PRACTICAL SESSION .1

Ex. 1

PARALLEL REACTIONS IN A LIQUID CSTR

$$\begin{cases} A \xrightarrow{k_1} B \\ A \xrightarrow{k_2} C \end{cases}$$

B is the desired product

$k_1 = 0.5\ \text{min}^{-1}$
$k_2 = 0.1\ \text{min}^{-1}$

$Q = 4\ \frac{l}{min}$
$C_A^0 = 2\ \frac{mol}{l}$

SELECTIVITY

$$\sigma_B = \frac{C_B}{C_A^0 - C_A}$$

YIELD

$$Y_B = \frac{C_B}{C_B^0}$$

$$\dot{F}_A^0 - \dot{F}_A + R_A V = 0$$

$$\dot{Q}\,(C_A^0 - C_A) + (-K_1 C_A - K_2 C_A)\,V = 0$$

$$\dot{Q}\,(C_A^0 - C_A^0(1-X)) - C_A^0(1-X)\,(k_1+k_2)\,V = 0 \qquad k_1 + k_2 \overset{def}{=} K$$

$$\dot{Q}X - (1-X)\,kV = 0 \qquad \tau \overset{def}{=} \frac{V}{Q}$$

$$\tau = \frac{1}{K}\,\frac{X}{1-X} \qquad \tau(95\%) = 31.67\ \text{min}$$

$$C_A^0 - C_A - K\tau C_A \Rightarrow C_A(\tau) = \frac{C_A^0}{1+K\tau}$$

$$C_B^0 - C_B + k_1\tau C_A \Rightarrow C_B(\tau) = k_1\tau C_A(\tau) = \frac{K_1\tau C_A^0}{1+k\tau}$$

$$C_B(\tau) = k_2\tau C_A(\tau) = \frac{k_2\tau C_A^0}{1+k\tau}$$

$$\sigma_B = \frac{C_B}{C_A^0 - C_A} = \dots = \frac{K_1}{K_{tot}} \qquad \sigma_C = \frac{k_2}{K_{tot}}$$

$$Y_B = \frac{C_B}{C_{A0}} = \frac{K_1\tau}{1+k\tau} \qquad Y_C = \frac{k_2\tau}{1+K\tau}$$

$$\frac{dY_B}{d\tau} = \frac{K_1(1+k\tau) - K_1\tau K}{(1+k\tau)^2} = \frac{K_1 + k_1 k\tau - k_1\tau K}{(1+k\tau)^2} = \frac{K_1}{(1+k\tau)^2} \qquad \text{no max}$$

EX. 2

PARALLEL REACTIONS IN A BATCH REACTOR (constant density)

$$\begin{cases} A \rightarrow B \\ A \rightarrow C \end{cases} \qquad \begin{cases} k_1 = 1.64\ h^{-1} \\ k_2 = 0.13\ h^{-1} \end{cases} \qquad \begin{aligned} &\rho = 800\ kg/m^3 \\ &T = 100\,°C \\ &V = 1\ m^3 \\ &\tau_D = 1\,h \end{aligned}$$

$$\begin{cases} C_F(\tau) = C_1 \tau_D + C_2 \tau + C_3 \\ I(\tau) = 15\ \$/kg_B \end{cases}$$

$k_1 + k_2 \overset{\text{def}}{=} k$

$$\begin{cases} \dfrac{dC_A}{dt} = -k_1 C_A - k_2 C_A \\ \dfrac{dC_B}{dt} = k_1 C_A \\ \dfrac{dC_C}{dt} = k_2 C_C \end{cases} \qquad \frac{dC_A}{dt} = -(k_1+k_2) C_A = -k C_A$$

$$C_A = C_A^\circ \exp(-kt)$$

$$\frac{dC_B}{dt} = k_1 C_A = k_1 C_{A0} \exp(-kt)$$

$$C_B - C_B^\circ = k_1 C_{A0} \int_0^\tau \exp(-kt)\,dt = \ldots = \frac{k_1 C_{A0}}{k}\left(1 - \exp(-k\tau)\right)$$

solution
$$\begin{cases} C_A(\tau) = C_A^\circ \exp(-k\tau) \\ C_B(\tau) = \dfrac{k_1 C_{A0}}{k}\left(1 - \exp(-k\tau)\right) \\ C_C(\tau) = \dfrac{k_2 C_{A0}}{k}\left(1 - \exp(-k\tau)\right) \end{cases}$$

$$C_A(\tau) = C_A^\circ \exp(-k\tau) \Rightarrow \ln\frac{C_A}{C_A^\circ} = -k\tau \qquad \tau = -\frac{1}{k}\ln\frac{C_A}{C_{A0}}$$

$$\tau = -\frac{1}{k}\ln\frac{C_A^\circ(1-X)}{C_A^\circ} = \ldots = -\frac{1}{k}\ln(1-X)$$

$$\tau(98\%) = -\frac{\ln(1-X)}{k} = 2.18\,h \qquad \frac{N_{cycles}}{day} = \frac{24}{\tau+\tau_D} = 7.52$$

$$Pr_B = C_B V \frac{N_{cycles}}{day} = \ldots = 155.5\ \frac{kmol}{day}$$

Optimal economical conditions

MARGIN = INCOMES − COSTS

$M = I - C$ to be maximized

$$I = I(t) = P_B \cdot \text{Price}_B \cdot MW_B =$$

$$= \frac{24\,h}{\tau + \tau_D} \cdot C_B V \cdot \underset{\text{Kg}}{15\,\$} \cdot MW_B$$

$$\stackrel{!}{=} \frac{24 \cdot V \cdot 15\, MW_B}{\tau + \tau_D} C_B = 24 \cdot V \cdot 15 \cdot MW_B \frac{C_B}{\tau + \tau_D}$$

$$I(\tau) = 24 V \cdot 15 \cdot MW_B \cdot \frac{K_1 C_{A0}}{K} \cdot \frac{1 - \exp(-K_s \tau)}{\tau + \tau_D} = \beta \frac{1 - \exp(-K_s \tau)}{\tau + \tau_D}$$

$$C(\tau) = (C_1 \tau_D + C_2 \tau + C_3) \cdot \frac{\text{n cycles}}{\text{day}} = \frac{24}{\tau + \tau_D} (C_1 \tau_D + C_2 \tau + C_3)$$

$$M = I - C = \beta \frac{1 - \exp(-K_s \tau)}{\tau + \tau_D} - 24 \frac{(C_1 \tau_D + C_2 \tau + C_3)}{\tau + \tau_D}$$

$$\stackrel{!}{=} \frac{\beta(1 - \exp(-K_s \tau)) - 24(C_1 \tau_D + C_2 \tau + C_3)}{\tau + \tau_D}$$

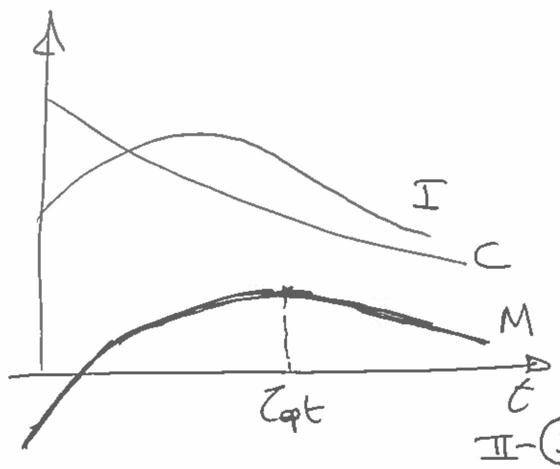
$$\frac{dM}{d\tau} = \frac{(\beta K_1 \exp(-K_1 \tau) - 24 C_2)(\tau + \tau_D) - (\beta(1 - \exp(-K\tau)) - 24(C_1 \tau_D + C_2 \tau + C_3))}{(\tau + \tau_D)^2} \stackrel{!}{=} 0$$

algebraic non linear equation $\quad 0 = (\beta K_1 \exp(-K_1 \tau) - 24 C_2)(\tau + \tau_D) - (\beta(1 - \exp(-K\tau)) - 24(C_1 \tau_D + C_2 \tau + C_3))$

↳ must be solved numerically

Alternatively, the M, I and C functions can be plotted and the maximum of M can be read directly

$$\tau_{opt} = 1.72\, h$$



EX 3.

NON ISOTHERMAL CSTR at constant density

$V = 1135\ \ell$

$A + B + (M) \longrightarrow C + (M)$

M does not partecipate to the reaction (it behaves like an inert species)

$$\begin{cases} \dot{Q}_A^\circ = 1360\ \ell/h \\ \dot{Q}_M^\circ = 1320\ \ell/h \\ \dot{Q}_B^\circ = 6600\ \ell/h \\ \dot{Q}_C^\circ = 0 \end{cases} \qquad \begin{cases} \dot{F}_A^\circ = 19.50\ kmol/h \\ \dot{F}_M^\circ = 32.60\ kmol/h \\ \dot{F}_B^\circ = 364\ kmol/h \end{cases}$$

$T_{in} = 29°C$ $\qquad A + B \xrightarrow{K} C \qquad r = K C_A^1 \cdot C_B^0 = K C_A^1 = K C_A$

$T_{MAX} = 53°C$ $\qquad A = 16.96 \cdot 10^{12}\ h^{-1}$, $E = 72000\ J/mol$ $\quad \} \ K = A \exp(-E/RT)$

Specific heats

$\widetilde{C_{pA}} = 146\ J/mol\,K$

$\widetilde{C_{pB}} = 75$ "

$\widetilde{C_{pC}} = 192$ "

$\widetilde{C_{pD}} = 82$ "

Formation enthalpies

$\widetilde{H_A^\circ}(T_0 = 20°C) = -148918\ J/mol$

$\widetilde{H_B^\circ} = -275000$ "

$\widetilde{H_C^\circ} = -505360$ "

$$\begin{cases} C_A^\circ = \dfrac{\dot{F}_A^\circ}{Q_{TOT}} = 2.11\ \dfrac{mol}{\ell} \\ C_B^\circ = \dfrac{\dot{F}_B^\circ}{Q_{TOT}} = 39.4\ " \\ C_M^\circ = \dfrac{\dot{F}_M^\circ}{Q_{TOT}} = 3.53\ " \end{cases}$$

MASS BALANCE $\quad \dot{Q}(C_A^\circ - C_A) + R_A V = 0$

ENERGY BALANCE $\quad X(T) = -\dfrac{\int_{T_{in}}^{T} \widetilde{C_{pin}}\, dT}{\widetilde{\Delta H_R}(T)}$

$\tau = \dfrac{V}{Q_{TOT}} = 0.123\ h$

$$\begin{cases} C_A^\circ X = K C_A^\circ (1 - X)\, \tau \\ X(T) = \dfrac{-\widetilde{C_{pin}}(T - T_{in})}{\widetilde{\Delta U_R^\circ} + \widetilde{\Delta C_p}(T - T_0)} \end{cases}$$

$\widetilde{C_{pin}} = \sum_{1}^{4}{}_j\, C_{pj}\, \vartheta_j^\circ \stackrel{!}{=} \widetilde{C_{pA}}\vartheta_A^\circ + C_{pB}\vartheta_B^\circ + C_{pM}\vartheta_M^\circ$

$\widetilde{\Delta C_p} = \sum_{1}^{3}{}_j\, \widetilde{C_{pj}}\, \nu_j = -\widetilde{C_{pA}} - \widetilde{C_{pB}} + \widetilde{C_{pC}}$

$\widetilde{\Delta U_R^\circ} = \sum_{1}^{3}{}_j\, \widetilde{H_R^\circ}\, \nu_j \stackrel{!}{=} -\widetilde{H_A^\circ} - \widetilde{H_B^\circ} + \widetilde{H_C^\circ}$

$$\begin{cases} X = \dfrac{K\tau}{1+K\tau} \\[2ex] X = \dfrac{-\tilde{C}_p^{in}(T-T_{in})}{\widetilde{\Delta H_R^0} + \widetilde{\Delta C_p}(T-T_0)} \end{cases} \qquad \begin{cases} X = \dfrac{A\exp(-E/RT)\,\tau}{1 + A\exp(-E/RT)\,\tau} \\[2ex] X = \dfrac{-\tilde{C}_p^{in}(T-T_{in})}{\widetilde{\Delta U_R^0} + \widetilde{\Delta C_p}(T-T_0)} \end{cases}$$

NLS of 2 equations (non linear)

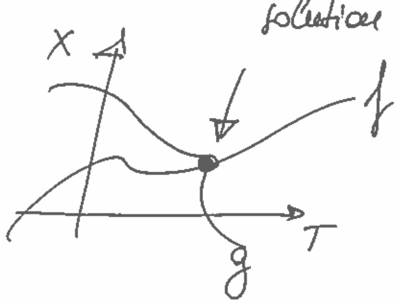
$$\underline{\underline{NLS}} \quad \begin{cases} X - \dfrac{A\exp(-E/RT)\,\text{Tau}}{1 + A\exp(-E/RT)\,\text{Tau}} = 0 \\[2ex] X + \dfrac{\tilde{C}_p^{in}(T-T_{in})}{\widetilde{\Delta U_R^0} + \widetilde{\Delta C_p}(T-T_0)} = 0 \end{cases} \qquad \begin{cases} f_1(X,T) = 0 \\[2ex] f_2(X,T) = 0 \end{cases}$$

It can be written as a single NL equation in T

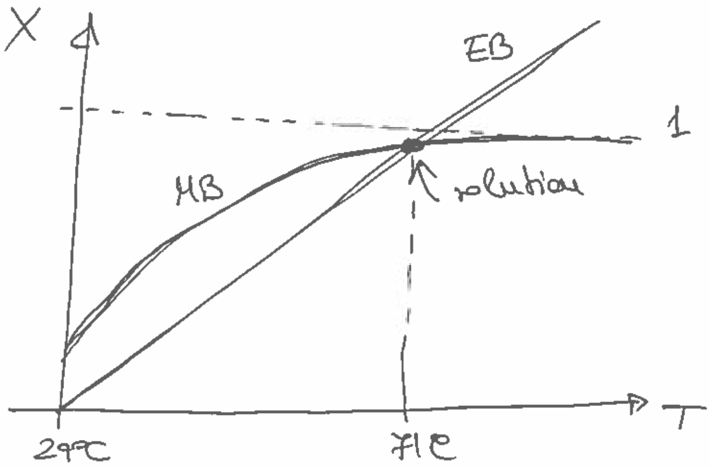
$$\frac{A\exp(-E/RT)\,\tau}{1 + A\exp(-E/RT)\,\tau} = -\frac{\tilde{C}_p^{in}(T-T_{in})}{\widetilde{\Delta U_R^0} + \widetilde{\Delta C_p}\,(T-T_0)}$$

Also a graphical procedure is possible, directly on the equations of the NLS in the following form:

$$\begin{cases} X = f(T) \\ X = g(T) \end{cases}$$



SOLUTION
$T = 71.9°C$
$X = 96.1\%$



EX 4

PLUG FLOW REACTOR with constant density

REACTIONS IN SERIES $A \xrightarrow{k_1} B \xrightarrow{k_2} C$

exactly same exercise PC 1 – 6

solution

$$\begin{cases} C_A(\tau) = C_A^0 \exp(-k_1 \tau) \\ C_B(\tau) = \dfrac{k_1 C_A^0}{k_2 - k_1}\left(\exp(-k_1\tau) - \exp(-k_2\tau)\right) \\ C_C(\tau) = C_A^0 - C_A(\tau) - C_B(\tau) \end{cases}$$

$$\begin{cases} k_1 = 1.99\ s^{-1} \\ k_2 = 1.86\ s^{-1} \end{cases} \qquad C_{A0} = 35.7\ \frac{mol}{m^3}$$

$$\tau = 3.23\ s \qquad \eta_C = \frac{C_C(\tau)}{C_A^0(\tau)} = 96.8\ \%$$

EX 5

Same exercise above, only different numbers

i) same analytical solution

ii) same MATLAB code

iii) quasi-steady state hypothesis for species B

if we look at the kinetic constants, we see that the second reaction is more than 1000 times FASTER than the first one

$$k_1 = 0.57\ s^{-1}$$
$$k_2 = 1114\ s^{-1}$$

$$\frac{k_2}{k_1} \gg 1$$

this means that B disappears as soon as it is produced

We can assume it is in quasi steady state

$$\frac{k_2}{k_1} \gg 1 \implies \frac{dC_B}{dt} \sim 0 \implies k_1 C_A(\tau) = k_2 C_B(\tau)$$

$$\implies C_B(\tau) = \frac{k_1}{k_2} C_A^0 \exp(-k_1 \tau)$$

Thus the problem is simplified because we have $C_B(\tau)$ explicitly and we do not have to solve the corresponding ODE equation

$$\begin{cases} \dfrac{dC_A}{d\tau} = -K_1 C_A \\[2ex] \dfrac{dC_C}{d\tau} = K_2 \cdot C_B = \dfrac{K_2 K_1}{K_2} C_A^\circ \exp(-K_1\tau) \end{cases}$$

$$\begin{cases} \dfrac{dC_A}{d\tau} = - K_1 C_A \\[2ex] \dfrac{dC_C}{d\tau} = K_1 C_A^\circ \exp(-K_1\tau) \end{cases}$$

2 equations

it is interesting to observe that K_2 does not appear explicitly

Indeed, if $K_2 \gg K_1$, the RDS (RATE DETERMING STEP) is the first reaction

$$\begin{cases} C_A(\tau) = C_A^\circ \exp(-K_1\tau) \\ C_C(\tau) = C_A^\circ (1 - \exp(-K_1\tau)) \\ C_B(\tau) = \dfrac{K_1}{K_2} C_A^\circ \exp(-K_1\tau) \end{cases}$$

since $\frac{K_1}{K_2} \ll 1$ it is always expected be very small

EX 6

SERIES REACTION IN A CONSTANT VOLUME BATCH REACTOR

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

$T = 500\,K$

$V = 0.50\ m^3$

$N_A^\circ = 20\ kmol$

$\tau_D = 1\,h$

$$C_{A0} = \frac{N_A^\circ}{V} = 40\ \frac{kmol}{m^3}$$

$$\begin{cases} k_1 = 1.75\ h^{-1} \\ k_2 = 0.0297\ h^{-1} \end{cases}$$

$$\begin{cases} \dfrac{dC_A}{dt} = -k_1 C_A \\ \dfrac{dC_B}{dt} = k_1 C_A - k_2 C_B \\ \dfrac{dC_C}{dt} = k_2 C_B \end{cases} \Longrightarrow \begin{cases} C_A(t) = C_A^\circ \exp(-k_1 t) \\ C_B(t) = \dfrac{k_1 C_A^\circ}{k_2 - k_1}\left(\exp(-k_1 t) - \exp(-k_2 t)\right) \\ C_C(t) = C_A^\circ - C_B - C_A \end{cases}$$

Max. yield of B $\quad y_B = \dfrac{C_B}{C_A^\circ} = \dfrac{k_1}{k_2 - k_1}\left(\exp(-k_1 t) - \exp(-k_2 t)\right)$

same position of maximum of B
see previous problem

$$t_{\substack{MAX \\ yield\ B}} = \frac{\ln \frac{k_2}{k_1}}{k_2 - k_1} \qquad t_{\substack{MAX \\ yield\ B}} = 2.37\,h$$

Max. production of B

$$P_B \overset{def}{=} N_B(\tau)\, N_{cycles/day} = C_B(\tau)\, V \cdot \frac{24\,h}{\tau + \tau_D}$$

$$= \frac{24 \cdot k_1 C_{A0} V}{k_2 - k_1}\left[\exp(-k_1 t) - \exp(-k_1 t)\right] / (\tau + \tau_D)$$

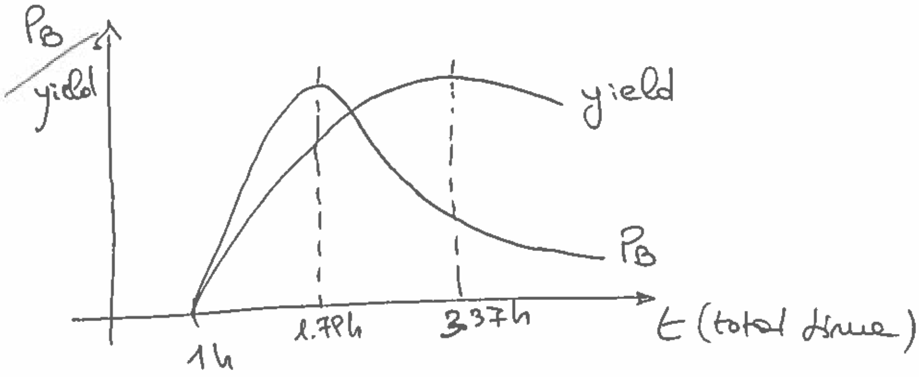
$$\propto \frac{\exp(-k_1 t) - \exp(-k_2 t)}{\tau + \tau_D}$$

$$\frac{dP_B}{d\tau} = \alpha\ \frac{\left(-k_1 \exp(-k_1 \tau) + k_2 \exp(-k_2 \tau)\right)(\tau + \tau_D) - \left(\exp(-k_1 \tau) - \exp(-k_2 \tau)\right)}{(\tau + \tau_D)^2} = 0$$

$$\left(-k_1 \exp(-k_1 \tau) + k_2 \exp(-k_2 \tau)\right)(\tau + \tau_D) - \left(\exp(-k_1 \tau) - \exp(-k_2 \tau)\right) = 0$$

non linear algebraic
equation to solve in τ

$$\left(-k_1 \exp(-k_1\tau) + k_2 \exp(-k_2\tau)\right)(\tau+\tau_D) - \left(\exp(-k_1\tau) - \exp(-k_2\tau)\right) = 0$$



instead of solving the equation above, one can plot the P_B function and find the maximum position on it

$$\tau_{\substack{max \\ production}} = 0.79\,h$$

EXERCISE 7

$$A \underset{k_b}{\overset{k_f}{\rightleftharpoons}} B$$

inlet mixture
$$\begin{cases} T_{in} = 330\,K \\ \dot{F}_{tot}^{in} = 163\ kmol/h \\ C_A^{in} = 9.30\ kmol/m^3 \\ x_A^{in} = 0.90 \\ x_B^{in} = 0 \\ x_I^{in} = 0.10 \end{cases}$$

thermodynamic data
$$\begin{cases} \Delta H_R(T_0) = \Delta H_R^0 = -6900\ J/mol \\ T_0 = 300\,K \\ C_{pA} = 131\ J/mol/K \\ C_{pB} = 171\ J/mol/K \\ C_{pI} = 161\ J/mol/K \end{cases}$$

kinetics
$$\begin{cases} k_f(T_f^*) = 31.1\ 1/h \\ T_f^* = 360\,K \\ E = 65700\ J/mol \end{cases}$$

thermodynamic equilibrium
$$\begin{cases} K_{eq}(T_{eq}^*) = 3.03 \\ T_{eq}^* = 60\,°C = 333\,K \end{cases}$$

SOLUTION

a) Preliminary calculations

$$\begin{cases} \dot{F}_A^{in} = x_A^{in} \cdot \dot{F}_{tot}^{in} = 146.7\ kmol/h \\ \dot{F}_B^{in} = x_B^{in} \cdot \dot{F}_{tot}^{in} = 0 \\ \dot{F}_I^{in} = x_I^{in} \cdot \dot{F}_{tot}^{in} = 16.3\ kmol/h \end{cases} \qquad \begin{cases} \vartheta_A = \dfrac{x_A^{in}}{x_A^{in}} = 1 \\ \vartheta_B = \dfrac{x_B^{in}}{x_A^{in}} = 0 \\ \vartheta_I = \dfrac{x_I^{in}}{x_A^{in}} = 0.111 \end{cases}$$

$$\tilde{C}_p^{in} = \sum_j \vartheta_j \tilde{C}_{pj} = 148.8\ J/mol/K$$

$$\Delta\tilde{C}_p = \tilde{C}_{pB} - \tilde{C}_{pA} = 40\ J/mol/K$$

$$\Delta H_R^{in} = \Delta H_R^0 + \Delta\tilde{C}_p (T_{in} - T_0) = -5700\ J/mol$$

$$\Delta H^{*} = \Delta H(T_{eq}^{*}) = \widetilde{\Delta H}_R^{\circ} + \Delta\widetilde{C}_p (T_{eq}^{*} - T_0) = -5580 \ \mathrm{J/mol}$$

b) <u>kinetics</u>

$$K_f(T_f^{*}) = K_f^{*} = A \exp\left(-\frac{E}{RT_f^{*}}\right)$$

$$K_f(T) = A \exp\left(-E/RT\right)$$

$$\boxed{K_f(T) = K_f^{*} \exp\left(-\frac{E}{R}\left(\frac{1}{T} - \frac{1}{T_f^{*}}\right)\right) \qquad (E1)}$$

c) <u>equilibrium constant</u>

$$\frac{d \ln K_{eq}}{dT} = \frac{\Delta\widetilde{H}_R}{RT^2} \qquad \Delta\widetilde{H}_R = \Delta H_R^{\circ} + \Delta\widetilde{C}_p (T - T_0)$$

$$\ln K_{eq} = C' + \frac{\Delta\widetilde{C}_p}{R} \ln T - \frac{\Delta\widetilde{H}_0^{R} - \Delta\widetilde{C}_p T_0}{RT}$$

$$\ln \frac{K_{eq}}{K_{eq}^{*}} = -\left(\frac{\Delta H_R^{*}}{R} - \frac{\Delta\widetilde{C}_p}{R} T^{*}\right)\left(\frac{1}{T} - \frac{1}{T^{*}}\right) + \frac{\Delta\widetilde{C}_p}{R} \ln \frac{T}{T^{*}}$$

$$\boxed{K_{eq} = K_{eq}^{*} \cdot \exp\left[-\left(\frac{\Delta H_R^{*}}{R} - \frac{\Delta\widetilde{C}_p}{R} T^{\circ}\right)\left(\frac{1}{T} - \frac{1}{T^{\circ}}\right) + \frac{\Delta\widetilde{C}_p}{R} \ln \frac{T}{T^{\circ}}\right] \qquad (E2)}$$

in case $\Delta\widetilde{C}_p = 0 \implies K_{eq} = K_{eq}^{*} \exp\left(-\frac{\Delta H_R^{\circ}}{R}\left(\frac{1}{T} - \frac{1}{T^{\circ}}\right)\right)$

Van't Hoff

d) <u>formation rate of reference species</u>

$$R_A = -K_f \cdot C_A + K_b C_B = -K_f C_A^{\circ}(1 - X) + K_b C_A^{\circ} X$$

$$= \ldots = K_f C_A^{\circ}\left(-1 + X\left(1 + \frac{1}{K_{eq}}\right)\right) \qquad K_b = \frac{K_f}{K_{eq}}$$

$$\boxed{-R_A = K_f C_A^{\circ}\left(1 - \left(1 + \frac{1}{K_{eq}}\right) X\right) \qquad (E3)}$$

e) equilibrium conversion (useful)

$$k_f \cdot C_A^{eq} = k_b C_B^{eq} \rightarrow k_f C_A^{\circ}(1-X_{eq}) = k_b C_A^{\circ} X_{eq}$$

$$X_{eq} = \frac{K_{eq}}{1+K_{eq}}$$! be careful $K_{eq} = K_{eq}(T) \rightarrow X_{eq} = X_{eq}(T)$

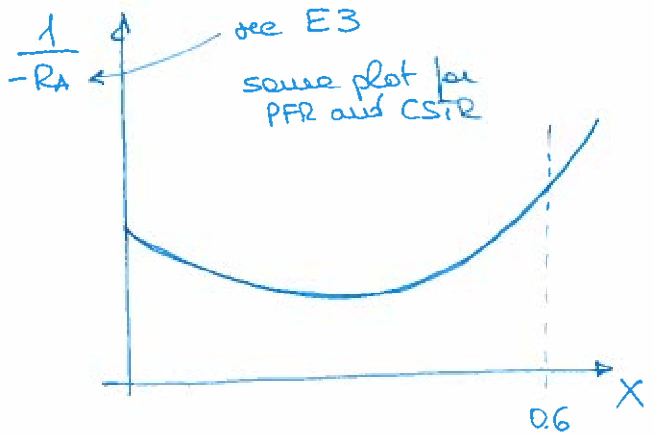
f) ENERGY BALANCE EQUATION

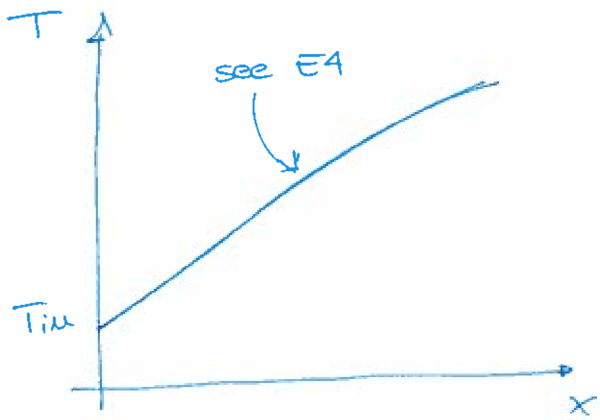
$$\int_{T_{in}}^{T} \widetilde{C_p}^{in}\, dT + \widetilde{\Delta H_R}(T) X = 0$$

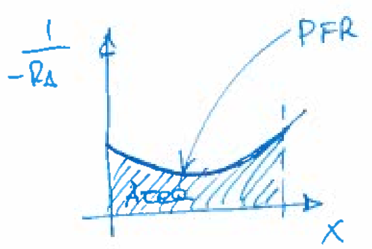
$$C_{p_j} = \text{const} \Longrightarrow \boxed{T = T_{in} + \frac{-\widetilde{\Delta H_R}^{in} X}{\widetilde{C_p}_{in} + \Delta \widetilde{C_p} X} \qquad (E\,4)}$$

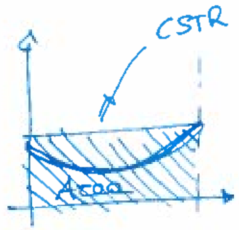
this is the same for PFR and CSTR cases

g) LEVENSPIEL'S PLOT









PFR

$V = \text{Area} \cdot F_A^{in}$

CSTR

$V = \text{Area} \cdot F_A^{in}$

PFR $\quad \text{Area} = \displaystyle\int_0^{X_f = 0.60} \frac{dX}{-R_A}$

X_f = is the desired final conversion

CSTR $\quad \text{Area} = \dfrac{1}{-R_A^f} \cdot X_f$

$$
\text{PFR}\quad \begin{cases} V = \dot{F}_A^{in} \displaystyle\int_0^{X_1} \frac{dX}{-\dot{R}_A} \\[2ex] T = T_{in} + \dfrac{-\widetilde{\Delta H}_R^{in}\, X}{\widetilde{Cp}_{in} + \Delta \widetilde{Cp}\, X} \end{cases}
$$

Alternative approach:

$$
\text{PFR}\quad \begin{cases} \dfrac{dX}{dV} = -\dfrac{\dot{R}_A}{\dot{F}_A^{in}} \\[2ex] \dfrac{dT}{dV} = \dfrac{\dot{Q}_R}{\dot{F}_A^{in}\left(\widetilde{Cp}^{in} + \Delta \widetilde{Cp}\, X\right)} \end{cases} \quad + \ \text{ICs} \begin{cases} X(V=0) = 0 \\ T(V=0) = T_{in} \end{cases}
$$

$$
\dot{Q}_R = \text{heat release} = \dot{R}_A\, \widetilde{\Delta H}_R
$$