HW 1 The Optimal Connectivity Determination in Uncertain Graphs

Determining the source-destination connectivity in uncertain graphs has wide applications in real life, e.g., routing detection, information diffusion control, etc. For an uncertain graph, each edge exists independently with some probability, and the existence of each edge can be unraveled through edge testing with a certain cost. Our goal is to determine whether source s and destination t are connected or not with the minimum expected cost.

Given the following several kinds of uncertain graphs, please provide the corresponding optimal strategy with the minimum expected cost, and prove its optimality, respectively.

1. For the pyramid graph in Figure 1, each edge exists with probability p, and the cost of testing the existence of each edge is 1.

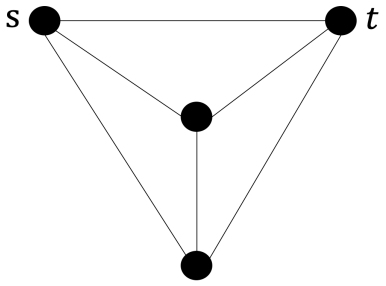

Figure 1: Pyramid Graph

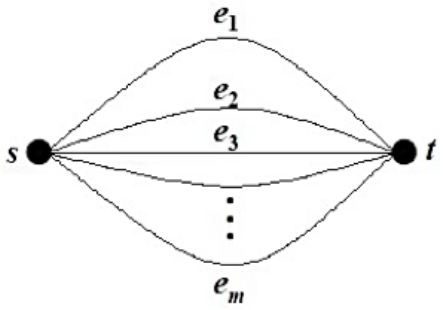

Figure 2: Parallel Graph

2. For the graph with m parallel edges between nodes s and t, edges are labeled e_1 through e_m as shown in Figure 2. The probability that edge e_i $(i = 1, 2, \cdots, m)$ exists is p_i. And the test cost of edge e_i is t_i.

3. **(Optional, with additional bonus up to 2% of your final course score.)** For the series-parallel graph consisting of n parallel graph labeled $P^1_{m_1}, \cdots, P^n_{m_n}$, arranged in series. Let the i-th edge in the j-th parallel graph be represented as e_{ij} $(1 \leq j \leq n, 1 \leq i \leq m_j)$. Edge e_{ij} exists with probability p_{ij} and has test cost c_{ij}.

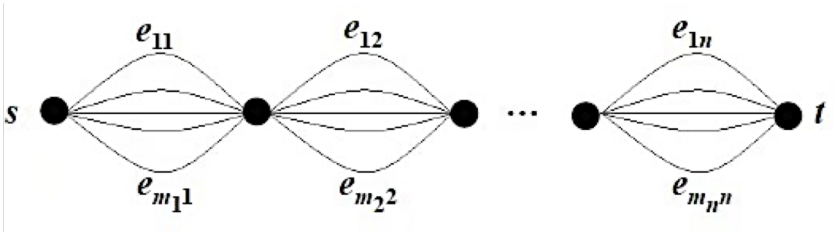

Figure 3: Serirs-Parallel Graph