Homework Number 6
Ae 121b, Winter Term 2017
Due: Wednesday, March 8, 2017

General Instructions:

1. Homework solutions should be neat and logically presented. Show all steps of your work so that we can follow what you are doing. This helps particularly in assigning partial credit if you do not get the correct final answer.

2. For problems which involve a control volume analysis, include a sketch of the flow/system, and indicate clearly your choice of control surface as necessary.

3. If a problem calls for a qualitative plot of something you may hand-sketch it, otherwise it should be a quantitatively correct, computer-generated plot.

4. Always state and justify any assumptions you make. For instance, if you use a 1D equation, indicate why it should apply in this case.

5. If you use any results or equations that were derived or presented in class, you do not need to derive them. However, if you use results or equations from other references you must show how they are derived and justify their use (i.e. describe the assumptions that go into the derivation and show that those assumptions are valid for the problem).

6. If the problem requires writing a program, please submit a hardcopy of your code.

7. You may not use Ae121 materials from previous years.

Homework Problems:

Problem 1: Combustion Chamber Model

Assume you have a combustion chamber for a LOX-methane rocket with a cross-sectional area of 0.157 m^2, a length of 0.75 m, and an total fuel injector cross-sectional area of 0.0157 m^2. The overall equivalence ratio is 1.139 (slightly fuel-rich). Assume that the oxidizer is injected as a gas and the fuel partly as a gas (with an initial equivalence ratio of 0.45) and partly as a liquid with an injection velocity of 10 m/s (use this also as the initial droplet velocity). The inlet gas temperature is 600K and the combustion chamber pressure is 3.4474 MPa.

a) For initial (injected) droplet diameters of 30, 50, 80, 100, and 200 μm calculate the droplet diameter, gas temperature, equivalence ratio, gas and droplet velocities, and liquid and vapor flow rates as a function of position along the axis of the combustion chamber. Plot these parameters and identify the droplet size that gives the best balance between good combustion efficiency and heat load on the injector.

This problem involves solving the equations developed in class along the axis of the combustion chamber. There are several algebraic relations that are required, two derivatives for which there are algebraic equations (dm_l/dx and $d\phi/dx$), and a system of three ode's that can be solved with a routine such as ode45 in Matlab. The ode's are written in terms of the derivatives dD^2/dx, dv_d/dx, and dT_g/dx. The following pseudocode will give you a start on constructing the solution. Use GRI30 as the gas model. It may also be useful to create functions for the gas enthalpy h_g and viscosity μ that return these properties as a function of T_g, P, ϕ.

b) (will not be graded, but submit plots). Vary the droplet injection velocity in the code for a few of the droplet diameters to see what effect this has on the distance required to evaporate the droplets.

Algorithm 1 Combustion Chamber Model

procedure MAIN
  Define globals
  Define constants and conditions at inlet
  Compute flow rates at inlet
  Equilibrate gas with inlet stoichiometry to get inlet T_g
  Call ode45 or equivalent to solve system of 3 ode's
  Plot results
end procedure

procedure FUNCTION FOR RHS OF ODE'S TO USE WITH ODE45
  if Droplet diameter > 0 **then**
    Compute flow rates
    Get gas properties from Cantera
    Calculate v_g, Re, and C_D
    Calculate evaporation constant K (implement as a separate function)
    Calculate dm_l/dx and $d\phi/dx$ (algebraic relations)
    Set RHS's for ODE's dD^2/dx, dv_d/dx, and dT_g/dx (requires functions for $dh_g/d\phi$ and $d\phi/dT_g$)
  else
    Set RHS's to zero
  end if
end procedure

procedure FUNCTION FOR EVAPORATION CONSTANT $K(T_g, P, \phi)$
  Define constants for methane
  Calculate T_b from Clausius-Clapeyron equation
  Calculate average temperature $\bar{T}$ (use a reasonable guess for T_f)
  Set the state of the fuel and oxidizer at $\bar{T}$ and P
  Calculate k_g and C_p
  Calculate B_oq
  Calculate K
end procedure

procedure FUNCTION FOR $dh_g/d\phi(T_g, P, \phi)$
  Define a ϕ_1 and ϕ_2 that are slightly above and below ϕ
  Set gas to these compositions in Cantera and equilibrate (use TP)
  Calculate gas enthalpies at these two conditions
  Calculate estimate of $dh_g/d\phi$ from slope over this interval
end procedure

procedure FUNCTION FOR $dh_g/dT(T_g, P, \phi)$
  Define a T_{g1} and T_{g2} that are slightly above and below T_g
  Equilibrate in Cantera using the current composition at these two values of T_g
  Calculate gas enthalpies at these two conditions
  Calculate estimate of dh_g/dT from slope over this interval
end procedure