

# MULTI STEADY STATE CSTR - RECAP

ASSUMPTION: SINGLE REACTION in a CSTR

At steady state: the heat generated must be equal to the heat removed

$$\widetilde{\Delta H}_R^{in} \frac{\dot{R}_A V}{\dot{F}_A^{in}} = \widetilde{C}_P^* (1 + \kappa) (T - T^*)$$



$$\begin{cases} G(T) = \widetilde{\Delta H}_R^{in} \frac{\dot{R}_A V}{\dot{F}_A^{in}} \\ R(T) = \widetilde{C}_P^* (1 + \kappa) (T - T^*) \end{cases}$$

In the exercise considered, the generated heat is

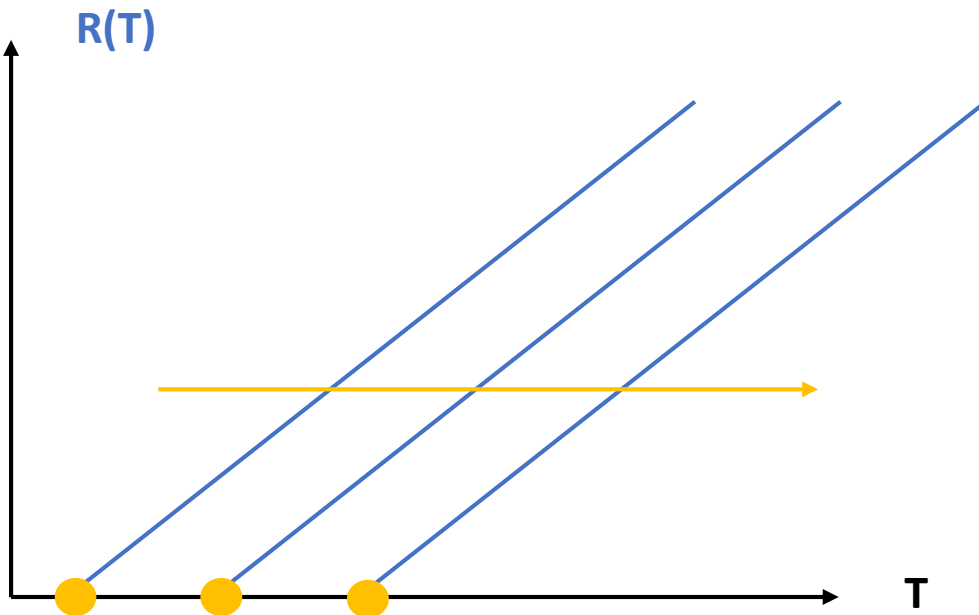
$$\frac{V}{\dot{F}_A^{in}} (r_1 \widetilde{\Delta H}_{R1} + r_2 \widetilde{\Delta H}_{R2})$$

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$T^*$  : intercept

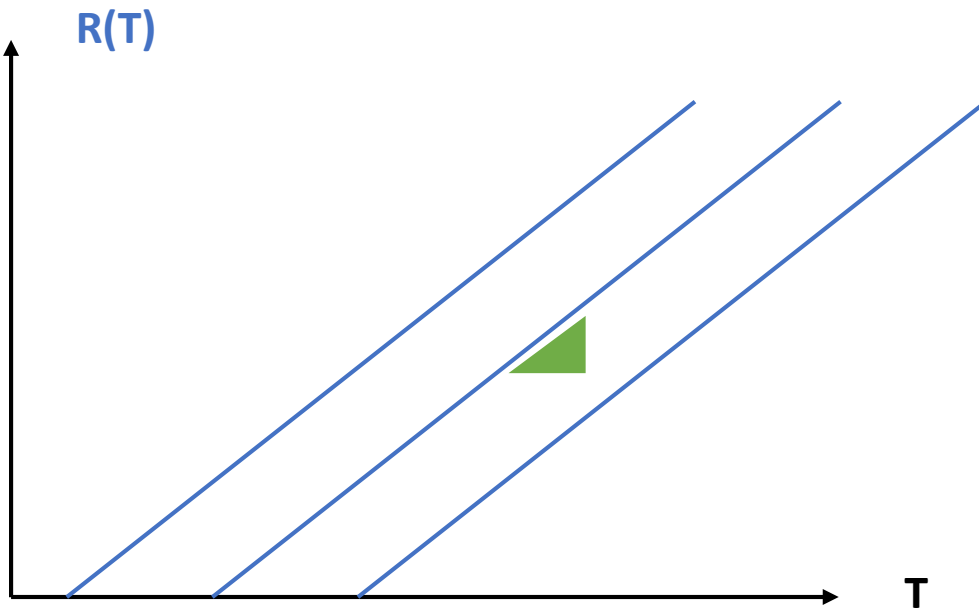
Removed heat decreases with increasing  $T^*$

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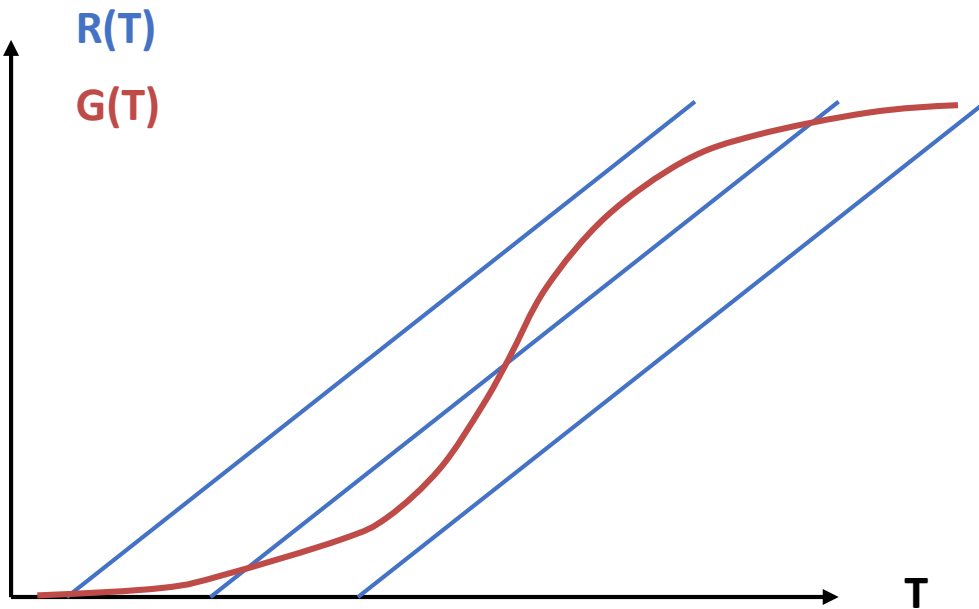
$\widetilde{C}_P^* (1 + \kappa)$  : slope of the curve  
Removed heat increases with increasing slope

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Heat generated: it depends on the shape of the conversion  $\rightarrow$  it depends on the reaction rate ( $\dot{R}_A$ ) !

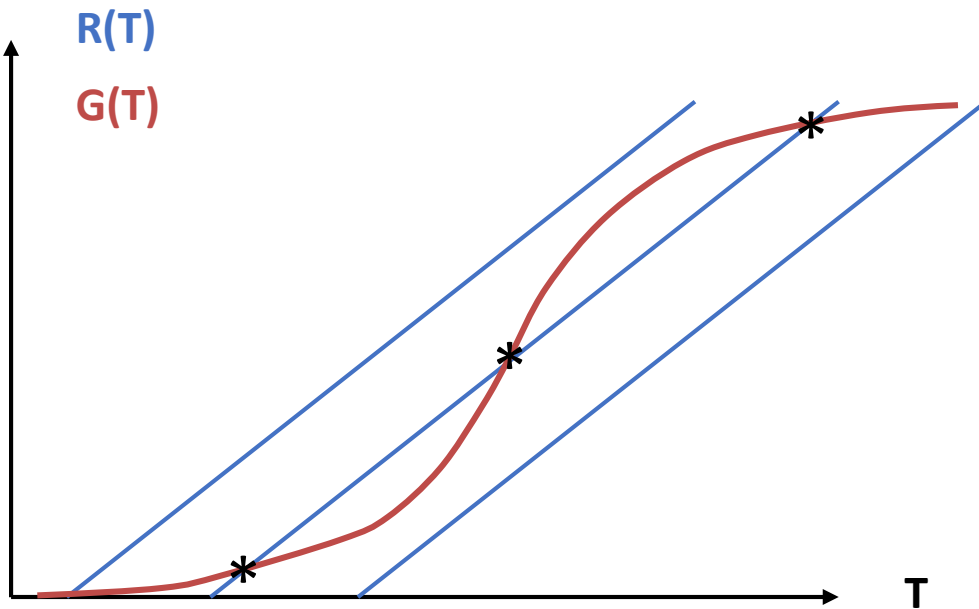
This is the shape for an irreversible reaction

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-X

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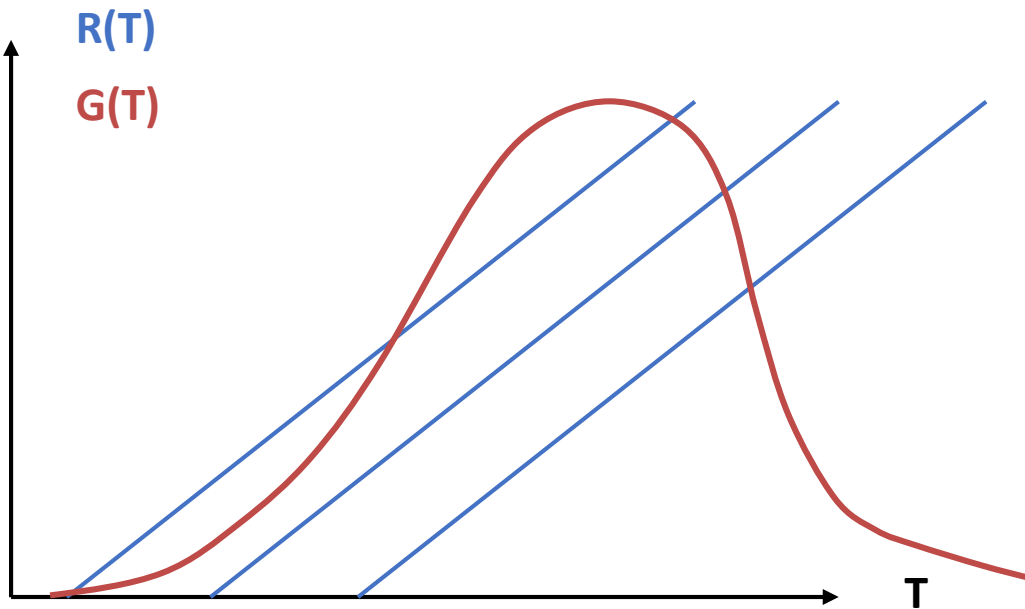
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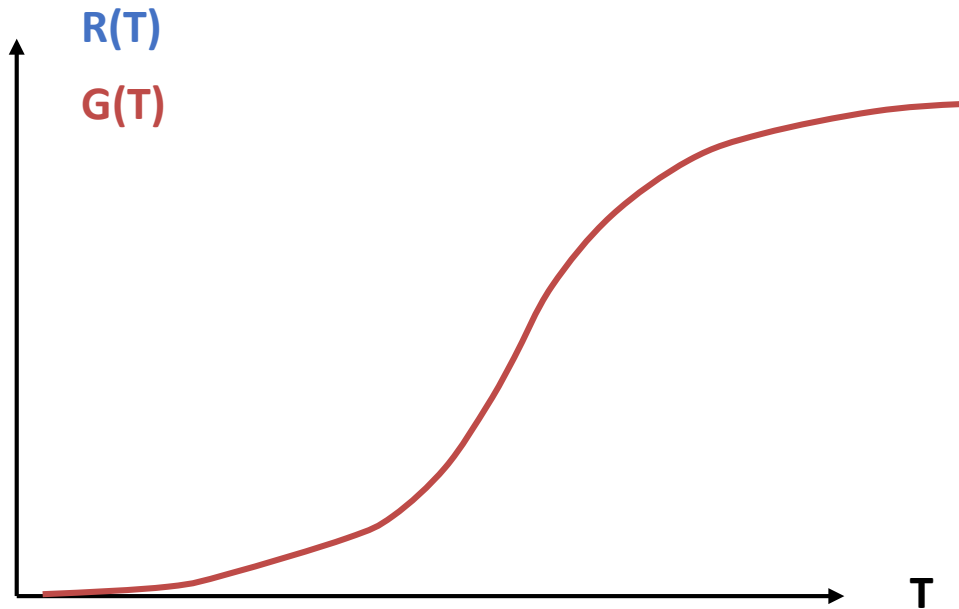
Heat generated: it depends on the shape of the conversion  $\rightarrow$  it depends on the reaction rate ( $\dot{R}_A$ ) !

Reversible exothermic reactions can have maxima in the generated heat

# MULTI STEADY STATE CSTR – STATIC STABILITY ANALYSIS

Basic case of single irreversible reaction

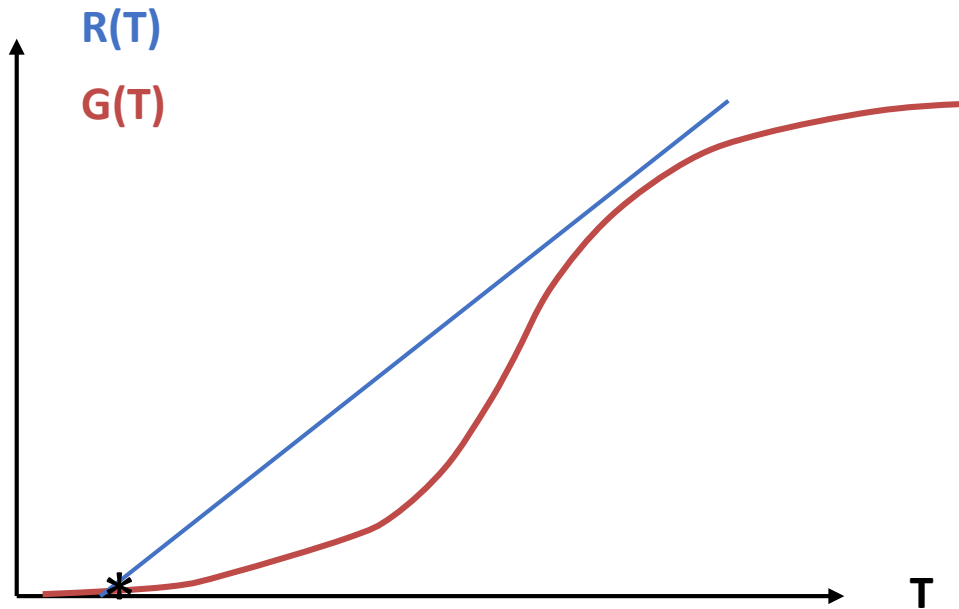
What happens if you vary  $T$  in (and therefore  $T^*$ ) ?  $\longrightarrow T^* = \frac{\kappa T_{ext}^{in} + T_{in}}{\kappa + 1}$



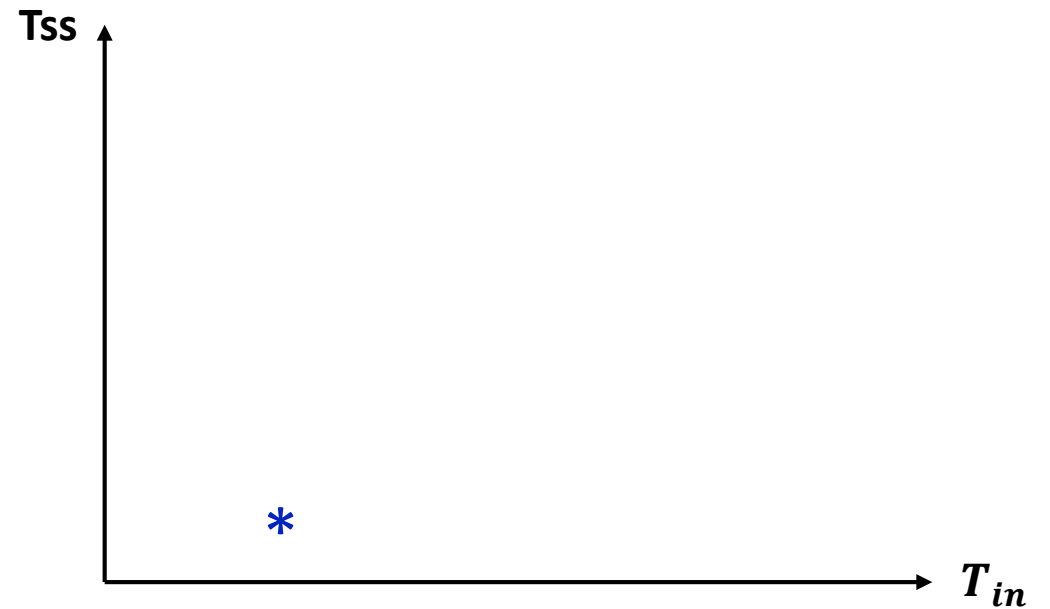
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Low temperature steady state solution on the low branch



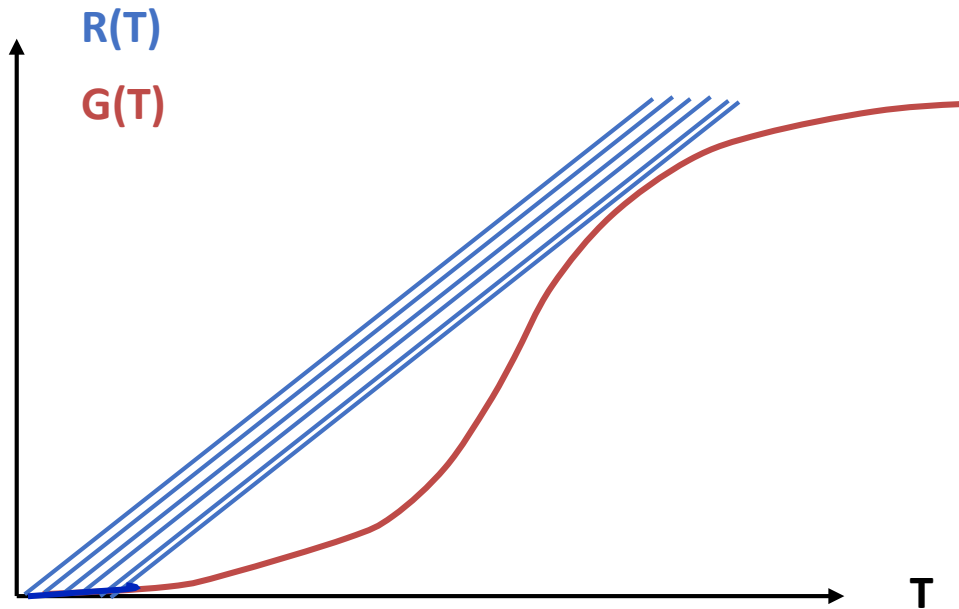
You find a certain steady state  $T$  based on your inlet temperature



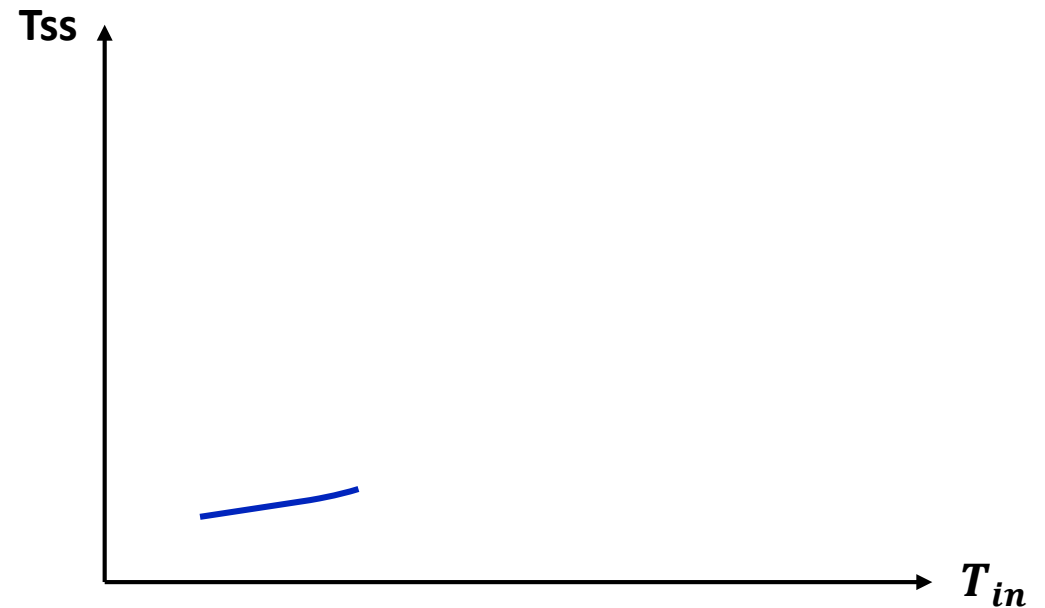
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What happens if you vary  $T$  in (and therefore  $T^*$ ) ?  $\longrightarrow T^* = \frac{\kappa T_{ext}^{in} + T_{in}}{\kappa + 1}$



Low temperature steady state solution on the low branch: consider different  $T$  in

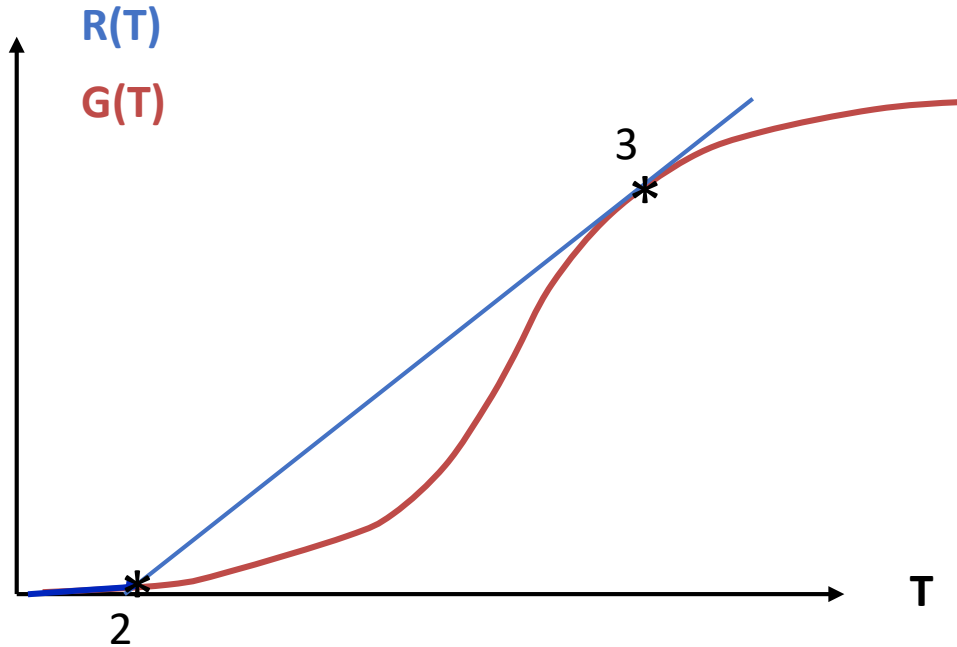


Consider all the  $T$  values of the steady states of the different inlet temperature: obtain the lower branch

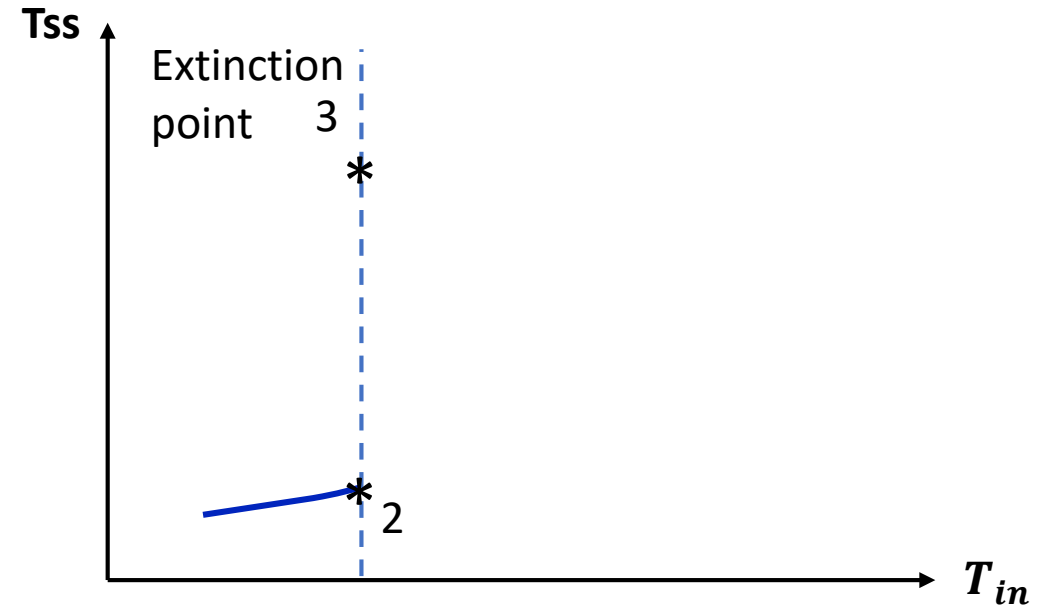
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The curve  $R(T)$  is tangent to  $G(T)$ :  
2 steady state solutions

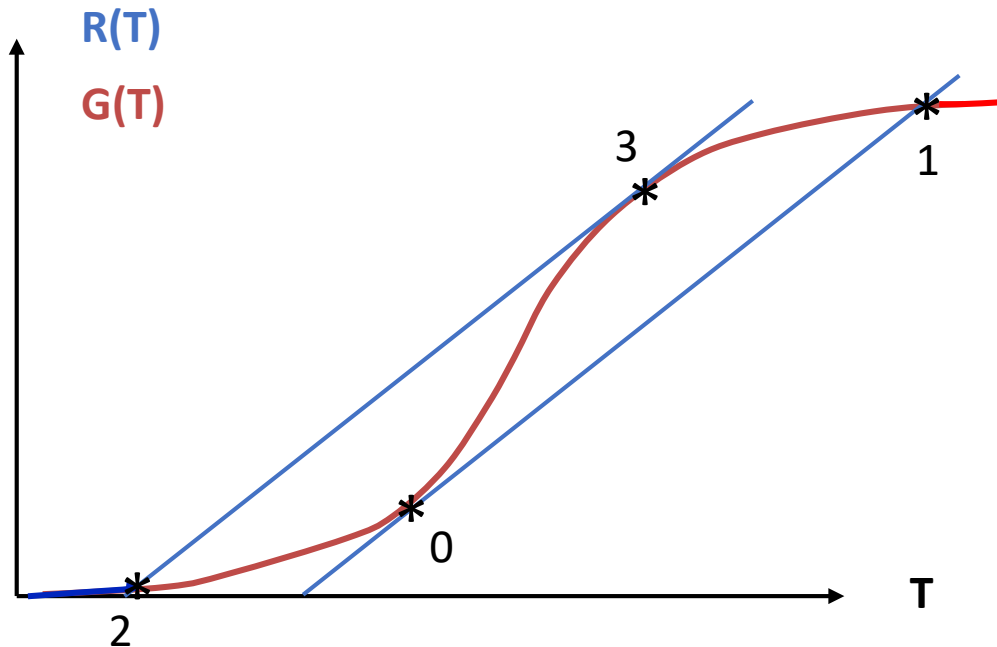


The same inlet value corresponds to 2 different steady state temperatures

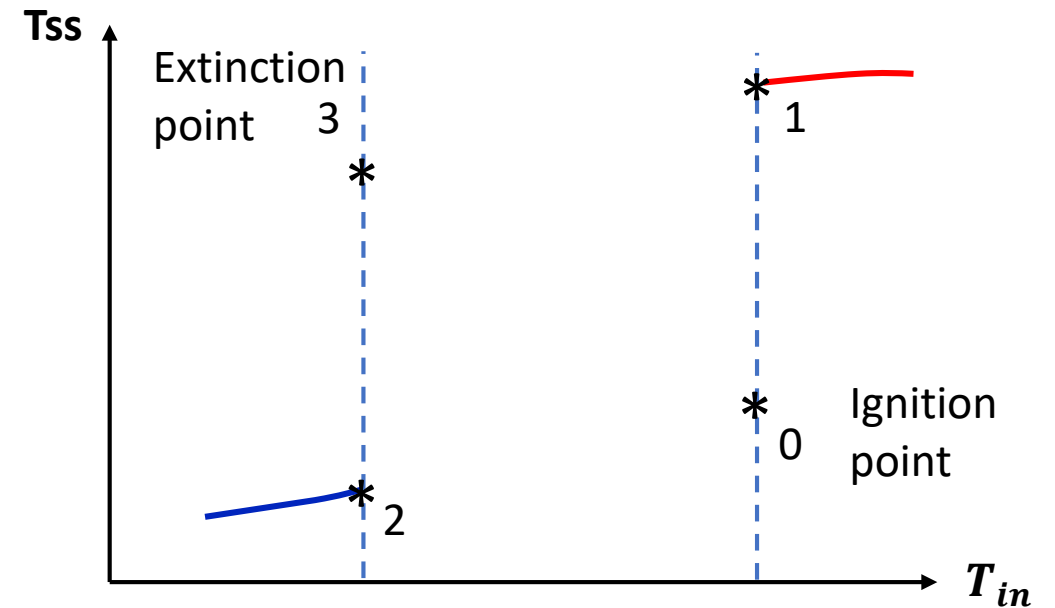
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Basic case of single irreversible reaction

What happens if you vary  $T$  in (and therefore  $T^*$ ) ?  $\longrightarrow T^* = \frac{\kappa T_{ext}^{in} + T_{in}}{\kappa + 1}$



Same way of reasoning from very high inlet  $T$  to smaller values



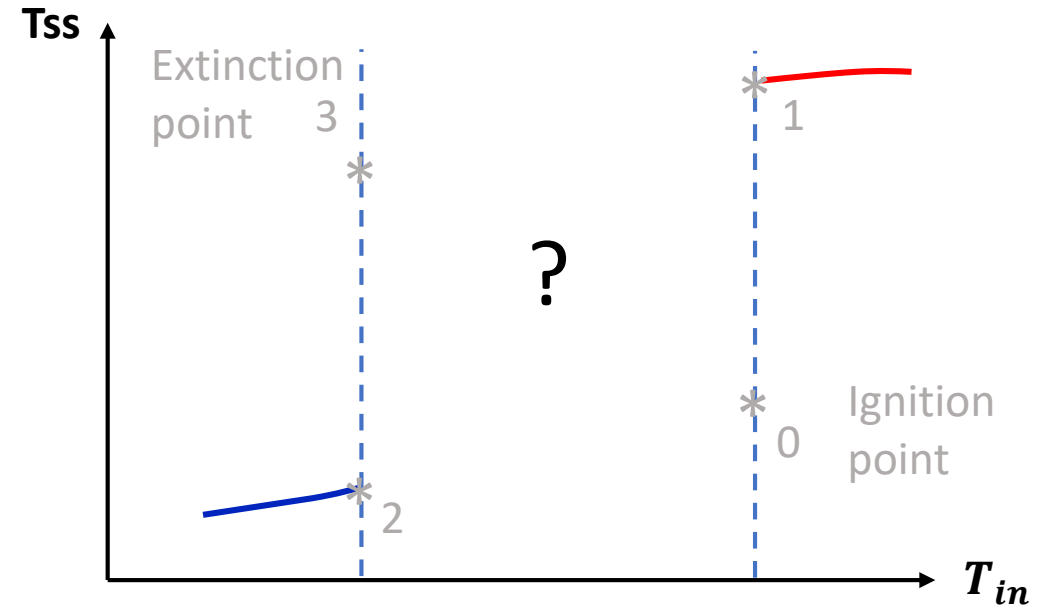
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POLITECNICO MILANO 1863

What happens in between?  
Multiple steady state  $\rightarrow$   
You need to know if they are stable or not

What happens if you vary  $T$  in (and therefore  $T^*$ ) ?  $\longrightarrow$

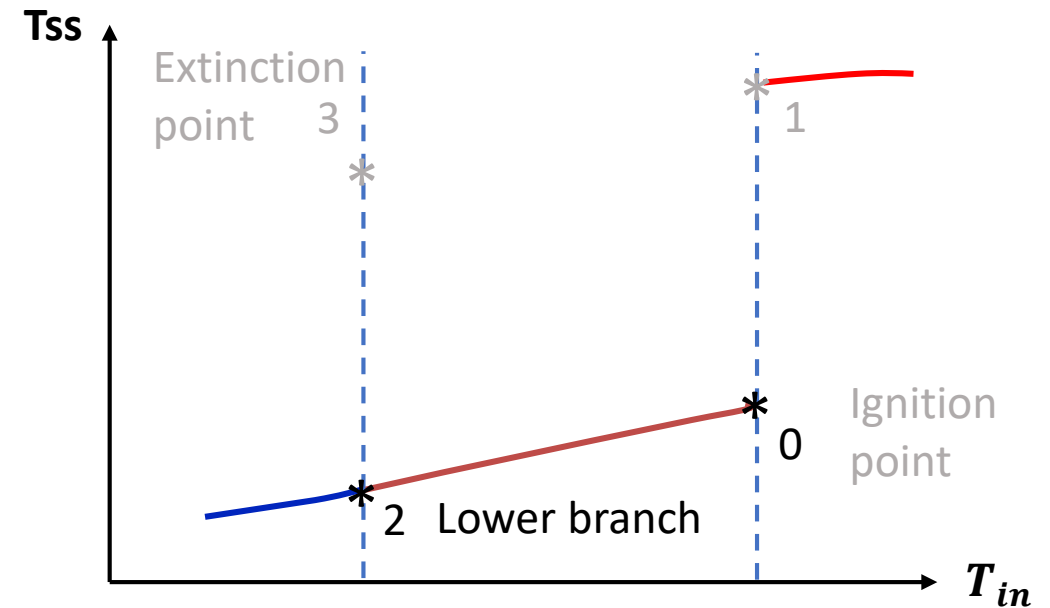
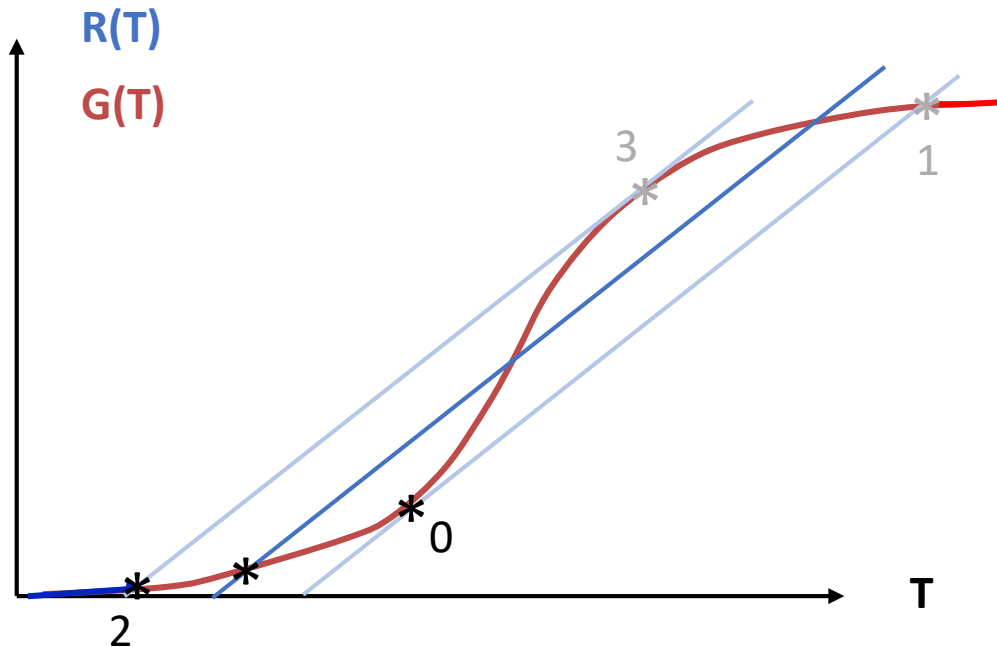
$$T^* = \frac{\kappa T_{ext}^{in} + T_{in}}{\kappa + 1}$$



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Basic case of single irreversible reaction

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From 2 to 0:

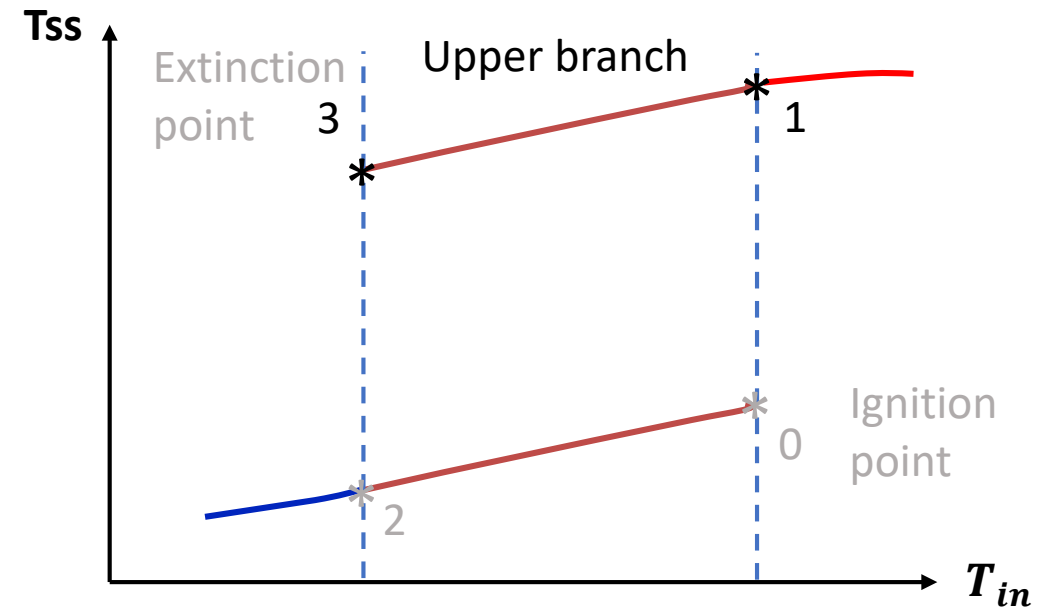
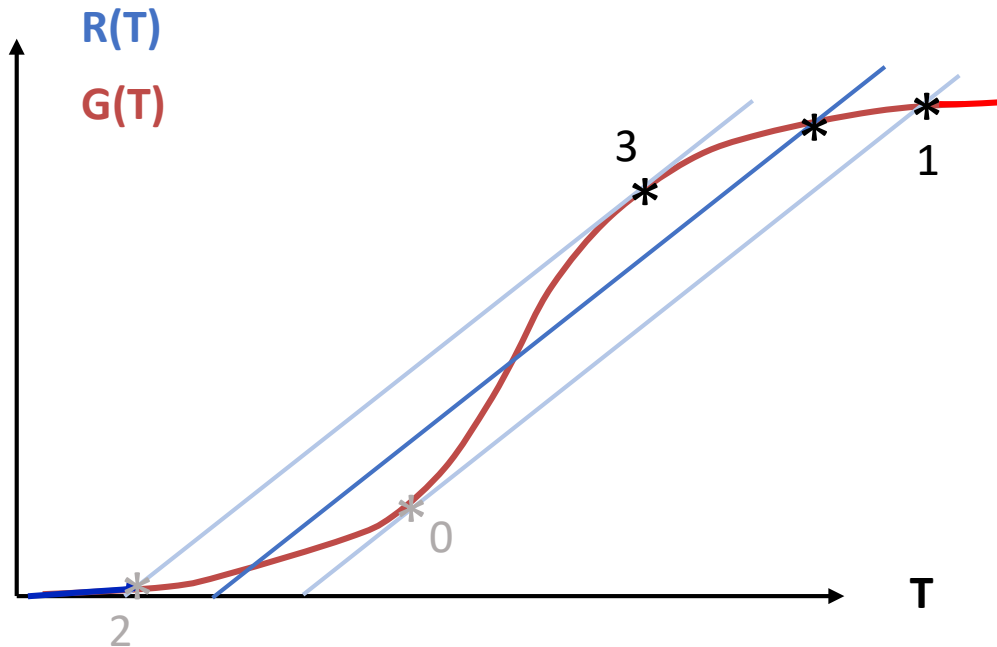
$$\frac{dR}{dT} > \frac{dG}{dT}$$

In case of both negative and positive temperature perturbation, you go back to the initial state  $*$   $\rightarrow$  STABLE

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From 3 to 1:

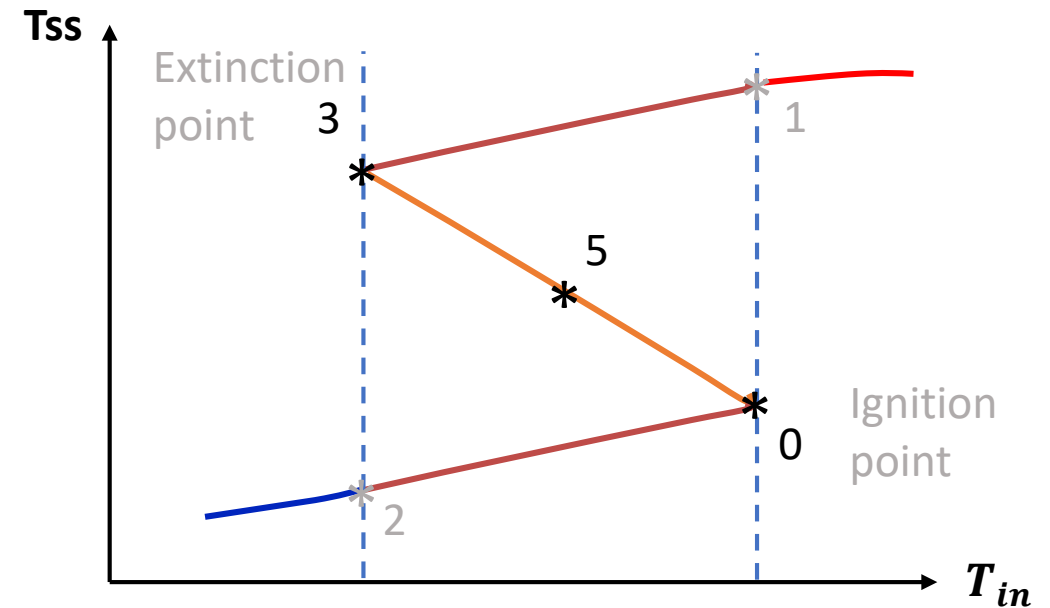
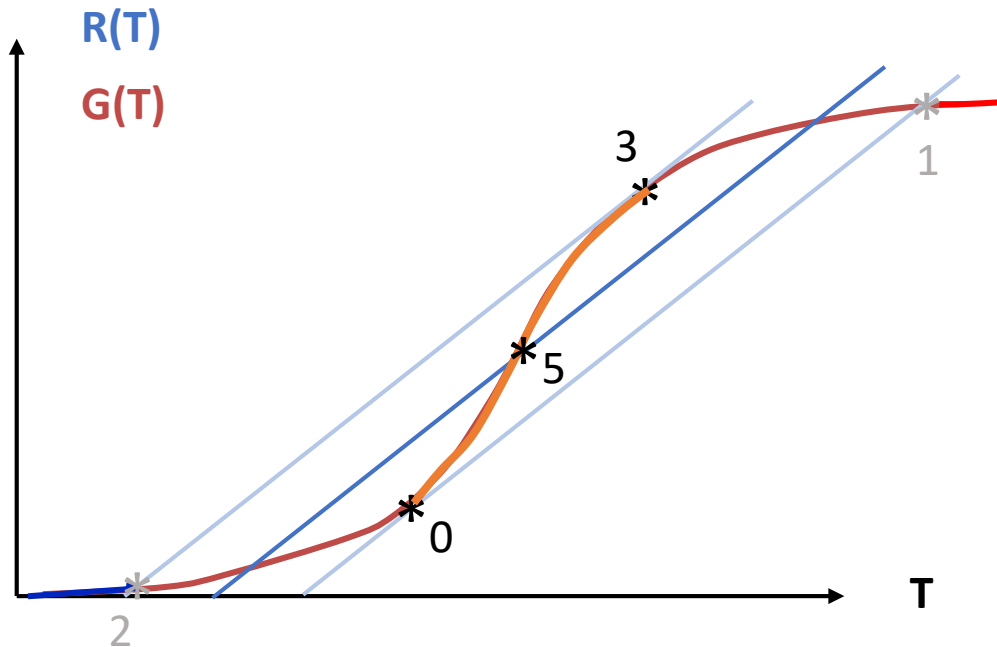
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From 0 to 3:

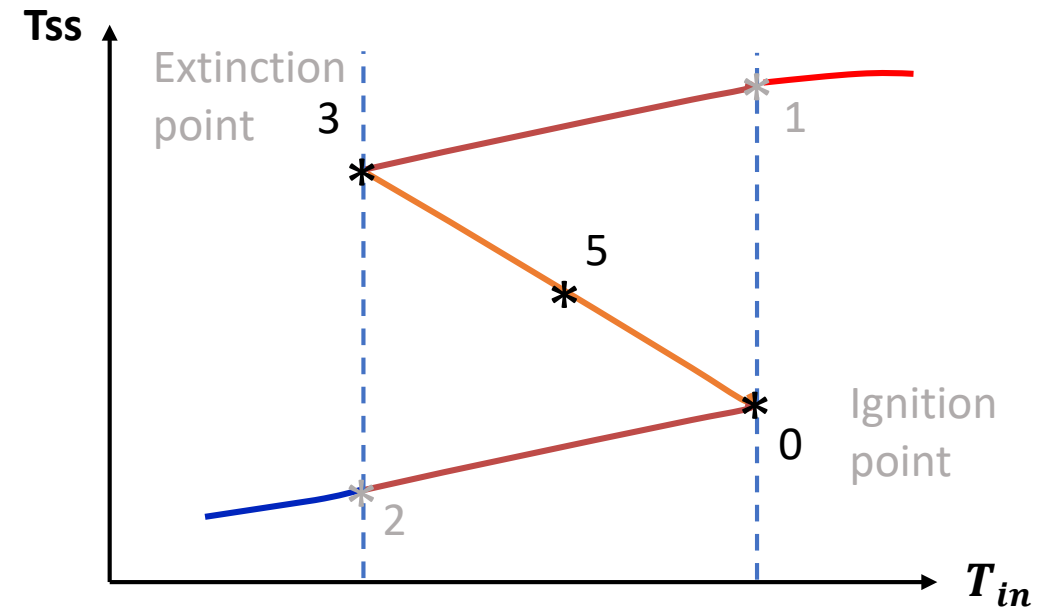
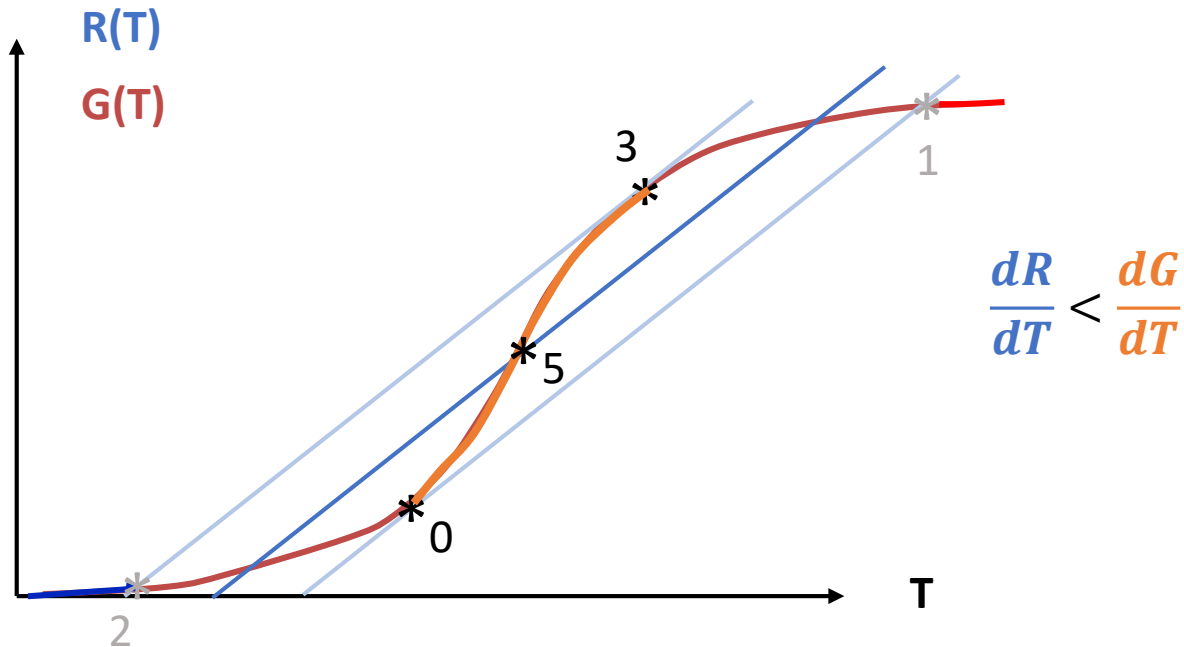
$$\frac{dR}{dT} < \frac{dG}{dT}$$

The heat removal cannot compensate for the heat generation in case of both negative and positive  $T$  perturbation  $\rightarrow$  UNSTABLE

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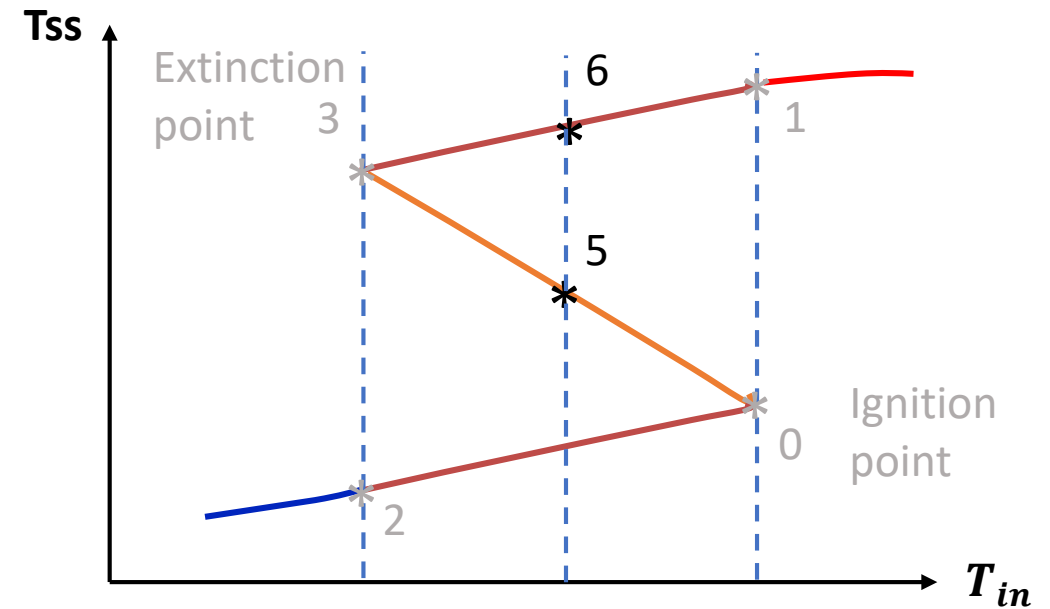
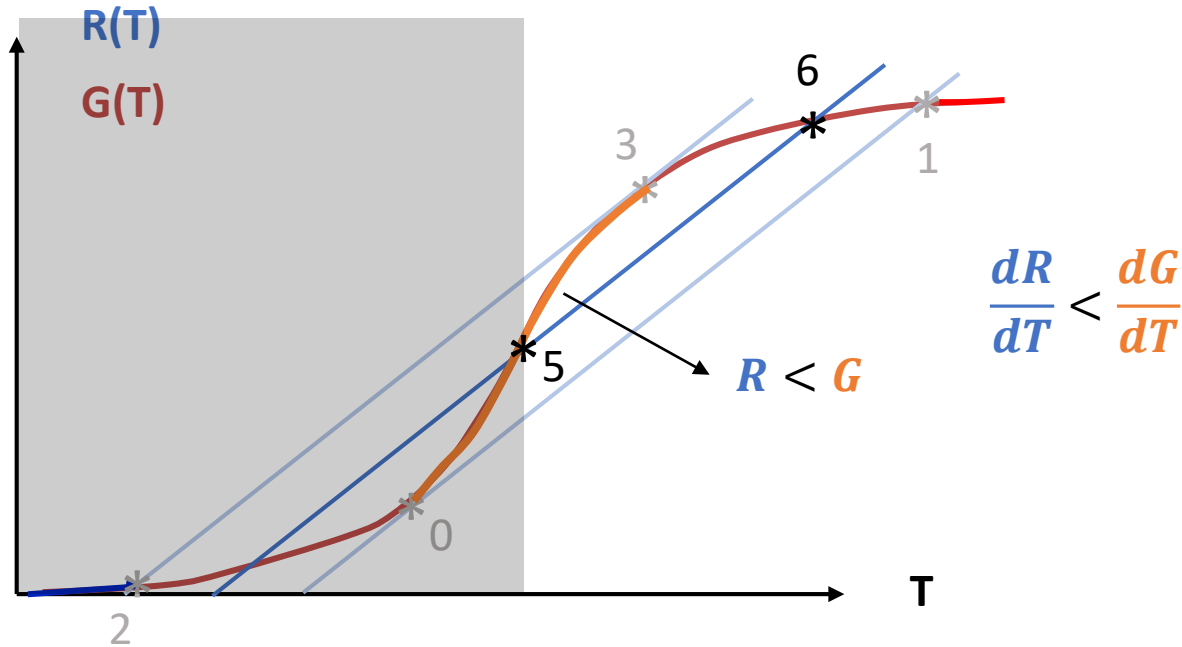
Note that: if we have a SS between 3 and 0, the SS temperature DECREASES with increasing inlet temperature



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If we are at point \*5:

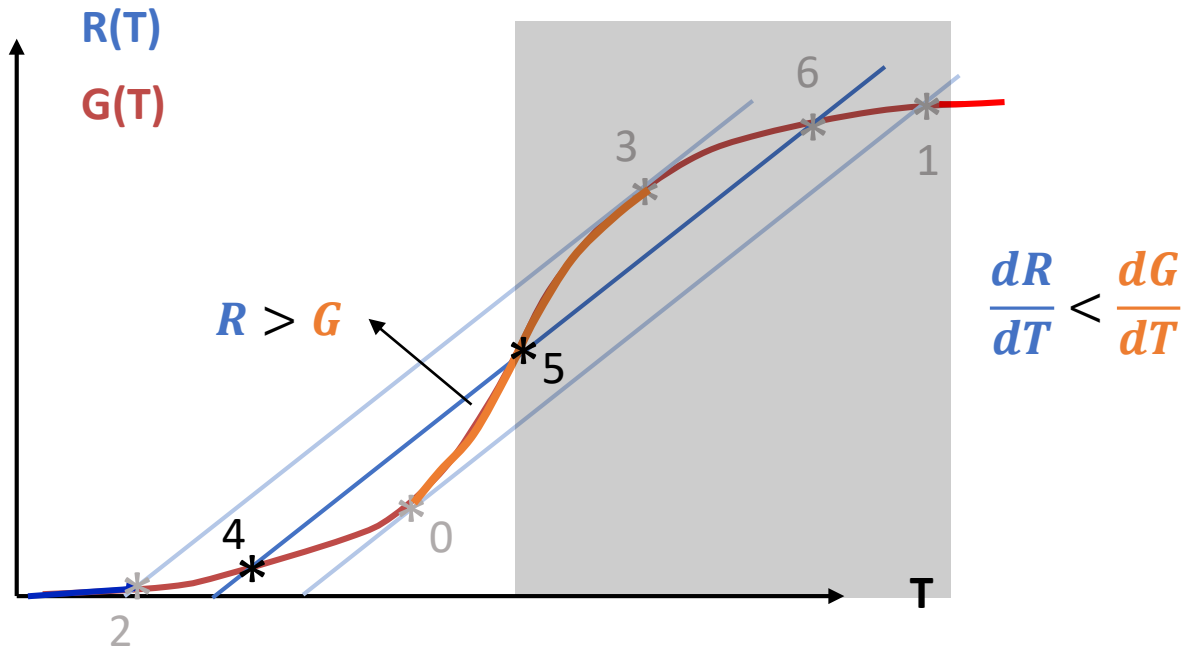
$dT > 0$  positive perturbation:

The heat generated is higher than the heat removed and increases faster  $\rightarrow$  go to SS \*6 (upper branch)

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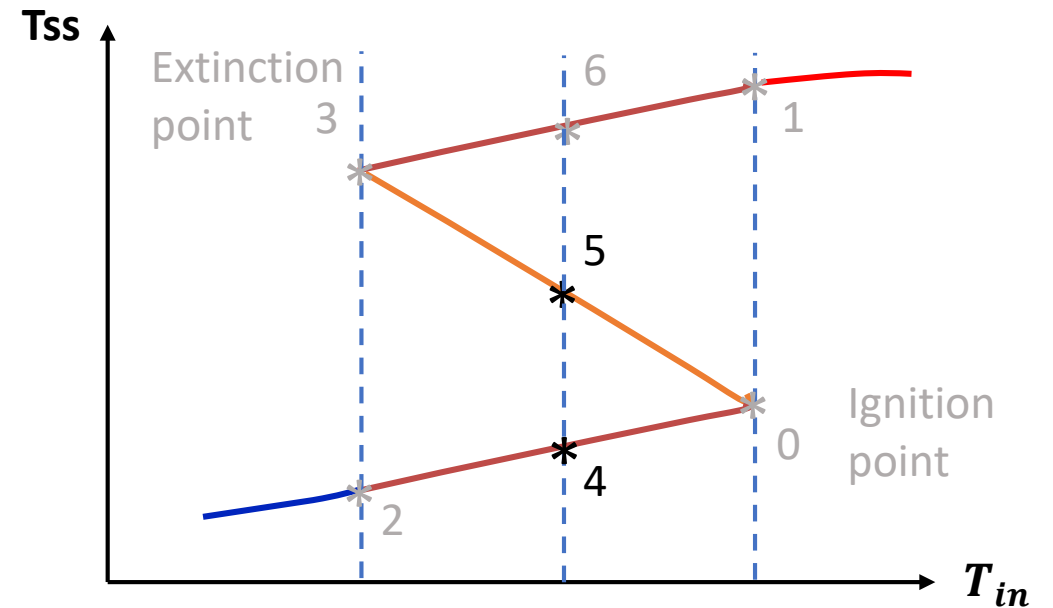
Basic case of single irreversible reaction

What happens if you vary  $T$  in (and therefore  $T^*$ ) ?  $\longrightarrow T^* = \frac{\kappa T_{ext}^{in} + T_{in}}{\kappa + 1}$



If we are at point \*5:

$dT < 0$  positive perturbation:  
The heat generated is smaller than the heat removed and increases faster  $\rightarrow$  go to SS \*4 (lower branch)



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