Homework sheet 4

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Let $\Omega := [0,1] \times [0,1]$

- Solve the 2D Helmholtz equation on Ω with homogeneous boundary conditions:

  $$\Delta u + u = (1 + 2\pi^2)\cos(\pi x)\cos(\pi y)$$

  $$\nabla u \cdot \eta = 0$$

  [Hint: you may wish to study the code used in the 2D Poisson equation example in the lecture notes, although it can be significantly shortened by assembling the linear system using matrix commands such as "kron" and "eye"]

  What can be done in the formulation of the Neumann problem to assure that the final linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is symmetric?

- Use forward and backward (as opposed to central) difference quotients to approximate the Neumann boundary condition. [Hint: we need to be able to define values such as $U_{-1,0}$, so the choice of difference quotient (forwards or backwards) will change depending on which part of the boundary is considered]

  What happens to the order of convergence of the method?

- Solve the 2D heat equation

  $$u_t - \Delta u = 0$$

  $$u(x,0) = \exp\left(\left(x_1 - \frac{1}{2}\right)^2 + \left(x_2 - \frac{1}{2}\right)^2\right)$$

  on $\Omega \times [0,5]$ with a zero Dirichlet boundary condition using an *implicit* Euler scheme. Plot the evolving solution using your favourite method.