Pingala Series

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Contents

***Abstract*—This manual provides a simple introduction to Transforms**

1 JEE 2019

Let

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \tag{1.1}$$

$$b_n = a_{n-1} + a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \tag{1.2}$$

Verify the following using a python code.

1.1
$$\sum_{k=1}^{n} a_k = a_{n+2} - 1, \quad n \geq 1 \tag{1.3}$$

1.2
$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \tag{1.4}$$

1.3
$$b_n = \alpha^n + \beta^n, \quad n \geq 1 \tag{1.5}$$

1.4
$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \tag{1.6}$$

Solution:

```
wget https://github.com/Galahad7377/
    EE3900/blob/main/pingala/codes/1.py
```

2 Pingala Series

2.1 The *one sided* Z-transform of $x(n)$ is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad z \in \mathbb{C} \tag{2.1}$$

2.2 The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n), \quad x(0) = x(1) = 1, n \geq 0 \tag{2.2}$$

Generate a stem plot for $x(n)$.
Solution:

```
wget https://github.com/Galahad7377/
    EE3900/blob/main/pingala/codes/2.2.
    py
```

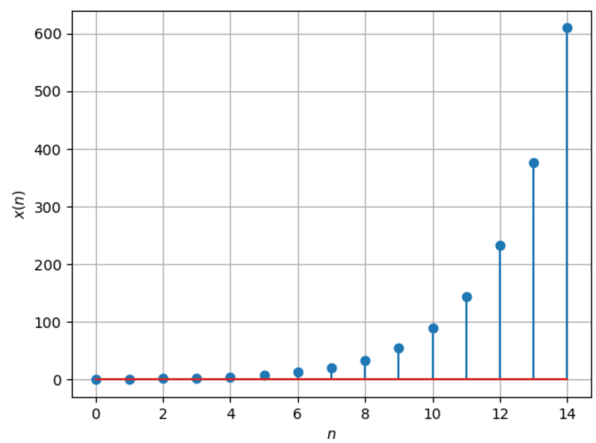

Fig. 2.2: Pingala series

2.3 Find $X^+(z)$.
Solution:

$$x(n+2) = x(n+1) + x(n)$$

$$\implies \mathcal{Z}^+\{x(n+2)\} = \mathcal{Z}^+\{x(n+1)\} + \mathcal{Z}^+\{x(n)\}$$

$$\implies z^2 X^+(z) - z^2 - z = zX^+(z) - z + X^+(z)$$

$$\implies X^+(z) = \frac{1}{1 - z^{-1} - z^{-2}}$$

2.4 Find $x(n)$.
Solution:

$$\frac{1}{1 - z^{-1} - z^{-2}} = \frac{1}{(1 - \alpha z^{-1})(1 - \beta z^{-1})}$$

$$X^+(z) = \left(\frac{z}{\alpha - \beta}\right)\left(\frac{1}{1 - \alpha z^{-1}} - \frac{1}{1 - \beta z^{-1}}\right)$$

using GP summation

$$= \sum_{n=0}^{\infty} \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} z^{-n}$$

$$\implies x(n) = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} u(n) = a_{n+1} u(n)$$

2.5 Sketch

$$y(n) = x(n-1) + x(n+1), \quad n \geq 0 \qquad (2.3)$$

Solution:

```
wget https://github.com/Galahad7377/
    EE3900/blob/main/pingala/codes/2.5.
    py
```

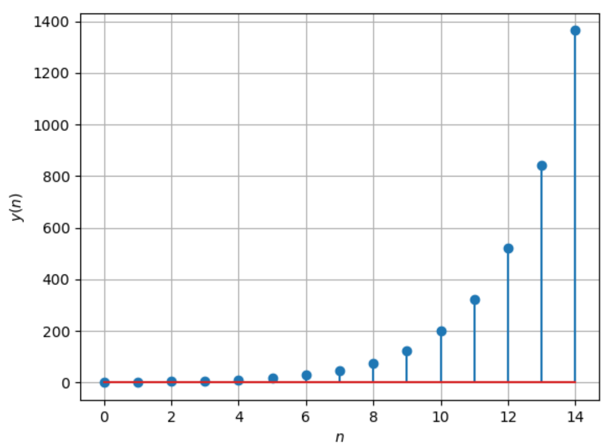

Fig. 2.5: y(n)

2.6 Find $Y^+(z)$.

Solution:

$$\mathcal{Z}^+\{y(n)\} = \mathcal{Z}^+\{x(n+1)\} + \mathcal{Z}^+\{x(n-1)\}$$

$$Y^+(z) = zX^+(z) - z + z^{-1}X^+(z)$$

$$X^+(z) = \frac{1}{1 - z^{-1} - z^{-2}}$$

$$\implies Y^+(z) = \frac{1 + 2z^{-1}}{1 - z^{-1} - z^{-2}}$$

2.7 Find $y(n)$.

Solution:

$$Y^+(z) = X^+(z) + \frac{2}{(z-\alpha)(z-\beta)}$$

$$= X^+(z) + \frac{2}{\alpha - \beta}\left(\sum_{n=0}^{\infty} (\alpha^n + \beta^n) z^{-n}\right)$$

$$\implies y(n) = a_{n+1}u(n) + 2\frac{\alpha^n + \beta^n}{\alpha - \beta}u(n)$$

$$= \frac{\alpha^{n+1} - \beta^{n+1} + 2\alpha^n + 2\beta^n}{\alpha - \beta}u(n)$$

using the fact that $\alpha\beta = 1$ and $\alpha + \beta = -1$

$$\implies y(n) = \alpha^{n+1} + \beta^{n+1}$$

3 Power of the Z transform

3.1 Show that

$$\sum_{k=1}^{n} a_k = \sum_{k=0}^{n-1} x(n) = x(n) * u(n-1) \qquad (3.1)$$

3.2 Show that

$$a_{n+2} - 1, \quad n \geq 1 \qquad (3.2)$$

can be expressed as

$$[x(n+1) - 1]\, u(n) \qquad (3.3)$$

3.3 Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{x(k)}{10^k} = \frac{1}{10} X^+(10) \qquad (3.4)$$

3.4 Show that

$$\alpha^n + \beta^n, \quad n \geq 1 \qquad (3.5)$$

can be expressed as

$$w(n) = \left(\alpha^{n+1} + \beta^{n+1}\right) u(n) \qquad (3.6)$$

and find $W(z)$.

3.5 Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{y(k)}{10^k} = \frac{1}{10} Y^+(10) \qquad (3.7)$$

3.6 Solve the JEE 2019 problem.