Problem 3

a). $f(x,y,z) = (x+y+z)^2$

let $g(x,y,z) = x^2 + xy^2 + 3z^2$

to find extreme $\nabla f = \lambda \nabla g$

$2(x+y+z) = \lambda 2x$,

$2(x+y+z) = \lambda \cdot 4y$

$2(x+y+z) = \lambda \cdot 6z$

$x = xy = 3z$, where $x^2 + xy^2 + 3z^2 = 1$

$x^2 + \frac{x^2}{2} + \frac{x^2}{3} = \frac{11}{6}x^2 = 1$

$(x, y, z) = \left(\sqrt{\frac{6}{11}}, \sqrt{\frac{6}{22}}, \sqrt{\frac{2}{33}}\right)$

$f\left(\sqrt{\frac{6}{11}}, \sqrt{\frac{6}{22}}, \sqrt{\frac{2}{33}}\right) = \frac{11}{6}$ ✱

b) $f(x,y,z) = xy + z^2$

let $g(x,y,z) = x^2 + y^2 + z^2$

to find extreme, $\nabla f = \lambda \nabla g$

$y = 2x\lambda = 4y\lambda^2$

$x = 2y\lambda$

$2z = \lambda 2z$,

where $x^2 + y^2 + z^2 = 1$

$\lambda = 0 \Rightarrow x = y$, $z = 1$

if $\lambda = 1$ or $\frac{1}{4}$, x, y or z but exist ✓

$f(0,0,1) = 1$ ✱