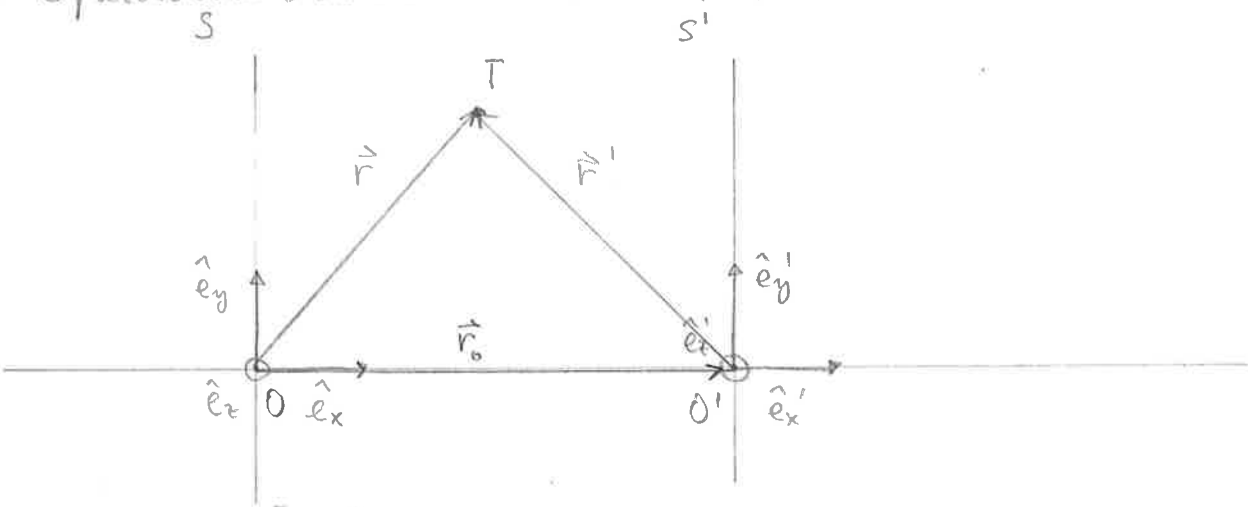
VI. POSEBNA TEORIJA RELATIVNOSTI

1) Galilejeve transformacije (GT)

- Opazovalna sistema S in S' (nepospešena)



$$\vec{r}_0 = v_0 t\,\hat{e}_x = \begin{bmatrix} v_0 t \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vec{r}\,' = x'\hat{e}_x' + y'\hat{e}_y' + z'\hat{e}_z' = x'\hat{e}_x + y'\hat{e}_y + z'\hat{e}_z = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Očitno velja: $\vec{r} = \vec{r}_0 + \vec{r}\,' \Rightarrow \boxed{\vec{r}\,' = \vec{r} - \vec{r}_0}$ ..... (1)

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} v_0 t \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} x' &= x - v_0 t \\ y' &= y \\ z' &= z \end{aligned}}$$

Če dodamo temu (na prvi pogled očitno) predpostavko, da čas v obeh sistemih teče enako hitro, $\boxed{t' = t}$, dobimo Galilejeve transformacije:

G.T.: $\boxed{\begin{aligned} t' &= t \\ x' &= x - v_0 t \\ y' &= y \\ z' &= z \end{aligned}}$ ali $\boxed{\begin{aligned} c_0 t' &= c_0 t \\ x' &= x - \beta_0 c_0 t \\ y' &= y \\ z' &= z \end{aligned}}$; $\boxed{\begin{aligned} c_0 &= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \\ \beta_0 &= \frac{v_0}{c_0} \end{aligned}}$

Recimo, da je hitrost $\vec{v}$ točke T za opazovalca v S

$$\vec{v} = \frac{d\vec{r}}{dt} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ hitrost T za opazovalca v S'

$$\vec{v}\,' = \frac{d\vec{r}\,'}{dt'} = \frac{d\vec{r}\,'}{dt} = \begin{bmatrix} \frac{d}{dt}(x - v_0 t) \\ \frac{dy}{dt} \\ \frac{d}{dt} z \end{bmatrix} = \begin{bmatrix} v - v_0 \\ 0 \\ 0 \end{bmatrix}$$

- zdi se smiselno
- vendar GT (zelo elementarna matematika - seštevanje in odštevanje vektorjev ter odvajanje - in predpostavka $t' = t$) ne opišejo vedno dobro narave

Primer: denimo, da je T svetlobni blisk, $v = c_0$

$\Rightarrow \boxed{c_0' = c_0 - v_0}$

kar je v nasprotju z ugotovitvijo, da je c_0 enaka v vseh opazovalnih sistemih (!?!)

② Lorentzove transformacije (LT)

Za pravilen opis narave moramo GT popraviti

GT		LT
$c_0 t' = c_0 t$	$\longrightarrow$	$c_0 t' = \gamma_0 (c_0 t - \beta_0 x)$
$x' = x - \beta_0 c_0 t$		$x' = \gamma_0 (x - \beta_0 c_0 t)$
$y' = y$		$y' = y$
$z' = z$		$z' = z$

$$\gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}}$$

$$\beta_0 = \frac{v_0}{c_0}$$

Ko GT nadomestimo z LT, iz klasične mehanike (kinematike) preidemo v posebno teorijo relativnosti (PTR).

Komentarji k PTR:

(1) Če $v_0 \ll c_0 \Rightarrow \beta_0 \ll 1 \; (\to 0) \Rightarrow \gamma_0 \simeq 1$
$\Rightarrow \beta_0 x \ll c_0 t$ $\Bigg\}$ $\Rightarrow$ LT zvezno preidejo v GT

(2) $\beta_0^2 \geq 0 \Rightarrow \gamma_0 \geq 1$

(3) $v_0 \to c_0 \Rightarrow \beta_0 \to 1 \Rightarrow \gamma_0 \to \infty$

(4) LT: $c_0 t' = \gamma_0 (c_0 t - \beta_0 x)$

$\Rightarrow t' = \gamma_0 \left(t - \frac{\beta_0}{c_0} x\right)$: v splošnem $t' \neq t$ (?!?) (glej še v nadaljevanju)

(5) LT: simetrija med transformacijo x in $c_0 t$ $\Rightarrow$ vpeljemo 4D vektor

(četverec) prostor-čas

$$X^\mu \equiv \begin{bmatrix} c_0 t \\ x \\ y \\ z \end{bmatrix} \equiv \begin{bmatrix} X^0 \\ X^1 \\ X^2 \\ X^3 \end{bmatrix}$$

časovna komponenta (dimenzija) (1)

prostorske komponente (dimenzije) (3)

Pozor: x^μ vpeljemo izključno na podlagi simetrije (podobnosti) med LT; to ne pomeni da si moramo (moremo) kaj predstavljati v 4 (prostorskih) dimenzijah!

(6) z vpeljavo x^μ lahko LT zapišemo v matrični obliki:

$$x'^\mu = L x^\mu \; ; \; x'^\mu = \begin{bmatrix} c_0 t' \\ x' \\ y' \\ z' \end{bmatrix} \; ; \; L = \begin{bmatrix} \gamma_0 & -\beta_0\gamma_0 & 0 & 0 \\ -\beta_0\gamma_0 & \gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(7) Matrika L je obrnljiva:

$$L^{-1} = \begin{bmatrix} \gamma_0 & \beta_0\gamma_0 & 0 & 0 \\ \beta_0\gamma_0 & \gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(za d.n. preveri da velja $L^{-1}L = LL^{-1} = \mathbb{1}_{4\times 4}$)

$\Rightarrow L^{-1} \{ x'^\mu = L x^\mu$

$\Rightarrow L^{-1} x'^\mu = x^\mu$

$\Rightarrow \boxed{x^\mu = L^{-1} x'^\mu}$: obratna LT (OLT)

OLT po komponentah:

$$\boxed{\begin{aligned} c_0 t &= \gamma_0 (c_0 t' + \beta_0 x') \\ x &= \gamma_0 (x' + \beta_0 c_0 t') \\ y &= y' \\ z &= z' \end{aligned}}$$

③ PTR in transformacija hitrosti

$$\vec{v} \equiv \frac{d\vec{r}}{dt} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \text{ ; hitrost v } S$$

Skladno s tem:

$$\vec{v}\,' \equiv \frac{d\vec{r}\,'}{dt'} = \begin{bmatrix} \frac{dx'}{dt'} \\ \frac{dy'}{dt'} \\ \frac{dz'}{dt'} \end{bmatrix} = \begin{bmatrix} v_x' \\ v_y' \\ v_z' \end{bmatrix} \text{ ; hitrost v } S'$$

$$\left.\begin{aligned} v_x &= \frac{dx}{dt} \\ x &= x[t'(t)] \end{aligned}\right\} \Rightarrow \text{posredno odvajanje: } \boxed{\frac{dx}{dt} = \frac{dx}{dt'}\cdot\frac{dt'}{dt}}$$

OLT: $\boxed{\frac{dx}{dt'}} = \frac{d}{dt'}\left\{\gamma_0\,(x' + \beta_0 c_0 t')\right\} = \gamma_0 \frac{d}{dt'}(x' + \beta_0 c_0 t') = \gamma_0\left(\frac{dx'}{dt'} + \beta_0 c_0\right)$

$$= \boxed{\gamma_0\,(v_x' + v_0)}$$

LT: $\boxed{\frac{dt'}{dt}} = \frac{1}{c_0}\frac{d}{dt}(c_0 t') = \frac{1}{c_0}\frac{d}{dt}\left\{\gamma_0\,(c_0 t - \beta_0 x)\right\} = \frac{\gamma_0}{c_0}\frac{d}{dt}(c_0 t - \beta_0 x)$

$$= \frac{\gamma_0}{c_0}\left(c_0 - \beta_0 \frac{dx}{dt}\right) = \boxed{\gamma_0\left(1 - \frac{v_0 v_x}{c_0^2}\right)}$$

$$\Rightarrow v_x = \gamma_0^2\,(v_x' + v_0)\left(1 - \frac{v_0 v_x}{c_0^2}\right)$$

$$\Rightarrow v_x' + v_0 = \frac{1}{\gamma_0^2}\,\frac{v_x}{\left(1 - \frac{v_0 v_x}{c_0^2}\right)} = \frac{v_x\left(1 - \frac{v_0^2}{c_0^2}\right)}{\left(1 - \frac{v_0 v_x}{c_0^2}\right)} = \frac{v_x\,(c_0^2 - v_0^2)}{(c_0^2 - v_0 v_x)}$$

$$\Rightarrow \boxed{v_x'} = \frac{v_x\,(c_0^2 - v_0^2)}{(c_0^2 - v_0 v_x)} - v_0 = \frac{v_x\,(c_0^2 - v_0^2) - v_0\,(c_0^2 - v_0 v_x)}{(c_0^2 - v_0 v_x)}$$

$$= \frac{v_x c_0^2 - \cancel{v_x v_0^2} - v_0 c_0^2 + \cancel{v_0^2 v_x}}{c_0^2 - v_0 v_x} = \boxed{\frac{v_x - v_0}{\left(1 - \frac{v_0 v_x}{c_0^2}\right)}} \quad \ldots\ldots \quad (Lv_x)$$

Podobno dobimo (izpeljite za d. n.):

$$\boxed{\begin{aligned} v_y' &= \frac{1}{\gamma_0}\,\frac{v_y}{1-\frac{v_x v_0}{c_0^2}} \quad \ldots\ldots \quad (Lv_y) \\ v_z' &= \frac{1}{\gamma_0}\,\frac{v_z}{1-\frac{v_x v_0}{c_0^2}} \quad \ldots\ldots \quad (Lv_z) \end{aligned}}$$

(Lv_x), (Lv_y) in (Lv_z): LT hitrosti.

Primer: $\vec{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} c_0 \\ 0 \\ 0 \end{bmatrix}$

$$\left.\begin{aligned} &\Rightarrow v_x' = \frac{c_0 - v_0}{1-\frac{c_0 v_0}{c_0^2}} = \frac{c_0 - v_0}{1-\frac{v_0}{c_0}} = \frac{c_0\,(c_0 - v_0)}{(c_0 - v_0)} = c_0 \\ &\Rightarrow v_y' = 0 \\ &\Rightarrow v_z' = 0 \end{aligned}\right\} \vec{v}\,' = \vec{v} \quad (\checkmark)$$