Question 3

a)

```
> a := expand( (x^4-3*x^3*y-x^2-y)*(8*x-4*y+12)*(2*y^2-2) )
```

$$a := 16x^5y^2 - 56x^4y^3 + 24x^3y^4 + 24x^4y^2 - 72x^3y^3 - 16x^5 + 56x^4y - 40x^3y^2 + 8x^2y^3 - 24x^4 + 72x^3y - 24x^2y^2 - 16xy^3 + 8y^4 + 16x^3 - 8x^2y - 24y^3 + 24x^2 + 16xy - 8y^2 + 24y \quad (1)$$

```
> gcd(coeff(a, x));
```

$$16y^3 - 16y \quad (2)$$

c)

```
> a := expand( (x^4-3*x^3*y-x^2-y)*(2*x-y+3)*(8*y^2-8) );
```

$$a := 16x^5y^2 - 56x^4y^3 + 24x^3y^4 + 24x^4y^2 - 72x^3y^3 - 16x^5 + 56x^4y - 40x^3y^2 + 8x^2y^3 - 24x^4 + 72x^3y - 24x^2y^2 - 16xy^3 + 8y^4 + 16x^3 - 8x^2y - 24y^3 + 24x^2 + 16xy - 8y^2 + 24y \quad (3)$$

```
> b := expand( (x^3*y^2+x^3+x^2+3*x+y)*(2*x-y+3)*(12*y^3-12));
```

$$b := 24x^4y^5 - 12x^3y^6 + 36x^3y^5 + 24x^4y^3 - 12x^3y^4 - 24x^4y^2 + 72x^3y^3 - 12x^2y^4 - 36x^3y^2 + 108x^2y^3 - 12xy^4 - 12y^5 - 24x^4 + 12x^3y + 108xy^3 + 36y^4 - 60x^3 + 12x^2y - 108x^2 + 12xy + 12y^2 - 108x - 36y \quad (4)$$

```
> c := gcd(content(a, x), content(b,x ). x);
  r0 := primpart(a, x);
  r1 := primpart(b, x);
```

$$c := 1$$

$$r0 := 2x^5 - 7x^4y + 3x^3y^2 + 3x^4 - 9x^3y - 2x^3 + x^2y - 3x^2 - 2xy + y^2 - 3y$$

$$r1 := 2x^4y^2 - x^3y^3 + 3x^3y^2 + 2x^4 - x^3y + 5x^3 - x^2y + 9x^2 - xy - y^2 + 9x + 3y \quad (5)$$

```
> while r1 <> 0 do
      r := prem(r0, r1, x):
      r0, r1 := r1, primpart(r, x):
      print(r0);
      print(r1);
  od:
```

$$2x^4y^2 - x^3y^3 + 3x^3y^2 + 2x^4 - x^3y + 5x^3 - x^2y + 9x^2 - xy - y^2 + 9x + 3y$$

$$-2x^3y^4 + x^2y^5 + 6x^3y^3 - 6x^2y^4 - 2xy^5 + y^6 - 10x^3y^2 + 30x^2y^3 - 2xy^4 - 6y^5 + 6x^3y - 18x^2y^2 + 20xy^3 + 11y^4 - 6x^3 + 28x^2y - 2xy^2 - 9y^3 - 3x^2 + 21xy + 9y^2 + 9x$$

$$-2x^3y^4 + x^2y^5 + 6x^3y^3 - 6x^2y^4 - 2xy^5 + y^6 - 10x^3y^2 + 30x^2y^3 - 2xy^4 - 6y^5 + 6x^3y - 18x^2y^2 + 20xy^3 + 11y^4 - 6x^3 + 28x^2y - 2xy^2 - 9y^3 - 3x^2 + 21xy + 9y^2 + 9x$$

$$-2x^2y^7 + xy^8 + 12x^2y^6 - 9xy^7 - 30x^2y^5 + 17xy^6 + 8y^7 + 176x^2y^4 - 85xy^5 - 48y^6 - 52x^2y^3 + 258xy^4 + 88y^5 + 200x^2y^2 - 130xy^3 - 72y^4 + 18x^2y + 291xy^2 + 72y^3 + 90x^2 - 9y^2 + 135x + 27y$$

$$-2x^2y^7 + xy^8 + 12x^2y^6 - 9xy^7 - 30x^2y^5 + 17xy^6 + 8y^7 + 176x^2y^4 - 85xy^5 - 48y^6 - 52x^2y^3 + 258xy^4 + 88y^5 + 200x^2y^2 - 130xy^3 - 72y^4 + 18x^2y + 291xy^2 + 72y^3 + 90x^2 - 9y^2 + 135x + 27y$$

$$-2x + y - 3$$

$$-2x+y-3$$

$$0 \qquad \textbf{(6)}$$

```
> c;
  g := c*r0;
```

$$1$$

$$g := -2x+y-3 \qquad \textbf{(7)}$$

```
> normal(g);
```

$$-2x+y-3 \qquad \textbf{(8)}$$