ASSIGNMENT 1

CS21BTECH11020

PROBLEM 6b (2018) : If $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$,

Find $\mathbf{AC} + \mathbf{B^2} - \mathbf{10C}$.

SOLUTION: We have,

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \tag{3}$$

Since we know the identities,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{pmatrix} \tag{5}$$

Using Identity (4), we have

$$\mathbf{AC} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \tag{6}$$

$$\mathbf{AC} = \begin{pmatrix} (2 \times 1) + (3 \times (-1)) & (2 \times 0) + (3 \times 4) \\ (5 \times 1) + (7 \times (-1)) & (5 \times 0) + (7 \times 4) \end{pmatrix} \tag{7}$$

$$\mathbf{AC} = \begin{pmatrix} 2 - 3 & 0 + 12 \\ 5 - 7 & 0 + 28 \end{pmatrix} \tag{8}$$

$$\boxed{\mathbf{AC} = \begin{pmatrix} -1 & 12 \\ -2 & 28 \end{pmatrix}} \tag{9}$$

$$\mathbf{B^2} = \mathbf{BB} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} \tag{10}$$

$$\mathbf{B^2} = \begin{pmatrix} (0 \times 0) + (4 \times (-1)) & (0 \times 4) + (4 \times 7) \\ ((-1) \times 0) + (7 \times (-1)) & ((-1) \times 4) + (7 \times 7) \end{pmatrix} \tag{11}$$

$$\mathbf{B^2} = \begin{pmatrix} 0 - 4 & 0 + 28 \\ 0 - 7 & -4 + 49 \end{pmatrix} \tag{12}$$

$$\boxed{\mathbf{B^2} = \begin{pmatrix} -4 & 28 \\ -7 & 45 \end{pmatrix}} \tag{13}$$

$$\mathbf{10C} = (\mathbf{10I})\mathbf{C} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \tag{14}$$

$$\mathbf{10C} = \begin{pmatrix} (10 \times 1) + (0 \times (-1)) & (10 \times 0) + (0 \times 4) \\ (0 \times 1) + (10 \times (-1)) & (0 \times 0 + 10 \times 4) \end{pmatrix} \tag{15}$$

$$\mathbf{10C} = \begin{pmatrix} 10 + 0 & 0 + 0 \\ 0 - 10 & 0 + 40 \end{pmatrix} \tag{16}$$

$$\boxed{\mathbf{10C} = \begin{pmatrix} 10 & 0 \\ -10 & 40 \end{pmatrix}} \tag{17}$$

Using Identity (5) and values from (9), (13) and (17), We have

$$\mathbf{AC} + \mathbf{B^2} - \mathbf{10C} = \begin{pmatrix} -1 & 12 \\ -2 & 28 \end{pmatrix} + \begin{pmatrix} -4 & 28 \\ -7 & 45 \end{pmatrix} - \begin{pmatrix} 10 & 0 \\ -10 & 40 \end{pmatrix} \tag{18}$$

$$= \begin{pmatrix} (-1) + (-4) - (10) & (12) + (28) - (0) \\ (-2) + (-7) - (-10) & (28) + (45) - (40) \end{pmatrix} \tag{19}$$

$$\boxed{\mathbf{AC} + \mathbf{B^2} - \mathbf{10C} = \begin{pmatrix} \mathbf{-15} & \mathbf{40} \\ \mathbf{1} & \mathbf{33} \end{pmatrix}} \tag{20}$$