ASSIGNMENT 1

CS21BTECH11020

(3)

PROBLEM 6b (2018) : If $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$,

Find $AC + B^2 - 10C$.

SOLUTION: We have,

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$$

Since we know the identities,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$$
(4)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{pmatrix}$$
 (5)

Using Identity (4), we have

$$\mathbf{AC} = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{pmatrix} \tag{7}$$

$$\mathbf{AC} = \begin{pmatrix} -1 & 12 \\ -2 & 28 \end{pmatrix} \tag{8}$$

$$\mathbf{B^2} = \mathbf{BB} = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} \tag{9}$$

$$= \begin{pmatrix} 0 - 4 & 0 + 28 \\ 0 - 7 & -4 + 49 \end{pmatrix} \tag{10}$$

$$\mathbf{B^2} = \begin{pmatrix} -4 & 28 \\ -7 & 45 \end{pmatrix} \tag{11}$$

and,

$$\mathbf{10C} = (\mathbf{10I})\mathbf{C} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 10+0 & 0+0\\ 0-10 & 0+40 \end{pmatrix} \tag{13}$$

$$\mathbf{10C} = \begin{pmatrix} 10 & 0 \\ -10 & 40 \end{pmatrix} \tag{14}$$

(1) Using Identity (5) and values from (8), (11) and (14), We have

(2)
$$\mathbf{AC} + \mathbf{B^2} - \mathbf{10C} = \begin{pmatrix} -1 & 12 \\ -2 & 28 \end{pmatrix} + \begin{pmatrix} -4 & 28 \\ -7 & 45 \end{pmatrix} - \begin{pmatrix} 10 & 0 \\ -10 & 40 \end{pmatrix}$$

$$AC + B^{2} - 10C = \begin{pmatrix} -15 & 40 \\ 1 & 33 \end{pmatrix}$$
 (16)