ASSIGNMENT 1

CS21BTECH11020

PROBLEM 6b (2018) : If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find $AC + B^2 - 10C$.

SOLUTION : We have,

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

Since we know the identities,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix} \tag{1}$$

and,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \pm \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{bmatrix} \tag{2}$$

Using Identity (1), we have

$$AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$\implies AC = \begin{bmatrix} (2 \times 1) + (3 \times (-1)) & (2 \times 0) + (3 \times 4) \\ (5 \times 1) + (7 \times (-1)) & (5 \times 0) + (7 \times 4) \end{bmatrix}$$

$$\implies AC = \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix}$$

$$\boxed{AC = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}} \tag{3}$$

$$B^2 = BB$$

$$\implies B^2 = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

$$\implies B^2 = \begin{bmatrix} (0 \times 0) + (4 \times (-1)) & (0 \times 4) + (4 \times 7) \\ ((-1) \times 0) + (7 \times (-1)) & ((-1) \times 4) + (7 \times 7) \end{bmatrix}$$

$$\implies B^2 = \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix}$$

$$\boxed{B^2 = \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}} \tag{4}$$

$$10C = (10I)C$$

$$\implies 10C = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$\implies 10C = \begin{bmatrix} (10 \times 1) + (0 \times (-1)) & (10 \times 0) + (0 \times 4) \\ (0 \times 1) + (10 \times (-1)) & (0 \times 0 + 10 \times 4) \end{bmatrix}$$

$$\implies 10C = \begin{bmatrix} 10+0 & 0+0 \\ 0-10 & 0+40 \end{bmatrix}$$

$$\boxed{10C = \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}} \tag{5}$$

Using Identity (2) and values from (3), (4) and (5), We have

$$AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$\implies \begin{bmatrix} (-1)+(-4)-(10) & (12)+(28)-(0) \\ (-2)+(-7)-(-10) & (28)+(45)-(40) \end{bmatrix}$$

$$\boxed{\mathbf{AC + B^2 - 10C} = \begin{bmatrix} \mathbf{-15} & \mathbf{40} \\ \mathbf{1} & \mathbf{33} \end{bmatrix}} \tag{6}$$