$$\begin{aligned}\frac{\partial y}{\partial x} &= \frac{\partial y}{\partial w_1}\frac{\partial w_1}{\partial x} \\ &= \left(\frac{\partial y}{\partial w_2}\frac{\partial w_2}{\partial w_1}\right)\frac{\partial w_1}{\partial x} \\ &= \left(\left(\frac{\partial y}{\partial w_3}\frac{\partial w_3}{\partial w_2}\right)\frac{\partial w_2}{\partial w_1}\right)\frac{\partial w_1}{\partial x} \\ &= \cdots\end{aligned}$$

$$f(x) + f(y) = f(x + y)$$

$$\begin{aligned}f(0) &= 0 \\ f(x) + f(y) &= f(x + y)\end{aligned}$$

$$\mathrm{Wrap}^2[P]\ x$$

$$\begin{aligned}\mathrm{Interleave}^2 &: O(\mathrm{size}(x)) \\ \mathbf{D}^2_\sigma[P] &: O(\mathrm{cost}(P\ x)) \\ \mathrm{Deinterleave}^2 &: O(\mathrm{size}(P\ x))\end{aligned}$$

$$\mathrm{total} : O(\mathrm{cost}(P\ x) + \mathrm{size}(x))$$

$$\mathrm{snd}(\mathrm{Wrap}^2[P]\ x)\ dy$$

$$O(\mathrm{cost}(P\ x) + \mathrm{size}(x))$$

$$\begin{aligned}(\underline{\lambda}(z : \mathbb{R}).\mathrm{SCotan}(0, \ldots, 0, z, 0, \ldots, 0)) &: O(\mathrm{size}(x)) \\ (\underline{\lambda}(z : \mathbb{R}).0_{\mathrm{Staged}}) &: O(\mathrm{size}(x)) \\ (\underline{\lambda}(z : \mathbb{R}).\mathrm{SCall}\ d_1\ (\partial_1 op(\ldots)(z)) +_{\mathrm{Staged}} \cdots +_{\mathrm{Staged}}\ \mathrm{SCall}\ d_n\ (\partial_n op(\ldots)(z))) &: O(\mathrm{size}(x))\end{aligned}$$

$$\begin{aligned}&(M, 0, +) \\ &(M \to M, \mathrm{id}, \circ)\end{aligned}$$

$$(\text{Staged } c, 0_{\text{Staged}}, +_{\text{Staged}})$$
$$(\text{Staged } c \to \text{Staged } c, \text{id}, \circ)$$

$$O((\text{cost}(P\ x) + \text{size}(x)) \log(\text{cost}(P\ x) + \text{size}(x)))$$