Homework Set 3

Problem 1 (Overfitting)

The aim of this exercise is to visualize the phenomena of overfitting. Recall the normal distribution with mean μ and variance σ^2:

$$N_{\mu,\,\sigma^2}(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

Define $g_{\sigma^2}(t) = sin^2(2\pi t) + N_{0,\,\sigma^2}$ a random function with normal noise. Create a time sequential signal S that contains 30 data points $(t_1, \ldots, t_{30})$ in the period $\mathrm{t} = [0, 1]$ with uniform period:

$$S_{\sigma^2=0.07} = \{(t_1, g_{0.07}(t_1)), \ldots, (t_{30}, g_{0.07}(t_{30}))\}$$

a) Fit the signal with a polynomial bases of dimensions $k = [2\ 5\ 10\ 14\ 18]$. Plot the 5 curves superimposed over a plot of the data points.

b) Let the $e_k(S)$ denote the mean squared training error of the fitting of the data set S with polynomial basis of dimension k. Plot the log of the training error $e_k(S)$ versus the polynomial dimension $k = 1, \ldots, 18$.

Generate a test signal T of a thousand points: $T_{\sigma^2=0.07} = \{(t_1, g_{0.07}(t_1)), \ldots, (t_{1000}, g_{0.07}(t_{1000}))\}$.

c) Let $e_k(S, T)$ denote the mean squared “test” error of the test signal T on the polynomial of dimension k fitted from training set S. Plot the log of the test error versus the polynomial dimension $k = 1, \ldots, 18$.

d) Compare the training error with the test error. Describe what you observe.

Problem 2 (AIC and BIC)

Given the data:

x_i	0.2	0.3	0.6	0.9	1.1	1.3	1.4	1.6
y_i	0.050446	0.098426	0.33277	0.7266	1.0972	1.5697	1.8487	2.5015

Construct the least squares polynomial model of degree 1, compute the residual sum of squares, and AIC and BIC value for the model. Repeat this for the polynomial of degree 2 and 3. Plot the data with the models. What model should be chosen based on AIC and BIC?

Problem 3 (Lagrange Multiplier)

a) Find the minimum value of the function $f(x, y, z) = (x + y + z)^2$, subject to the constraint $x^2 + 2y^2 + 3z^2 = 1$.

b) Find the minimum value of the function $f(x, y, z) = xy + z^2$, subject to the constraint $x^2 + y^2 + z^2 - 1 = 0$.