Question 1

a)

```
> my_gcdex := proc(a::polynom, b::polynom)
      local s0, s1;
      local t0, t1;
      local r0, r1;
      local k;
      local q;
      local g;
      s0, s1 := 1, 0;
      t0, t1 := 0, 1;
      r0, r1 := a, b;
      k := 1;
      while r1 <> 0 do
         q := quo(r0, r1, x);
         r0, r1 := r1, expand(r0 - r1 * q);
         s0, s1 := s1, expand(s0 - s1 * q);
         t0, t1 := t1, expand(t0 - t1 * q);
         print(r1);
         print(s1);
         print(t1);
      od;
      g := expand(s0*a + t0*b);
      g := g / lcoeff(g, x);
      return (s0, t0, g);
  end;
```

```
my_gcdex := proc(a::polynom, b::polynom)
    local s0, s1, t0, t1, r0, r1, k, q, g;
    s0, s1 := 1, 0;
    t0, t1 := 0, 1;
    r0, r1 := a, b;
    k := 1;
    while r1 <> 0 do
        q := quo(r0, r1, x);
        r0, r1 := r1, expand(r0 - r1 * q);
        s0, s1 := s1, expand(s0 - s1 * q);
        t0, t1 := t1, expand(t0 - t1 * q);
        print(r1);
        print(s1);
        print(t1)
    end do;
    g := expand(s0 * a + t0 * b);
    g := g / lcoeff(g, x);
    return s0, t0, g
end proc
```

(1)

```
>  a := expand((x+1)*(2*x^4-3*x^3+5*x^2+3*x-1));
```

$$a := 2x^5 - x^4 + 2x^3 + 8x^2 + 2x - 1 \tag{2}$$

```
> b := expand((x+1)*(7*x^4+5*x^3-2*x^2-x+4));
```

$$b := 7x^5 + 12x^4 + 3x^3 - 3x^2 + 3x + 4 \tag{3}$$

```
> my_g := my_gcdex(a, b);
```

$$-\frac{31}{7}x^4 + \frac{8}{7}x^3 + \frac{62}{7}x^2 + \frac{8}{7}x - \frac{15}{7}$$

$$1$$

$$-\frac{2}{7}$$

$$\frac{19761}{961}x^3 + \frac{819}{31}x^2 + \frac{3052}{961}x - \frac{2576}{961}$$

$$\frac{2996}{961} + \frac{49x}{31}$$

$$\frac{105}{961} - \frac{14x}{31}$$

$$\frac{14177633}{18595101}x^2 - \frac{9106436}{18595101}x - \frac{23284069}{18595101}$$

$$-\frac{667895}{18595101} + \frac{906223}{6198367}x + \frac{961}{2823}x^2$$

$$-\frac{1995997}{6198367} + \frac{3227999}{18595101}x - \frac{1922}{19761}x^2$$

$$\frac{13051647258853\,x}{209162619649} + \frac{13051647258853}{209162619649}$$

$$\frac{1042546734299}{209162619649} - \frac{1056158348231}{209162619649}x - \frac{4525452540168}{209162619649}x^2 - \frac{130165707}{14177633}x^3$$

$$\frac{3523548498288}{209162619649} - \frac{165062513210}{209162619649}x + \frac{78080829099}{209162619649}x^2 + \frac{37190202}{14177633}x^3$$

$$0$$

$$\frac{836650478596}{13051647258853} - \frac{209162619649}{13051647258853}x - \frac{418325239298}{13051647258853}x^2 + \frac{1045813098245}{13051647258853}x^3 + \frac{209162619649}{1864521036979}x^4$$

$$\frac{209162619649}{13051647258853} - \frac{627487858947}{13051647258853}x - \frac{1045813098245}{13051647258853}x^2 + \frac{627487858947}{13051647258853}x^3 - \frac{418325239298}{13051647258853}x^4$$

$$my\_g := \frac{1042546734299}{209162619649} - \frac{1056158348231}{209162619649}x - \frac{4525452540168}{209162619649}x^2 - \frac{130165707}{14177633}x^3,\ \frac{3523548498288}{209162619649} - \frac{165062513210}{209162619649}x + \frac{78080829099}{209162619649}x^2 + \frac{37190202}{14177633}x^3, x + 1 \tag{4}$$

```
> g := gcdex(a, b, x, 's', 't')
```

$$g := x + 1 \tag{5}$$

b)

```
> a := x^3 - 1; b := x^2 + 1; c := x^2;
```

$$a := x^3 - 1$$
$$b := x^2 + 1$$
$$c := x^2 \quad \textbf{(6)}$$

```
> g := gcdex(a, b, x, 's', 't');
  s, t;
```

$$g := 1$$
$$\frac{x}{2} - \frac{1}{2}, -\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2} \quad \textbf{(7)}$$

```
> d := c/g;
  q := quo(d*s, b/g, x);
```

$$d := x^2$$
$$q := \frac{x}{2} - \frac{1}{2} \quad \textbf{(8)}$$

```
> sigma := expand(d*s - (b/g) * q);
  degree(sigma)
```

$$\sigma := -\frac{x}{2} + \frac{1}{2}$$
$$1 \quad \textbf{(9)}$$

```
> tau := expand(d*t + q*a);
  degree(tau);
```

$$\tau := -\frac{1}{2}x + \frac{1}{2} + \frac{1}{2}x^2$$
$$2 \quad \textbf{(10)}$$

```
> expand(sigma*a + tau* b) - c
```

$$0 \quad \textbf{(11)}$$