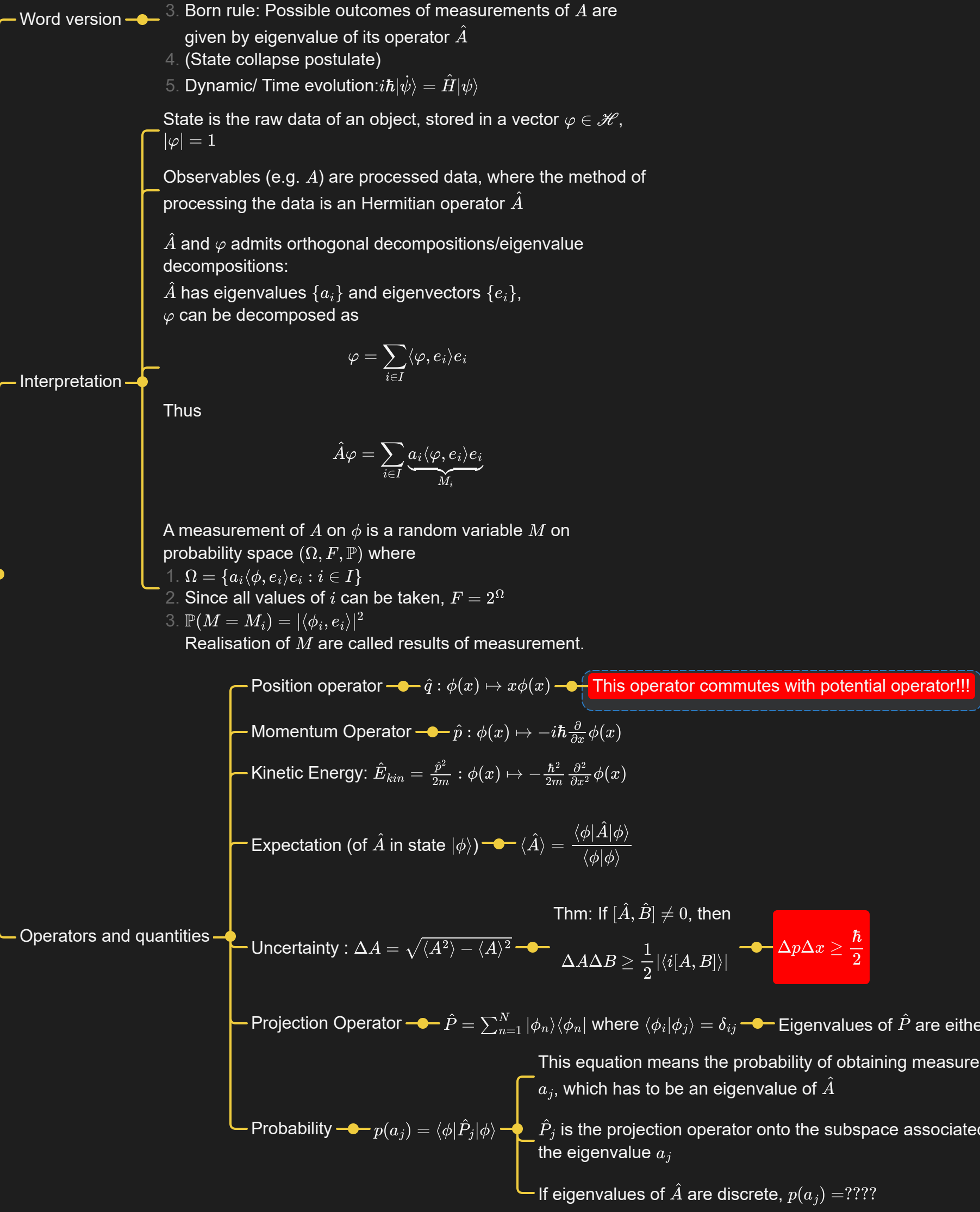


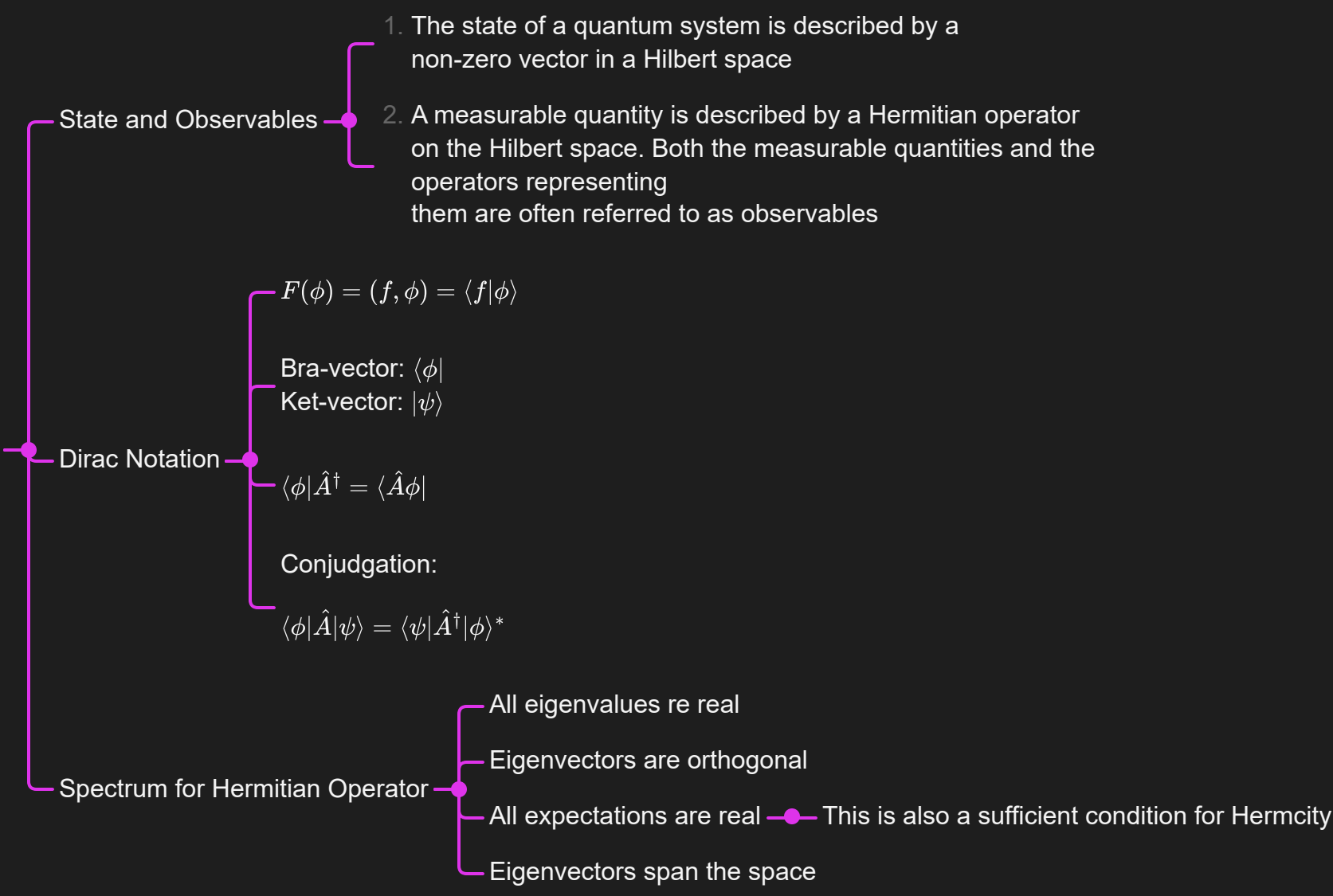
QM

Five Principles

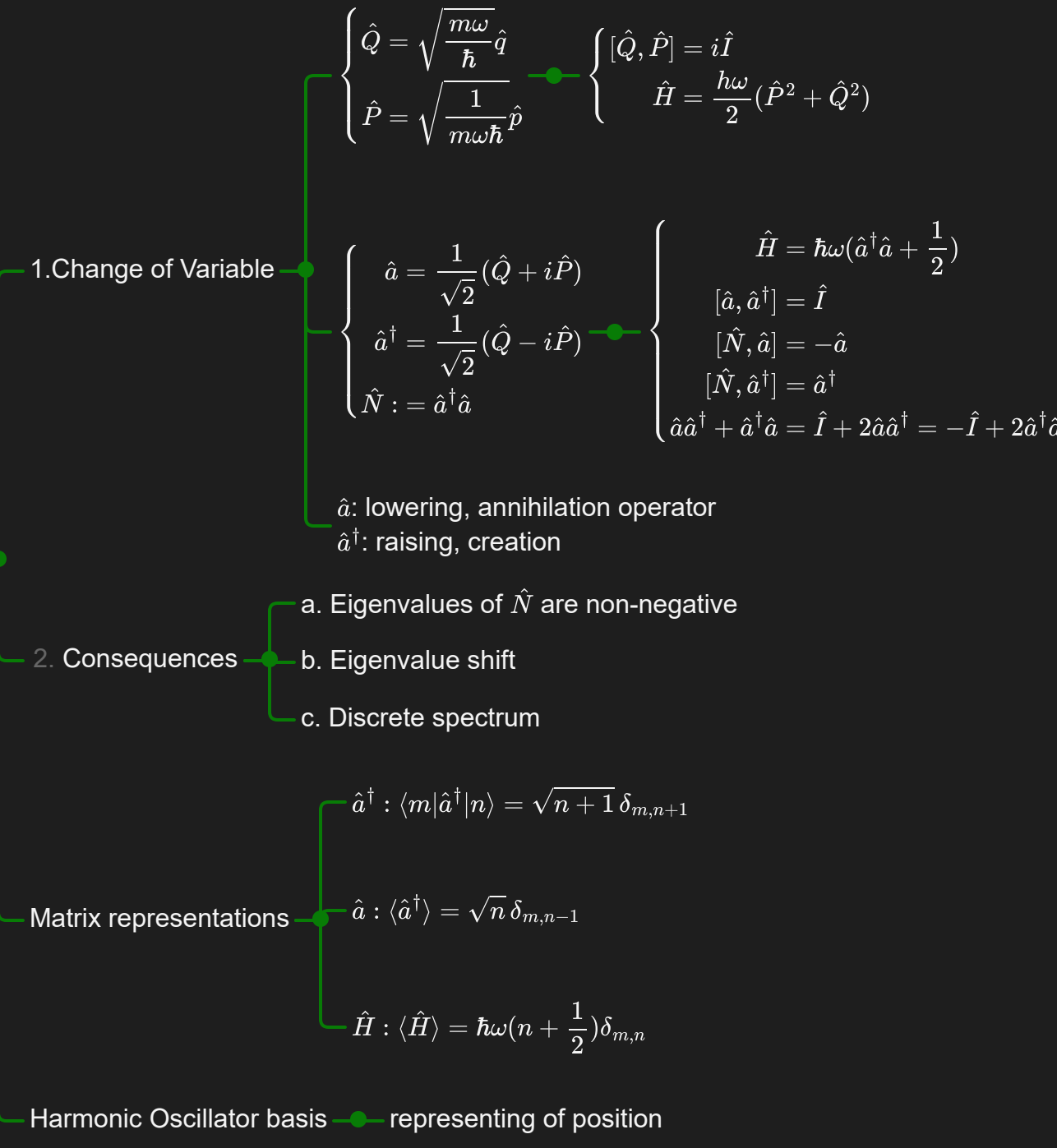


Mathematical Foundation

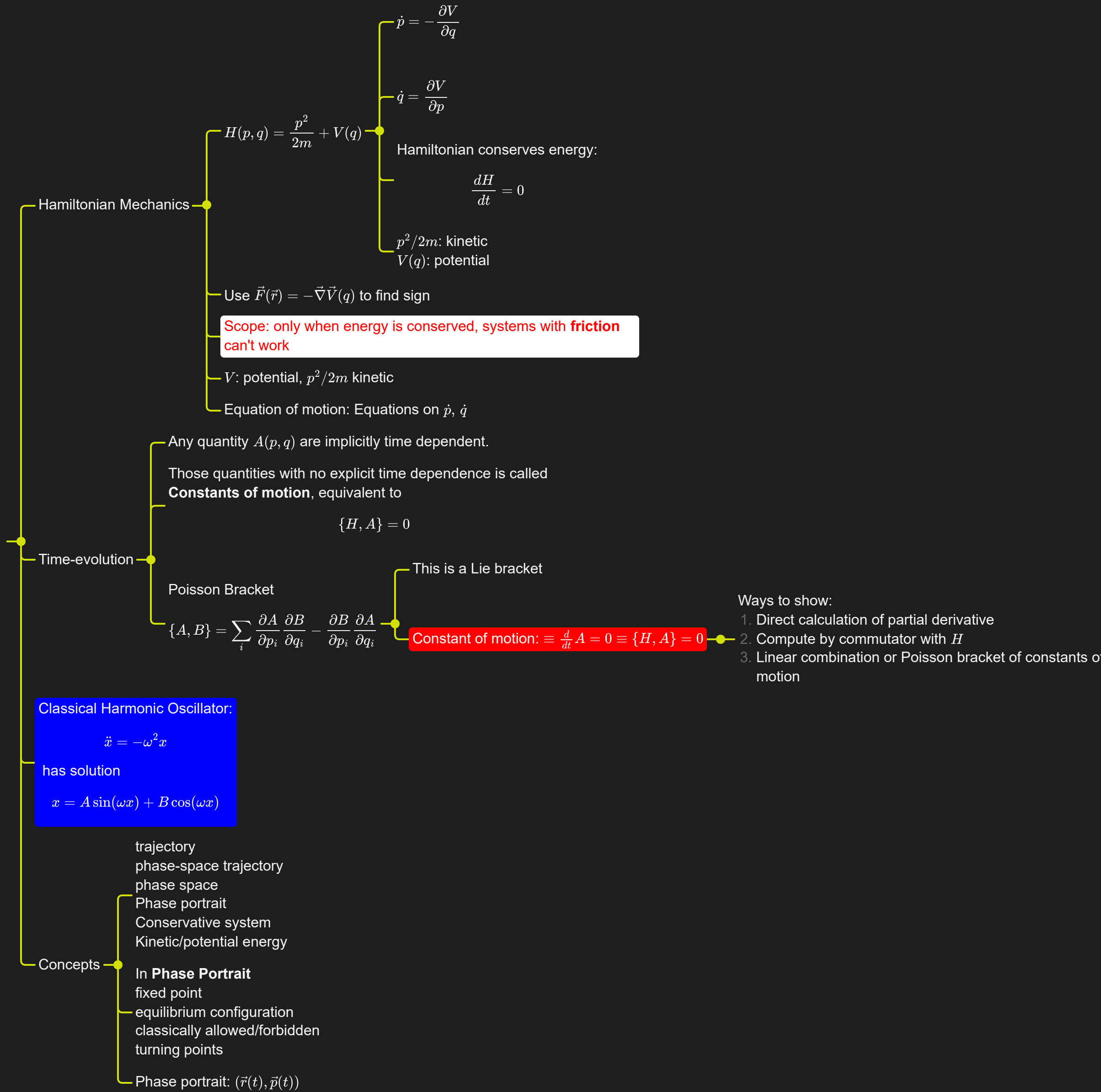
Mathematical Background



Quantum Harmonic Oscillator



Classic Dynamic Results



Models

Representations

Spectra

Angular Momentum

$$[\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z$$
$$[\hat{J}_x, \hat{J}_z] = i\hbar \hat{J}_y$$
$$[\hat{J}^2, \hat{J}_i] = 0$$

$$\hat{J}^2 = \hat{J}_1^2 + \hat{J}_2^2 + \hat{J}_3^2$$

$|\beta, m\rangle$ : Simultaneous eigenvector

$$\hat{J}^2 |\beta, m\rangle = \beta(\beta, m), \quad \hat{J}_3 |\beta, m\rangle = \hbar m |\beta, m\rangle$$

Symmetry gives  $\hat{J}_1$  and  $\hat{J}_2$  same eigenvalues

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(x)$$

$$\langle \hat{H} \rangle = \int_{\mathbb{R}^3} \phi^*(x, t) (\hat{H} \phi(x)) dx$$

For 1-d,  $\phi$  is a solution  $\implies \phi^*$  is a solution

$\psi$  interpreted as probability **amplitude**

$|\psi|^2$  is probability density, after normalisation

All functions are assumed to be  $L^2$  thus decay to zero at infinity

Also assumed  $\phi$  twice differentiable.

$\psi$  can be normalised  $\iff \psi \in L^2$

Superposition principle: If  $\phi_1$  and  $\phi_2$  are solutions, then  $c_1 \phi_1 + c_2 \phi_2$  is still a solution.

$$\text{Usually } \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$$

For conjugation  $\psi^*$ :

$$-i\hbar \psi^*(\vec{r}, t) = \hat{H} \psi^*(\vec{r}, t)$$

$$i\hbar \psi^*(\vec{r}, t) = \hat{H} \psi(\vec{r}, t)$$

$$\langle \hat{H} \rangle = \frac{\int_{-\infty}^{\infty} \phi^*(x, t) \hat{H} \phi(x, t) dx}{\int_{-\infty}^{\infty} |\phi(x, t)|^2 dx} = \langle \phi, \hat{H} \phi \rangle = \langle \hat{H} \rangle: \text{expectation value}$$

$$\hat{H} \phi_E(\vec{r}) = E \phi_E(\vec{r})$$

Eigenvalues/eigenenergies of Hamiltonian:  $E$

Eigenvalues/eigenfunctions/engensates:  $\phi_E(\vec{r})$

Eigenstates form orthogonal basis:

$$\int_{\text{whole space}} \phi_E(\vec{r}) \phi_{E'}(\vec{r}) dV = \delta_{EE'}$$

Stationary solution method for time-independent equation: TBF

Start with seperable assumption:

$$\psi(x, t) = \phi(x) \chi(t)$$

Stationary States

$$\psi(x, t) = \chi(t) e^{-iEt/\hbar} \phi_E(x)$$

End up having basis of solutions

$$\phi(t, x) = \sum_n a_n e^{iE_n t/\hbar} \phi_{E_n}(x)$$

where  $\phi_E$  is the eigen function corresponding to value  $E$

Heisenberg's equation

Ehrenfest theorem

Hadamard Lemma

Relation with Lie theory

Dynamics of Expectations