Homework 10

CS307-Operating System (D), Chentao Wu, Spring 2020.

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- (10.5) Consider the page table for a system with 12-bit virtual and physical addresses and 256-byte pages.

Page	Page Frame
0	–
1	2
2	C
3	A
4	–
5	4
6	3
7	–
8	B
9	0

The list of free page frames is D, E, F (that is, D is at the head of the list, E is second, and F is last). A dash for a page frame indicates that the page is not in the memory.

Convert the following virtual addresses to their equivalent physical addresses in hexadecimal. All numbers are given in hexadecimal.

i. $9EF$;

ii. 111;

iii. 700;

iv. $0FF$.

Solution. According to the problem descriptions, the address is 12-bit, which is 3-hexadecimal-bit; and the page offset is 8-bit, which is 2-hexadecimal-bit. Therefore, the page number and the page frame number is 4-bit, which is 1-hexadecimal-bit. Therefore, we can convert the virtual addresses according to the given mapping.

i. $9EF$ in virtual address corresponds to $0EF$ in physical address;

ii. 111 in virtual address corresponds to 211 in physical address;

iii. Page 7 does not correspond to any page frame in the mapping, so we get a free page frame from the given list and add a mapping $7 \to D$ in the mapping; therefore, 700 in virtual address corresponds to $D00$ in physical address;

iv. Page 0 does not correspond to any page frame in the mapping, so we get a free page frame from the given list and add a mapping $0 \to E$ in the mapping; therefore, $0FF$ in virtual address corresponds to EFF in physical address.

□

- (10.7) Consider the two-dimensional array `A`

```
int A[][] = new int [100][100];
```

where `A[0][0]` is at location 200 in a paged memory system with pages of size 200. A small process that manipulates the matrix resides in page 0 (locations 0 to 199). Thus, every instruction fetch will be from page 0.

For three page frames, how many page faults are generated by the following array-initialization loops? Use LRU replacement, and assume that page frame 1 contains the process and the other two are initially empty.

a. The first array-initialization loop is as follows.

```
for (int j = 0; j < 100; j++)
  for (int i = 0; i < 100; i++)
    a[i][j] = 0;
```

b. The second array-initialization loop is as follows.

```
for (int i = 0; i < 100; i++)
  for (int j = 0; j < 100; j++)
    a[i][j] = 0;
```

Solution. Here are the solutions to the two sub-questions.

a. For every `j`, when accessing `a[0][j], a[1][j], ..., a[99][j]`, exactly 50 page faults will happen, because:

   - When `i` is even, such as `a[0][j], a[2][j], ..., a[98][j]`, the accesses are missed and page faults occur;
   - When `i` is odd, such as `a[1][j], a[3][j], ..., a[99][j]`, the accesses are hit, because when page fault occurs in `i-1`, we also put the content of `a[i][j]` into the memory since the page size is 200.

   Therefore, $50 \times 100 = 5000$ page faults will happen in total.

b. For every even `i`, when accessing `a[i][0], a[i][1], ..., a[i][99]`, only 1 page fault will happen, because when a page fault happens when accessing `a[i][0]`, we put `a[i][0], a[i][1], ..., a[i][99], a[i+1][0], a[i+1][0], ..., a[i+1][99]` in the memory since the page size is 200. Hence, for every odd `i`, when accessing the `a[i][0], a[i][1], ..., a[i][99]`, no page fault will happen because the elements has been fetched when accessing `a[i-1][0]`. Therefore, only 50 page faults will happen in total.

□

- (10.8) Consider the following page reference string:

  $$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6$$

  How many page faults would occur for the following replacement algorithms, assuming one, two, three, four, five, six, and seven frames? Remember that all frames are initially empty, so your first unique pages will cost one fault each.

  - LRU replacement;
  - FIFO replacement;
  - Optimal replacement.

  Solution. Here are the answers to all questions.

Number of frames	LRU replacement	FIFO replacement	Optimal replacement
1	20 page faults	20 page faults	20 page faults
2	18 page faults	18 page faults	15 page faults
3	15 page faults	16 page faults	11 page faults
4	10 page faults	14 page faults	8 page faults
5	8 page faults	10 page faults	7 page faults
6	7 page faults	10 page faults	7 page faults
7	7 page faults	7 page faults	7 page faults

  Here are some specific explanations. We assume in every hit/miss sequence, the accesses to red-colored, bolded and underlined elements are hit and the others accesses are missed.

  - **LRU replacement with 1 frame**: no access is hit, and totally 20 page faults.

    $$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6$$

  - **FIFO replacement with 1 frame**: no access is hit, and totally 20 page faults.

    $$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6$$

  - **Optimal replacement with 1 frame**: no access is hit, and totally 20 page faults.

    $$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6$$

  - **LRU replacement with 2 frames**: 2 accesses are hit, and totally 18 page faults.

    $$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, \underline{\mathbf{2}}, 3, 7, 6, 3, 2, 1, \underline{\mathbf{2}}, 3, 6$$

  - **FIFO replacement with 2 frames**: 2 accesses are hit, and totally 18 page faults.

    $$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, \underline{\mathbf{2}}, 3, 7, 6, 3, 2, 1, \underline{\mathbf{2}}, 3, 6$$

  - **Optimal replacement with 2 frames**: 5 accesses are hit, and totally 15 page faults.

    $$1, 2, 3, 4, \underline{\mathbf{2}}, 1, 5, 6, \underline{\mathbf{2}}, 1, \underline{\mathbf{2}}, 3, 7, 6, \underline{\mathbf{3}}, 2, 1, \underline{\mathbf{2}}, 3, 6$$

  - **LRU replacement with 3 frames**: 5 accesses are hit, and totally 15 page faults.

    $$1, 2, 3, 4, \underline{\mathbf{2}}, 1, 5, 6, 2, 1, \underline{\mathbf{2}}, 3, 7, 6, \underline{\mathbf{3}}, 2, 1, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 6$$

- **FIFO replacement with 3 frames**: 4 accesses are hit, and totally 16 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, 1, 5, 6, 2, 1, \underline{\mathbf{2}}, 3, 7, 6, \underline{\mathbf{3}}, 2, 1, \underline{\mathbf{2}}, 3, 6$$

- **Optimal replacement with 3 frames**: 9 accesses are hit, and totally 11 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, 2, 1, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 6$$

- **LRU replacement with 4 frames**: 10 accesses are hit, and totally 10 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 7, 6, \underline{\mathbf{3}}, \underline{\mathbf{2}}, 1, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **FIFO replacement with 4 frames**: 6 accesses are hit, and totally 14 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, 2, 1, \underline{\mathbf{2}}, 3, 7, 6, \underline{\mathbf{3}}, 2, 1, \underline{\mathbf{2}}, 3, \underline{\mathbf{6}}$$

- **Optimal replacement with 4 frames**: 12 accesses are hit, and totally 8 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, 1, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **LRU replacement with 5 frames**: 12 accesses are hit, and totally 8 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **FIFO replacement with 5 frames**: 10 accesses are hit, and totally 10 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, 2, 1, \underline{\mathbf{2}}, 3, \underline{\mathbf{6}}$$

- **Optimal replacement with 5 frames**: 13 accesses are hit, and totally 7 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **LRU replacement with 6 frames**: 13 accesses are hit, and totally 7 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **FIFO replacement with 6 frames**: 10 accesses are hit, and totally 10 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, 1, 2, 3, \underline{\mathbf{6}}$$

- **Optimal replacement with 6 frames**: 13 accesses are hit, and totally 7 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **LRU replacement with 7 frames**: 13 accesses are hit, and totally 7 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **FIFO replacement with 7 frames**: 13 accesses are hit, and totally 7 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

- **Optimal replacement with 7 frames**: 13 accesses are hit, and totally 7 page faults.

$$1, 2, 3, 4, \underline{\mathbf{2}}, \underline{\mathbf{1}}, 5, 6, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, 7, \underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{2}}, \underline{\mathbf{1}}, \underline{\mathbf{2}}, \underline{\mathbf{3}}, \underline{\mathbf{6}}$$

□

- (10.9) Consider the following page reference string:

  $$7, 2, 3, 1, 2, 5, 3, 4, 6, 7, 7, 1, 0, 5, 4, 6, 2, 3, 0, 1$$

  Assuming demand paging with three frames, how many page faults would occur for the following replacement algorithms?

  - LRU replacement;
  - FIFO replacement;
  - Optimal replacement.

  Solution. Here are the answers to all questions.

LRU replacement	FIFO replacement	Optimal replacement
18 page faults	17 page faults	13 page faults

  Here are some specific explanations. We assume in every hit/miss sequence, the accesses to red-colored, bolded and underlined elements are hit and the others accesses are missed.

  - **LRU replacement**: 2 accesses are hit, and totally 18 page faults.

    $$7, 2, 3, 1, \underline{\mathbf{2}}, 5, 3, 4, 6, 7, \underline{\mathbf{7}}, 1, 0, 5, 4, 6, 2, 3, 0, 1$$

  - **FIFO replacement**: 3 accesses are hit, and totally 17 page faults.

    $$7, 2, 3, 1, \underline{\mathbf{2}}, 5, \underline{\mathbf{3}}, 4, 6, 7, \underline{\mathbf{7}}, 1, 0, 5, 4, 6, 2, 3, 0, 1$$

  - **Optimal replacement**: 7 accesses are hit, and totally 13 page faults.

    $$7, 2, 3, 1, \underline{\mathbf{2}}, 5, \underline{\mathbf{3}}, 4, 6, 7, \underline{\mathbf{7}}, \underline{\mathbf{1}}, 0, \underline{\mathbf{5}}, 4, 6, 2, 3, \underline{\mathbf{0}}, \underline{\mathbf{1}}$$

  □