_g + U_{L,1}' = 0} \quad \ldots\ldots (1) \quad \Rightarrow \frac{N_2}{N_1} U_g + \frac{N_2}{N_1} U_{L,1}' = 0 \Rightarrow \boxed{\frac{N_2}{N_1} U_g + U_{L,2}' = 0} \ldots (3)$$

2. Kirchhoffov izrek za sekundarni tokokrog:

$$\boxed{U_{L,2}' + U_{R_2}' = 0} \quad \ldots\ldots (2)$$

Enačbo (2) odštejemo od enačbe (3):

$$\frac{N_2}{N_1} U_g - U_{R_2}' = 0$$

$$\Rightarrow \frac{N_2}{N_1} U_g + I_2 R_2 = 0$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} U_0 \sin(\omega t) = -R_2 I_{2,0} \sin(\omega t - \delta_2) = -R_2 I_{2,0}\left[\sin(\omega t)\cos\delta_2 - \cos(\omega t)\sin\delta_2\right] \; ; \forall t}$$

$t_1 = 0$: $\omega t_1 = 0 \Rightarrow \sin(\omega t_1) = 0, \cos(\omega t_1) = 1$

$$\Rightarrow 0 = -R_2 I_{2,0} \sin\delta_2 \Rightarrow \boxed{\delta_2 = 0, \pi}$$

$t_2 = \frac{t_0}{4}$: $\omega t_2 = \frac{\pi}{2} \Rightarrow \sin(\omega t_2) = 1, \cos(\omega t_2) = 0$

$$\Rightarrow \frac{N_2}{N_1} U_0 = -R_2 I_{2,0} \cos\delta_2$$

a) $\delta_2 = 0 \Rightarrow \cos\delta_2 = 1 \Rightarrow I_{2,0} = -\frac{N_2}{N_1}\frac{U_0}{R_2} < 0$ //

b) $\boxed{\delta_2 = \pi} \Rightarrow \cos\delta_2 = -1 \Rightarrow \boxed{I_{2,0} = \frac{N_2}{N_1}\frac{U_0}{R_2}}$ $(> 0 \checkmark)$

$$\Rightarrow \boxed{I_2 = I_{2,0} \sin(\omega t - \delta_2) = -\frac{N_2}{N_1}\frac{U_0}{R_2}\sin(\omega t)}$$

$$\Rightarrow \boxed{\mathring{I}_2 = -\underbrace{\frac{N_2}{N_1}\frac{U_0}{R_2}}_{I_{2,0}}\,\omega\cos(\omega t)}$$

Iskati velja tudi:

$$\boxed{\mathring{I}_1 = I_{1,0}\,\omega\cos(\omega t - \delta_1)}$$

$$\Rightarrow \boxed{U_{L_1}'} = -L_1\mathring{I}_1 - \frac{N_1}{N_2}L_2\mathring{I}_2$$

$$= -L_1\omega I_{1,0}\cos(\omega t - \delta_1) - \frac{N_1}{N_2}L_2\omega\left(-\frac{N_2}{N_1}\frac{U_0}{R_2}\right)\cos(\omega t)$$

$$= -L_1\omega I_{1,0}\cos(\omega t - \delta_1) + L_2\omega\frac{U_0}{R_2}\cos(\omega t)$$

$$\boxed{= -L_1\omega I_{1,0}\left[\cos(\omega t)\cos\delta_1 + \sin(\omega t)\sin\delta_1\right] + L_2\omega\frac{U_0}{R_2}\cos(\omega t)}$$

To vstavimo v enačbo (1):

$$\boxed{U_0\sin(\omega t) - L_1\omega I_{1,0}\left[\cos(\omega t)\cos\delta_1 + \sin(\omega t)\sin\delta_1\right] + L_2\omega\frac{U_0}{R_2}\cos(\omega t) = 0;\ \forall t}$$

$t_1 = 0$: $-L_1\omega I_{1,0}\cos\delta_1 + L_2\omega\frac{U_0}{R_2} = 0 \Rightarrow \boxed{\omega L_2\frac{U_0}{R_2} = \omega L_1 I_{1,0}\cos\delta_1}$ ....(4)

$t_2 = \frac{t_0}{4}$: $U_0 - L_1\omega I_{1,0}\sin\delta_1 = 0 \Rightarrow \boxed{U_0 = \omega L_1 I_{1,0}\sin\delta_1}$

$$\Rightarrow \boxed{\operatorname{tg}\delta_1} = \frac{U_0 R}{\omega L_2 U_0} = \boxed{\frac{R_2}{\omega L_2}}$$

Limitna primera:

a) $R_2 \gg \omega L_2$: $\frac{R}{\omega L_2} \to \infty \Rightarrow \delta_1 \to \frac{\pi}{2}$

b) $R_2 \ll \omega L_2$: $\frac{R}{\omega L_2} \to 0 \Rightarrow \delta_1 \to 0$

Splošno: $\operatorname{tg}\delta_1 = \frac{\sin\delta_1}{\cos\delta_1} = \frac{\sqrt{1-\cos^2\delta_1}}{\cos\delta_1} \Rightarrow \operatorname{tg}^2\delta_1 = \frac{1-\cos^2\delta_1}{\cos^2\delta_1}$

$\Rightarrow \cos^2\delta_1\,\operatorname{tg}^2\delta_1 = 1-\cos^2\delta_1 \Rightarrow \cos^2\delta_1(1+\operatorname{tg}^2\delta_1) = 1 \Rightarrow \boxed{\cos\delta_1} = \frac{1}{\sqrt{1+\operatorname{tg}^2\delta_1}} = \ldots$

$$\ldots = \frac{1}{\sqrt{1+\frac{R_2^2}{\omega^2L_2^2}}} = \boxed{\frac{\omega L_2}{\sqrt{\omega^2L_2^2+R_2^2}}}$$

Iz enačbe (4) sledi:

$$\cancel{L_2}\frac{U_0}{R} = L_1 I_{1,0}\frac{\omega \cancel{L_2}}{\sqrt{\omega^2L_2^2+R^2}}$$

$$\Rightarrow \frac{U_0}{R} = I_{1,0}\frac{\omega L_1}{\sqrt{\omega^2L_2^2+R^2}}$$

$$\Rightarrow \frac{N_2}{N_1}\frac{U_0}{R} = I_{1,0}\frac{N_2}{N_1}\frac{\omega L_1}{\sqrt{\omega^2L_2^2+R^2}}$$

$$\Rightarrow \boxed{I_{2,0} = I_{1,0}\frac{N_2}{N_1}\frac{\omega L_1}{\sqrt{\omega^2L_2^2+R^2}}} \quad \text{ali} \quad \boxed{\frac{I_{1,0}}{I_{2,0}} = \frac{N_1}{N_2}\frac{\sqrt{\omega^2L_2^2+R^2}}{\omega L_1}}$$

To je splošen izraz, v približku $R_2 \ll \omega L_2$ (kratko-sklenjenega sekundarnega kroga) pa:

$$\omega^2L_2^2+R^2 \simeq \omega^2L_2^2$$

$$\Rightarrow \sqrt{\omega^2L_2^2+R_2^2} \simeq \sqrt{\omega^2L_2^2} \simeq \omega L_2$$

$$\Rightarrow \frac{N_1}{N_2}\frac{\sqrt{\omega^2L_2^2+R_2}}{\omega L_1} \simeq \frac{N_1}{N_2}\frac{\cancel{\omega}L_2}{\cancel{\omega}L_1} = \frac{N_1}{N_2}\frac{N_2^2}{N_1^2} = \frac{N_2}{N_1}$$

$$\Rightarrow \boxed{\frac{I_1}{I_2} \simeq \frac{N_2}{N_1}} \quad \text{ali} \quad \boxed{I_2 = I_1\frac{N_1}{N_2}}$$

- $N_2 > N_1$: $I_2 < I_1$, $U_2 > U_1$
- $N_2 < N_1$: $I_2 > I_1$, $U_2 < U_1$
- uporaba velikih tokov: elektroplavži

V nasprotnem skrajnem približku $R_2 \gg \omega L_2$:

$$\boxed{\frac{I_1}{I_2} = \frac{N_1}{N_2}\frac{R}{\omega L_1}}$$