Euler Potential Adjustments for Coupling with the SWMF

Current Euler Potential Method

The current Euler potential method uses the assumption that the magnetic field at the surface of the Earth is a dipole, and picks equally spaced foot points to trace from one hemisphere to the other. Because the potentials are fixed along a field line this gives us a set of equally spaced potentials which correspond to field lines that span our required domain. Using this method we stick to the standard dipolar definition of Euler potentials at the surface of the Earth

$$\alpha = A(r = 1)\,(\sin\theta)^2\,, \qquad \beta = \varphi$$

where *A* is related to the field strength and radial distance, which for the surface of the Earth in a dipole is a constant.

SWMF Euler Potential Method

Because the tracing from the SWMF is given from the equator to the surface of the Earth at regularly spaced spatial positions assigning Euler potentials is trickier. Ideally the potentials would work the same way as the currently used method, however, any error in the tracing introduces a large error in the potentials, making this highly dependent on the size of the cells and the tracing accuracy of the SWMF. To help with this we use the fact that the Euler potential definition

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

allows for adjustments to the potentials akin to the gauge transformations of the magnetic vector potential. Stern (1967) has a detailed description of this, while Zaharia (2008) gives an example directly related to the SCB code. We adjust the Euler potential equations to

$$\alpha = A(r)\,(\sin\theta)^2 + F(\beta), \qquad \beta = \varphi + G(\alpha)$$

where *F* and *G* are some function and *A* is now dependent on the radial position and the magnetic field strength at that position. For a dipole *F* and *G* would be zero. By using this assumption it is possible to adjust each field line independently, thus taking equally spatially spaced equatorial values and translating them into equally potential spaced surface values. Engel (2015) has a similar example using an IGRF field. With equally potential spaced surface values we can apply the current Euler potential method and proceed with the force balancing as usual. Note that the assumptions made here require that the minimum B point along a field line is mono-valued and that *F* and *G* are “well behaved”. For *F* this means periodicity in the azimuthal direction for each radial ring, and for *G* it means a continuous radial function at each azimuthal angle (e.g. no X-lines). Since the SCB code requires each of these to be true as well we disregard all field lines from the SWMF that don’t meet these criteria and construct the SCB domain based on the field lines left.

Stern, D. (1967), Geomagnetic Euler potentials, *J. Geophys. Res.*, 72(15), 3995– 4005, doi:10.1029/JZ072i015p03995.

Zaharia, S. (2008), Improved Euler potential method for three-dimensional magnetospheric equilibrium, *J. Geophys. Res.*, 113, A08221, doi:10.1029/2008JA013325.

Engel, M. A., Kress, B. T., Hudson, M. K., and Selesnick, R. S. (2015), Simulations of inner radiation belt proton loss during geomagnetic storms, *J. Geophys. Res. Space Physics*, 120, 9323– 9333, doi:10.1002/2015JA021568.