CX 4640 Homework 4

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1

(a)

$$g_1'(x) = 1 - 2x$$

The iteration is locally convergent when $|1 - 2x| < 1$, or equivalently, $0 < x < 1$, Conversely, when $x \leq 0$ or $x \geq 1$, the iteration is locally divergent.

(b)

$$g_2'(x) = 1 - \frac{2}{3}x$$

Following the same reasoning as (a), when $0 < x < 3$ the iteration is locally convergent and when $x \leq 0$ or $x \geq 3$, the iteration is locally divergent.

(c)

$$f'(x) = 2x$$

The function is given by

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - y}{2x} = \frac{1}{2}\left(x + \frac{y}{x}\right)$$

2

(a)

The iteration function of this scheme is

$$g(x) = x - f(x)/d$$

It's locally convergent when

$$|g'(x^*)| = |1 - f'(x^*)/d| < 1$$

(b)

$$g'(x) = 1 - f'(x)/d$$

$$x_{n+1} = g(x_n) = r + g'(r)(x_n - r) + \frac{g''(c)}{2}(x_n - r)^2$$

$$x_{n+1} - r = g'(r)(x_n - r) + \frac{g''(c)}{2}(x_n - r)^2$$

$$\frac{x_{n+1} - r}{x_n - r} = g'(r) + \frac{g''(c)}{2}(x_n - r)$$

$$\lim_{n\to\infty} \frac{x_{n+1} - r}{x_n - r} = g'(r) + 0 = g'(r) = 1 - f'(r)/d$$

In general, the convergence rate is $1 - f'(r)/d$, where r is the fixed point

(c)

To achieve quadratic convergence, we can set $g'(r) = 0$.
Proof: when $g'(r) = 0$,

$$\begin{aligned}
x_{n+1} &= g(x_n) \\
&= r + g'(r)(x_n - r) + \frac{g''(r)}{2}(x_n - r)^2 + \frac{g'''(c)}{6}(x_n - r)^3 \\
&= r + \frac{g''(r)}{2}(x_n - r)^2 + \frac{g'''(c)}{6}(x_n - r)^3
\end{aligned}$$

$$\frac{x_{n+1} - r}{(x_n - r)^2} = \frac{g''(r)}{2} + \frac{g'''(c)}{6}(x_n - r)$$

$$\lim_{n\to\infty} \frac{x_{n+1} - r}{(x_n - r)^2} = \frac{g''(r)}{2}$$

Therefore d needs to satisfy

$$g'(r) = 1 - f'(r)/d = 0$$

$$d = f'(r)$$

3

(a)

(1)

$$
\begin{aligned}
LHS &= R_{k+1} \\
&= I - AX_{k+1} \\
&= I - A(X_k + X_k(I - AX_k)) \\
&= I - A(2X_k - X_kAX_k) \\
&= I - 2AX_k + (AX_k)^2
\end{aligned}
$$

$$
\begin{aligned}
RHS &= R_k^2 \\
&= (I - AX_k)^2 \\
&= I - 2AX_k + (AX_k)^2
\end{aligned}
$$

Therefore

$$R_{k+1} = R_k^2$$

(2)

$$
\begin{aligned}
LHS &= E_{k+1} \\
&= A^{-1} - X_{k+1} \\
&= A^{-1} - (X_k + X_k(I - AX_k)) \\
&= A^{-1} - (2X_k - X_kAX_k) \\
&= A^{-1} - 2X_k + X_kAX_k)
\end{aligned}
$$

$$
\begin{aligned}
RHS &= E_kAE_k \\
&= (A^{-1} - X_k)A(A^{-1} - X_k) \\
&= (A^{-1} - X_k)(I - AX_k) \\
&= A^{-1} - 2X_k + X_kAX_k
\end{aligned}
$$

Therefore

$$E_{k+1} = E_kAE_k$$

(b)

See `MyInverse.m`

4

$$f'(x) = 2n \cdot x^{2n-1}$$

The iteration scheme is

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^{2n} - a^n}{2n \cdot x_k^{2n-1}}$$

For $n = 1$,

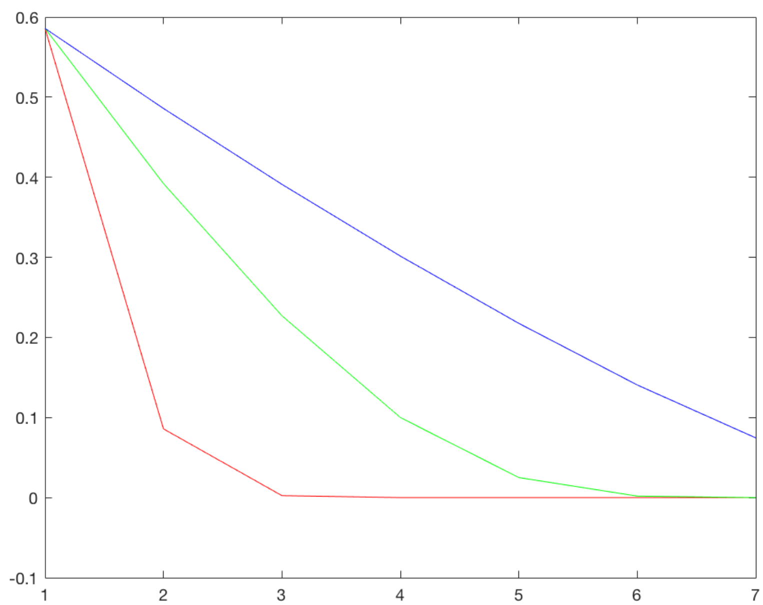
$$x_{k+1} = x_k - \frac{x_k^2 - a}{2x_k}$$

For $n = 5$,

$$x_{k+1} = x_k - \frac{x_k^{10} - a^5}{10x_k^9}$$

For $n = 10$,

$$x_{k+1} = x_k t - \frac{x_k^{20} - a^{10}}{20x_k^{19}}$$



The smaller n is, the faster the iteration converges.

5

See `Newton.m`

6

See `Symmetrize.m`, `Diagonalize.m` and `MySVD.m`