Assignment2

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Problem 2.25a

Let the signal

$$y[n] = x[n] * h[n]$$

where

$$x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]$$

and

$$h[n] = \left(\frac{1}{4}\right)^n u[n+3]$$

Determine $y[n]$ without using the distributive property of convolution.
Solution:
$u[-n-1]$ will only be non-zero for $n \leq -1$

$$\therefore x[n] = \left(\frac{1}{3}\right)^{|n|}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|k|} \left(\frac{1}{4}\right)^{n-k} u[n-k+3]$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{k} \left(\frac{1}{4}\right)^{n-k} u[n-k+3] + \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k} \left(\frac{1}{4}\right)^{n-k} u[n-k+3]$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{k} \left(\frac{1}{4}\right)^{n-k} u[n-k+3] + \frac{1}{12}\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{k} \left(\frac{1}{4}\right)^{n+k} u[n+k+4]$$

using summation of geometric progression formulae we can say that,

$$\implies y(n) = \begin{cases} \frac{1}{4^n.11} + \frac{4^4}{3^{n-1}} - \frac{3}{4^n} & n \geq -3 \\ \frac{4^4}{11} & n = -4 \\ \frac{12^4.3^n}{11} & n < -4 \end{cases}$$