Where is Helvellyn? Fuzziness of multi-scale landscape morphometry

Peter Fisher*, Jo Wood† and Tao Cheng‡

The landscape in which people live is made up of many features, which are named and have importance for cultural reasons. Prominent among these are the naming of upland features such as mountains, but mountains are an enigmatic phenomenon which do not bear precise and repeatable definition. They have a vague spatial extent, and recent research has modelled such classes as spatial fuzzy sets. We take a specifically multi-resolution approach to the definition of the fuzzy set membership of morphometric classes of landscape. We explore this idea with respect to the identification of culturally recognized landscape features of the English Lake District. Discussion focuses on peaks and passes, and the results show that the landscape elements identified in the analysis correspond to well-known landmarks included in a place name database for the area, although many more are found in the analysis than are named in the available database. Further analysis shows that a richer interrogation of the landscape can be achieved with Geographical Information Systems when using this method than using standard approaches.

key words fuzzy sets landforms morphometry multi-scale analysis mountains the Lake District

*Department of Geography, University of Leicester, Leicester LE1 7RH
email: pete.fisher@le.ac.uk
†School of Informatics, City University, London
‡Advanced Centre for Spatial Information Technology, The Hong Kong Polytechnic University, Kowloon, Hong Kong

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Introduction

'Where is a mountain?' is a simple question to ask, but it is not easy to give a consistent, precise response. When people look at a landscape which is made up of the surface of the earth continuously varying in elevation, together with natural and built phenomena superimposed on it, they tend to divide it into different regions or features, many of which are given names. In an upland area one group of important features is the mountains (Fisher and Wood 1998). But where are they? What is their extent?

It is easy to give a trivial answer: the mountain is the highest point, the summit. In this interpretation, the location of the mountain seems quite specific, but in many cases even this is open to interpretation, experience, opinion, discussion and even argument (Varzi 2001; Fisher and Wood 1998). If the summit is the mountain, then to have visited the mountain, you have to have walked to that specific point. If you walked near it, then you cannot say that you have been to the mountain. However, most people do not consider this a necessary or acceptable definition of the mountain, and 'summit' is not usually considered a synonym for 'mountain'. For most people therefore, the mountain is not a point, it is a region which to some degree they can visit. If they go to the summit (or near it), then they will feel that they have definitely visited the mountain (and generally others would agree). They may still feel that they have visited the mountain, however, if they only get close to the summit, although someone who has been to that, or another summit, may disagree. The mountain has a spatial extent, but that extent is a matter of degree; some areas are definitely the mountain, others are to a lesser extent part of the mountain, and different people may recognize different extents of the mountain.

The mountain is not a real (*bona fide*) feature; it can be interpreted as simply a region in the continuum

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of variation of the surface of the earth, which people choose to identify and name. It can be interpreted as a *fiat* object (*sensu* Smith and Varzi 2000) which only exists in the human understanding and division of the landscape. Furthermore, the features are vague in a philosophical sense (Williamson 1994; Sainsbury 1989 1995, 25), which is to say that either the boundary condition is not well defined (semantic vagueness as defined in the sorites paradox or paradox of the heap; Sainsbury 1995), or the class to which a location is allocated may vary with perception (epistemic vagueness). Indeed, mountains in particular have formed one focus for the philosophical literature on vagueness, and the existence of vague objects (Varzi 2001; Burgess 1990; Sainsbury 1989). In essence, those authors pose the question 'where is Snowdon (Sainsbury 1989) or Everest (Varzi 2001)?' They argue that no meaningful answer can be clear-cut, and in the philosophy literature the argument persists as to whether this is due to human perception dividing a landscape into features called mountains, or whether the mountains actually exist as vague objects (Burgess 1990; Sainsbury 1989 1995; Evans 1978).

A similar argument is appropriate to other landscape features, particularly ridges or valleys. As with mountains, it is easy to specify where these features are in a trivial sense (a valley is defined by the line which joins the lowest points on any cross-section), but to describe or understand the spatial extent of (or region associated with) the feature which people agree to give a particular label is much harder, but most have a spatial extent to some degree. At the location of the core concept they are definite, but that core concept fails to capture their full identity, and they have a spatial extent beyond that core area, where most people would to some extent say they exist.

Next, we review the theoretical framework of the current research, and then formally state the method we propose for examining the vagueness in feature definition. The actual study area and methodology used are discussed before the results are introduced. These are analysed with respect to named features, and the application of these results to visibility analysis is explored. The paper concludes with some suggestions for further work.

Morphometry and vagueness

The attempt to compute the landscape has given rise to an area of research known as geomorphometry; the measurement of the shape of the earth's surface. In geomorphometry it is possible to generate descriptive statistics of the shape of the surface or to assign a location in a landscape to an exhaustive set of features based on the local form of the land surface (Pike 2000; Wood 1996b; Evans 1980; Peucker and Douglas 1975). Among the simpler, geometric (and therefore computable) set of forms is the assignment of a location to one of six morphometric classes: pit, peak, pass, channel, ridge and plane (Wood 1996b; Evans 1980; Peucker and Douglas 1975). These six classes are illustrated in Figure 1, and have very obvious correspondence with the expected form of features which people recognize in the landscape, namely, peak with mountain, channel with valley, and ridge and pass are often named as such. A few other morphometric divisions of a landscape have been suggested, but either they include more classes than this and are therefore more complex (Dikau 1989), or they invoke process as well as form (Pike 2000; Pellegrini 1995). Several researchers, however, have discussed the fact that many such classes in the landscape are hard to define meaningfully in terms of either their elevation or their spatial extent (Fisher and Wood 1998; Usery 1996; Wood 1996a 1996b). Thus if we consider a regular grid of elevations stored as a Digital Elevation Model (DEM), a grid cell (or pixel) in the DEM can be assigned to the morphometric class peak, but questions remain: What does *peak* mean? Where *is* the peak other than within the single pixel which contains the feature? When is a location identified as a peak, not a peak (if ever)? Is the statement that this location is a peak correct under all observations (Skidmore 1990)? These questions recall those asked about a mountain in the introduction to this paper, and like a mountain the concept of a peak is vague, especially in its spatial extent. The vagueness of these features is probably best matched to the epistemic model of vagueness (Sainsbury 1995; Williamson 1994). That is to say that there is a precise definition of the different morphometric classes, and so any location can be allocated to a specific class, but the class to which a location is assigned by this precise process varies due to the scale of measurement giving rise to ambiguity as to the correct classification and so vagueness.

Several researchers have introduced the idea that the vagueness in geography may be appropriate for analysis by fuzzy sets (Robinson 1988 2003; Fisher 2000a 2000b; Leung 1987 1988; Gale 1972;

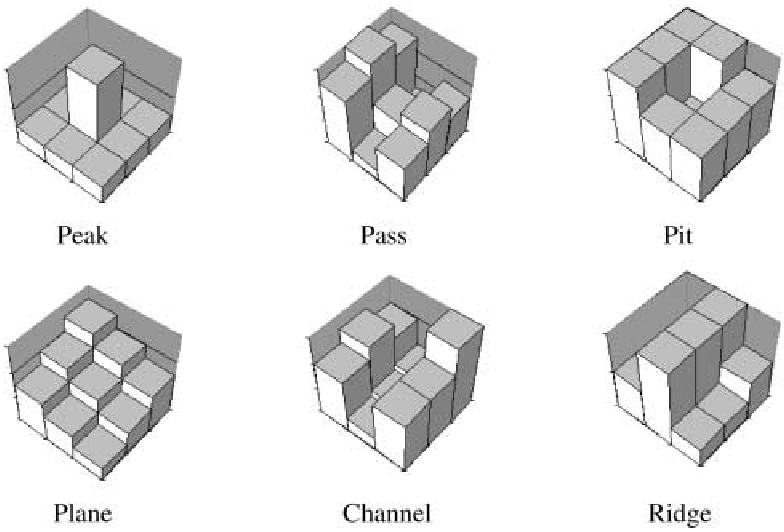

Figure 1 The six morphometric classes as represented by a gridded elevation model

Zadeh 1965). In classic set theory, if an object belongs to a set it is assigned an integer value 1 as membership of that set. If it does not belong, it is assigned membership 0. This form of assignment is sometimes referred to as Boolean or Cantor set theory. In fuzzy set theory, a core concept is defined and objects which exactly match that core concept are assigned a class membership of 1. The membership is assigned a reducing real number for objects as they are increasingly dissimilar from that core concept until they have no similarity to the core concept, when the membership is assigned a value 0.

Fuzzy set theory has been used in studies to the analysis of vague landforms. Two methods for defining the membership values of the fuzzy sets have been used (Robinson 1988 2003; Fisher 2000a 2000b). In the first approach, *a priori* knowledge is used to assign a value of fuzzy membership to a landscape feature with a particular metric property such as height. This approach is called a semantic import model. Thus Usery (1996) determined a fuzzy extent to Stone Mountain in Georgia, using the height above a certain elevation as a membership function with membership increasing with height. Similarly, Cheng and Molenaar (1999a 1999b) used height to determine membership functions of separate elements of dynamic beach landforms. The second approach uses surface derivatives, such as slope and curvature, as input to a multivariate fuzzy classification which yields the membership values (Burrough *et al*. 2000 2001; MacMillan *et al.* 2000; Irvin *et al.* 1997). This is called the similarity relation model. These methods have only been applied at the resolution of the DEM, although Arrell *et al*. (in preparation) examine the variation of fuzzy membership from clustering with the spatial resolution of the DEM. These methods are comparable to methods used widely in fuzzy set research (Klir and Yuan 1995; Kruse *et al*. 1994).

In the case of morphometry, however, one reason the morphometric class is vague is the geographical scale of measurement. By scale, we mean a combination of both spatial *extent* and spatial *detail* or resolution (Tate and Wood 2001), and in the present research, we use the variation in the extent over which the feature is defined as the basis of the fuzzy membership. What may be a channel at one scale may be another morphometric class at another scale, say a ridge (Figure 2). So is it a channel or a ridge? Clearly it is to some degree both. Indeed, as demonstrated by Wood (1996a 1996b), many locations may be classed as all of the above-mentioned six morphometric classes, depending solely on the scale of measurement. Wood also showed that by analysis of the influence of the region on a location at different scales, a much greater amount of information could be extracted from a DEM about

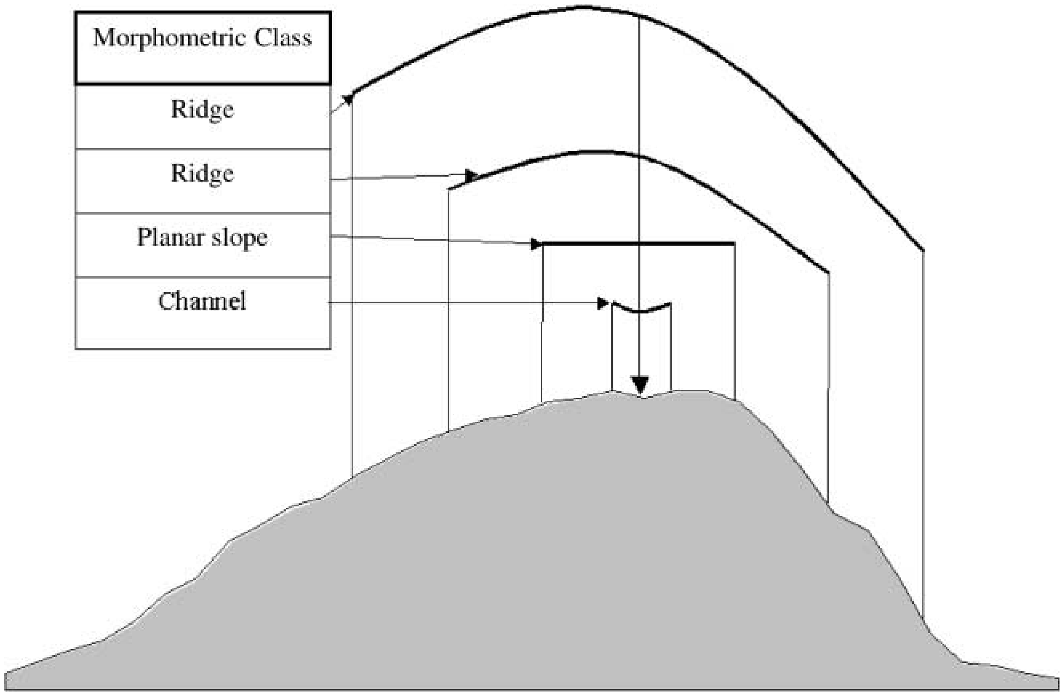

Figure 2 The morphometric class at the point indicated by the vertical arrow varies as shown with the scale over which it is measured

the spatial characteristics of a phenomenon. In this paper, we propose an approach based on the multi-scale analysis of the landscape, from which we can derive fuzzy memberships of morphometric classes, and we explore the consequences of this by looking at regional scale morphometry in the Lake District of northwest England.

Multi-scale morphometry as fuzziness

Let us accept that at any particular scale the landscape can be assigned to one morphometric class as a Boolean or crisp set. Thus the landform, L, at a location, x, can belong to the set of possible morphometric classes (ridge, peak, pass, channel, pit, planar) (Figure 1). These six classes are widely used in geomorphometry when defining local surface form (Wood 1996b; Evans 1980; Peucker and Douglas 1975), and represent all the permutations of the first and second derivatives of a surface in two orthogonal directions. We indicate this set of six classes by the symbol [A]. Thus

$$L_x = [\mathrm{A}] \quad (1)$$

Since there are six possible valuations of A it follows that for five values of A, the membership of the Boolean set of that class (where memberships can be integers of value either 1 or 0 indicating belonging and not belonging to the set respectively) is given as:

$$m_{Ax} = 0 \quad (2)$$

and for only one possible valuation of A the membership of the Boolean set is

$$m_{Ax} = 1 \quad (3)$$

However, we have argued that one of the reasons for the vagueness of the assignment of any location to a morphometric class is that landform class is not necessarily stable under repeated observation at different scales. Thus if $m_{Ax|s1} = 1$ for a particular landform class, it does not follow that either $m_{Ax|s2} = 1$ or that $m_{Ax|s3} = 1$, where $s1$, $s2$ and $s3$ indicate different scales of measurement (Figure 2).

If this is the case, then it follows that the fuzzy membership of a morphometric class, A, at location x can be given by a weighted average of the

Boolean memberships of that class over the scales of measurement:

$$\mu_{Ax} = \frac{\sum_{i=1}^{n} w_i \, m_{Ax \mid s_i}}{n \sum w_i} \quad (4)$$

for each of A, where n is the number of scales of analysis, and w is a weight applied to each scale. In the work reported here we have used equal weights for all resolutions so that all the w terms can be ignored. A fruitful area for future research is to investigate the possibility of differential weight matrices and the idea that different features should be analysed with different weight matrices appropriate to their scale of expression in the landscape.

If we agree that the morphometric classes can be defined, then we can reasonably ask what is the class at a location. If vagueness exists, then a reasonable answer is to take the modal class at a location, but to acknowledge the variance in that class. Thus it is possible to define the dominant or modal class, modA_x, as:

$$\operatorname{mod} A_x = \max_{j=1}^{6}(\mu_{A_{jx}}) \quad (5)$$

where $\mu_{A_{jx}}$ is the fuzzy membership in each of the six morphometric classes at a location, x. Variation is then measured by the entropy, given in this case by:

$$E_x = \frac{-\sum_{j=1}^{6} \mu_{A_{jx}} \cdot \log[\mu_{A_{jx}}]}{-\log\left(\frac{1}{6}\right)} \quad (6)$$

where E_x is the entropy at x, and is scaled between 0 (classification agreement at all scales) and 1 (observations split equally between all classes) by dividing by the maximum entropy generated by six classes.

The characterization of scale-based uncertainty so far has been described independently of the model of the surface and indeed any operational definition of scale itself. To execute the multi-scale analysis we have used the method of Wood (1996a 1996b), where the surface is modelled as a gridded DEM and then generalized to alternative scales by fitting a local quadratic surface centred on an expanding window or filter. Morphometric analysis is then performed on the generalized surface over a range of window sizes. The resulting morphometric classifications remain at the original scale of the DEM, but are executed for progressively generalized versions of the DEM.

Methods

A geographical database of elevations in the Lake District of northwest England (Figure 3) is readily available from the Ordnance Survey using their Panorama™ product, which is a 50 m resolution gridded DEM of the whole country. Delivered in 20 × 20 km tiles, this dataset requires nine tiles to include the whole of the Lake District. The dataset is derived from contours digitized from 1:50 000 maps. In some figures, the Ordnance Survey's Panorama™ contour data are also used. The area for which we present detailed discussion focuses on the central part of the Lake District, including many of the classic mountains of the area, especially Sca Fell and Helvellyn (Figure 3) and therefore is just a subset from the centre of a larger area. These data were supplemented by a toponym (place name) dataset of named topographic locations, which includes summits and passes, drawn from the Bartholomew 1:250 000 topographic dataset. The scale mismatch between the DEM and the toponym dataset is regrettable, but the only available place name dataset which has a nominal scale equivalent to that of the DEM (Ordnance Survey, 1:50 000 Gazetteer) records the locations with a 1 km precision and is therefore of little practical use for this study. The Bartholomew data are used in the production of a series of motoring atlases by Harper Collins Publisher. The summits included in the dataset therefore are likely to be chosen with two principal considerations: the toponyms must be clear and uncluttered in the Bartholomew 1:250 000 series maps, and they should reflect places for which people are likely to want to know the location. Among the possible summits this means that the data are likely to include a selection of the more important summits as decided by the creator of the dataset, but passes are more likely to be included if they form important communication routes. Other possible datasets which could have been used and which may be the basis of future research include GPS waypoint files placed on the World Wide Web by fell walkers and climbers.

The DEM was processed as outlined above using Landserf (Wood 2002a). Analyses were conducted using filter sizes of from 3 × 3 to 75 × 75 pixels, representing features with planimetric sizes from 100 m to 3700 m: relatively local to regional features. The analysis of each filter size classified every pixel into one of six morphometric classes and was saved. In the Idrisi32 GIS (Eastman 1999), the datasets

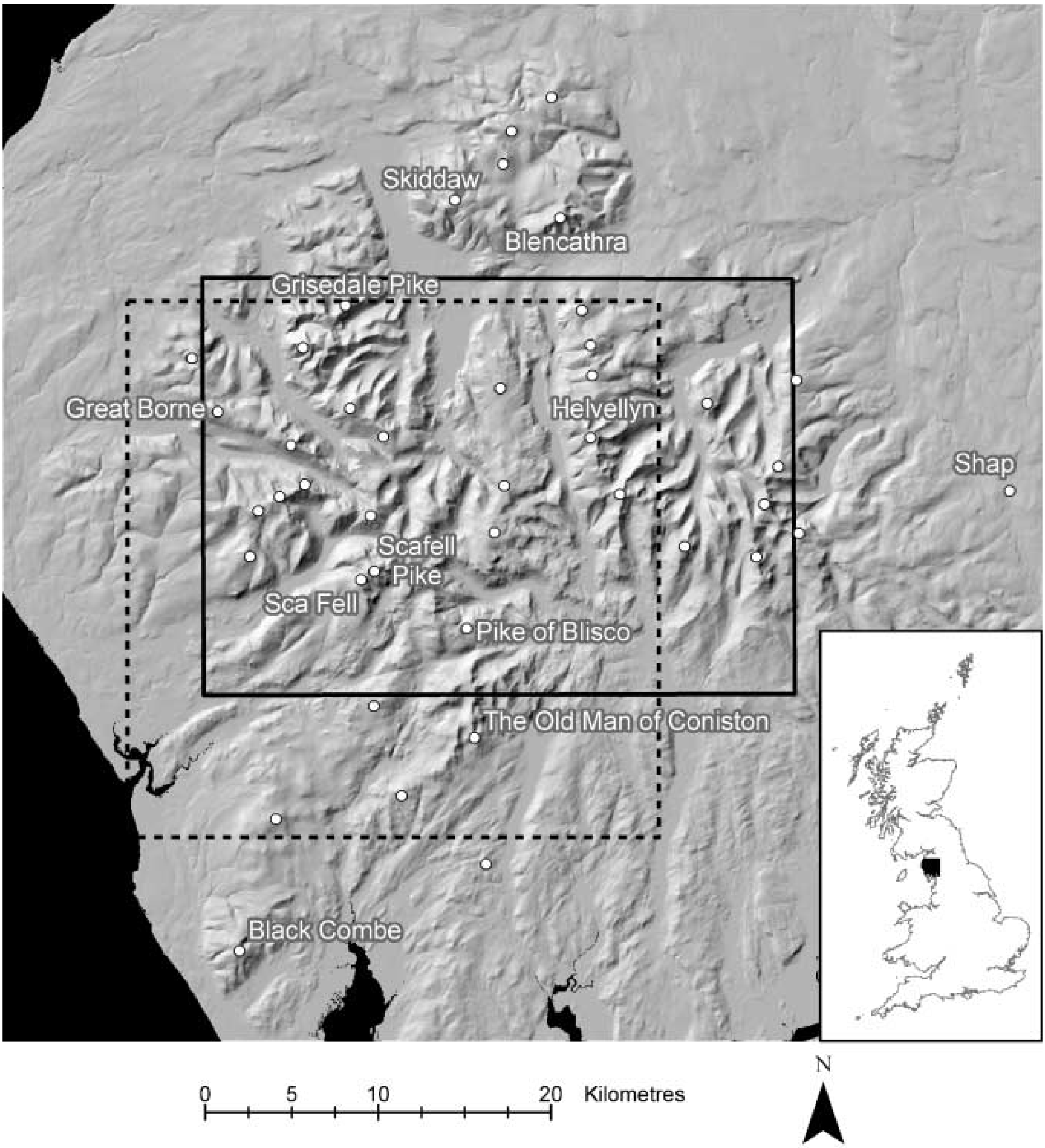

Figure 3 The physiography of the English Lake District in northwest England (© Crown Copyright Ordnance Survey. An EDINA Digimap/JISC supplied service). The small white dots indicate positions of peaks named in the toponym data. The rectangular area shown in the middle is the area shown again in Figures 4, 5 and 6. The rectangle with dashed outline is shown in Figures 11 and 12

for each morphometric class were separated, and added to the corresponding dataset at other scales by raster overlay. Maps of the results were prepared for this article with the ArcGIS version 8.2 software.

Derivation of pits, peaks and passes is not simple. Over a 3 × 3 window of pixels there is usually a slope, and that slope means that if the criteria of allocation are used without amendment, locations are very rarely allocated to one of these three categories. In Landserf, this slope can be compensated by defining a slope threshold. The larger the threshold the more likely a location is to be identified as a pit, peak or pass (Wood 1996b 1998). In the research reported, the effect of two threshold values were explored, 1° and 4°.

Results: fuzzy morphometric classes

Following the approach identified above, Figure 4 shows the distributions of the fuzzy memberships of each of the six morphometric classes over an area of the central Lake District indicated in Figure 3. The values in Figure 4 are based on a threshold

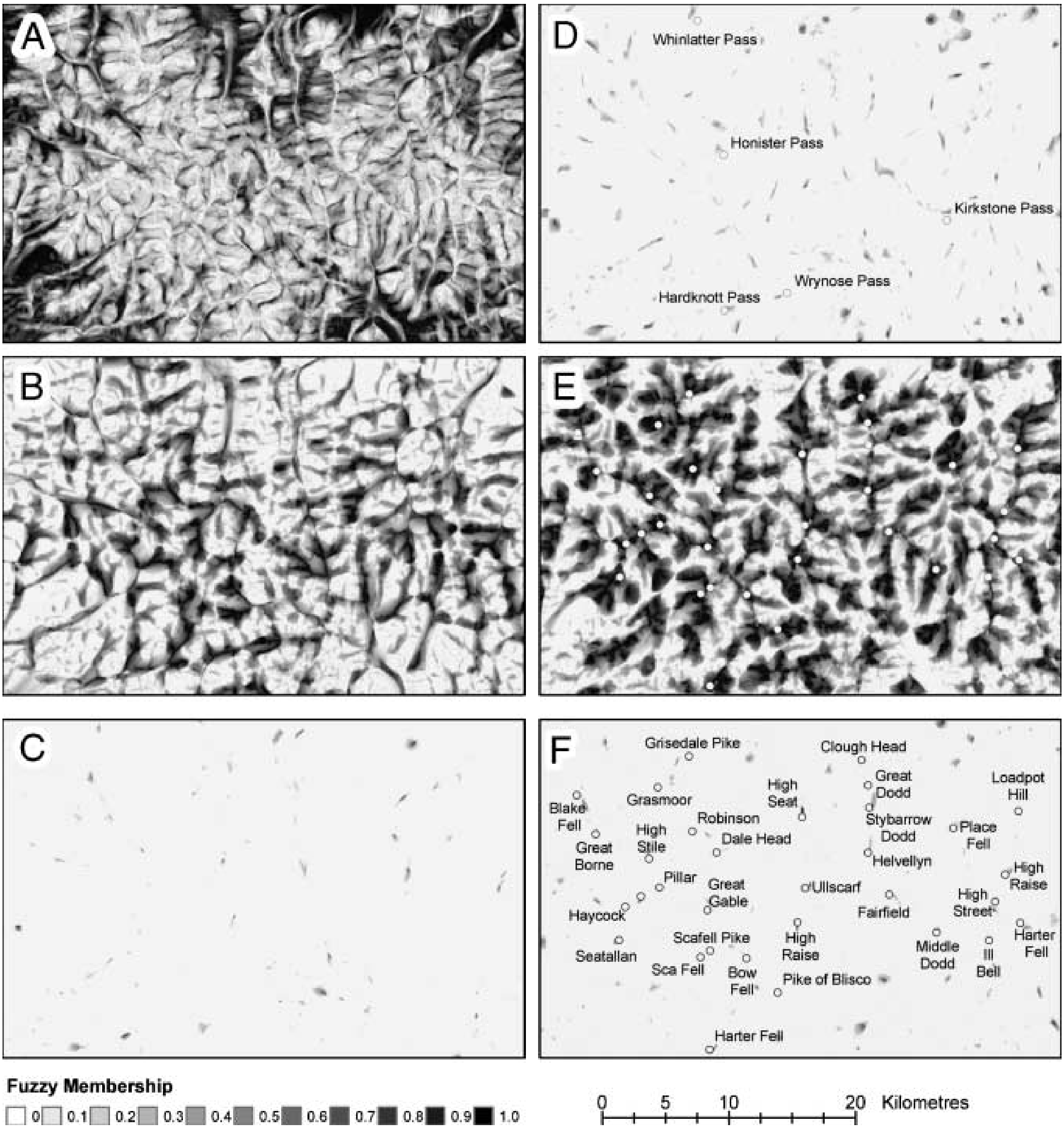

Figure 4 Fuzzy memberships of the six morphometric landforms: (A) planarity; (B) channelness; (C) pitness; (D) passness; (E) ridgeness; (F) peakness. The locations of toponyms for passes and summits are shown on (D) and (F) (Copyright © Bartholomew, 2002). The feature extractions shown in this figure used a threshold of 1° for the peaks

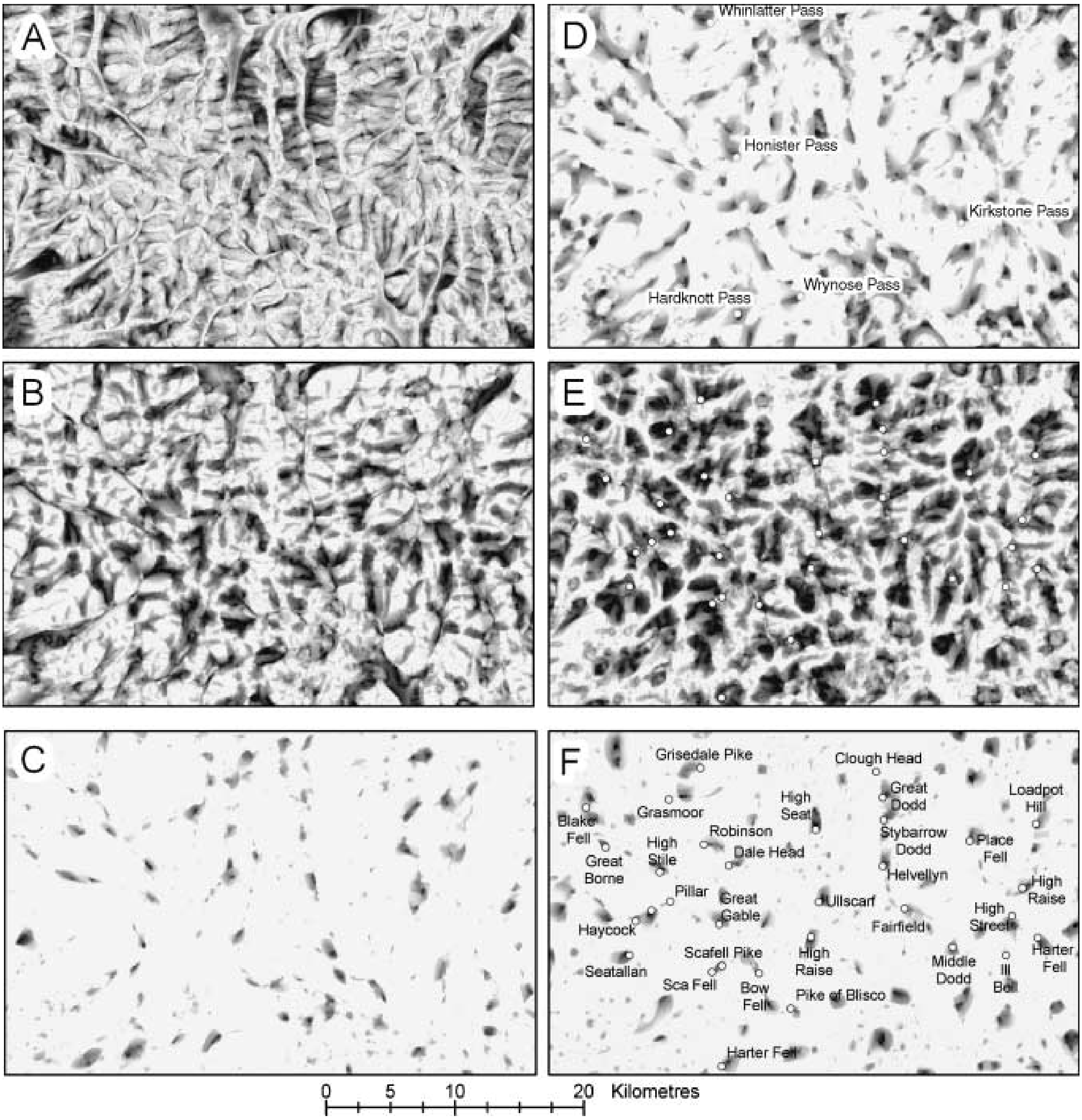

Figure 5 Fuzzy memberships of the six morphometric landforms: (A) planarity; (B) channelness; (C) pitness; (D) passness; (E) ridgeness; (F) peakness. The locations of toponyms for passes and summits are shown on (D) and (F) (Copyright © Bartholomew, 2002). The feature extractions shown in this figure used a threshold of 4˚ for the peaks

of 1° for inclusion in the categories peak, pit and pass. Figure 5 shows the same extents when the threshold is raised to 4°. The differences in the distributions in Figures 4 and 5 are in the extent of the pits, peaks and passes, as might be expected. With the threshold set to 1°, all three are very inextensive; nowhere is always either a peak (the maximum number of times a grid cell is a peak is 31 of 37 possible realizations) or a pass (35 of 37). Indeed, the modal image (Figure 6A) shows hardly any locations as anything but ridge, channel or planar slopes. Using the 4° threshold these classes are all much more extensive, and some locations are always both peak, pass and pit. The 4° threshold

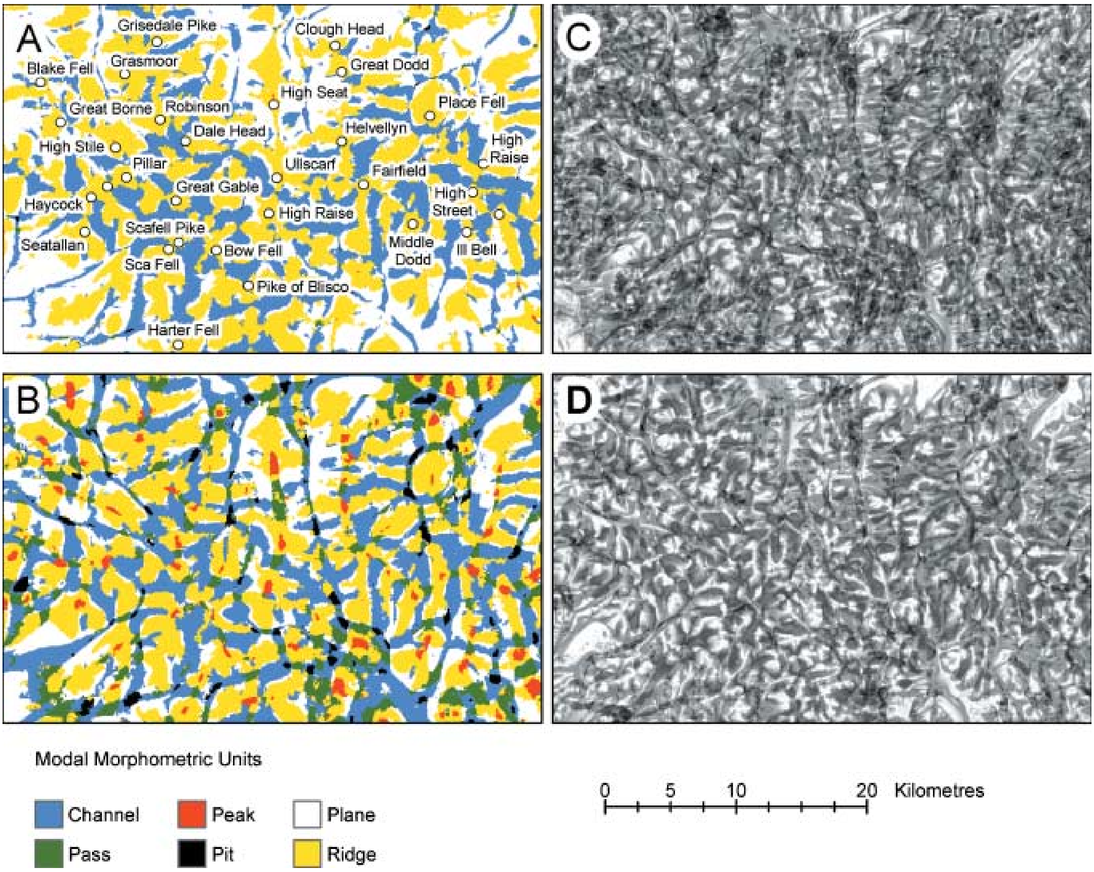

Figure 6 (A) and (B) show modal classes and (C) and (D) the corresponding entropy values when thresholds of 1° and 4° respectively were used in extraction of morphometric features

also corresponds to the earlier reported morphometric classification of the central Lake District by Wood (1996a, Plate 3). Most subsequent discussion is therefore based on the result from this coarser threshold.

The entropy images (Figure 6C and D) both show that the core areas of the modal landforms (Figure 6A and B) are repeatedly identified as that landform with increasing unreliability at the transition to another modal landform. The modal areas of pitness are identified with valley bottoms and line areas of channelness, frequently at valley confluences. This effect was noted by Wood (2002b) and he suggested that, given this relationship between pit and valley confluences, pits in this location are an artefact of the DEM, rather than an error as they are frequently treated.

Naming the landscape

Peaks

In Figure 5F it is possible to see a pronounced association of the locations of toponyms with large values of peakness. For example, the locations of the Ullscarf and High Raise toponyms are clearly located within the footprint of areas of large values of peakness (Figure 7A, B and C). In the result of the 4° threshold analysis (Figure 7B) the toponym for High Raise, for example, is very precisely located, and the Ullscarf toponym is within the footprint of the peak feature. On the other hand, in the result of the 1° threshold analysis (Figure 7C) neither High Raise nor Ullscarf toponyms are clearly located within the feature. In the same area a third peak is apparent, especially in Figure 7B, which is

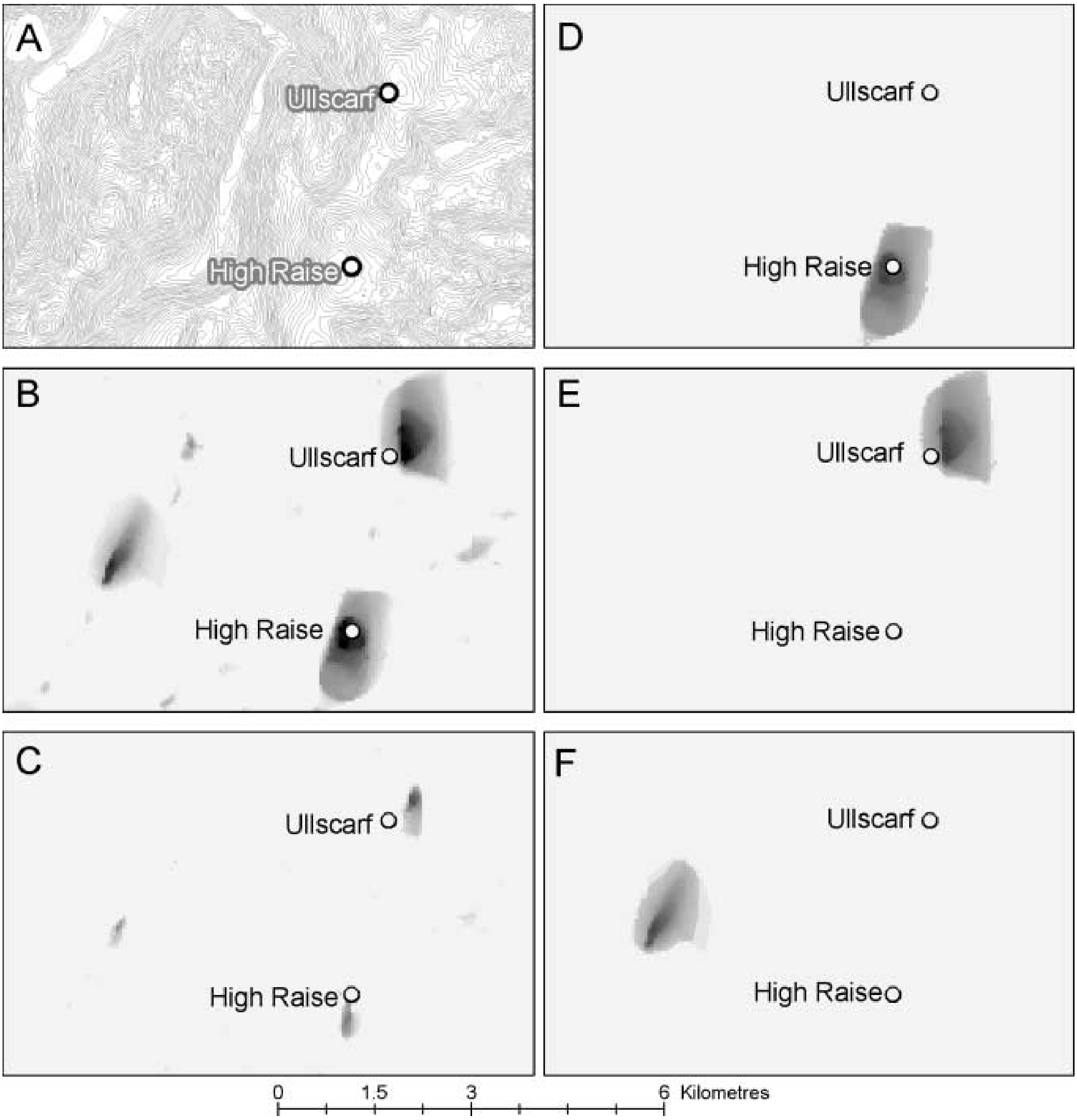

Figure 7 The High Raise, Ullscarf and Glaramara cluster of peaks. (A) The contour map (© Crown Copyright Ordnance Survey. An EDINA Digimap/JISC supplied service), and the degree of peakness with (B) 4° and (C) 1° threshold used in extraction. Each peak can be extracted as a separate feature, (D) High Raise, (E) Ullscarf and (F) Glaramara, the last of which is unnamed in the database of summits, but clearly identifiable in 1:50 000 and 1:25 000 maps of the area. Toponym locations are from the Bartholomew 1:250 000 topographic database (Copyright © Bartholomew, 2002)

not named in the database of toponyms, although it can clearly be identified as Glaramara from 1:50 000 and 1:25 000 Ordnance Survey maps of the area. The association of the peakness zone with the toponym seems uncontroversial, especially as any imprecision in the location of the toponym with respect to the zone of peakness is not surprising given the difference in nominal scales of the two databases (1:50 000 for the contour and DEM databases and 1:250 000 for the toponyms), combined with the fact that the toponym database is a set of cartographic points used to label the features in relatively small scale maps. Despite not appearing on the Bartholomew toponym database, Glaramara is well known to walkers in the region. The guru of walkers, Wainwright (1960) describes it thus:

> Prominent in the mid-Borrowdale scene is the bulky fell of Glaramara, which, with an ally in Rosthwaite Fell, seems, on the approach from the north, to throw a great barrier across the valley . . . The ancient and beautiful name really applies only to the grey turret of rock at the summit but happily has been commonly adopted for the fell as a whole. (Wainwright 1960 Glaramara 2)

Across the Lake District many peaks, unnamed like Glaramara, can be seen, as can the general association of the toponyms that are included in the database with zones of peakness (Figures 4F and 5F).

Scafell Pike is the highest mountain in England, and is located next to its twin peak Sca Fell (Figure 8A). Scafell Pike has a clearly associated zone of peakness, although the maximum membership is only 0.730 (Table I). Sca Fell, on the other hand, is practically non-existent as a morphometric feature. It is unequivocally located in the database of toponyms, but it is not well identified as a peak. However, the toponym is located on a ridge (Figure 8D). Fisher and Wood (1998) have previously discussed the problem of the mountainness of Sca Fell and Scafell Pike. They point out that Sca Fell is really a region, rather than a mountain, and the paper mapping of the area shows Scafell Pike as a peak and Sca Fell as a zone (Ordnance Survey 1998). Although Sca Fell is the second highest mountain in England, it has no peak character of mountainness. Indeed, we can see that morphometrically Sca Fell and Scafell Pike form part of the same ridge, with Scafell Pike being a peak on that ridge. They are probably separated culturally because of the difficulty in walking directly between them across Mickledore and climbing Broad Stand (Fisher and Wood 1998; Ordnance Survey 1998).

The location and peakness of Helvellyn is also of interest. The toponym location is placed to the southwest of the actual peak (Figure 8B) most probably for cartographic reasons, leaving room on the map for a greater density of information to the northeast. Again, its degree of peakness is not unity (maximum membership value 0.946, Table I). Figure 8E shows the degree of ridgeness in the vicinity, and it is clear that Helvellyn lies at the intersection of three ridges, from the northwest, northeast and south. Interestingly, one of the best known ridges in the region, Striding Edge and its northern component Swirral Edge, is only marginally associated with the NE ridged region identified here (Figure 8E). While they are both significant to the walker, here the features appear fused as part of a more regional feature. The degree of peakness of Helvellyn itself is really only an integral part of this ridge system.

Finally, The Old Man of Coniston (Figure 8C) is one of the best-known peaks of the Lake District. The

Table I The fuzzy memberships and other statistics for a selection of the peak objects found in the Lake District. Heights are from the Bartholomew attribute table of summits (copyright © Bartholomew, 2002)

	Maximum membership of peakness	*Height/metres*	*Relative importance*	*Area of the fuzzy peak/hectare*	*Area weighted fuzzy membership of peakness*
Blake Fell	0.892	572	3	1724	0.2265
Bow Fell	0.459	902	2	677	0.1887
Dalehead	0.649	754	2	550	0.2132
Grasmoor	0.865	851	2	746	0.2573
Great Borne	1.000	616	3	236	0.4269
Great Gable	0.730	899	2	789	0.2336
Grisedale Pike	0.595	790	2	860	0.2611
Helvellyn	0.946	949	1	1724	0.2766
High Raise	1.000	803	2	669	0.3517
High Stile	0.919	806	2	654	0.2319
Old Man of Coniston	0.703	803	2	584	0.2819
Pike of Blisco	0.541	702	3	727	0.2560
Robinson	0.649	737	3	644	0.2055
Scafell Pike	0.730	977	1	299	0.3048
Steeple	0.946	841	2	894	0.2726
Ullscarf	1.000	726	2	687	0.3291

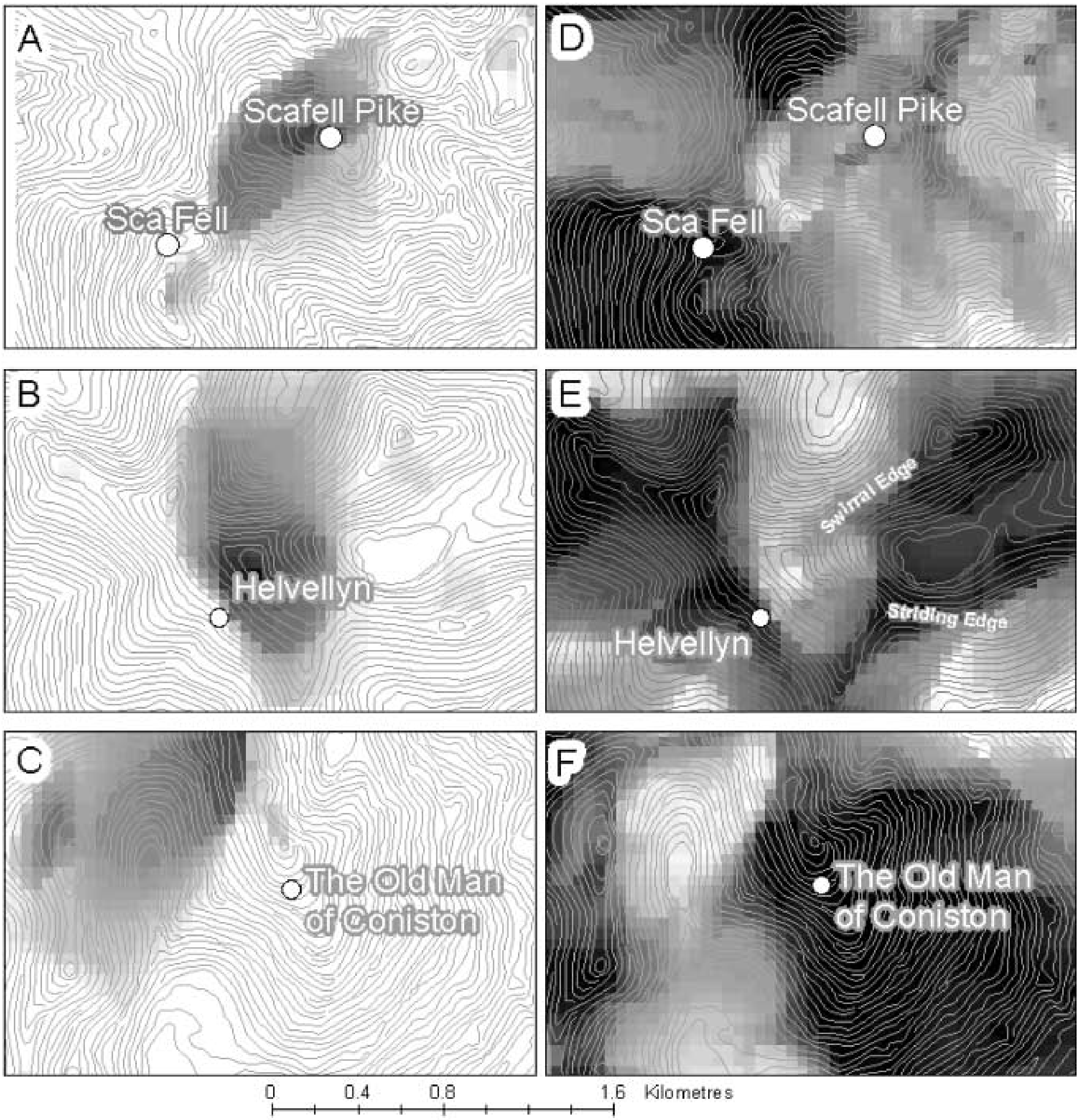

Figure 8 Detail maps of peakness in the vicinity of (A) Sca Fell and Scafell Pike, (B) Helvellyn and (C) The Old Man of Coniston, and (D) to (F) show the corresponding maps of ridgeness in the vicinity. The diagrams show the degree of peakness or ridgeness as greyscale images with the contour lines from the Ordnance Survey Panorama database (© Crown Copyright Ordnance Survey. An EDINA Digimap/JISC supplied service) together with toponyms from the Bartholomew 1:250 000 topographic database (Copyright © Bartholomew, 2002)

maximum fuzzy membership again is low (Table I), and the feature is linked to a ridge (Figure 8F). Many other mountains in the Lake District are morphometrically a mixture of peak and ridge (e.g. Pike of Blisco, Clough Head, Grisedale Pike), and these mountains may appear as named summits on ridges, rather than as peaks in their own right. Whether they are separated in the analysis presented here is related to the degree to which the peak dominates. Every named mountain or hill in the toponym database is associated with either a zone of raised peakness, or of high ridgeness values.

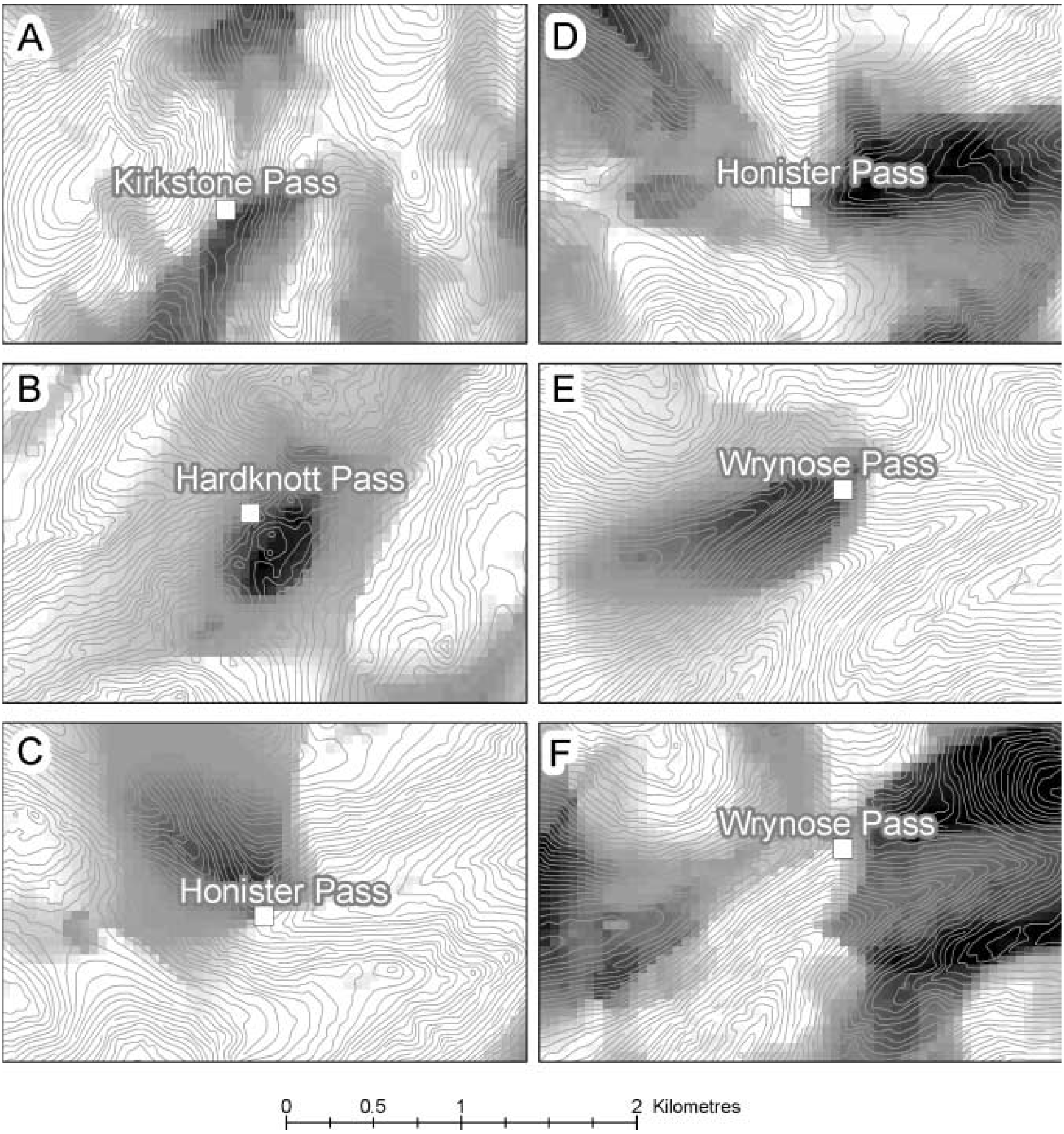

Figure 9 Detail maps of passness in the vicinity of (A) Kirkstone Pass, (B) Hardknott Pass, (C) Honiston Pass and (E) Wrynose Pass. The degree of channelness is shown in the vicinity of (D) Honiston Pass and (F) Wrynose Pass. The diagrams show the degree of passness or channelness as greyscale images with the contour lines from the Ordnance Survey Panorama database (© Crown Copyright Ordnance Survey. An EDINA Digimap/JISC supplied service) together with toponyms from the Bartholomew 1:250 000 topographic database (Copyright © Bartholomew, 2002)

Passes

The toponym dataset has passes as named locations, as well as hills and mountains. Five passes are identified in the study area (Figures 4D and 5D). Each is located close to a zone of elevated passness. There are, however, many morphometric passes which are not named in the toponym database. As with the peaks these features have a much greater

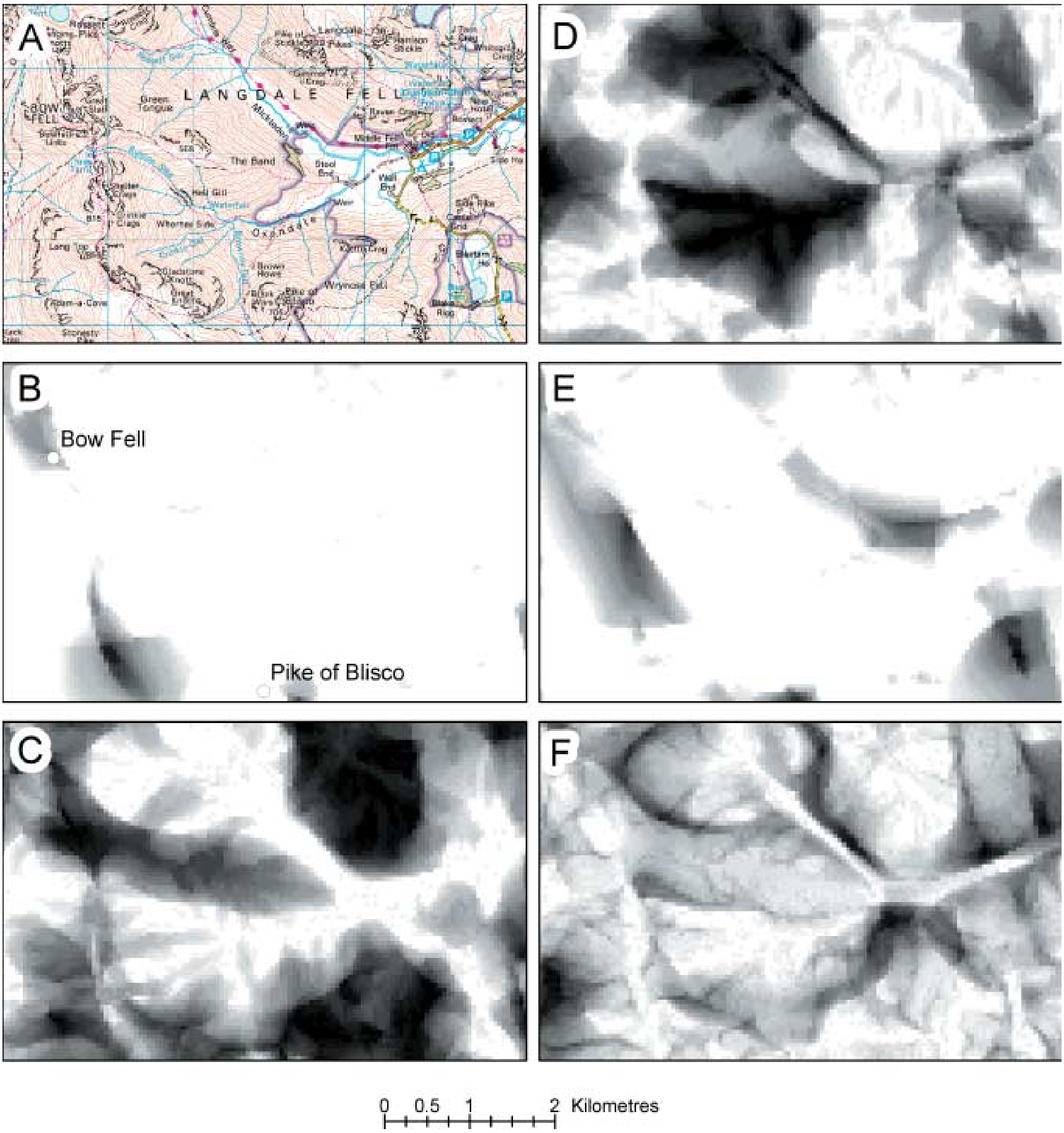

Figure 10 The Langdale valley and environs. (A) A portion of the 1:50 000 raster scanned map of the area (© Crown Copyright Ordnance Survey. An EDINA Digimap/JISC supplied service); details of the area can be seen on the Ordnance Survey Landranger map number 90; (B) the degree of peakness; (C) the degree of ridgeness, (D) the degree of channelness; (E) the degree of passness and (F) the planarity over the same area. For legend see Figure 4

geographical extent in the version of extracted features which used the threshold of 4° (Figure 5D). The toponym database is part of a larger database which is used in preparing road atlases. Therefore it records features which have cultural significance or serve a purpose in vehicle navigation. Thus hills, mountains and passes are included, but passes without roads are not, and even then to be included the passes need to have some cultural significance.

Figure 9 shows detail for four of these passes (Figure 9A, B, C and E). Kirkstone, Hardknott, Honister and Wrynose Passes are all identifiably associated with zones of high passness, which can all be seen in the pattern of the contours to correspond to passes at these locations.

Wrynose Pass is very skewed, with a shallow descent to the west, and a steep descent to the east. The analysis has picked out the trough as the area of the pass at multiple scales, while the toponym is located at the crest of the steep part to the east. In this instance, there is clearly some doubt about where exactly should be named as the pass. Maps of the area (Ordnance Survey 1998) place the name of the pass along the road in the area of the trough identified here as the morphometric pass. This ambiguity in location is likely to be due, in part, to the importance of the pass as an east–west orientated road connecting the central and western areas of the Lake District. The point that perhaps best represents the morphometric centre of the pass is located at the Three Shires Stone, where the road intersects the north–south ridge separating drainage basins to the east and west.

As with the relationship between peak and ridge, we can also see a pronounced relationship between channel and pass. This is exemplified here by the detailed maps of Honister Pass showing the degree of passness (Figure 9C) and channelness (Figure 9D), and the corresponding maps for Wrynose Pass (Figure 9E and F, respectively). The zone of high values of passness can be seen to complement the spatial extent of the zones of channelness.

Other landscape features

Other landscape morphometric classes, like ridges and channels, can also be associated with identified and named landscape features. Thus looking at the Langdale area of the Lake District (Figure 10A), it is only possible to see the peak of Bow Fell, and the named but poorly identified Pike of Blisco (Figure 10B). There are at least two other well-known peaks in the area, Pike of Stickle and Harrison Stickle, two of the Langdale Pikes (north central in Figure 10A). These do not appear at all in the map of peakness, but it is possible to see an extensive zone of high values of ridgeness, which can be associated with the Langdale Pikes (Figure 10C). Again these features have a cultural significance when viewed from the populated and accessible Langdale Valley because they are part of a south-facing buttress of the massif to the north, and they sit high above it. From the north, a relatively unpopulated area, the Pikes are barely noticeable. Also in Figure 10C, it is possible to see the ridge which forms part of the Bow Fell and Pike of Blisco peaks. Interestingly, it is also possible to detect in the peakness and ridgeness images the north–south running ridge from Bow Fell associated with Long Top and Crinkle Crags, as well as the ridges which run to the east from Bow Fell known as The Band and the less prominent Green Tongue. Zones of large values of channelness can be associated with the principal streams in the area, Mickleden and Oxendale, as well as the head streams of both (Figure 10D). At the confluence of Oxendale and Mickleden, it is also possible to see a zone of elevated degrees of passness (Figure 10E) as well as two main zones of passness in the mountain areas, one between Bow Fell and Crinkle Crags (known as Three Tarns) and one associated with the entry to Blea Tarn in the southeast corner of the map (Figure 10E). Planarity (Figure 10F) is hard to associate with any particular features, being more to do with the interface between the valley features and the ridges and peaks. In Langdale, a pronounced ring of elevated membership of planarity runs around Rossett Gill, the headwater to the north of Langdale, and to a lesser degree a similar ring runs around Oxendale.

Although not named in any available toponym database, it is possible to consider the morphometric classes in other areas and interpret them in terms of named topographic features. People do not see morphometric classes in the landscape, but their perception may partly be based on morphometry. The correspondence between the derived zones of elevated memberships and the named features is remarkable.

Named peaks

Having associated place names with peaks and passes, and because fuzzy memberships reduce to zero over most of the study area, it is possible to form distinct named geographic objects which themselves contain the fuzzy memberships of the degree of belonging to that named feature. Figure 7D–F shows each of the three objects for High Raise, Ullscarf and Glaramara, respectively. The separation of objects as fields is discussed by Cova and Goodchild (2002). In the present work, the fuzzy image of peaks was converted to a vector theme and then the vector polygons associated with each peak isolated.

Having extracted such fuzzy footprints of named peaks of the Lake District, it is possible to make some observations. The range of values of the maximum fuzzy membership of peakness in the named mountains and hills is large (Table I; from 0.459 to 1.000), and the most peaked hills are not the best known or tallest mountains. Indeed, they are typically quite low mountains. Thus Great Borne, High Raise and Ullscarf all have maximum fuzzy membership within the footprint equal to 1.0 (Table I); within the footprint of these peaks there is some location which is a peak at all resolutions. These, however, are not the most important peaks in many people's conception of the Lake District, rather Helvellyn and Scafell Pike are generally considered more important (maximum memberships 0.946 and 0.730, respectively), but these are not very peak-like summits in contrast to the others. Indeed, some of the peaks which are not included in the database of toponyms have higher fuzzy memberships of peakness than many of the named hills and mountains.

From the fuzzy memberships of individual peaks, it is possible to derive different interpretations of the memberships, making them dependent on local context or on evaluating them in other ways. For example, memberships can be re-scaled so that the maximum degree of membership within any one footprint of a peak is always 1, no matter how many times it is a peak in the analysis. Alternatively it may be scaled so that if the named peaks are ranked by conceived importance in the landscape, then fuzzy memberships within the most important class of peaks can be scaled so that it has a maximum membership of 1, but the maximum of less important classes of peaks can be scaled to smaller maximum values (0.9, 0.8, etc. perhaps, following a ranking such as that suggested in Table I).

It is also possible to determine an area weighted fuzzy membership where the value is equal to the sum of the products of the membership and the area of the footprint at that membership. This gives a measure of the impact of the peak on the landscape (Table I). Thus it would appear that Great Borne has the greatest impact as a peak on the landscape (area weighted fuzzy sum = 0.4269), while High Raise is the second most important (area weighted fuzzy sum = 0.3517). On the other hand, Scafell Pike and Helvellyn are well down any ordering of the list of peaks judged on this basis (having values of 0.3048 and 0.2766, respectively). Note that only a small selection of peaks is listed in Table I.

The analysis presented here gives equal weight to all scales of analysis, but the relevance of a peak at one scale may be very different from that at another, thus landmarks like Helvellyn and Scafell Pike may not be peaks at fine resolutions of observation, but certainly should be peaks at coarser resolutions. This aspect of the identification is not developed here, but will be the subject of further work.

But what peaks can you see from Helvellyn?

Having a database of named objects corresponding to specific mountains, it is now possible to answer some questions which are part of peoples' everyday discussions, but which have not previously been addressable in any meaningful sense from spatial databases. For example, if the question 'which mountains can be seen from a point in the landscape?' is posed, until now it has only been possible to state which named summits (or other named representative points) are visible from that point. We should note that the question is really: 'looking at the landscape from this point which mountains can be recognized?' With the results outlined, it is possible to go much further in stating which peaks will be recognizable: answering the real question. Not only is it possible to identify mountains and hills which will be upstanding, and so recognizable, in the view even if the summit of a named representative point is not visible, it is also possible to state to what degree that mountain is recognizable compared to how it might be recognizable from another location, or compared to the degree to which another mountain might be recognizable.

For example, if the visible area is determined from the named location of Helvellyn, the visible area shown in Figure 11 is identified. In a standard analysis it would be possible only to conclude that the five named summits listed in Table II are visible, being the toponym locations which are within the visible area, and to say little more about them. From a comparison with the fuzzy memberships of peakness (Figure 11B), it can be seen that some part of the footprint of many more hills and mountains (all peaks to some degree) are actually recognizable, whether named or not in the toponym database. The fuzzy memberships of peaks and peaks in-view, as well as the area of the fuzzy peaks and the area of the fuzzy peak which is visible, are listed in Table III together with differences and ratios, respectively. Taking the fuzzy membership of peakness together with the difference between the peak and the amount of the peak in-view shows

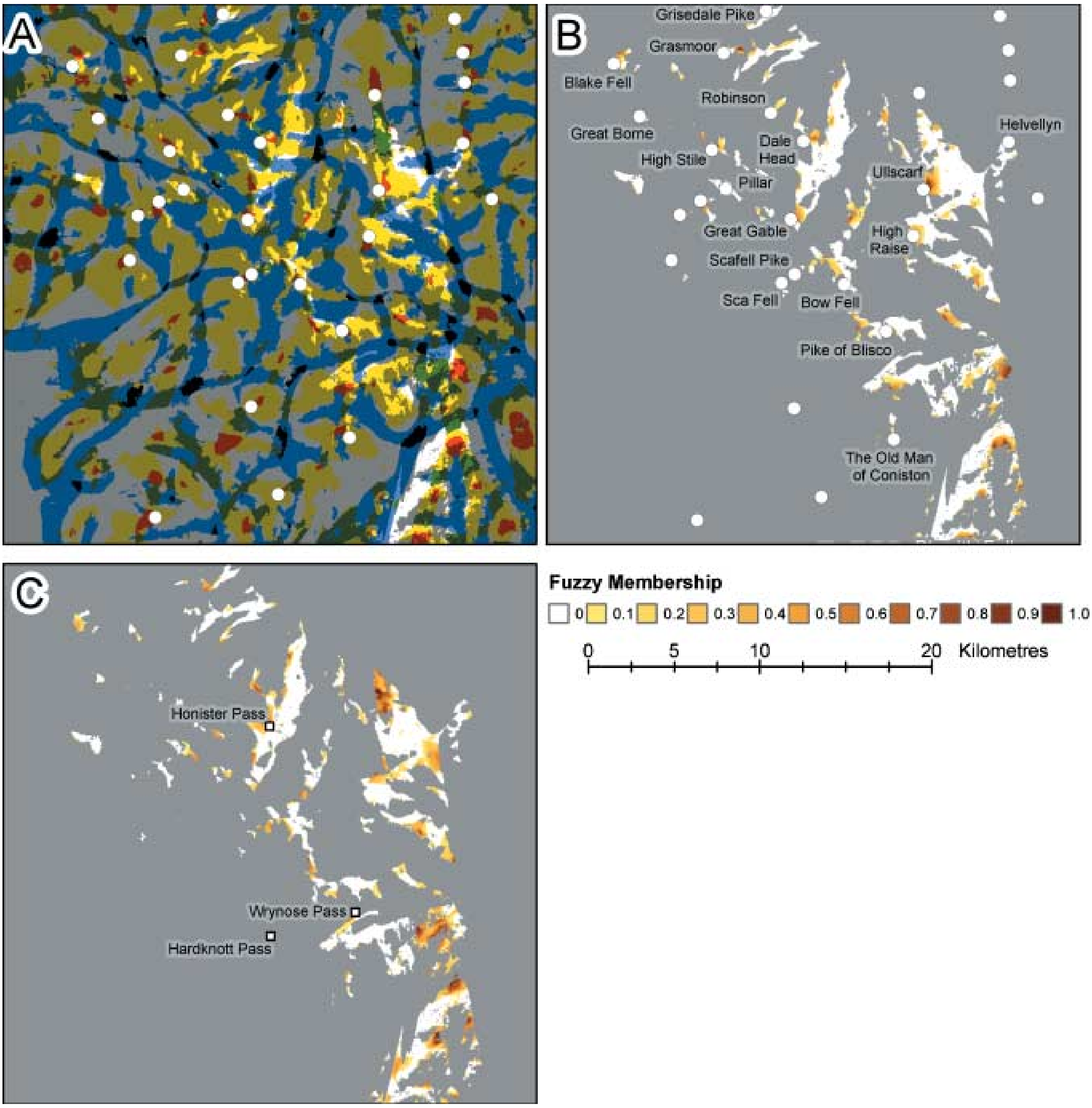

Figure 11 The area visible from Helvellyn with (A) the modal landform classes shown with the area not visible shown as semi-transparent grey (for legend of classes see Figure 6), (B) the fuzzy membership of peakness shown in the background and the area invisible as solid grey, and (C) the fuzzy membership of passness shown in the background and the area invisible in solid grey. Toponym locations are from the Bartholomew 1:250 000 topographic database (Copyright © Bartholomew, 2002)

that Blake Fell, for example, is relatively important in the view because it has a peak with maximum fuzzy membership 0.89, and the highest value in-view is 0.76, which gives only a small difference, meaning that a definite peak feature should be visible. On the other hand, the Old Man of Coniston (maximum membership within the area of the peak = 0.7) is relatively unrecognizable as a peak in the visible area (maximum membership in-view = 0.38). The ratio of the area of both Blake Fell and the Old Man which is included in the visible area is very small, which may mean that neither is actually

Table II Named summits in-view from Helvellyn

Great Borne
Great Gable
High Raise
Scafell Pike
Steeple

recognizable. The maximum fuzzy memberships of many peaks are wholly unaffected by the viewshed; at least one location of maximum membership is visible for each of Bow Fell, Dalehead, Grasmoor, Great Borne, High Raise, High Stile, Steeple, Ullscarf and most of the unnamed peaks, although the maximum membership of peakness for some of these peaks is small (Bow Fell, for example).

Of the named peaks, the area of the fuzzy peak Ullscarf is most in-view, but many of the unnamed peaks are also largely in-view. The only peak whose whole area is in-view is High Spy, which, of course, means that its fuzzy membership is equal in the visible area and total footprint. These peaks, where the area in-view is much the same as the total area of the peak, are much lower in elevation than the viewpoint, and so they are looked down on; no part of it is shielded by other parts of the landscape. Because the whole area of the peak is in-view, this peak may be difficult to see in the visible area from Helvellyn, because it cannot form part of the horizon, which means that it will have little prominence in the visible field. Ullascarf, High Man, Glaramara and Brund Fell all have the same characteristic.

Table III Comparisons of the fuzzy membership of peakness and the area of the peak fuzzy objects with the fuzzy membership and area of the peak which is in-view from Helvellyn

	Maximum fuzzy membership of peakness (A)	*Maximum fuzzy membership of peakness in-view (B)*	*A–B*	*Total area of fuzzy peak (C)*	*Area of fuzzy peak in-view (D)*	*D/C*
Peaks (mountains and hills) named in the database of toponyms						
Blake Fell	0.89	0.76	0.14	1724	168	0.10
Bow Fell	0.46	0.46	0.00	677	487	0.72
Crinkle Crags	0.92	0.65	0.27	727	465	0.64
Dalehead	0.65	0.65	0.00	550	318	0.58
Grasmoor	0.86	0.86	0.00	746	301	0.40
Great Borne	1.00	1.00	0.00	236	121	0.51
Great Gable	0.73	0.59	0.14	789	406	0.51
Grisedale Pike	0.59	0.54	0.05	860	152	0.18
Helvellyn	0.95	0.84	0.11	598	168	0.28
High Raise	1.00	1.00	0.00	669	37	0.06
High Stile	0.92	0.92	0.00	654	147	0.22
Old Man of Coniston	0.70	0.38	0.32	584	85	0.15
Pike of Blisco	0.54	0.49	0.05	196	40	0.20
Robinson	0.65	0.51	0.14	644	346	0.54
Scafell Pike	0.73	0.68	0.05	299	57	0.19
Steeple	0.95	0.95	0.00	894	240	0.27
Ullscarf	1.00	1.00	0.00	687	607	0.88
Peaks (mountains and hills) unnamed in the database of toponyms						
Arnsbarrow Hill	0.97	0.97	0.00	1101	629	0.57
Brund Fell	0.54	0.54	0.00	151	138	0.91
Carron Crag	0.68	0.68	0.00	415	208	0.50
Glaramara	0.86	0.86	0.00	613	529	0.86
Grey Knotts	0.62	0.54	0.08	339	101	0.30
Grizedale	0.76	0.76	0.00	464	165	0.36
High Arnside	0.97	0.97	0.00	1224	812	0.66
High Man	0.92	0.92	0.00	853	832	0.98
High Spy	0.49	0.49	0.00	84	84	1.00
Hill Fell	0.43	0.43	0.00	208	152	0.73
Lingmoor Fell	0.95	0.95	0.00	619	525	0.85
Maiden Moor	0.65	0.65	0.00	300	115	0.38
Swirl How	0.78	0.73	0.05	979	168	0.17

Given that the maximum fuzzy memberships are very variable, and considering a person who is unfamiliar with the landmarks in this landscape, it is possible to state that if both the fuzzy membership of the concept of peak and the fuzzy membership of the peak in-view is large, and the ratio of the area of the peak in-view to the area of the peak is about 0.5, then that person may be able to see and recognize the peak from the view point. If the ratio is small or large they may not, and if there is a large difference they may not. Taking a cut-off of 0.6 as defining a large membership of peakness, 0.2 as an acceptable difference between the full peak and the peak in-view, and 0.3–0.7 as defining a recognizable range of ratios, then from Helvellyn such a person will be able to see: Grasmoor, Great Borne, High Raise and Steeple of the peaks in the toponym database, and of the remaining peaks Arnsbarrow Hill, Carron Crag, Grizedale and Maiden Moor. This list includes fewer of the named peaks than those given in Table II, some named peaks which were not considered to be in-view in that analysis, and some not identified in the toponym database. Someone who is familiar with the landmarks of the area would be able to recognize peaks where the criteria are relaxed and the change in the maximum fuzzy membership of the concept of peak is small. Bow Fell, Crinkle Crags, Dalehead, Great Gable, High Raise, High Stile, Robinson and Scafell Pike, as well as Grey Knotts, Hill Fell and Swirl How. The most experienced fell walker would, of course, have the potential to identify all the peaks named.

Table IV lists the area weighted fuzzy memberships for the peaks in-view and compares the area weighted values for the peaks with the amount of the peak in-view. Two different approaches are used. Values in columns A and B are calculated from the maximum membership for the peak listed in Table III columns A and B, respectively. The maximum membership of each peak when treated in columns C and D is stretched so that it is always 1 to give a more local assessment, which also means that the weighted membership of the peak in-view may actually be larger than the peak on its own, as actually happens in most instances (ratio greater than 1).

Again the ratios closest to 1 indicate peaks which are as definite within this viewshed as they are in the general landscape. Those with large deviations indicate large changes, and where there are large deviations in both they are especially unlikely to be recognizable in the landscape. The first summary shows the peak compared with the full range of scale realizations (Table IV columns A and B and their ratio). This analysis confirms the problems of recognizing Blake Fell, The Old Man of Coniston, Helvellyn, Pike of Blisco, Grisedale Pike, Grey Knotts and Scafell Pike. There may also be problems with recognizing Swirl How, High Stile and Robinson. The second summary (Table IV, columns C and D and their ratio) picks out Helvellyn as particularly affected, the local appearance of peakness being enhanced in the portion of the peak which remains in-view.

In addition to peaks, it is possible to determine the degree to which the other fuzzy phenomena are visible, thus Wrynose Pass (Figure 11C) is very slightly visible, and Honister Pass is quite clearly visible. To identify either is likely to take the eye of a knowledgeable fell walker, because they do not have the presence in the view of a peak. Other passes which are unnamed in the database are also visible very clearly.

And from where can you see Helvellyn?

It is also possible to conduct a reverse visibility analysis of a peak. This is achieved by taking the area of the peak as the object extracted from the peakness image, and determining for each location in the full study area the maximum fuzzy membership of the peak that can be seen. This process will record 0 where the peak is invisible, and higher values where the peak can be seen to some degree. The net result is a viewshed with fuzzy membership values which record the maximum degree of peakness of the named peak which is visible from each location (Figure 12).

The reverse fuzzy viewshed of Helvellyn therefore delimits those areas from which it is possible to get a good view of the peak. It was calculated for the degree of peakness extracted with a 4° threshold for peak identification (Figure 12A). The area which can see Helvellyn is dominated by locations which have a good view of the mountain. The dominant membership is as high as the higher membership of the degree of peakness. Rather fewer areas have reduced memberships in the visibility of peakness, and can be seen to be in locations where visibility is sheltered by the landscape. This shows the areas from which it is possible to get a good view of the mountain.

To take analysis a stage further, it is possible to use the simple fuzzy intersection to determine those

Table IV Comparisons of the area weighted fuzzy membership of peakness of the peak fuzzy objects and area of peak in-view from Helvellyn

	Area weighted fuzzy membership of peakness standardized against total area of peak (A)	*Area weighted fuzzy membership of peakness in-view standardized against area of peak in-view (B)*	*A/B*	*Area weighted fuzzy membership of peakness (all with maximum 1) standardized against total area of peak (C)*	*Area weighted fuzzy membership of peakness (all with maximum 1) in-view standardized against area of peak in-view (D)*	*C/D*
Peaks (mountains and hills) named in the database of toponyms						
Blake Fell	0.2265	0.0224	10.12	0.2540	0.3039	0.84
Bow Fell	0.1887	0.1402	1.35	0.4106	0.4243	0.97
Crinkle Crags	0.2238	0.1290	1.74	0.2436	0.2576	0.95
Dalehead	0.2132	0.1597	1.33	0.3287	0.4255	0.77
Grasmoor	0.2573	0.1140	2.26	0.2975	0.3268	0.91
Great Borne	0.4269	0.2602	1.64	0.4269	0.5083	0.84
Great Gable	0.2336	0.1203	1.94	0.3201	0.3934	0.81
Grisedale Pike	0.2611	0.0465	5.62	0.4391	0.4847	0.91
Helvellyn	0.2766	0.0341	8.11	0.3199	0.6342	0.50
High Raise	0.3517	0.2122	1.66	0.3517	0.3702	0.95
High Stile	0.2319	0.0642	3.61	0.2524	0.3113	0.81
Old Man of Coniston	0.2819	0.0288	9.79	0.4011	0.5209	0.77
Pike of Blisco	0.2560	0.0371	6.91	0.3789	0.3760	1.01
Robinson	0.2055	0.0636	3.23	0.3168	0.2307	1.37
Scafell Pike	0.3048	0.0691	4.41	0.4177	0.5393	0.77
Steeple	0.2726	0.0980	2.78	0.2882	0.3857	0.75
Ullscarf	0.3291	0.3156	1.04	0.3291	0.3570	0.92
Peaks (mountains and hills) unnamed in the database of toponyms						
Arnsbarrow Hill	0.3595	0.2271	1.58	0.3695	0.4086	0.90
Brund Fell	0.2940	0.2873	1.02	0.5439	0.5815	0.94
Carron Crag	0.2682	0.1380	1.94	0.3970	0.4071	0.98
Glaramara	0.1796	0.1587	1.13	0.2077	0.2127	0.98
Grey Knotts	0.3217	0.0704	4.57	0.5176	0.4365	1.19
Grizedale	0.2877	0.1274	2.26	0.3802	0.4732	0.80
High Arnside	0.2978	0.2460	1.21	0.3148	0.3065	1.03
High Man	0.3985	0.3438	1.16	0.4337	0.3834	1.13
High Spy	0.1916	0.1916	1.00	0.3938	0.3938	1.00
Hill Fell	0.2250	0.1742	1.29	0.7568	0.7992	0.95
Lingmoor Fell	0.2795	0.2280	1.23	0.2955	0.2941	1.00
Maiden Moor	0.2244	0.1049	2.14	0.3460	0.4212	0.82
Swirl How	0.2684	0.0700	3.83	0.2921	0.2491	1.17

locations which would be called peaks from which it is possible to see the peak Helvellyn. Membership of the intersection of two fuzzy sets, P and Q, is given by the minimizing operation (Fisher 2000a; Leung 1988; Robinson 1988; Zadeh 1965)

$$\mu_{P \cap Q} = \min(\mu_P, \mu_Q) \tag{7}$$

The intersection between the reverse fuzzy viewshed and the degree of peakness is shown in Figure 12B, and the maximum fuzzy memberships within the areas of peaks named in the toponym database are listed in Table V. From these it is clear that if a viewer wishes to see Helvellyn from another peak, the best views can be achieved from High Raise, Ullscarf or Middle Dodd. From the mapping (Figure 12B) we can see that the first two provide the largest range of locations with large membership values. Furthermore, High Seat and Skiddaw may provide reasonable views too. Skiddaw is sufficiently far away from Helvellyn, and that peak is visible from only a small area of that peak and its actual visibility must be considered in doubt given the established unreliability of viewshed determination (Fisher 1993). Clearly, the eastern High Raise, Pike of Blisco and Bow Fell are not places to visit if what you want to get is a view of Helvellyn, but if you happen to be at one of these peaks

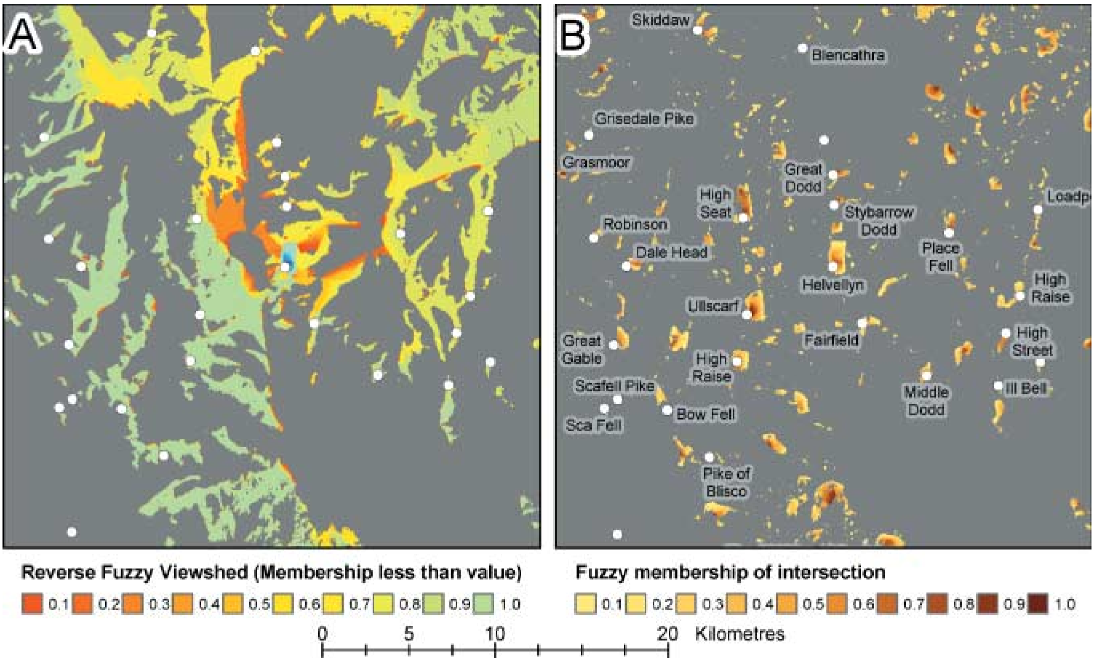

Figure 12 **(A) The areas which can see Helvellyn and the degree to which it is possible to see Helvellyn from them (the reverse fuzzy viewshed) based on the 4° threshold for extraction, and (B) the intersection of the reverse fuzzy viewshed of Helvellyn with the degree of peakness, showing the distributions of areas where peaks occur from which it is possible to see Helvellyn**

Table V **Named peaks identified as locations from which it is possible to see Helvellyn**

	Fuzzy membership of the intersection of the reverse fuzzy viewshed and the degree of peakness
High Raise	0.946
Middle Dodd	0.946
Ullscarf	0.946
High Seat	0.784
Skiddaw	0.784
Great Dodd	0.730
Great Gable	0.730
Place Fell	0.730
Fairfield	0.703
High Street	0.703
Stybarrow Dodd	0.703
Dale Head	0.649
Blencathra	0.486
Robinson	0.486
High Raise (eastern)	0.351
Pike of Blisco	0.351
Bow Fell	0.297

Helvellyn may be recognizable, but perhaps only to an experienced fell walker.

Conclusion

We have demonstrated one answer to a fundamental issue of spatial information processing: knowledge of the spatial extent of named, but indistinct, geographical locations. Research presented in this paper has shown the possibility and utility of defining landscape phenomena using a novel method of multi-scale analysis, which can be used to model objects which are vague for scale reasons. It was found that the morphometric analysis used does not map directly into these concepts, but that the class peak corresponds to many known mountains and hills. Similarly, passes are identifiably associated with landscape objects identified and named by people, and included in national toponym databases. Channels and ridges can also be associated with named landscape features, but a useable database of toponyms was not available to this research. For peaks, it was possible to identify an approximate spatial extent together with fuzzy memberships for the named locations, and it is then possible to give meaningful answers to typical questions people may ask about the landscape together with approximate compensation for the individual's familiarity with the landscape. One concern with the analysis presented is the problem of peaks named and unnamed in the toponym database, and the lack of association of important summits with peaks at least in the area examined here. Peak, however, is not a synonym for either summit or mountain, rather, just as meaningful peaks only arise from a multi-scale analysis, inclusion of the toponym of a summit in a database reflects recognition through a multifaceted cultural and technical process. Future work should examine the integration of more strands of information in recognizing the spatial extents of mountains.

The method articulated here has been shown to be successful in the limited contexts of the analysis presented, providing new insights that are possible and statements that can be made about the area. There remain problems with the method, however. Primary among these is parameterizing the morphometric extraction function, the weighting of the results from different scales and the decision of the scales over which to perform the analysis. For the Lake District there is a wealth of qualitative writing, and the Wainwright Guides (Wainwright 1960), in particular, provide extensive narrative description of the landscape. Future work will attempt a more thorough integration of the analysis with the description, but some ideas on further developments are shown at http://www.soi.city.ac.uk/~jwo/tibg/.

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