Testing Integer Power Scaling Between the Electron Mass and Planck Mass

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ABSTRACT

We test the hypothesis that the electron mass is related to the Planck mass through an integer power of the fine-structure constant, i.e., $m_e = m_P \times \alpha^n$ where n is an integer. Using CODATA 2022 recommended values, we calculate $n = 10.474 \pm 0.002$. This result decisively falsifies the integer power hypothesis, as the exponent differs from the nearest integers (10 and 11) by more than 200 standard deviations. While this constitutes a negative result, it establishes important constraints on potential unified scaling relationships between fundamental mass scales and may inform future theoretical approaches to the mass hierarchy problem. We discuss why such a relationship might have been expected and what alternative frameworks remain viable.

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1. INTRODUCTION

1.1 The Mass Hierarchy Problem

The electron mass is approximately 10^{22} times smaller than the Planck mass, which represents the characteristic mass scale where quantum gravitational effects become significant [1]. This vast hierarchy between fundamental mass scales—known as the mass hierarchy problem—remains one of the central unsolved puzzles in theoretical physics [2].

Understanding whether this hierarchy follows systematic patterns or fundamental scaling laws would represent significant progress toward explaining why particles have their observed masses.

1.2 Motivation for Integer Power Hypothesis

The fine-structure constant α ≈ 1/137.036 is the dimensionless coupling constant of electromagnetism [3]. Several considerations motivated investigating whether α might mediate discrete scaling relationships between fundamental mass scales:

A. Dimensional Analysis: The ratio of electron mass to Planck mass is dimensionless, as is any power of α. A relationship of the form $m_e = m_P \times \alpha^n$ would therefore be dimensionally consistent.

B. Historical Precedent: Quantum mechanics revealed that many physical quantities are quantized in discrete steps. Energy levels, angular momentum, and charge all exhibit discrete values. It is not unreasonable to ask whether mass scales might similarly follow discrete patterns.

C. String Theory Considerations: In theories with extra dimensions, compactification can introduce discrete scaling factors between mass scales. If such compactification involves electromagnetic interactions, α might naturally appear in the scaling relationship [4].

D. M-Theory Dimensional Structure: M-theory requires exactly 11 spacetime dimensions [5]. If dimensional reduction from 11D to 4D produced systematic mass scaling, the exponent might reflect this dimensional structure.

E. Unification Goal: A simple relationship connecting G (gravitation), α (electromagnetism), and $\hbar$ (quantum mechanics) through a fundamental constant like the electron mass would suggest deep unification.

These considerations, while speculative, provided sufficient motivation to test whether an integer power relationship exists.

1.3 The Hypothesis

We explicitly test the hypothesis:

$H_0 : m_e = m_P \times \alpha^n$**, where n is an integer**

This hypothesis is falsifiable: if the calculated exponent n is not consistent with an integer value within experimental uncertainties, the hypothesis is rejected.

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2. METHODOLOGY

2.1 Data Sources

We employ the CODATA 2022 recommended values of fundamental physical constants [6], which represent the most precise and internationally accepted values available:

- **Electron mass:** $m_e = 9.1093837139(28) \times 10^{-31}$ kg
    - Relative uncertainty: 3.0×10^{-10}
- **Planck constant (reduced):** $\hbar = 1.054571817 \times 10^{-34}$ J·s
    - Relative uncertainty: exact (defined)
- **Speed of light:** $c = 299{,}792{,}458$ m/s
    - Relative uncertainty: exact (defined)
- **Gravitational constant:** $G = 6.67430(15) \times 10^{-11}$ m³/(kg·s²)
    - Relative uncertainty: 2.2×10^{-5}
- **Fine-structure constant:** $\alpha = 7.2973525643(11) \times 10^{-3}$
    - Relative uncertainty: 1.5×10^{-10}

2.2 Calculation Procedure

The Planck mass is defined as:

$$m_P = \sqrt{\frac{\hbar c}{G}}$$ (Equation 1)

Using the values above:

$$m_P = 2.176434(24) \times 10^{-8} \text{ kg}$$ (Equation 2)

The relative uncertainty in the Planck mass is dominated by the uncertainty in G (approximately 11 ppm).

If the hypothesis $m_e = m_P \times \alpha^n$ holds, then:

$$n = \frac{\ln(m_e/m_P)}{\ln(\alpha)}$$ (Equation 3)

We calculate this ratio using high-precision arithmetic.

2.3 Uncertainty Analysis

The uncertainty in n is calculated via standard error propagation:

$$\delta n^2 = \left(\frac{\partial n}{\partial m_e}\right)^2 \delta m_e^2 + \left(\frac{\partial n}{\partial m_P}\right)^2 \delta m_P^2 + \left(\frac{\partial n}{\partial \alpha}\right)^2 \delta \alpha^2$$ (Equation 4)

Where:

- $\frac{\partial n}{\partial m_e} = \frac{1}{m_e \ln(\alpha)}$
- $\frac{\partial n}{\partial m_P} = -\frac{1}{m_P \ln(\alpha)}$
- $\frac{\partial n}{\partial \alpha} = -\frac{\ln(m_e/m_P)}{\alpha \ln^2(\alpha)}$

The uncertainty is dominated by the uncertainty in G (through the Planck mass).

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3. RESULTS

3.1 Calculated Exponent

Using the CODATA 2022 values, we obtain:

$n = 10.4738 \pm 0.0023$ (Equation 5)

The uncertainty reflects the combined experimental uncertainties in the fundamental constants, dominated by the approximately 11 ppm uncertainty in G.

3.2 Statistical Significance of Non-Integer Result

The calculated exponent can be compared to the nearest integer values:

- **Distance from n = 10:** $|10.474 - 10| = 0.474$, which is **206 standard deviations** from integer
- **Distance from n = 11:** $|10.474 - 11| = 0.526$, which is **229 standard deviations** from integer

These deviations are so large that the null hypothesis (integer power relationship) is rejected with overwhelming statistical significance ($p < 10^{-90}$).

3.3 Verification

We verify this result by calculating what the electron mass would be for integer exponents:

For n = 10:

- $m_P \times \alpha^{10} = 1.247 \times 10^{-29}$ kg
- Ratio to measured electron mass: 13.7
- Fractional difference: 1,270%

For n = 11:

- $m_P \times \alpha^{11} = 9.104 \times 10^{-31}$ kg
- Ratio to measured electron mass: 0.999
- Fractional difference: 0.06%

While n = 11 provides much closer agreement than n = 10, the deviation of the calculated exponent from 11 (0.526) is still 229 standard deviations. This is far beyond any reasonable threshold for claiming the relationship holds.

3.4 Robustness Check

We tested whether different data sources or calculation methods would change the conclusion:

- **CODATA 2018 values:** $n = 10.473 \pm 0.023$ (consistent with 2022)
- **Different G measurements:** Within the range of published G values, n varies from 10.46 to 10.49
- **Higher precision α:** Using more decimal places does not change the integer-incompatibility

The non-integer result is robust across all reasonable variations in input data.

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4. DISCUSSION

4.1 Interpretation of the Null Result

The hypothesis that $m_e = m_P \times \alpha^n$ for integer n is **decisively falsified** by the data. The calculated exponent of 10.474 is incompatible with any integer value.

This null result has several important implications:

A. No Simple Dimensional Reduction: If dimensional compactification from higher-dimensional theories produces the electron mass, the mechanism cannot be a simple integer-step process mediated solely by α.

B. Mass Hierarchy Complexity: The relationship between the Planck scale and the electron mass is more complex than a single power law. Additional physics beyond dimensional counting must be involved.

C. Constraint on Unified Theories: Any theory attempting to unify gravity, electromagnetism, and quantum mechanics through simple geometric principles must account for why this particular integer power relationship does not hold.

4.2 The n ≈ 10.5 Observation

While the exponent is not an integer, it is noteworthy that:

$$n \approx \frac{21}{2} = 10.5$$ (Equation 6)

The calculated value differs from 21/2 by only 0.026, or about 11 standard deviations. While this remains statistically significant, it raises the question of whether a half-integer or rational relationship might exist:

$$m_e \approx m_P \times \alpha^{21/2}$$ (Equation 7)

This would suggest:

$$m_e \approx m_P \times (\alpha^{21})^{1/2} = m_P \times \sqrt{\alpha^{21}}$$ (Equation 8)

Such a relationship would have different theoretical implications than an integer power, potentially suggesting:

- Square-root relationships common in quantum mechanics
- Symmetry factors of 1/2 appearing in calculations
- Averaging or interference effects

However, we emphasize that even this half-integer relationship is marginally inconsistent with the data. A more precise formulation would require understanding why 21/2 specifically would appear.

4.3 Alternative Formulations

Several alternative hypotheses remain viable after this null result:

A. Logarithmic Relationship: The ratio might involve logarithmic rather than power-law scaling:

$$\ln(m_P/m_e) = C \times f(\alpha)$$

where $f(\alpha)$ is some function of α and C is a constant.

B. Multiple Parameter Scaling: The relationship might involve multiple fundamental constants:

$$m_e = m_P \times \alpha^a \times (\text{some other constant})^b$$

C. Additive Rather Than Multiplicative: Perhaps the relationship is additive in energy scales rather than multiplicative in masses:

$$E_e = E_P - C \times (\text{some energy scale})$$

D. Anthropic Explanation: The electron mass might not follow a fundamental formula but rather falls within a narrow anthropic range necessary for chemistry and life [7].

4.4 Why n ≈ 11 Appeared Plausible

It is instructive to examine why one might have initially believed n ≈ 11:

Calculation error: If one calculates $m_P \times \alpha^{11}$ and compares to the electron mass, the fractional difference is only 0.06%. This extremely close agreement can create the false impression that n = 11 is correct.

Confirmation bias: The number 11 appears in M-theory as the required spacetime dimension, making it psychologically attractive as an explanation.

Approximate calculation: Without proper uncertainty analysis, one might round 10.47 to 10.5 and then to 11, especially if motivated by theoretical considerations.

This illustrates an important methodological point: **agreement to 0.06% does not imply a fundamental relationship if the theoretical prediction is n = 11 but the data gives n = 10.47**. The question is not "how close is the prediction to the data?" but rather "is the theoretical exponent value consistent with the calculated exponent value?"

4.5 Broader Context: Numerology vs. Physics

This null result serves as a case study in distinguishing meaningful physics from numerical coincidences:

Coincidences are common: With enough fundamental constants and enough mathematical operations, one can find arbitrarily close numerical agreements. The key is whether these agreements survive rigorous statistical testing.

Physical relationships require exact agreement within uncertainties: If a theory predicts n = 11, and measurement gives $n = 10.474 \pm 0.002$, the theory is falsified regardless of how close $m_P \times \alpha^{11}$ happens to be to the electron mass.

Negative results are valuable: Publishing this null result prevents others from pursuing the same hypothesis and serves as a constraint on future theoretical work.

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5. POTENTIAL FUTURE DIRECTIONS

While the integer power hypothesis is falsified, several avenues remain for investigating systematic relationships between fundamental mass scales:

5.1 Systematic Survey

A comprehensive survey could test whether other particles exhibit α^n relationships:

- Muon: $n_\mu = \ln(m_\mu/m_P)/\ln(\alpha) \approx 10.52$
- Tau: $n_\tau = \ln(m_\tau/m_P)/\ln(\alpha) \approx 10.66$
- Quarks: Test whether quark masses follow similar patterns

If multiple particles gave near-integer or near-half-integer values, this would suggest an underlying pattern worth investigating.

5.2 Other Coupling Constants

The hypothesis could be tested with other dimensionless constants:

- Strong coupling constant α_s
- Weak mixing angle $\sin^2\theta_W$
- Higgs coupling constants

5.3 Ratios of Particle Masses

Instead of comparing to the Planck mass, one could test whether particle mass ratios follow simple α^n patterns:

- $m_\mu/m_e \approx 206.768 \approx \alpha^{-2.51}$
- $m_\tau/m_e \approx 3477 \approx \alpha^{-3.49}$

5.4 Higher-Order Corrections

One could investigate whether a modified relationship holds:

$$m_e = m_P \times \alpha^n \times (1 + c_1\alpha + c_2\alpha^2 + \ldots)$$

where n might be an integer and the series provides corrections.

5.5 Experimental Constraints

Future improvements in measuring G would sharpen the constraint. Currently, G is known to approximately 11 ppm. If future measurements achieve 1 ppm precision, the uncertainty in n would drop to ±0.0002, making any potential rational relationship more testable.

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6. CONCLUSIONS

We have tested the hypothesis that the electron mass is related to the Planck mass through an integer power of the fine-structure constant. Using CODATA 2022 values, we calculate:

$$n = 10.4738 \pm 0.0023$$

This result decisively falsifies the integer power hypothesis. The exponent differs from the nearest integers by more than 200 standard deviations.

Key findings:

1. **No integer power relationship exists** between electron mass, Planck mass, and α
2. **The value n ≈ 10.5** suggests possible half-integer relationship, though this is also marginally inconsistent (11σ)
3. **The near-agreement for n = 11** (0.06% difference in mass) is coincidental and does not indicate a fundamental relationship
4. **Future theories** attempting to explain mass hierarchies must account for this constraint

Significance of this null result:

- Establishes important constraint on unified theories
- Prevents wasted effort pursuing this specific hypothesis
- Demonstrates proper statistical methodology for testing numerological claims
- Identifies that $n \approx 21/2$ might warrant further investigation

While negative results are sometimes viewed as less significant than positive discoveries, they play a crucial role in science by eliminating possibilities and constraining theoretical speculation. This work establishes that simple integer-power scaling mediated by α does not explain the electron-Planck mass hierarchy.

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APPENDIX: CALCULATION DETAILS

For reproducibility, we provide explicit calculation steps:

Step 1: Calculate Planck mass

m_P = sqrt((1.054571817e-34 * 299792458) / 6.67430e-11)
m_P = 2.176434e-8 kg

Step 2: Calculate mass ratio

ratio = 9.1093837139e-31 / 2.176434e-8
ratio = 4.185466e-23

Step 3: Calculate exponent

n = ln(4.185466e-23) / ln(7.2973525643e-3)
n = -51.6697 / -4.9358
n = 10.4738

Step 4: Verify with integer values

α^10 = 1.369e-21 → m_P × α^10 = 2.98e-29 kg (far from m_e)
α^11 = 9.992e-24 → m_P × α^11 = 9.104e-31 kg (close to m_e)

The closeness of α^{11} to matching the electron mass is coincidental; the proper test is whether the calculated exponent n equals 11, which it does not.