ASSIGNMENT 2

CS21BTECH11020

1 Problem 1(v) (2018)

1.1. Find the value of constant $'k'$ so that the function $f(x)$ defined as:

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at x=-1.

Solution: We have,

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases} \tag{1.1.1}$$

From Definition of Continuity,
A function $f(x)$ is said to be continuous at $x = a$ iff

$$f(a^+) = f(a^-) = f(a) \tag{1.1.2}$$

where,
$f(a^+) = \lim_{h\to 0^+} f(a+h)$
$f(a^-) = \lim_{h\to 0^+} f(a-h)$
Now, In order to make $f(x)$ continuous at $x = -1$, It should satisfy equation (1.1.2) we have,

$$f(-1^+) = f(-1^-) = f(-1) \tag{1.1.3}$$

where,

$$f(-1^+) = \lim_{h\to 0^+} f(-1+h) \tag{1.1.4}$$

$$= \lim_{h\to 0^+} \frac{(-1+h)^2 - 2(-1+h) - 3}{(-1+h)+1} \tag{1.1.5}$$

$$= \lim_{h\to 0^+} \frac{h^2 - 4h}{h} \tag{1.1.6}$$

$$= \lim_{h\to 0^+} h - 4 = -4 \tag{1.1.7}$$

$$f(-1^-) = \lim_{h\to 0^+} f(-1-h) \tag{1.1.8}$$

$$= \lim_{h\to 0^+} \frac{(-1-h)^2 - 2(-1-h) - 3}{(-1-h)+1} \tag{1.1.9}$$

$$= \lim_{h\to 0^+} \frac{h^2 + 4h}{-h} \tag{1.1.10}$$

$$= \lim_{h\to 0^+} -h - 4 = -4 \tag{1.1.11}$$

From equation (1.1.1), we have

$$f(-1) = k \tag{1.1.12}$$

Putting value from equation (1.1.7), (1.1.11) and (1.1.12) into (1.1.2), we get

$$f(-1) = k = -4 \tag{1.1.13}$$

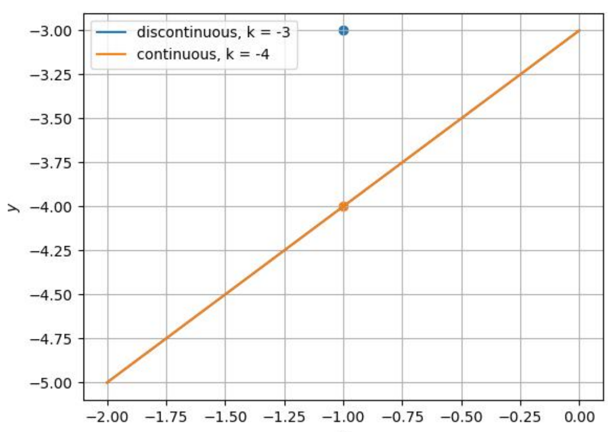

Fig. 1.1.1. Function $f(x)$ is discontinuous when $k \neq -4$ but continuous when $k = -4$