

Circuits and Transforms

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CONTENTS

Abstract—This manual provides a simple introduction to Transforms

1 DEFINITIONS

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \quad (1.1)$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st} dt \quad (1.2)$$

2 LAPLACE TRANSFORM

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1 \mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2 \mu C$.

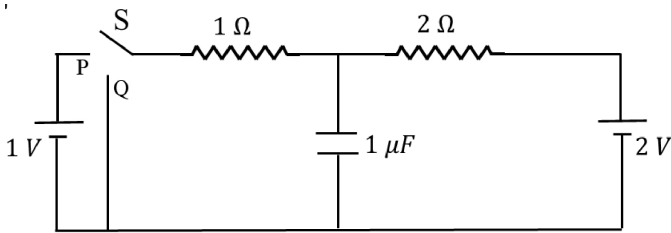


Fig. 2.1

2. Draw the circuit using latex-tikz.

3. Find q_1 .

Solution: No current will pass through the capacitor, hence

$$i = \frac{2 - 1}{2 + 1} = \frac{1}{3} A$$

$$V_c = 2 - 2i = \frac{4}{3} V$$

$$q_1 = \frac{4}{3} \mu C$$

4. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.

Solution:

$$\begin{aligned} U(s) &= \int_{-\infty}^{\infty} u(t)e^{-st} dt \\ &= \int_0^{\infty} e^{-st} dt = \frac{1}{s}, \quad s > 0 \end{aligned}$$

5. Show that

$$e^{-at}u(t) \xleftrightarrow{\mathcal{H}} L \frac{1}{s+a}, \quad a > 0 \quad (2.1)$$

and find the ROC.

Solution:

$$\begin{aligned} &= \int_{-\infty}^{\infty} u(t)e^{-at}e^{-st} dt \\ &= \int_0^{\infty} e^{-(s+a)t} dt \\ &= \frac{1}{s+a}, \quad s+a > 0 \end{aligned}$$

6. Now consider the following resistive circuit transformed from Fig. ?? where

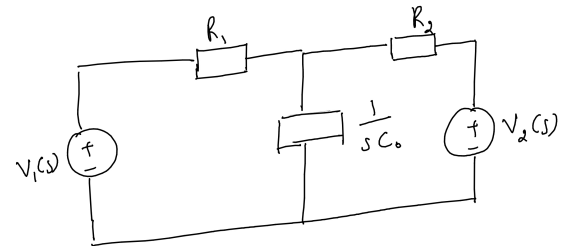


Fig. 2.2

$$u(t) \xleftrightarrow{\mathcal{H}} LV_1(s) \quad (2.2)$$

$$2u(t) \xleftrightarrow{\mathcal{H}} LV_2(s) \quad (2.3)$$

Find the voltage across the capacitor $V_{C_0}(s)$.

Solution: From Q2.4

$$V_1(s) = \frac{1}{s} \quad V_2(s) = \frac{2}{s}$$

$$\frac{V_1 - V_C}{R_1} + \frac{V_2 - V_C}{R_2} - V_C sC = 0$$

simplifying,

$$\Rightarrow V_C(s) = \frac{2R_1 + R_2}{R_1 + R_2} \left(\frac{1}{s} - \frac{1}{\frac{R_1 + R_2}{R_1 R_2 C} + s} \right)$$

7. Find $v_{C_0}(t)$. Plot using python.

Solution: Using the inverse Laplace transform from the obtained $V_C(s)$ we get,

$$v_C(t) = \frac{2R_1 + R_2}{R_1 + R_2} u(t) \left(1 - e^{-\frac{R_1 + R_2}{R_1 R_2 C} t} \right)$$

$$R_1 = 1\Omega \quad R_2 = 2\Omega \quad C = 1\mu F$$

$$v_C(t) = \frac{4}{3} u(t) \left(1 - e^{-1.5 \times 10^6 t} \right)$$

wget <https://github.com/Galahad7377/EE3900/blob/main/cktsig/codes/2.7.py>

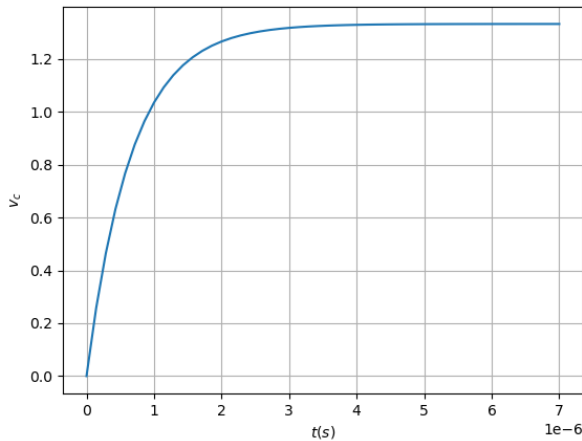


Fig. 2.3: $v_C(t)$

8. Verify your result using ngspice.

9. Obtain Fig. ?? using the equivalent differential equation.

Solution:

$$\frac{dq}{dt} = \frac{V_1 - V_C}{R_1} + \frac{V_2 - V_C}{R_2}$$

$$\frac{dq}{dt} = 1 - \frac{q}{C} + \frac{1}{2} \left(2 - \frac{q}{C} \right)$$

simplifying and integrating,

$$\int_0^{q_1} \frac{dq}{2 - 1.5q/C} = \int_0^t dt$$

$$\ln \frac{2 - 1.5q/C}{2} = 1.5 \frac{t}{C}$$

$$\Rightarrow q_1 = \frac{4}{3} \left(1 - e^{-1.5 \times 10^6 t} \right) \times 10^{-6}$$

3 INITIAL CONDITIONS

1. Find q_2 in Fig. ??.

Solution:

$$i = \frac{2}{1 + 2} = \frac{2}{3} A$$

$$V_C = 2 - 2i = \frac{2}{3} V$$

$$\Rightarrow q_2 = CV_C = \frac{2}{3} \mu C$$

2. Draw the equivalent s -domain resistive circuit when S is switched to position Q. Use variables R_1, R_2, C_0 for the passive elements. Use latex-tikz.

3. $V_{C_0}(s) = ?$

Solution:

$$\frac{V_C - 2/s}{R_2} + \frac{V_C}{R_1} + (V_C - 4/3s) sC = 0$$

$$V_C(s) = \frac{\frac{2}{sR_2} + \frac{4}{3}C}{\frac{1}{R_1} + \frac{2}{R_2} + sC}$$

4. $v_{C_0}(t) = ?$ Plot using python.

Solution: Further simplifying,

$$V_C(s) = \frac{4}{3} \left(\frac{1}{\frac{R_1 + R_2}{R_1 R_2 C} + s} \right) + \frac{2R_1 R_2}{R_2 (R_1 + R_2)} \left(\frac{1}{s} - \frac{1}{\frac{R_1 + R_2}{R_1 R_2 C} + s} \right)$$

Using the inverse Laplace transform from the obtained $V_C(s)$ we get,

$$v_C(t) = \frac{4}{3} e^{\frac{R_1 + R_2}{R_1 R_2 C} t} + \frac{2R_1 R_2}{R_2 (R_1 + R_2)} \left(1 - e^{\frac{R_1 + R_2}{R_1 R_2 C} t} \right)$$

$$R_1 = 1\Omega \quad R_2 = 2\Omega \quad C = 1\mu F$$

$$\Rightarrow v_C(t) = \frac{2}{3} u(t) \left(1 + e^{-1.5 \times 10^6 t} \right)$$

wget <https://github.com/Galahad7377/EE3900/blob/main/cktsig/codes/3.4.py>

5. Verify your result using ngspice.

6. Find $v_{C_0}(0-), v_{C_0}(0+)$ and $v_{C_0}(\infty)$.

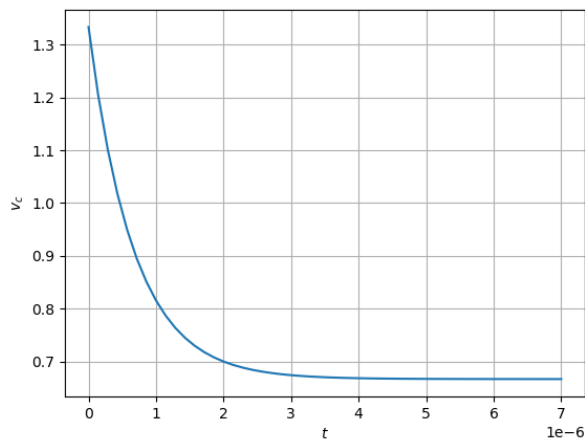


Fig. 3.1: $v_c(t)$

Solution: from section 2,

$$v_c(0-) = \frac{4}{3}V$$

from Q3.4

$$v_c(0+) = \frac{4}{3}V$$

$$v_c(\infty) = \frac{2}{3}V$$

7. Obtain the Fig. in problem ?? using the equivalent differential equation.

Solution:

$$\frac{dq}{dt} = \frac{2 - V_C}{2} - V_C$$

simplifying and integrating,

$$\int_{4/3\mu C}^q \frac{dq}{2 - 3q/C} = 2 \int_0^t dt$$

$$\ln \frac{2 - 3q/C}{-2} = -1.5 \frac{t}{C}$$

$$q = \frac{2}{3} \left(1 + e^{-1.5 \times 10^6 t} \right) \times 10^{-6}$$