

ASSIGNMENT 1

CS21BTECH11020

PROBLEM 6b (2018) : If $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$, $B^2 = BB = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$
 $\begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$,

Find $AC + B^2 - 10C$.

SOLUTION: We have,

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} (0 \times 0) + (4 \times (-1)) & (0 \times 4) + (4 \times 7) \\ ((-1) \times 0) + (7 \times (-1)) & ((-1) \times 4) + (7 \times 7) \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 0 - 4 & 0 + 28 \\ 0 - 7 & -4 + 49 \end{pmatrix}$$

$$\boxed{B^2 = \begin{pmatrix} -4 & 28 \\ -7 & 45 \end{pmatrix}} \quad (4)$$

$$10C = (10I)C = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$$

Since we know the identities,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{pmatrix} \quad (2)$$

$$10C = \begin{pmatrix} (10 \times 1) + (0 \times (-1)) & (10 \times 0) + (0 \times 4) \\ (0 \times 1) + (10 \times (-1)) & (0 \times 0) + (10 \times 4) \end{pmatrix}$$

$$10C = \begin{pmatrix} 10 + 0 & 0 + 0 \\ 0 - 10 & 0 + 40 \end{pmatrix}$$

Using Identity (1), we have

$$AC = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} (2 \times 1) + (3 \times (-1)) & (2 \times 0) + (3 \times 4) \\ (5 \times 1) + (7 \times (-1)) & (5 \times 0) + (7 \times 4) \end{pmatrix}$$

$$AC = \begin{pmatrix} 2 - 3 & 0 + 12 \\ 5 - 7 & 0 + 28 \end{pmatrix}$$

$$\boxed{AC = \begin{pmatrix} -1 & 12 \\ -2 & 28 \end{pmatrix}} \quad (3)$$

$$\boxed{10C = \begin{pmatrix} 10 & 0 \\ -10 & 40 \end{pmatrix}} \quad (5)$$

Using Identity (2) and values from (3), (4) and (5),
We have

$$AC + B^2 - 10C = \begin{pmatrix} -1 & 12 \\ -2 & 28 \end{pmatrix} + \begin{pmatrix} -4 & 28 \\ -7 & 45 \end{pmatrix} - \begin{pmatrix} 10 & 0 \\ -10 & 40 \end{pmatrix}$$

$$= \begin{pmatrix} (-1) + (-4) - (10) & (12) + (28) - (0) \\ (-2) + (-7) - (-10) & (28) + (45) - (40) \end{pmatrix}$$

$$\boxed{AC + B^2 - 10C = \begin{pmatrix} -15 & 40 \\ 1 & 33 \end{pmatrix}}$$