Homework sheet 5

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- Solve the 1D wave equation

$$u_{tt} = c^2 u_{xx}$$

on the interval $[0, L]$, where $u(0, t) = 0$ and $u(L, T) = 0$ using an *implicit* finite difference scheme in MATLAB.

Suppose the string is 'plucked' at the midpoint giving the initial conditions of

$$u(x, 0) = \begin{cases} \frac{2}{L} x & 0 \le x < \frac{L}{2} \\ \frac{2}{L}(L - x) & \frac{L}{2} \le x < L \end{cases}$$

$$u_t(x, 0) = 0$$

Plot your solutions at five different time points

Hint: one such implicit scheme is given by:

$$\frac{U_j^{n+1} - 2U_j^n + U_j^{n-1}}{\Delta t^2} = \frac{c^2}{2\Delta x^2} \left(U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1} + U_{j+1}^{n-1} - 2U_j^{n-1} + U_{j-1}^{n-1} \right)$$