

附录 1 初等数学常用公式

1. $|x + y| \leq |x| + |y|$.

2. $|x| - |y| \leq |x - y| \leq |x| + |y|$.

3. $\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$

4. 若 $|x| \leq a (a > 0)$, 则 $-a \leq x \leq a$.

5. 若 $|x| \geq b$ 且 $b > 0$, 则 $x \geq b$ 或 $x \leq -b$.

6. 设 $ax^2 + bx + c = 0$ 的判别式为 Δ (只就 $a > 0$ 的情形讨论).

(1) 当 $\Delta > 0$ 时, 方程有两个不等的实根 $x_1, x_2 (x_1 < x_2)$,

$$ax^2 + bx + c > 0 \text{ 的解集为 } \{x \mid x > x_2 \text{ 或 } x < x_1\};$$

$$ax^2 + bx + c < 0 \text{ 的解集为 } \{x \mid x_1 < x < x_2\};$$

(2) 当 $\Delta = 0$ 时, 方程有两个相等的实根 $x_1 = x_2$,

$$ax^2 + bx + c > 0 \text{ 的解集为 } \{x \mid x \in \mathbf{R}, \text{ 且 } x \neq x_1\};$$

(3) 当 $\Delta < 0$ 时, 方程无实根,

$$ax^2 + bx + c > 0 \text{ 的解集为 } \mathbf{R}.$$

7. $a^m \cdot a^n = a^{m+n}$.

8. $a^{-m} = \frac{1}{a^m}, a^m \div a^n = a^{m-n}$.

9. $(a^m)^n = a^{mn}$.

10. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.

11. $\log_a (M \cdot N) = \log_a M + \log_a N$.

12. $\log_a \frac{M}{N} = \log_a M - \log_a N$.

13. $\log_a M^N = N \log_a M$.

14. $\log_a M = \frac{\log_b M}{\log_b a}$.

15. $N = a^{\log_a N}$.

注: 假定第 7 ~ 15 中所有变量满足指数与对数函数的定义.

16. $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$.

17. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

18. $a + (a + d) + (a + 2d) + \cdots + [a + (n-1)d] = na + \frac{n(n-1)}{2}d$. (等差数列前 n 项和)

19. $a + aq + aq^2 + \cdots + aq^{n-1} = \frac{a(1-q^n)}{1-q} (q \neq 1)$. (等比数列前 n 项和)

20. $a^2 - b^2 = (a+b)(a-b)$.

21. $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

$$22. a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2).$$

$$23. (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3.$$

$$24. \sin^2 \alpha + \cos^2 \alpha = 1.$$

$$25. 1 + \tan^2 \alpha = \sec^2 \alpha = \frac{1}{\cos^2 \alpha}.$$

$$26. 1 + \cot^2 \alpha = \csc^2 \alpha = \frac{1}{\sin^2 \alpha}.$$

$$27. \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta.$$

$$28. \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta.$$

$$29. \tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \cdot \tan \beta}.$$

$$30. \sin 2\alpha = 2\sin \alpha \cos \alpha.$$

$$31. \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha.$$

$$32. \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)].$$

$$33. \cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)].$$

$$34. \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)].$$

$$35. \sin \alpha \sin \beta = -\frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)].$$

$$36. \text{扇形弧长 } l = r\theta, \text{扇形面积 } S = \frac{1}{2}rl = \frac{1}{2}r^2\theta. (\theta \text{ 为圆心角, 以弧度计})$$

$$37. \text{圆面积 } S = \pi r^2, \text{圆周长 } l = 2\pi r.$$

$$38. \text{圆锥体体积 } V = \frac{1}{3}\pi r^2 h.$$

$$39. \text{球体积 } V = \frac{4}{3}\pi r^3.$$

$$40. \text{球表面积 } S = 4\pi r^2.$$

