# PHYS 7125 Homework 3

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## 1

## 2

The only non-vanishing components of the metric are

$$g_{\psi\psi} = 1, \quad g_{\theta\theta} = \sin^2\psi, \quad g_{\phi\phi} = \sin^2\psi\sin^2\theta$$

Since the matrix is diagonal, the corresponding components of the inverse metric are simply

$$g^{\psi\psi} = 1, \quad g^{\theta\theta} = 1/\sin^2\psi, \quad g^{\phi\phi} = 1/\sin^2\psi\sin^2\theta$$

The only non-vanishing first derivatives of the metric components are

$$g_{\theta\theta,\psi} = 2\sin\psi\cos\psi, \quad g_{\phi\phi,\psi} = 2\sin\psi\cos\psi\sin^2\theta, \quad g_{\phi\phi,\theta} = 2\sin^2\psi\sin\theta\cos\theta$$

### a

The only non-vanishing Christoffel symbols are

$$\Gamma^\theta_{\theta\psi} = \Gamma^\theta_{\psi\theta} = \frac{1}{2}g^{\theta\theta}g_{\theta\theta,\psi} = \frac{2\sin\psi\cos\psi}{2\sin^2\psi} = \cot\psi$$

$$\Gamma^\psi_{\theta\theta} = -\frac{1}{2}g^{\psi\psi}g_{\theta\theta,\psi} = -\sin\psi\cos\psi$$

$$\Gamma^\phi_{\phi\psi} = \Gamma^\phi_{\psi\phi} = \frac{1}{2}g^{\phi\phi}g_{\phi\phi,\psi} = \frac{2\sin\psi\cos\psi\sin^2\theta}{2\sin^2\psi\sin^2\theta} = \cot\psi$$

$$\Gamma^\psi_{\phi\phi} = -\frac{1}{2}g^{\psi\psi}g_{\phi\phi,\psi} = -\sin\psi\cos\psi\sin^2\theta$$

$$\Gamma^\phi_{\phi\theta} = \Gamma^\phi_{\theta\phi} = \frac{1}{2}g^{\phi\phi}g_{\phi\phi,\theta} = \frac{2\sin^2\psi\sin\theta\cos\theta}{2\sin^2\psi\sin^2\theta} = \cot\theta$$

$$\Gamma^\theta_{\phi\phi} = -\frac{1}{2}g^{\theta\theta}g_{\phi\phi,\theta} = -\frac{2\sin^2\psi\sin\theta\cos\theta}{2\sin^2\psi} = -\sin\theta\cos\theta$$

**b**

There are $\frac{1}{12}\cdot 3^2\cdot(3^2-1)=6$ independent components (others can be obtained by symmetry)

$$\begin{aligned}
R_{\psi\theta\psi\theta} &= g_{\psi\psi}R^{\psi}_{\theta\psi\theta} = \sin^2\psi - \cos^2\psi - 0 + 0 - (-\cos^2\psi) = \sin^2\psi \\
R_{\psi\theta\psi\phi} &= g_{\psi\psi}R^{\psi}_{\theta\psi\phi} = 0 - 0 + 0 - 0 = 0 \\
R_{\psi\theta\theta\phi} &= g_{\psi\psi}R^{\psi}_{\theta\theta\phi} = 0 - 0 + 0 - 0 = 0 \\
R_{\psi\phi\psi\phi} &= g_{\psi\psi}R^{\psi}_{\phi\psi\phi} = (\sin^2\psi - \cos^2\psi)\sin^2\theta - 0 + 0 - (-\cos^2\psi\sin^2\theta) = \sin^2\psi\sin^2\theta \\
R_{\psi\phi\theta\phi} &= g_{\psi\psi}R^{\psi}_{\phi\theta\phi} = -2\sin\psi\cos\psi\sin\theta\cos\theta - 0 + \sin\psi\cos\psi\sin\theta\cos\theta - (-\sin\psi\cos\psi\sin\theta\cos\theta) = 0 \\
R_{\theta\phi\theta\phi} &= g_{\theta\theta}R^{\theta}_{\phi\theta\phi} = \sin^2\psi\Big[\sin^2\theta - \cos^2\theta - 0 + (-\cos^2\psi\sin^2\theta) - (-\cos^2\theta)\Big] = \sin^4\psi\sin^2\theta
\end{aligned}$$

$$\begin{aligned}
R_{\psi\psi} &= g^{\theta\theta}R_{\theta\psi\theta\psi} + g^{\phi\phi}R_{\phi\psi\phi\psi} = 2 \\
R_{\psi\theta} &= g^{\phi\phi}R_{\phi\psi\phi\theta} = 0 \\
R_{\psi\phi} &= g^{\theta\theta}R_{\theta\psi\theta\phi} = 0 \\
R_{\theta\theta} &= g^{\psi\psi}R_{\psi\theta\psi\theta} + g^{\phi\phi}R_{\phi\theta\phi\theta} = \sin^2\psi + \sin^2\psi = 2\sin^2\psi \\
R_{\theta\phi} &= g^{\psi\psi}R_{\psi\theta\psi\phi} = 0 \\
R_{\phi\phi} &= g^{\psi\psi}R_{\psi\phi\psi\phi} + g^{\theta\theta}R_{\theta\phi\theta\phi} = \sin^2\psi\sin^2\theta + \sin^2\psi\sin^2\theta = 2\sin^2\psi\sin^2\theta
\end{aligned}$$

$$R = g^{\psi\psi}R_{\psi\psi} + g^{\theta\theta}R_{\theta\theta} + g^{\phi\phi}R_{\phi\phi} = 2 + 2 + 2 = 6$$