ASSIGNMENT 5

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Outline

Problem Statement

Class 12th Probability Example 31

If a fair coin is tossed 10 times, find the probability of

1. exactly six heads
2. at least six heads
3. at most six heads

Solution

Binomial Distribution

The Binomial Distribution Formula for any Random Variable X is

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \tag{1}$$

where,
$n =$ Number of Experiments (Trails)
$p =$ Probability of Success
$x = 0, 1, 2, \ldots, n$

Solution

Part 1

Let the Random Variable X be the number of heads in coin toss.
we have,

$$X \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \tag{2}$$

$$\text{(Number of Trails) } n = 10 \tag{3}$$

$$\text{(Probability of getting head in one toss) } p = \frac{1}{2} \tag{4}$$

Using PMF from equation (1), we get

$$P(X = 6) = \binom{10}{6} \times p^6 \times (1 - p)^4 = 210 \times \frac{1}{2}^{10} = \frac{105}{512} \tag{5}$$

Soltuion

Part 2

$$P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) \tag{6}$$

$$P(X \geq 6) = \binom{10}{6}p^6(1-p)^4 + \binom{10}{7}p^7(1-p)^3 + \binom{10}{8}p^8(1-p)^2 \tag{7}$$

$$+ \binom{10}{9}p^9(1-p) + \binom{10}{10}p^{10} \tag{8}$$

$$P(X \geq 6) = 210 \times \frac{1}{2}^{10} + 120 \times \frac{1}{2}^{10} + 45 \times \frac{1}{2}^{10} + 10 \times \frac{1}{2}^{10} + 1 \times \frac{1}{2}^{10} \tag{9}$$

$$P(X \geq 6) = \frac{193}{512} \tag{10}$$

Solution

Part 3

$$P(X \leq 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \qquad (11)$$
$$+ P(X = 4) + P(X = 5) + P(X = 6) \qquad (12)$$

$$P(X \leq 6) = \binom{10}{0}p^0(1-p)^{10} + \binom{10}{1}p(1-p)^9 + \binom{10}{2}p^2(1-p)^8 \quad (13)$$

$$+ \binom{10}{3}p^3(1-p)^7 + \binom{10}{4}p^4(1-p)^6 + \binom{10}{5}p^5(1-p)^5 \quad (14)$$

$$+ \binom{10}{6}p^6(1-p)^4 \qquad (15)$$

$$P(X \leq 6) = 1 \times \frac{1}{2}^{10} + 10 \times \frac{1}{2}^{10} + 45 \times \frac{1}{2}^{10} + 120 \times \frac{1}{2}^{10} \qquad (16)$$

$$+ 210 \times \frac{1}{2}^{10} + 252 \times \frac{1}{2}^{10} + 210 \times \frac{1}{2}^{10} = \frac{848}{1024} = \frac{53}{64} \qquad (17)$$