

# Introduction to Artificial Intelligence

## Exercise 4: GraphPlan

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### Question 13 - max level and level sum (3 points)

#### Optimality:

Theoretically, for each of the heuristics - is its optimality guaranteed?

#### Zero Heuristic:

The zero heuristic is admissible because it always returns a heuristic value of zero, which is less than or equal to the actual cost to reach the goal in any state. Since it never overestimates the cost, it meets the criteria for admissibility in heuristic search algorithms.

#### Max Level:

The max level heuristic finds the earliest stage where all goal conditions can be met. It's considered admissible because it always underestimates the steps needed to achieve the goal, focusing on the earliest point where all goals can be achieved without considering obstacles (mutex). The planning graph simplifies the problem by ignoring these obstacles, ensuring any actual plan must be at least as long. This cautious approach guarantees the heuristic always provides a valid estimate of the minimum actions needed.

While the max level heuristic ensures finding the shortest possible plan, it may not always lead to the most efficient search in practice. This is because it tends to overlook how actions might interfere with each other, potentially causing the search algorithm to explore many paths before finding the best one.

#### Level Sum:

The level sum heuristic adds up the levels where each goal first appears in the planning graph. It treats each goal separately, which can lead to an overestimation of the total cost. This heuristic isn't admissible because it doesn't account for how achieving one goal might make it easier to achieve another. This oversight can result in redundant steps. Because it isn't admissible, the level sum heuristic doesn't guarantee the shortest possible plan.

**Empirically, what are the lengths of the plans you found for the DWR problem (in questions 11 and 12) with each of the heuristics? Include also the null heuristic results in your comparison. For each of these - is it an optimal plan?**

Heuristic	Actions	Optimality
Zero	6	Yes
Max Level	6	Yes
Level Sum	6	Yes

**Are the theoretical and empirical results consistent with each other? If so, explain. If not, explain how this is possible.**

All three heuristics (zero, max level, level sum) resulted in plans with the same length (6 actions) for the DWR problem.

Despite the level sum heuristic not being theoretically guaranteed to find the shortest plan, it happened to find an optimal plan in this particular scenario.

The zero and max level heuristics, which are theoretically guaranteed to be optimal, also found optimal plans empirically.

### **Running Time:**

**Theoretically, can we claim that one of the heuristics is guaranteed to expand less-or-equal nodes than the other heuristic (in the general case)?**

Therefore, both the theoretical claims of optimality and the empirical results of plan lengths align well in this evaluation of the DWR problem with these heuristics.

When comparing the efficiency of the zero, max level, and level sum heuristics in terms of node expansion, we look at how each guides the search:

The zero heuristic doesn't guide the search—it treats all states as equally far from the goal, resulting in a uniform-cost search.

The max level heuristic is admissible, aiming for the earliest level where all goal conditions are met in the planning graph. However, if goals appear early but are mutually exclusive (mutex), it might underestimate actions needed, potentially increasing node expansions in such cases.

The level sum heuristic isn't admissible but often gives practical guidance by summing the levels where each goal first appears.

The max level heuristic guarantees expanding fewer or an equal number of nodes compared to the zero heuristic due to its admissibility and informed guidance. By never overestimating the goal cost, it directs the search to more promising paths, reducing unnecessary expansions. In contrast, the zero heuristic gives no direction,

treating all paths equally likely to lead to the goal, leading to more node explorations.

The level sum heuristic's node expansion performance is less predictable than the zero and max level heuristics. Its inadmissibility means it may overestimate the goal cost by ignoring goal interdependencies, potentially inflating the heuristic value. While effective in practice and sometimes resulting in fewer node expansions, it doesn't ensure optimal solutions. Thus, its efficiency varies widely depending on specific problems and how well the heuristic aligns with actual costs, making it hard to predict compared to the more defined behaviours of the zero and max level heuristics.

**Empirically, how many search nodes were expanded with each one of the heuristics? So which one was more efficient in this case?**

Heuristic	Number of Search nodes	Plan Length	Optimality
Zero	49	6	Yes
Max Level	29	6	Yes
Level Sum	9	6	Yes

In this case, the Level Sum Heuristic was the most efficient in terms of node expansions, as it expanded the fewest number of nodes (9 nodes) compared to the Zero and Max Level heuristics. This efficiency can be attributed to the Level Sum Heuristic providing more informed guidance, despite not being theoretically guaranteed to always find the shortest path.

While all three heuristics found optimal plans of length 6 actions, the Level Sum Heuristic's fewer node expansions indicate that it directed the search more effectively towards the goal, leveraging the structure of the planning graph to prioritize promising states.

**Question 14 - set level (3 points)**

**Is the optimality of this heuristic (in the same sense as in the previous question) guaranteed?**

The level set heuristic is admissible, ensuring it never overestimates the cost to achieve the goal. It achieves this by only considering levels where all goal conditions can coexist without mutual exclusion, thereby providing a genuine lower bound on the necessary actions. This ensures an accurate estimate of the minimum steps required by avoiding mutex relations among evaluated goal conditions.

Because of its admissibility, the level set heuristic guarantees optimality when guiding an A\* search. This means the A\* algorithm, guided by the level set heuristic, will consistently find the shortest possible plan whenever such a plan exists.

### **What is the relation between this heuristic and the max level heuristic in terms of number of nodes expanded?**

The set level heuristic is typically more accurate than the max level heuristic, leading to fewer node expansions in an A\* search. While the max level heuristic looks for the earliest stage where all goal conditions are met, it doesn't consider situations where these goals might interfere with each other. This oversight can sometimes underestimate the actual number of steps needed, causing the search algorithm to explore more nodes. On the other hand, the set level heuristic ensures all goal conditions are achieved simultaneously at a level without any conflicts, giving a more precise estimate of the actions required. This accuracy helps the A\* search focus on better paths and avoid less promising ones, reducing the number of nodes expanded.

Both heuristics guarantee finding an optimal solution, but the set level heuristic outperforms the max level heuristic by providing clearer guidance. It's essentially a more detailed version of the max level heuristic because it adds the requirement of no conflicts between goals. This refinement means the set level heuristic will always expand fewer or an equal number of nodes compared to the max level heuristic, as the max level heuristic is a simplified form of the set level heuristic.

### **Is this heuristic perfect in the sense that it always returns the precise distance to the goal?**

The set level heuristic isn't flawless because it doesn't always give the exact distance to the goal. It's more precise than the max level heuristic by finding the earliest level where all goal conditions exist without any conflicts. This provides a minimum estimate of the actions needed to achieve the goal within the planning graph's simplified framework.

However, the set level heuristic doesn't consider all the complexities of the actual planning problem, such as specific sequences of actions that might be necessary due to interactions and constraints not fully captured in the planning graph.

So, while the set level heuristic improves search efficiency and gives a good estimate of the distance to the goal, it's not perfect. It doesn't always provide the exact number of steps required because it's based on a simplified model of the problem.