Lid-Driven Cavity Flow

Ruipengyu (Brandon) Li

August 15, 2017

1 Introduction

Lid-driven cavity flow is calculated using various methods.

2 Finite Difference Method

2.1 Description

Cell-centred staggered grid is used for this method as shown in Fig.1. Alternatively, cell-vertex grid layout (Fig.2) can be used and the difference is mainly the at the boundary. Depending on the numerical schemes, a few layers of ghost cells are needed outside the physical boundary. Indices start from 1 for all the variables. Central differencing requires one extra point beyond the boundary. This means that if the number of grid lines is (ni, nj), there will be $(ni, nj+1)$ u-nodes, $(ni+1, nj)$ v-nodes and $(ni+1, nj+1)$ p-nodes.

2.2 Governing Equations

The non-dimensional Navier-Stokes equations for incompressible flow assuming constant fluid properties reads:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial vv}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

2.3 Momentum Equation

Consider u-momentum equation. The unsteady term is discretized using explicit or implicit Euler scheme:

$$\frac{\partial u}{\partial t} = \frac{u_{i,j} - u^0_{i,j}}{\Delta t}$$

The first advection term is discretized using the 2nd-order central difference scheme. For implicit Euler, it is

$$\frac{\partial uu}{\partial x} = \frac{u^0_{i+1,j}u_{i+1,j} - u^0_{i-1,j}u_{i-1,j}}{2\Delta x}$$

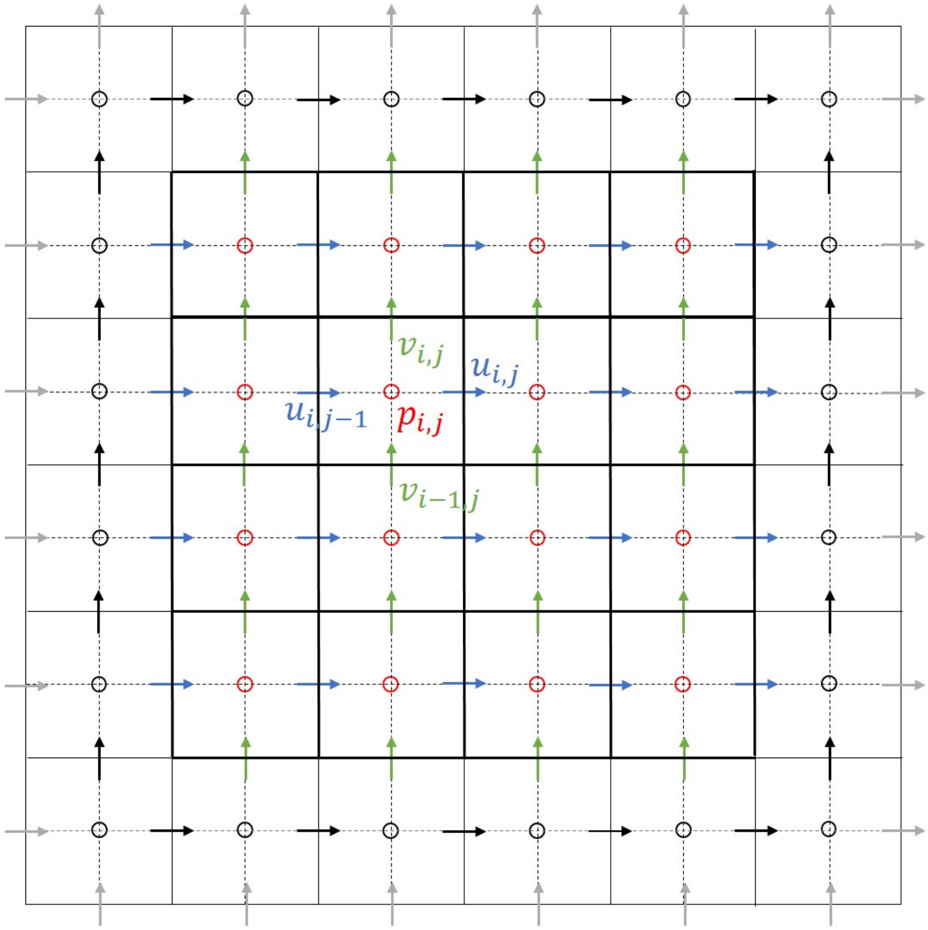

Figure 1: Cell-centred staggered grid layout

Similarly for the second advection term, gradient is evaluated using the values at north and south faces of a u-cell:

$$\frac{\partial uv}{\partial y} = \left(\frac{v^0_{i,j} + v^0_{i+1,j}}{2}\frac{u_{i,j} + u_{i,j+1}}{2} - \frac{v^0_{i,j-1} + v^0_{i+1,j-1}}{2}\frac{u_{i,j} + u_{i,j-1}}{2}\right)\frac{1}{\Delta y}$$
$$= \frac{1}{4\Delta y}\left[(v^0_{i,j} + v^0_{i+1,j})(u_{i,j} + u_{i,j+1}) - (v^0_{i,j-1} + v^0_{i+1,j-1})(u_{i,j} + u_{i,j-1})\right]$$

However, this inevitably includes interpolation using $u_{i,j}$ itself, which increases the complexity of the resulting algebraic equation. Here, a simple alternative is to evaluate the derivative using the adjacent available u points, i.e. $u_{i,j+1}$ and $u_{i,j-1}$, but keep the v value interpolated at the $u_{i,j}$ faces as it

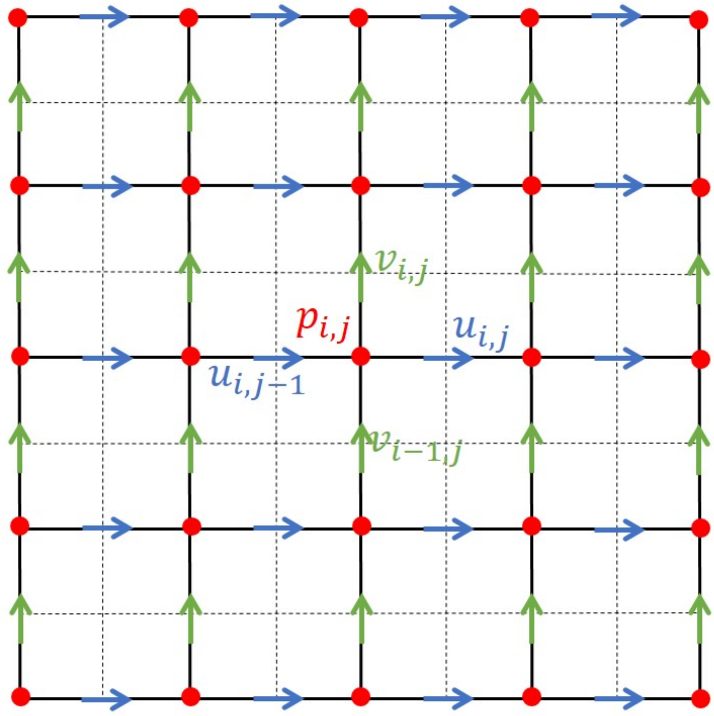

Figure 2: Cell-vertex staggered grid layout

is a known from the last iteration. Thus, the advection term reads:

$$\frac{\partial uv}{\partial y} = \frac{u_{i,j+1}v^0_{i+1/2,j} - u_{i,j-1}v^0_{i+1/2,j-1}}{2\Delta y}$$

where

$$v^0_{i+1/2,j} = \frac{v^0_{i,j} + v^0_{i+1,j}}{2}$$

$$v^0_{i+1/2,j-1} = \frac{v^0_{i,j-1} + v^0_{i+1,j-1}}{2}$$

and $u_{i,j}$ term is not present. The advantage is that the formula for pressure correction will be simplified. Pressure term is discretized as

$$-\frac{\partial p}{\partial x} = -\frac{p^0_{i+1,j} - p^0_{i,j}}{\Delta x}$$

Diffusion (viscous) term:

$$\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \frac{1}{Re}\left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{\Delta x^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\Delta y^2}\right)$$

Put all the terms together:

$$\frac{u_{i,j}-u^0_{i,j}}{\Delta t}+\frac{u^0_{i+1,j}u_{i+1,j}-u^0_{i-1,j}u_{i-1,j}}{2\Delta x}+\frac{v^0_{i+1/2,j}u_{i,j+1}-v^0_{i+1/2,j-1}u_{i,j-1}}{2\Delta y}$$
$$=-\frac{p^0_{i+1,j}-p^0_{i,j}}{\Delta x}+\frac{1}{Re}\left(\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{\Delta x^2}+\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{\Delta y^2}\right)$$

It may also be rearranged to the form following the compass notation

$$a^u_P u_{i,j}=a^u_E u_{i+1,j}+a^u_W u_{i-1,j}+a^u_N u_{i,j+1}+a^u_S u_{i,j-1}+\frac{p^0_{i,j}-p^0_{i+1,j}}{\Delta x}+b^u \quad (4)$$

where

$$a^u_E=-\frac{u^0_{i+1,j}}{2\Delta x}+\frac{1}{Re\Delta x^2} \quad (5a)$$

$$a^u_W=\frac{u^0_{i-1,j}}{2\Delta x}+\frac{1}{Re\Delta x^2} \quad (5b)$$

$$a^u_N=-\frac{v^0_{i+1/2,j}}{2\Delta y}+\frac{1}{Re\Delta y^2} \quad (5c)$$

$$a^u_S=\frac{v^0_{i+1/2,j-1}}{2\Delta y}+\frac{1}{Re\Delta y^2} \quad (5d)$$

$$a^u_P=\frac{1}{\Delta t}+\frac{2}{Re\Delta x^2}+\frac{2}{Re\Delta y^2} \quad (5e)$$

$$b^u=\frac{u^0_{i,j}}{\Delta t} \quad (5f)$$

with

$$v^0_{i+1/2,j}=\frac{v^0_{i,j}+v^0_{i+1,j}}{2}$$
$$v^0_{i+1/2,j-1}=\frac{v^0_{i,j-1}+v^0_{i+1,j-1}}{2}$$

The v-momentum equation can be obtained in a similar manner:

$$\frac{v_{i,j}-v^0_{i,j}}{\Delta t}+\frac{v^0_{i,j+1}v_{i,j+1}-v^0_{i,j-1}v_{i,j-1}}{2\Delta y}+\frac{u^0_{i,j+1/2}v_{i+1,j}-u^0_{i-1,j+1/2}v_{i-1,j}}{2\Delta x}$$
$$=-\frac{p^0_{i,j+1}-p^0_{i,j}}{\Delta y}+\frac{1}{Re}\left(\frac{v_{i-1,j}-2v_{i,j}+v_{i+1,j}}{\Delta x^2}+\frac{v_{i,j-1}-2v_{i,j}+v_{i,j+1}}{\Delta y^2}\right)$$

After assembly of coefficients:

$$a^v_P v_{i,j}=a^v_E v_{i+1,j}+a^v_W v_{i-1,j}+a^v_N v_{i,j+1}+a^v_S v_{i,j-1}+\frac{p^0_{i,j}-p^0_{i,j+1}}{\Delta y}+b^v \quad (6)$$

where

$$a_E^v = -\frac{u^0_{i,j+1/2}}{2\Delta x} + \frac{1}{Re\Delta x^2} \tag{7a}$$

$$a_W^v = \frac{u^0_{i-1,j+1/2}}{2\Delta x} + \frac{1}{Re\Delta x^2} \tag{7b}$$

$$a_N^v = -\frac{v^0_{i,j+1}}{2\Delta y} + \frac{1}{Re\Delta y^2} \tag{7c}$$

$$a_S^v = \frac{v^0_{i,j-1}}{2\Delta y} + \frac{1}{Re\Delta y^2} \tag{7d}$$

$$a_P^v = \frac{1}{\Delta t} + \frac{2}{Re\Delta x^2} + \frac{2}{Re\Delta y^2} \tag{7e}$$

$$b^v = \frac{v^0_{i,j}}{\Delta t} \tag{7f}$$

with

$$u^0_{i,j+1/2} = \frac{u^0_{i,j} + u^0_{i,j+1}}{2}$$

$$u^0_{i-1,j+1/2} = \frac{u^0_{i-1,j} + u^0_{i-1,j+1}}{2}$$

The momentum equations are discretized as above.

2.4 Pressure Correction

The SIMPLE algorithm states that the uncorrected and corrected velocities both satisfy the momentum equations. From this, we can get the expression for correction velocity by writing momentum equations in terms of u' and v' neglecting the contribution from neighbouring points:

$$u'_{i,j} = \frac{p'_{i,j} - p'_{i+1,j}}{a\Delta x} \tag{8a}$$

$$u'_{i-1,j} = \frac{p'_{i-1,j} - p'_{i,j}}{a\Delta x} \tag{8b}$$

$$v'_{i,j} = \frac{p'_{i,j} - p'_{i,j+1}}{a\Delta y} \tag{8c}$$

$$v'_{i,j-1} = \frac{p'_{i,j-1} - p'_{i,j}}{a\Delta y} \tag{8d}$$

where

$$a = a_P^u = a_P^v = \frac{1}{\Delta t} + \frac{2}{Re\Delta x^2} + \frac{2}{Re\Delta y^2}$$

Now, the continuity equation, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, is discretized to derive the expression for pressure correction around the scalar cell $p(i,j)$:

$$\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \frac{v_{i,j} - v_{i,j-1}}{\Delta y} = 0$$

$$\frac{u^*_{i,j} + (p'_{i,j} - p'_{i+1,j})/(a\Delta x) - u^*_{i-1,j} - (p'_{i-1,j} - p'_{i,j})/(a\Delta x)}{\Delta x} + \frac{v^*_{i,j} + (p'_{i,j} - p'_{i,j+1})/(a\Delta y) - v^*_{i,j-1} - (p'_{i,j-1} - p'_{i,j})/(a\Delta y)}{\Delta y} = 0 \quad (9)$$

It can be rearranged to

$$a^p_P p_{i,j} = a^p_E p_{i+1,j} + a^p_W p_{i-1,j} + a^p_N p_{i,j+1} + a^p_S p_{i,j-1} + b^p \quad (10)$$

where

$$a^p_E = \frac{1}{a\Delta x^2} \quad (11a)$$

$$a^p_W = \frac{1}{a\Delta x^2} \quad (11b)$$

$$a^p_N = \frac{1}{a\Delta y^2} \quad (11c)$$

$$a^p_S = \frac{1}{a\Delta y^2} \quad (11d)$$

$$a^p_P = a^p_E + a^p_W + a^p_N + a^p_S \quad (11e)$$

$$b^p = \frac{u^*_{i-1,j} - u^*_{i,j}}{\Delta x} + \frac{v^*_{i,j-1} - v^*_{i,j}}{\Delta y} \quad (11f)$$

with

$$a = \frac{1}{\Delta t} + \frac{2}{Re\Delta x^2} + \frac{2}{Re\Delta y^2}$$

Note that b^p can be seen as the mass source and $b^p \to 0$ indicates the convergence of velocity fields. The above is the *pressure correction formula.* And it is essentially a central difference formulation of the *Poisson equation* in terms of p':

$$\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} = Q$$

when $Q = -ab$.

2.5 Boundary Condition

For cell-centred scheme with staggered grid, u-velocity needs to be interpolated on top and bottom of the domain since u-nodes are not on the physical boundary.

$$\frac{u_{i,1} + u_{i,2}}{2} = 0$$

$$\frac{u_{i,nj+1} + u_{i,nj}}{2} = U_{top}$$

$$u_{1,j} = 0$$

$$u_{ni,j} = 0$$

Similar treatment is done for v-velocity on the left and right boundaries.

$$\frac{v_{1,j} + u_{2,j}}{2} = 0$$

$$\frac{v_{ni,j} + u_{ni+1,j}}{2} = 0$$

$$v_{i,1} = 0$$

$$v_{i,nj} = 0$$

Boundary conditions for p' must also be given. At all the walls, we can assume zero gradient, e.g.

$$\left(\frac{\partial p}{\partial x}\right)_w = 0$$

Note that boundary conditions for the cell-vertex layout are straightforward as velocity nodes coincide with the physical boundaries.

3 Finite Volume Method

3.1 Grid Layout

Velocity indices start from 2. And a velocity node $u(i,j)$ or $v(i,j)$ points towards the scalar node with the same index. Velocity control volumes at boundaries are extended as shown in Fig.3.

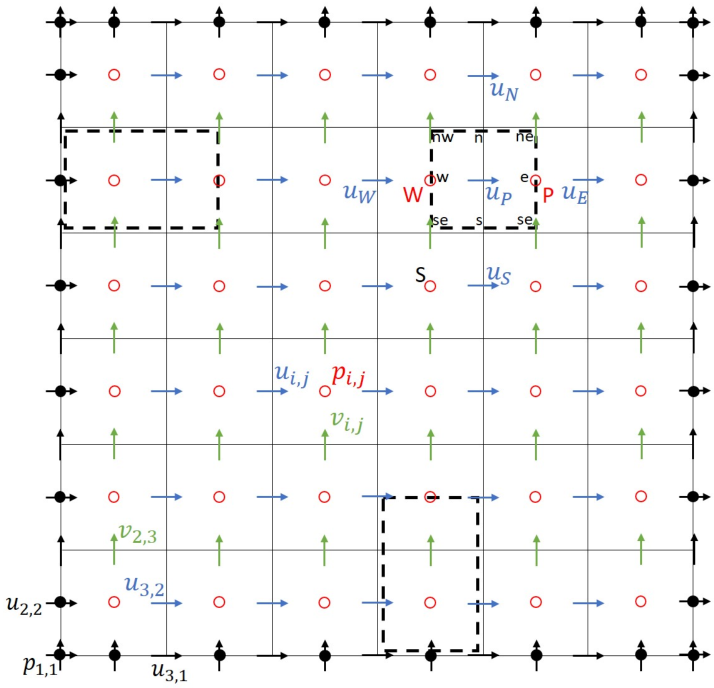

Figure 3: Cell-centred staggered grid layout

3.2 Integral Form of Governing Equations

The governing differential equations for momentum is recast as

$$\frac{\partial u_i}{\partial t} + \nabla \cdot (u_i \mathbf{u}) = -\nabla \cdot (p\mathbf{i}_i) + \frac{1}{Re}\nabla \cdot (\nabla u_i) \tag{12}$$

Integrate the equation over the control volume and apply Gauss's theorem:

$$\frac{\partial}{\partial t}\int_\Omega u_i dV + \int_S u_i \mathbf{u} \cdot \mathbf{n} dS = -\int_S p\mathbf{i}_i \cdot \mathbf{n} dS + \frac{1}{Re}\int_S \nabla u_i \cdot \mathbf{n} dS \tag{13}$$

For the purpose of demonstration, we integrate from the differential equation of velocity component u:

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{14}$$

Integral of u-equation over the control volume Ω:

$$\begin{aligned}\int_\Omega \frac{\partial u}{\partial t} dV + \int_\Omega \frac{\partial uu}{\partial x} dV + \int_\Omega \frac{\partial vu}{\partial y} dV = -\int_\Omega \frac{\partial p}{\partial x} dV \\ + \frac{1}{Re}\left(\int_\Omega \frac{\partial^2 u}{\partial x^2} dV + \int_\Omega \frac{\partial^2 u}{\partial y^2} dV\right)\end{aligned} \tag{15}$$

These six terms need to be evaluated considering $\int_\Omega (...)dV = \int_s^n \int_w^e (...)dxdy$ and the mid-point rule in terms of u_P in a uniform grid.

3.3 Discretisation

3.3.1 U-Momentum

- Unsteady term using Implicit Euler Scheme:

$$\begin{aligned}\int_\Omega \frac{\partial u}{\partial t} dV &\approx \frac{du_P}{dt}\Omega_P \\ &\approx \frac{u_P - u_P^0}{\Delta t}\Delta x \Delta y\end{aligned}$$

  and time discretisation will be included later.

- Advection term 1 using the known velocity:

$$\begin{aligned}\int_\Omega \frac{\partial uu}{\partial x} dV &= \int_s^n \int_w^e \frac{\partial uu}{\partial x} dxdy \\ &\approx \left[\left(u^0 u\right)_e - \left(u^0 u\right)_w\right]\Delta y\end{aligned}$$

  For CDS:

$$\begin{aligned}\left(u^0 u\right)_e &= \frac{u_e^0}{2}(u_P + u_E) \\ \left(u^0 u\right)_w &= \frac{u_w^0}{2}(u_W + u_P)\end{aligned}$$

  For UDS:

$$\begin{aligned}\left(u^0 u\right)_e &= max(u_e^0, 0)u_P - max(-u_e^0, 0)u_E \\ \left(u^0 u\right)_w &= max(u_w^0, 0)u_W - max(-u_w^0, 0)u_P\end{aligned}$$

  with flux term evaluated from the velocity of last iteration

$$\begin{aligned}u_e^0 &= \frac{u_P^0 + u_E^0}{2} \\ u_w^0 &= \frac{u_W^0 + u_P^0}{2}\end{aligned}$$

- Advection term 2:

$$\begin{aligned}\int_\Omega \frac{\partial vu}{\partial y} dV &= \int_w^e \int_s^n \frac{\partial uv}{\partial y} dydx \\ &\approx \left[\left(v^0 u\right)_n - \left(v^0 u\right)_s\right]\Delta x\end{aligned}$$

For CDS:

$$\left(v^0u\right)_n = \frac{v_n^0}{2}(u_P + u_N)$$

$$\left(v^0u\right)_s = \frac{v_s^0}{2}(u_S + u_P)$$

For UDS:

$$\left(v^0u\right)_n = max(v_n^0, 0)u_P - max(-v_n^0, 0)u_N$$

$$\left(v^0u\right)_s = max(v_s^0, 0)u_S - max(-v_s^0, 0)u_P$$

with flux term evaluated from the velocity of last iteration

$$v_n^0 = \frac{v_{nw}^0 + v_{ne}^0}{2}$$

$$v_s^0 = \frac{v_{sw}^0 + v_{se}^0}{2}$$

- Pressure gradient term:

$$\begin{aligned} -\int_\Omega \frac{\partial p}{\partial x} dV &= -\int_s^n \int_w^e \frac{\partial p}{\partial x} dxdy \\ &\approx -\left(p_P - p_W\right)\Delta y \end{aligned}$$

- Diffusion term 1:

$$\begin{aligned} \frac{1}{Re}\int_\Omega \frac{\partial^2 u}{\partial x^2} dV &= \frac{1}{Re}\int_s^n \int_w^e \frac{\partial^2 u}{\partial x^2} dxdy \\ &\approx \frac{1}{Re}\left[\left(\frac{\partial u}{\partial x}\right)_e - \left(\frac{\partial u}{\partial x}\right)_w\right]\Delta y \\ &\approx \frac{1}{Re}\left(\frac{u_E - u_P}{\Delta x} - \frac{u_P - u_W}{\Delta x}\right)\Delta y \\ &= \frac{\Delta y}{Re\Delta x}\left(u_E - 2u_P + u_W\right) \end{aligned}$$

- Diffusion term 2:

$$\begin{aligned} \frac{1}{Re}\int_\Omega \frac{\partial^2 u}{\partial y^2} dV &= \frac{1}{Re}\int_w^e \int_s^n \frac{\partial^2 u}{\partial y^2} dydx \\ &\approx \frac{1}{Re}\left[\left(\frac{\partial u}{\partial y}\right)_n - \left(\frac{\partial u}{\partial y}\right)_s\right]\Delta x \\ &\approx \frac{\Delta x}{Re\Delta y}\left(u_N - 2u_P + u_S\right) \end{aligned}$$

Bring them all together, For CDS:

$$\frac{u_P - u_P^0}{\Delta t}\Delta x\Delta y + \frac{u_e^0 \Delta y}{2}(u_P + u_E) - \frac{u_w^0 \Delta y}{2}(u_W + u_P) + \frac{v_n^0 \Delta x}{2}(u_P + u_N)$$
$$-\frac{v_s^0 \Delta x}{2}(u_S + u_P) = (p_W^0 - p_P^0)\Delta y + \frac{u_E - 2u_P + u_W}{Re\Delta x}\Delta y + \frac{u_N - 2u_P + u_S}{Re\Delta y}\Delta x$$

Rearrange to

$$a_P^u u_P = a_E^u u_E + a_W^u u_W + a_N^u u_N + a_S^u u_S + (p_W^0 - p_P^0)\Delta y + b^u$$
$$a_E^u = \left(\frac{1}{Re\Delta x} - \frac{u_e^0}{2}\right)\Delta y$$
$$a_W^u = \left(\frac{1}{Re\Delta x} + \frac{u_w^0}{2}\right)\Delta y$$
$$a_N^u = \left(\frac{1}{Re\Delta y} - \frac{v_n^0}{2}\right)\Delta x$$
$$a_S^u = \left(\frac{1}{Re\Delta y} + \frac{v_s^0}{2}\right)\Delta x$$
$$b^u = \frac{u_P^0}{\Delta t}\Delta x\Delta y$$
$$a_P^u = \frac{\Delta x\Delta y}{\Delta t} + a_E^u + a_W^u + a_N^u + a_S^u + (u_e^0 - u_w^0)\Delta y + (v_n^0 - v_s^0)\Delta x$$

For UDS:

$$a_P^u u_P = a_E^u u_E + a_W^u u_W + a_N^u u_N + a_S^u u_S + (p_W^0 - p_P^0)\Delta y + b^u$$
$$a_E^u = \left(\frac{1}{Re\Delta x} + max(-u_e^0, 0)\right)\Delta y$$
$$a_W^u = \left(\frac{1}{Re\Delta x} + max(u_w^0, 0)\right)\Delta y$$
$$a_N^u = \left(\frac{1}{Re\Delta y} + max(-v_n^0, 0)\right)\Delta x$$
$$a_S^u = \left(\frac{1}{Re\Delta y} + max(v_s^0, 0)\right)\Delta x$$
$$b^u = \frac{u_P^0}{\Delta t}\Delta x\Delta y$$
$$a_P^u = \frac{\Delta x\Delta y}{\Delta t} + a_E^u + a_W^u + a_N^u + a_S^u + (u_e^0 - u_w^0)\Delta y + (v_n^0 - v_s^0)\Delta x$$

3.3.2 V-Momentum

The algebraic equations of v-momentum using CDS are

$$a_P^v v_P = a_E^v v_E + a_W^v v_W + a_N^v v_N + a_S^v v_S + (p_S^0 - p_P^0)\Delta x + b^v$$

$$a_E^v = \left(\frac{1}{Re\Delta x} - \frac{u_e^0}{2}\right)\Delta y$$

$$a_W^v = \left(\frac{1}{Re\Delta x} + \frac{u_w^0}{2}\right)\Delta y$$

$$a_N^v = \left(\frac{1}{Re\Delta y} - \frac{v_n^0}{2}\right)\Delta x$$

$$a_S^v = \left(\frac{1}{Re\Delta y} + \frac{v_s^0}{2}\right)\Delta x$$

$$b^v = \frac{v_P^0}{\Delta t}\Delta x\Delta y$$

$$a_P^v = \frac{\Delta x\Delta y}{\Delta t} + a_E^v + a_W^v + a_N^v + a_S^v + (u_e^0 - u_w^0)\Delta y + (v_n^0 - v_s^0)\Delta x$$

For UDS:

$$a_P^v v_P = a_E^v v_E + a_W^v v_W + a_N^v v_N + a_S^v v_S + (p_S^0 - p_P^0)\Delta x + b^v$$

$$a_E^v = \left(\frac{1}{Re\Delta x} + max(-u_e^0, 0)\right)\Delta y$$

$$a_W^v = \left(\frac{1}{Re\Delta x} + max(u_w^0, 0)\right)\Delta y$$

$$a_N^v = \left(\frac{1}{Re\Delta y} + max(-v_n^0, 0)\right)\Delta x$$

$$a_S^v = \left(\frac{1}{Re\Delta y} + max(v_s^0, 0)\right)\Delta x$$

$$b^v = \frac{v_P^0}{\Delta t}\Delta x\Delta y$$

$$a_P^v = \frac{\Delta x\Delta y}{\Delta t} + a_E^v + a_W^v + a_N^v + a_S^v + (u_e^0 - u_w^0)\Delta y + (v_n^0 - v_s^0)\Delta x$$

3.3.3 Pressure Correction

In terms of a u-CV, the velocity correction using the SIMPLE algorithm is

$$u'_P = \frac{(p'_W - p'_P)\Delta y}{a^u_P}$$

Similarly for a v-CV,

$$v'_P = \frac{(p'_P - p'_N)\Delta x}{a}$$

Or in terms of the main control volume:

$$u'_e = \frac{(p'_P - p'_E)\Delta y}{a}$$

$$u'_w = \frac{(p'_W - p'_P)\Delta y}{a}$$

$$v'_n = \frac{(p'_P - p'_N)\Delta x}{a}$$

$$v'_s = \frac{(p'_S - p'_P)\Delta x}{a}$$

where $a = a^u_P = a^v_P$. To derive the expression for pressure correction, the differential continuity equation is integrated over the scalar CV:

$$\int_s^n \int_w^e \frac{\partial u}{\partial x} dxdy + \int_w^e \int_s^n \frac{\partial v}{\partial y} dydx = 0$$

$$(u_e - u_w)\Delta y + (v_n - v_s)\Delta x = 0$$

$$[(u^*_e + u'_e) - (u^*_w + u'_w)]\Delta y + [(v^*_n + v'_n) - (v^*_s + v'_s)]\Delta x = 0$$

$$\begin{aligned} &[u^*_e + (p'_P - p'_E)\Delta y/a]\,\Delta y - [u^*_w + (p'_W - p'_P)\Delta y/a]\,\Delta y \\ &\quad + [v^*_n + (p'_P - p'_N)\Delta x/a]\,\Delta x - [v^*_s + (p'_S - p'_P)\Delta x/a]\,\Delta x = 0 \end{aligned} \tag{20}$$

Rearrange to

$$a_P p'_P = a_E p'_E + a_W p'_W + a_N p'_N + a_S p'_S + b$$

$$a_E = \Delta y^2/a$$

$$a_W = \Delta y^2/a$$

$$a_N = \Delta x^2/a$$

$$a_S = \Delta x^2/a$$

$$a_P = a_E + a_W + a_N + a_S$$

$$b = (u^*_e - u^*_w)\Delta y + (v^*_n - v^*_s)\Delta x$$

3.4 Boundary Condition

Dirichlet b.c. for velocities and zero gradient for pressure.