Labwork 2: Image Filtering

Task 1: $I = \begin{bmatrix} 23 & 12 & 34 & 22 \\ 89 & 35 & 25 & 78 \\ 67 & 34 & 21 & 98 \\ 57 & 76 & 36 & 54 \end{bmatrix}$ and the following kernel k: $\frac{1}{37} * \begin{bmatrix} 23 & 12 & 34 \\ 89 & 35 & 25 \\ 67 & 34 & 21 \end{bmatrix}$

+ Filter image I using k and 0-padding technique

zero padding

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 23 & 12 & 34 & 22 & 0 \\ 0 & 89 & 35 & 25 & 78 & 0 \\ 0 & 67 & 34 & 21 & 98 & 0 \\ 0 & 57 & 76 & 36 & 54 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 286 & 478 \\ 344 & 397 \end{bmatrix}$$

Let stride $= 3$:

$$\longrightarrow \begin{bmatrix} 255 & 255 \\ 255 & 255 \end{bmatrix}$$

+ Filter image using the median filter 3x3 and 0-padding.

Let stride $= 2$

Filter: $\begin{bmatrix} 0 & 23 & 22 & 0 \\ 23 & 34 & 34 & 22 \\ 35 & 36 & 36 & 25 \\ 0 & 34 & 34 & 0 \end{bmatrix}$ $\begin{bmatrix} 255 & 255 \\ 255 & 255 \end{bmatrix}$

Task 2: $J = \begin{bmatrix} 123 & 127 & 128 & 119 & 115 & 130 \\ 140 & 145 & 148 & 153 & 167 & 172 \\ 133 & 154 & 183 & 192 & 194 & 191 \\ 194 & 199 & 207 & 210 & 198 & 195 \\ 164 & 170 & 175 & 162 & 173 & 151 \end{bmatrix}$

+ Average filter:

stride = 4

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 123 & 127 & 128 & 119 & 115 & 130 & 0 \\ 0 & 140 & 145 & 148 & 153 & 167 & 172 & 0 \\ 0 & 133 & 154 & 183 & 192 & 194 & 191 & 0 \\ 0 & 194 & 199 & 207 & 210 & 198 & 195 & 0 \\ 0 & 164 & 170 & 175 & 162 & 173 & 151 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ \\ \\ \end{bmatrix}$$

Avg Filter: $\begin{bmatrix} 183 \\ \\ \end{bmatrix}$ $\dfrac{\sum \text{matrix}}{9}$

Laplacian filter $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

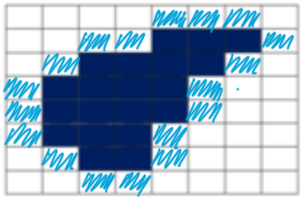
Sobel : $x \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

$Y = x^T$

Labwork 3 :

Given the following image I and the structuring element S.

Image I:



Structuring element S:

- Compute the erosion of image I by the structuring element S
- Compute the dilation of image I by the structuring element S
- Compute the opening of image I by the structuring element S
- Compute the closing of image I by structuring element S

Erosion :

$$g(x,y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{other} \end{cases}$$

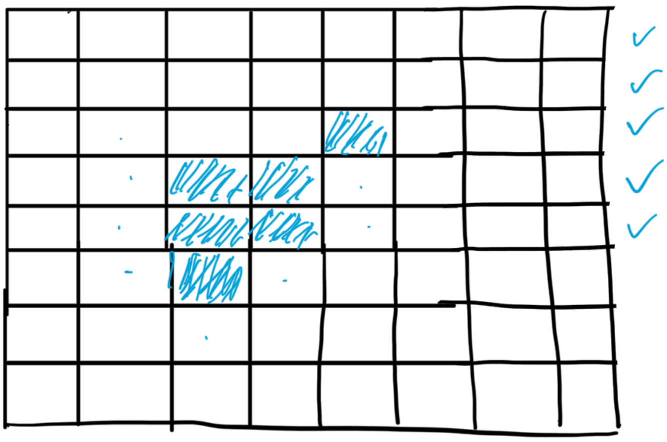
Dilation :

$$g(x,y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{other} \end{cases}$$

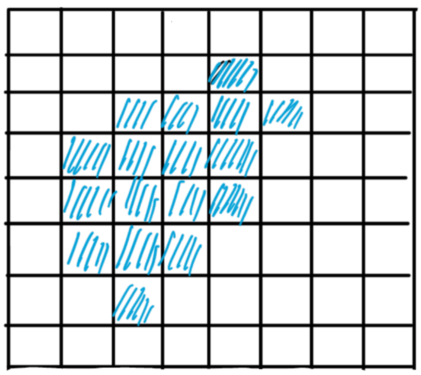
Opening of img : Erosion → dialation

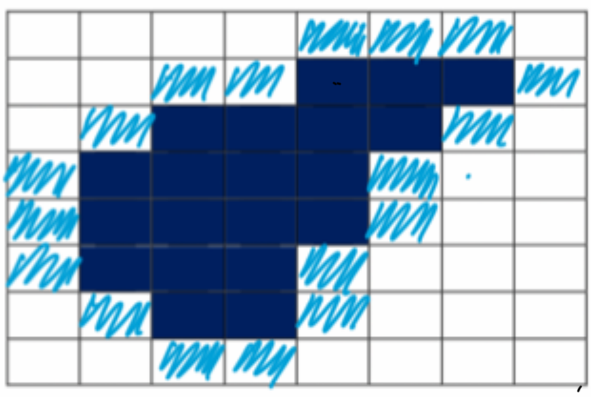
Closing of img : Dialation → Erosion

Erosion

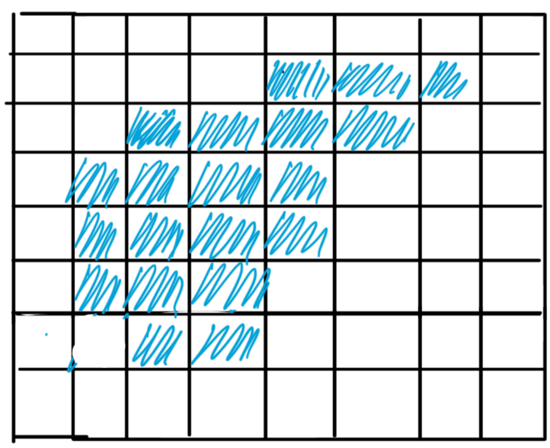


Opening →





Closing →



1. Histogram : Distribution of gray intensity over all pixels in the imgs => Using for img enhancement

making img more useful < Highlight detail, Remove noise, More visually appealing

↓

Normalize histogram : divide its value to the total number of pixels.

↓

Cummulative distribution function : total number of pixels w/ intensity up to r

$$cdf(r) = \sum_{i=0}^{r} h(i)$$

+ Histogram equalization : Histogram → N.Histogram → cdf → N.cdf → N.cdf * L (max intensity) → New intensity (làm tròn cái trc)

+ Negative image : $s = L - r$ $(L = 2^m - 1$ where $2^m > max(I))$

↳ Enhancing white / grey detail in dark regions of img

+ Thresholding : $s = \begin{cases} 1 & r > threshold \\ 0 & r \le threshold \end{cases}$ | use for segmentation → isolate an obj of interest from background

+ Log transformation : $s = c . \log(1 + r)$

+ Power law transformation : $s = c . r^{\gamma}$ < $\gamma < 1$: brighter output value; $\gamma = 1$: same; $\gamma > 1$: darker output value

→ Grey level transformation.

Point processing operation processed an img pixel and produce an img pixel

Task 1: Given the following image I:

3	3	2
1	1	0
2	2	2

N = 9

- Calculate and draw histogram, normalized histogram of image I.
- Calculate and display negative image of I.

Intensity	0	1	2	3
Histogram	1	2	4	2
N. Histogram	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{2}{9}$

Negative img : $2^m > 3 \Rightarrow m = 2$

$\Rightarrow$ 2 bit image. $L = 2^2 - 1 = 3$

$\Rightarrow s = L - r = 3 - r$

$$\begin{bmatrix} 0 & 0 & 1 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Task 2: Given the following image J:

0	2	1	7
3	2	5	2
1	1	7	6
5	0	0	3

N = 16

$L = 2^m - 1 = 7$

$2^m > 7$

$2^3 > 7$

- Calculate and draw histogram, normalized histogram of image J.
- Convert image J to a binary image called B using the thresholding technique where the predefined threshold k is the pixel which appears most frequency in the image.

k = 2

Intensity	0	1	2	3	4	5	6	7
Histogram	3	3	3	2	0	2	1	2
N. Histogram	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	0	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$

$$S = \begin{cases} 1 & (r > 2) \\ 0 & (r \le 2) \end{cases}$$

J =

0	2	1	7
3	2	5	2
1	1	7	6
5	0	0	3

$$\Rightarrow B = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Task 3 : + Histogram equalization : $\begin{bmatrix} 2 & 5 & 7 & 6 \\ 4 & 5 & 4 & 7 \\ 6 & 5 & 6 & 4 \\ 0 & 7 & 5 & 6 \end{bmatrix}$

$N = 16$

$L = 2^m - 1 = 8 - 1 = 7$

$2^m > 7 \Rightarrow m = 3$

Intensity	0	1	2	3	4	5	6	7
Histogram	1	0	1	0	3	4	4	3
N. Histo	$\frac{1}{16}$	0	$\frac{1}{16}$	0	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{3}{16}$
CDF	1	1	2	2	5	9	13	16
N. CDF	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{5}{16}$	$\frac{9}{16}$	$\frac{13}{16}$	$\frac{16}{16}$
L * N. CDF	0.4375	0.4375	0.875	0.875	2.1875	3.9375	5.6875	7
New intensity	0	0	1	1	2	4	6	7

$\Rightarrow$ New img : $\begin{bmatrix} 1 & 4 & 7 & 6 \\ 2 & 4 & 2 & 7 \\ 6 & 4 & 6 & 2 \\ 0 & 7 & 4 & 6 \end{bmatrix}$

+ Thresholding (k is the median value) : $k = 3.6 \Rightarrow s = \begin{cases} 1 & , r > 3.5 \\ 0 & , r \le 3.5 \end{cases}$

$$B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

2. Spatial Filtering

Neighbourhood operations

+ Operate on a larger neighbourhood of pixels than point operations
+ Simple neighbourhood operations: min / max / median

Spatial filtering (kernel)

+ An operation take a pixel w/ some neighbor pixels and produces one output pixel
+ Average filter: smoothing operation → Not preserve edges. remove noise / highliting gross detail

  How: average all of pixels from neighbourhood around a central value

$$\begin{bmatrix} 104 & 100 & 108 \\ 99 & 106 & 98 \\ 95 & 90 & 85 \end{bmatrix} * \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} \Rightarrow \text{New pixel } e: \; e = \frac{1}{9}(104 + 100 + \cdots + 85)$$

  ↳ Weighted average filter

Correlation: don't need to flip the kernel but directly filtering

+ Median Filtering: removing noise while preserving edges

  How: replacing pixel with median pixel value around

Convolution: flip the kernel twice then filtering.

+ Similar w/ filtering ⇒ Turn the filter 90° to the left

$$\underbrace{\begin{bmatrix} a & b & c \\ d & e & e \\ f & g & h \end{bmatrix}}_{\text{Original}} * \underbrace{\begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}}_{\text{filter}} \Rightarrow \text{New } e = az + by + cx + dw + ev + eu + ft + gs + hr$$

$$\begin{bmatrix} z & y & x \\ w & v & u \\ t & s & r \end{bmatrix}$$

+ Sharpening spatial filters: remove bluring / highlight edge
  + Laplacian filter: $g(x,y) = f(x,y) - \nabla^2 f$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{Laplacian of Gaussian}} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

(In frequency domain only)

+ Low pass filter: $H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$ ⇒ Img smoothing

+ Sobel filter: $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ (Y direction) $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ (X direction)

+ Prewitt filter : $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

+ Roberts : $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Task 1: Given the following image I:

23	12	34	22
89	35	25	78
67	34	21	98
57	76	36	54

And the following kernel K:

1/37 *

23	12	34
89	35	25
67	34	21

- Filter image I using the kernel K and the 0-padding technique.
- Filter image I using the median filter of size 3x3 and the 0-padding technique.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 23 & 12 & 34 & 22 & 0 \\ 0 & 89 & 35 & 25 & 78 & 0 \\ 0 & 67 & 34 & 21 & 98 & 0 \\ 0 & 57 & 76 & 36 & 54 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Filter image $\longrightarrow$ $\begin{bmatrix} 0 & 23 & 25 & 0 \\ 23 & 34 & 34 & 22 \\ 35 & 36 & 36 & 25 \\ 0 & 34 & 34 & 0 \end{bmatrix}$

Task 2: Given the following image J:

123	127	128	119	115	130
140	145	148	153	167	172
133	154	183	192	194	191
194	199	207	210	198	195
164	170	175	162	173	151

- Smooth image J using the averaging filter of size 3x3 and the 0-padding technique.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 123 & 127 & 128 & 119 & 115 & 130 & 0 \\ 0 & 140 & 145 & 148 & 153 & 167 & 172 & 0 \\ 0 & 133 & 154 & 183 & 192 & 194 & 191 & 0 \\ 0 & 194 & 199 & 207 & 210 & 198 & 195 & 0 \\ 0 & 164 & 170 & 175 & 162 & 173 & 151 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\longrightarrow$ $\begin{bmatrix} 59 & 90 & 91 & 92 & 95 & 65 \\ 91 & 142 & 150 & 155 & 159 & 108 \\ 107 & 167 & 177 & 184 & 186 & 124 \\ 113 & 175 & 184 & 188 & 185 & 122 \\ 81 & 123 & 125 & 125 & 121 & 80 \end{bmatrix}$

+ Global thresholding (T) : select initial estimate for T (avg grey level)

$\downarrow$

Segment img using T $\Rightarrow$ 2 groups of pixels : G_1 , G_2

$\downarrow$

Compute avg in G_1 , G_2 $\Rightarrow$ μ_1 , μ_2

$\downarrow$

New threshold : $T = \dfrac{\mu_1 + \mu_2}{2}$

$\downarrow$

Compute until T satisfy.

+ Adaptive thresholding : Divide image into sub img $\rightarrow$ thresholding each of these.

3. Morphological Image Processing : techniques that deal with shape of features

↳ remove imperfections during segmentation

+ Erosion : $g(x,y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{other} \end{cases}$ ⇒ split apart joined obj / strip away extrusions

+ Dilation : $g(x,y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{other} \end{cases}$ → repair breaks / repair instusions

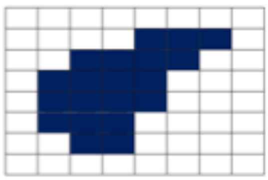
+ Opening : Erosion → Dilation

+ Closing : Dilation → Erosion.

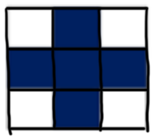
Exercise

Given the following image I and the structuring element S.

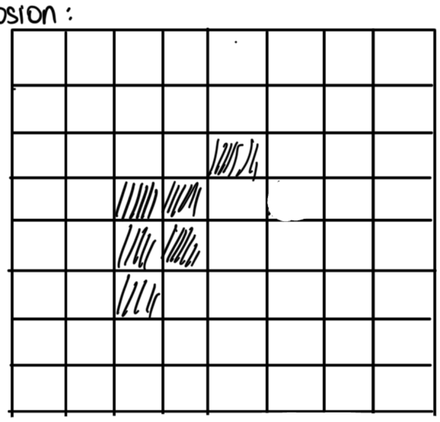
Image I:



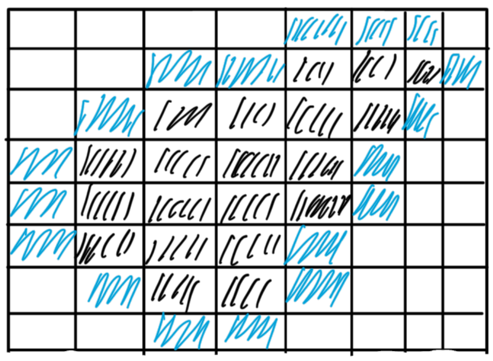
Structuring element S:



+ Erosion:

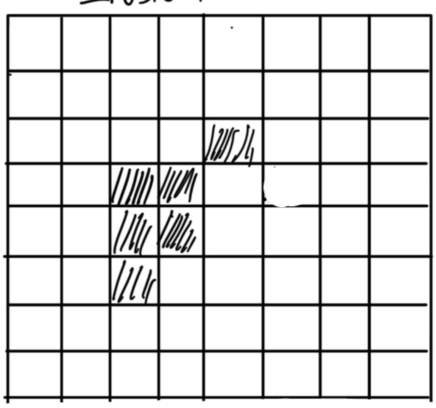


+ Dilation

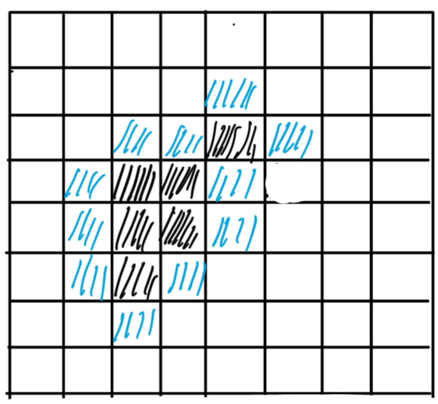


+ Opening :

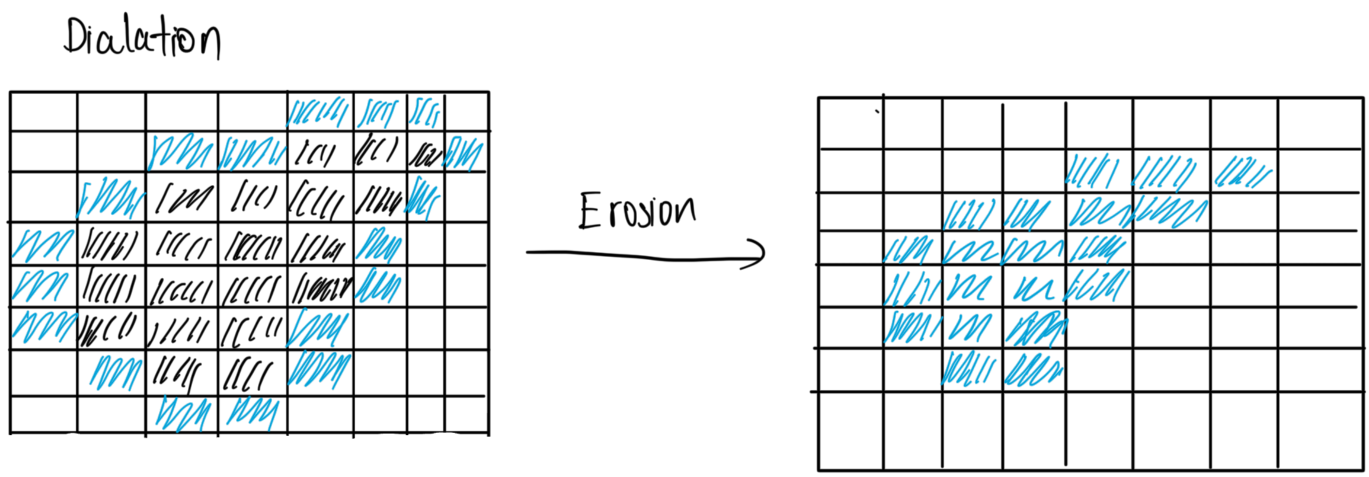
Erosion



Dilation →



+ Closing :



+ Segmentation : divide an img → different regions based on the characteristics of pixels

↓

How : dis continuity detection :

- Points : Laplacian of Gaussian filter + Thresholding .
- Lines : $\begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$
- Edges : Derivative : Sobel , Prewitt , Roberts filter

↓

Problems : too much detail → smooth imgs before edge detection

Edge detection ← Laplacian filter combine with smoothing Gaussian → remove noise .

Hit or Miss ?