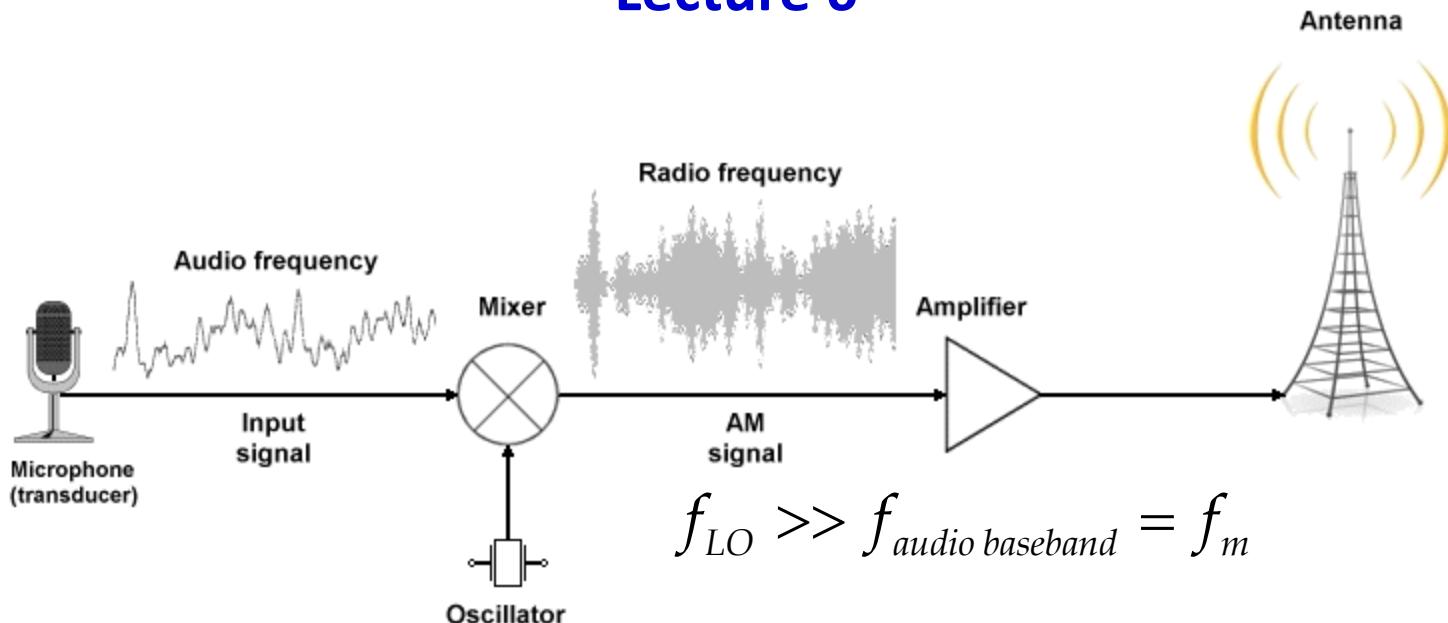


# Amplitude Modulation and demodulation

## Amplitude Modulation – Early Radio EE 442 – Spring Semester Lecture 6



[http://www.technologyuk.net/telecommunications/telecom\\_principles/amplitude\\_modulation.shtml](http://www.technologyuk.net/telecommunications/telecom_principles/amplitude_modulation.shtml)

## Why Use Modulation?

Most input signals, commonly created by transducers, can't be sent directly over the communication channel. We refer to these signals as **baseband signals** (*i.e.*, messages or information).

Instead, a carrier wave, whose properties are better suited to the transmission requirements, is modified (modulated) to represent the signal.

**Modulation** is the systematic alteration of the carrier wave so that it carries the message or information intended to be communicated.

A. Bruce Carlson, *Communication Systems: An Introduction to Signals and Noise in Electrical Communication*, 2<sup>nd</sup> ed., McGraw-Hill Book Company, New York, 1975; pp. 5-7.

# Reasons for Using Modulation

**Modulation for ease of radiation** – Antennas must be greater than one-tenth of the wavelength ( $\lambda/10$ ); thus, low frequency baseband signals would require overly large antennas.

**Modulation for frequency assignment** – The FCC assigns frequency bands to each radio application. Different carrier frequencies are used to meet this need.

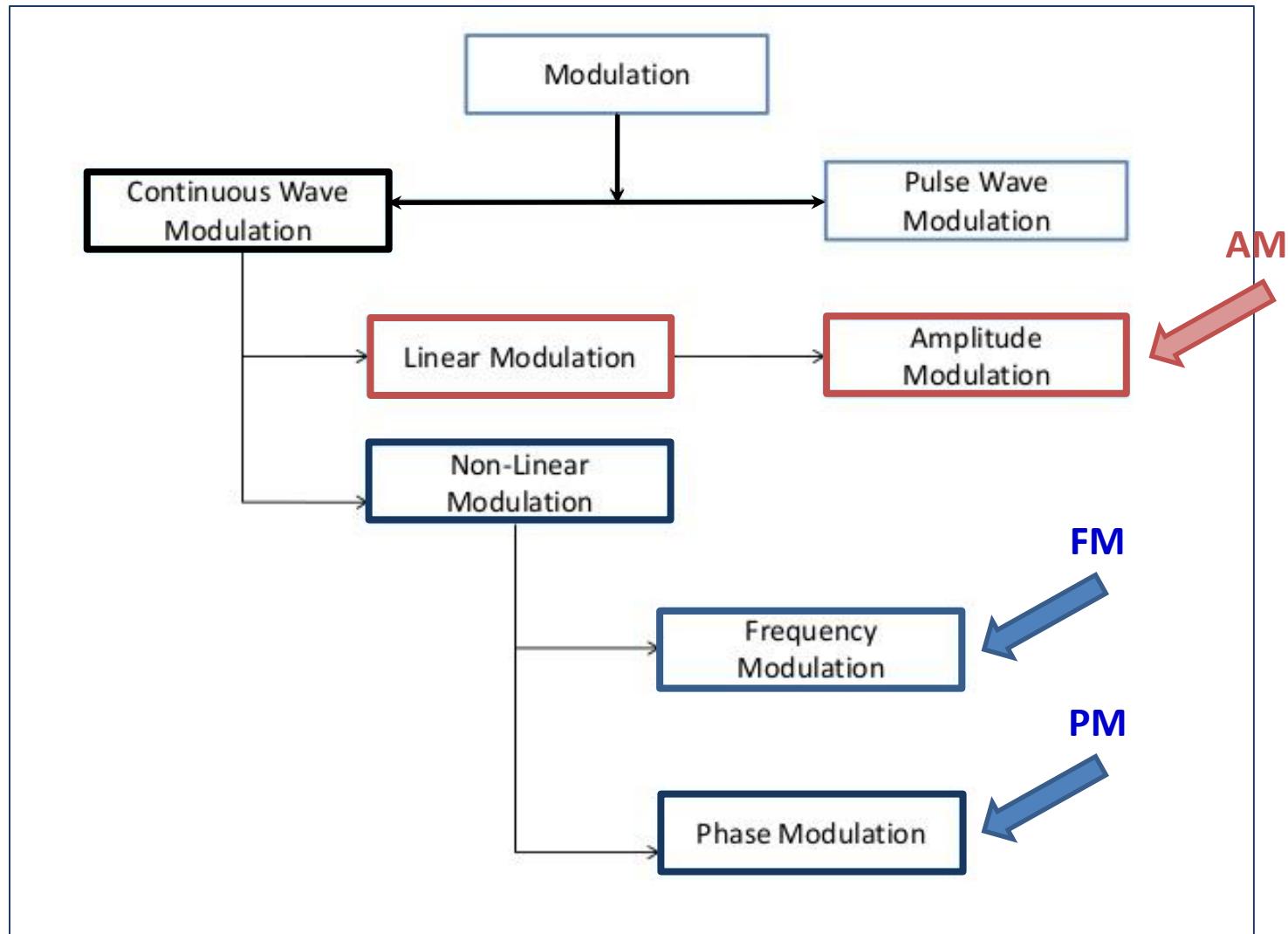
**Modulation for multiplexing** – This allows for multiple signals to be carried on a single transmission medium (multiplexing is one form of modulation).

**Modulation to overcome equipment limitations** – Modulation is used to place signals in a portion of the spectrum where equipment limitations are minimal or most easily met.

**Modulation to reduce noise and interference** – Some types of modulation are useful for reducing noise and interference.

A. Bruce Carlson, *Communication Systems: An Introduction to Signals and Noise in Electrical Communication*, 2<sup>nd</sup> ed., McGraw-Hill Book Company, New York, 1975; pp. 5-7.

# Modulation Options



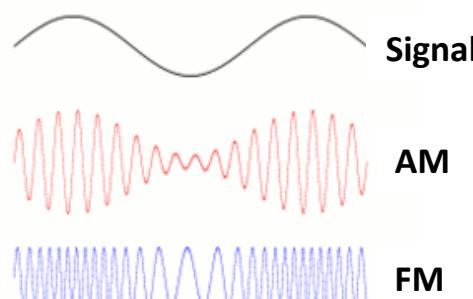
# Baseband versus Carrier Communication

**Baseband communication** is the transmission of a message as generated is transmitted without frequency translation.

**Carrier communication** requires the modulation of the message onto a carrier signal to transmit it over a different frequency band. We use modulators to do this frequency translation.

(Note: “Pulse modulated” signals, such as PAM, PWM, PPM, PCM and DM are actually baseband digital signal coding (and **not** the result of frequency conversion).

**Use of Sinusoidal Carrier Signal:** Using a sine waveform there are three parameters which we can use to “modulate” a message onto the carrier – they are the **amplitude, frequency and phase** of the sinusoidal carrier.



AM Modulation -- Radio

## Amplitude Modulation Description

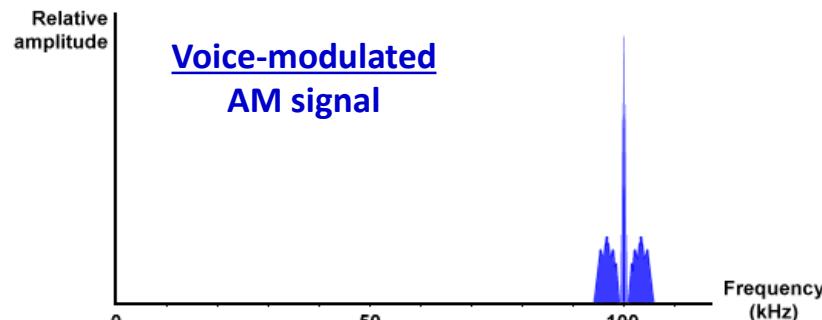
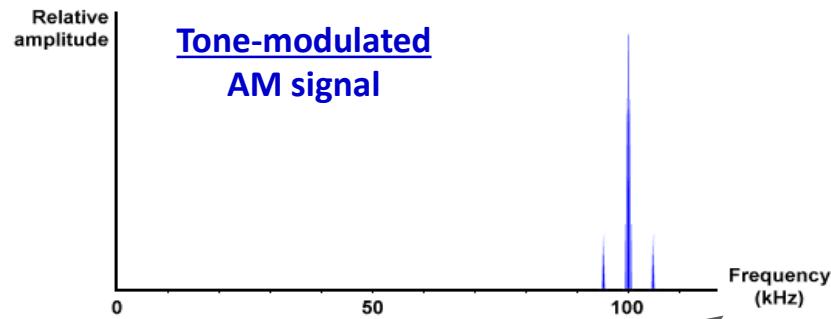
***Amplitude modulation (AM)*** is a modulation technique where the *amplitude* of a high-frequency sine wave (called a *radio frequency*) is varied in direct proportion to the *modulating signal  $m(t)$* . The modulating signal contains the intended message or information – sometimes consisting of audio data, as in AM radio broadcasting, or two-way radio communications.

The high-frequency sinusoidal waveform (*i.e., carrier*) is modulated by combining it with the message signal using a ***multiplier*** or ***mixer*** (Note: mixing is a nonlinear operation because it generates new frequencies).

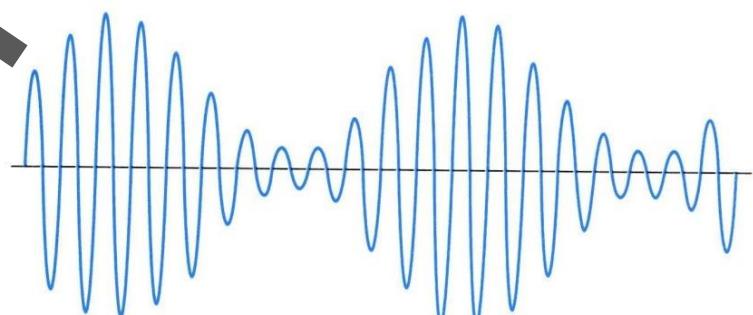
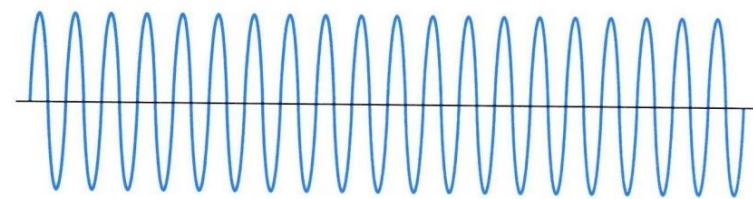
**Agbo & Sadiku; Section 3.2, pp. 84 to 99**

# Amplitude Modulation in Pictures

Frequency Domain



Time Domain



# **Amplitude Modulation Advantages & Disadvantages**

As with any technology there are advantages and disadvantages to be considered.

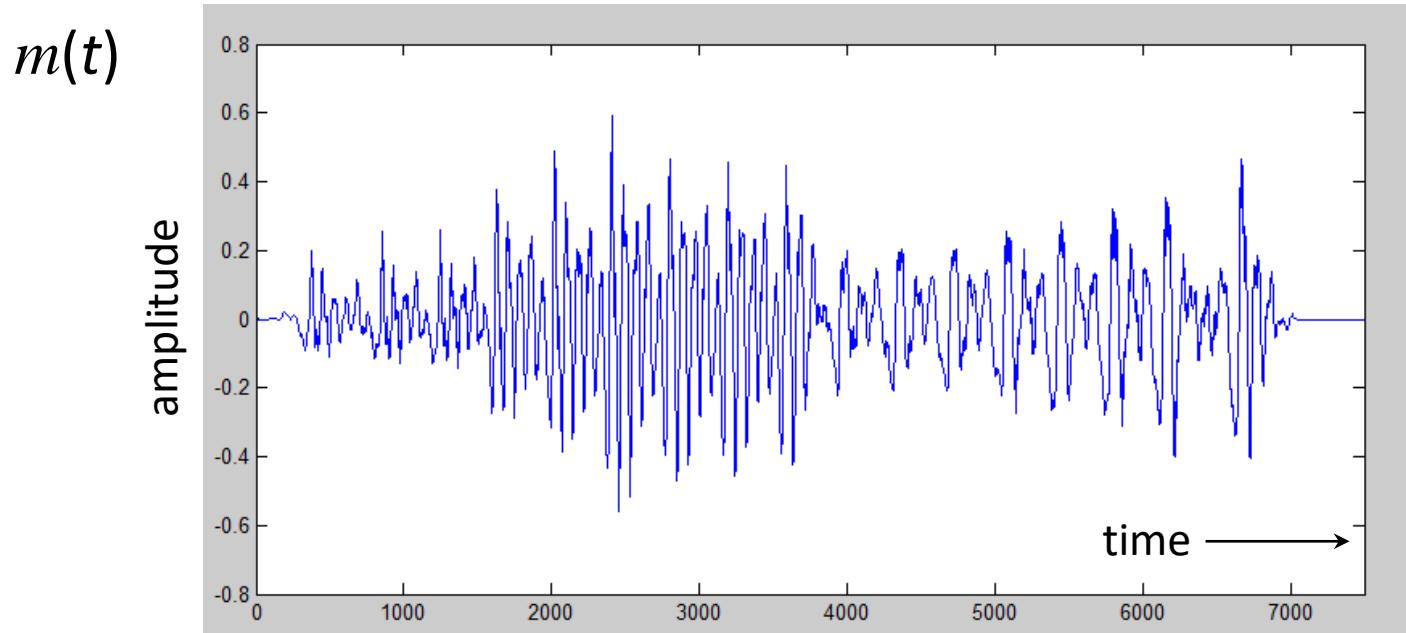
## **Advantages**

- It is simple to implement
- It can be demodulated using a circuit consisting of very few components
- AM receivers are inexpensive because no specialized components are required

## **Disadvantages**

- It is not efficient with respect to power usage
- It is not efficient in bandwidth; requires a bandwidth equal to twice the highest audio frequency
- It is prone to high levels of noise because most noise is amplitude based and AM detectors are sensitive to it

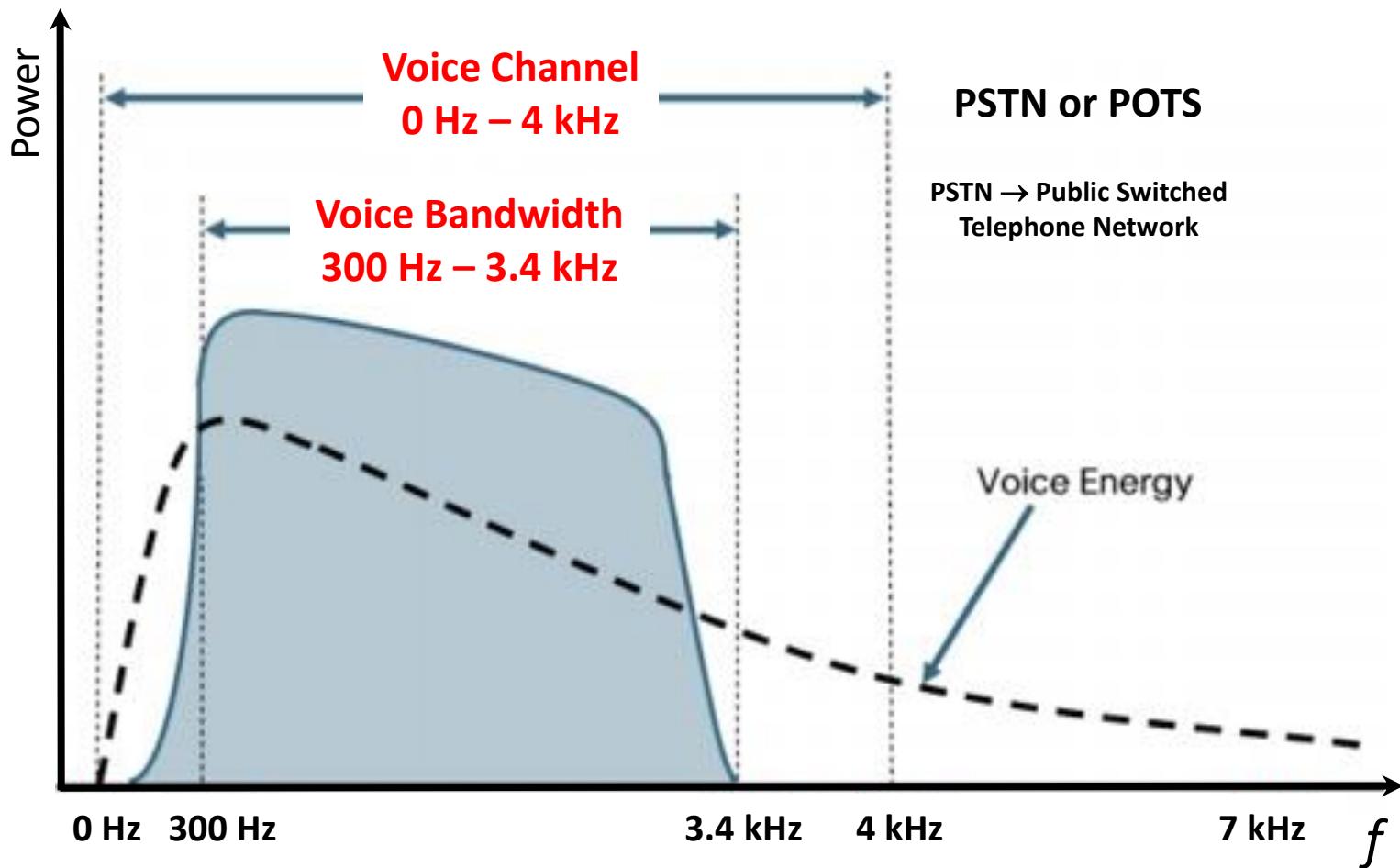
## Example: Voice Signal – 300 Hz to 3400 Hz Baseband



Symbol  $m(t)$  represents the source's message signal.

### Time Domain Display

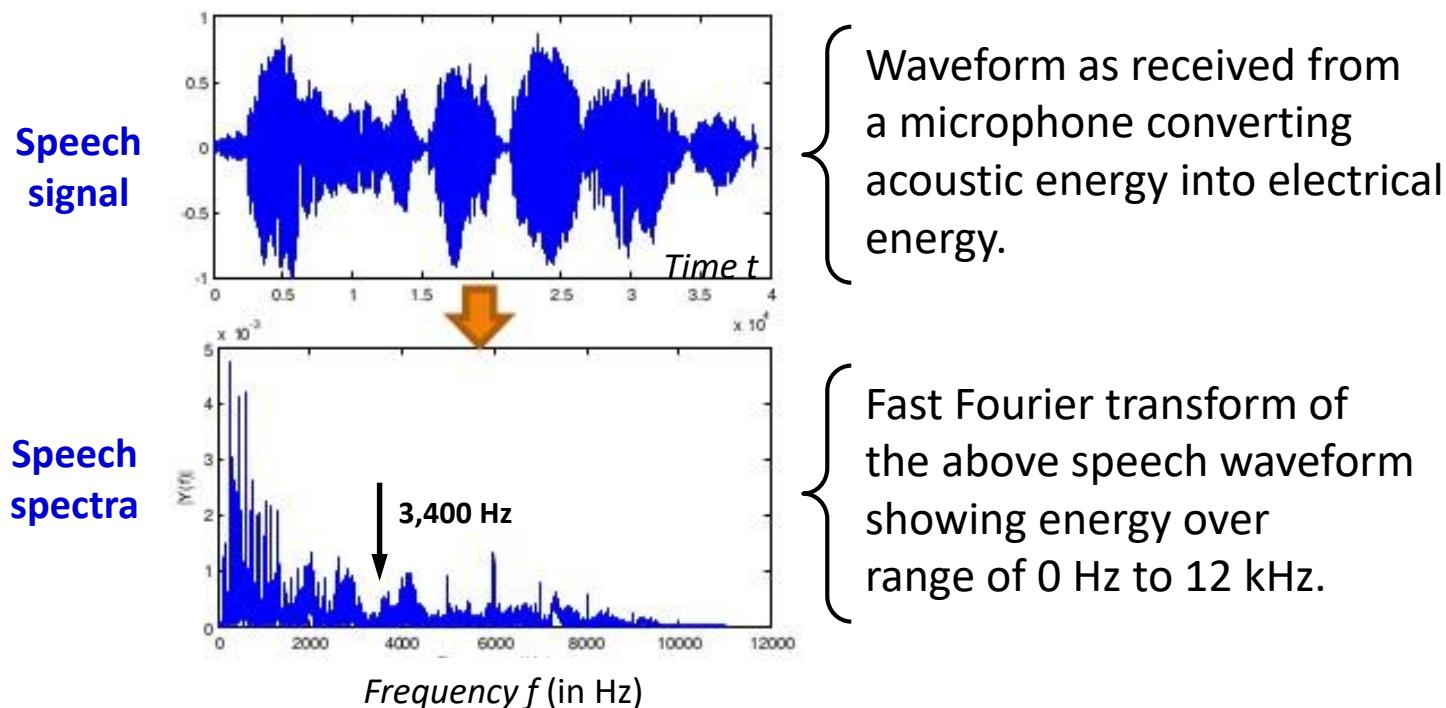
# Voice Band for Telephone Communication



Frequency Domain

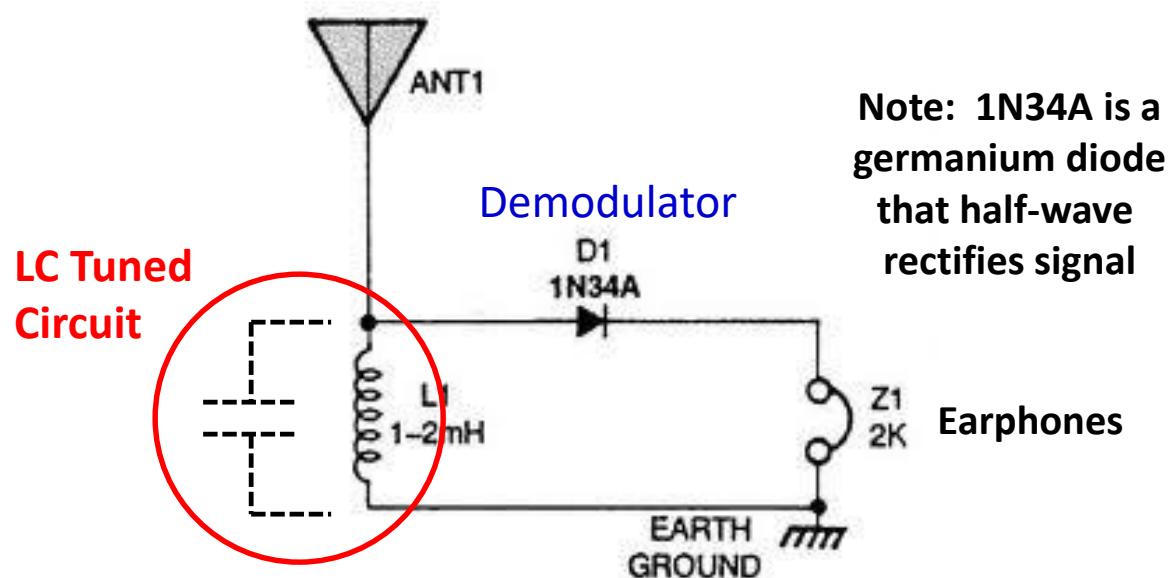
# Representative Voice Spectrum for Human Speech

For the telephone AT&T determined many years ago that speech could be easily recognized when the lowest frequencies and frequencies above 3.4 kilohertz were cutoff.

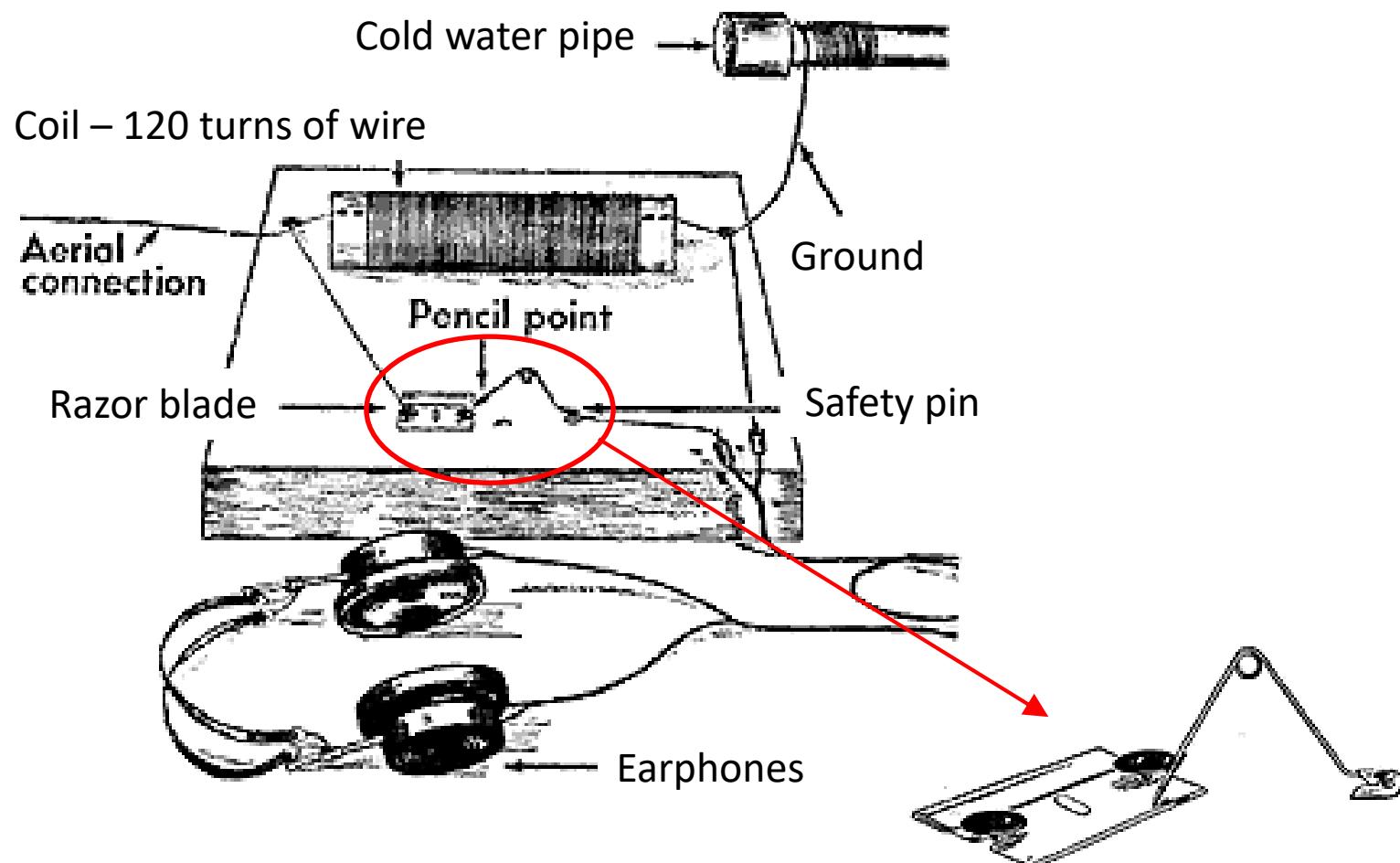


# Early AM Crystal Radio Receiver (Minimalist Radio)

A **crystal radio receiver**, also called a **crystal set** or **cat's whisker receiver**, is a very simple radio receiver, popular in the early days of radio. It needs no other power source but that received solely from the power of radio waves received by a wire antenna. It gets its name from its most important component, known as a crystal detector, originally made from a piece of crystalline mineral such as galena. This component is now called a diode.



## Foxhole Radio (as used in World war I)



<http://bizarrelabs.com/foxhole.htm>

## Foxhole Radio (as built in a modern shop today)



<http://www.peeblesoriginals.com/projects/images/foxhole1.jpg>

## Crystal Radio Receiver from 1922

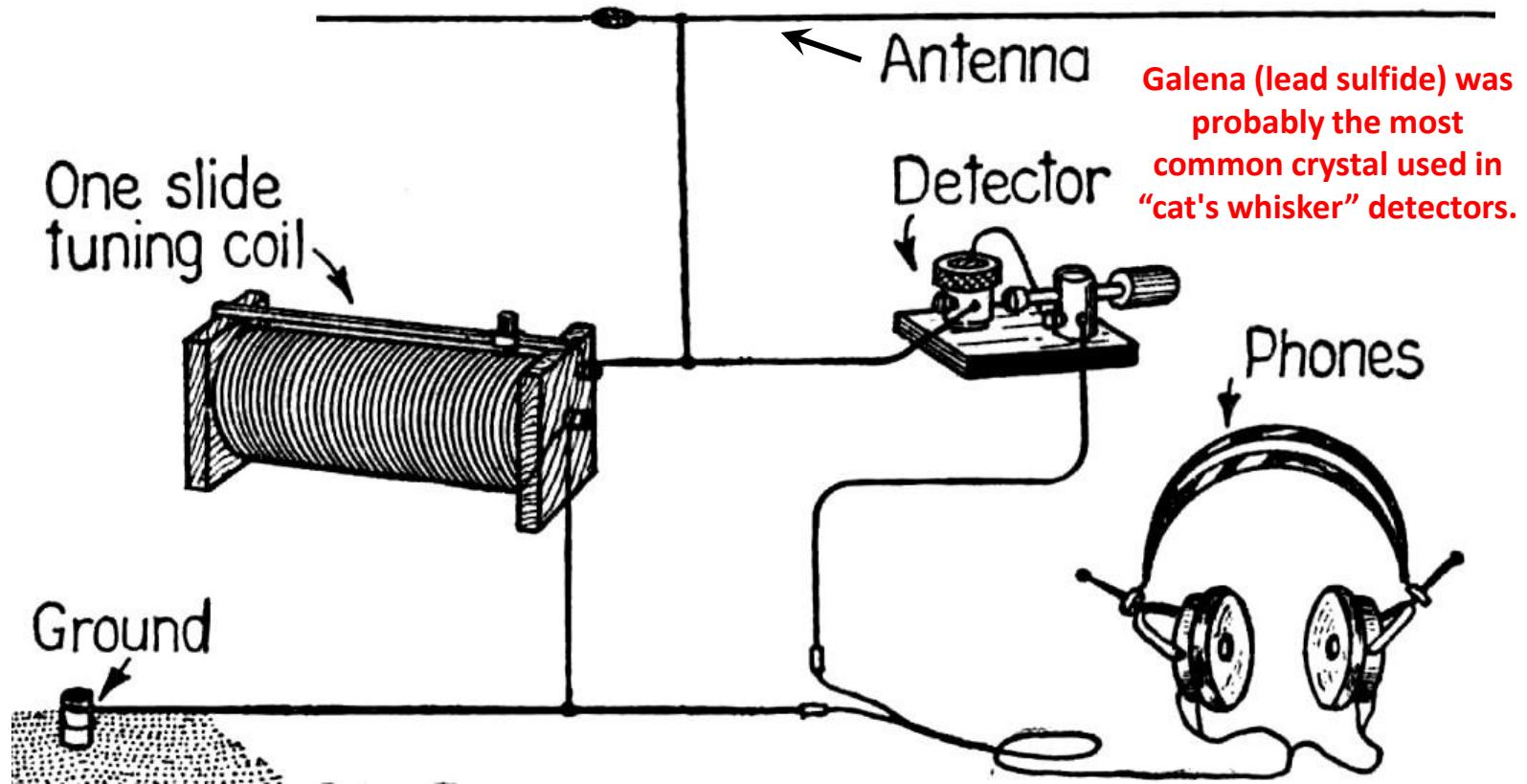


Diagram from 1922 showing the circuit of a crystal radio. This common circuit did not use a tuning capacitor, but used the capacitance of the antenna to form the tuned circuit with the coil.

[https://en.wikipedia.org/wiki/Crystal\\_radio](https://en.wikipedia.org/wiki/Crystal_radio)

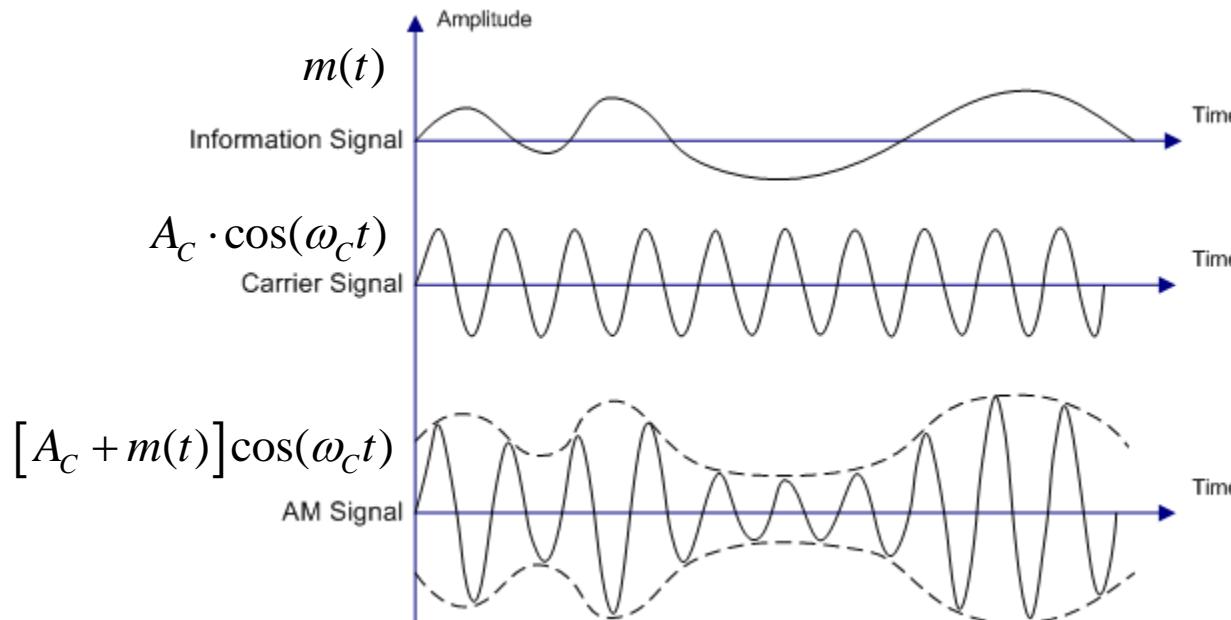
## Amplitude Modulation (DSB with Carrier)

Amplitude Modulation: The amplitude of a carrier signal is varied linearly with a time-varying message signal.

Carrier signal:  $c(t) = A_C \cdot \cos(\omega_C t + \theta)$  {Note: Keep  $\omega_C$  &  $\theta$  fixed.}

Only amplitude  $A_C$  is allowed to vary in AM:  $A_C \Rightarrow [A_C + m(t)]$

$$\phi_{AM}(t) = [A_C + m(t)] \cdot \cos(\omega_C t) = A_C \cdot \cos(\omega_C t) + m(t) \cdot \cos(\omega_C t)$$



## Expressions for AM-DSB with Carrier

Carrier signal:  $c(t) = A_C \cdot \cos(\omega_C t + \theta)$  {Note: Keep  $\omega_C$  &  $\theta$  fixed.}

Only amplitude  $A_C$  is allowed to vary in AM:  $A_C \Rightarrow [A_C + m(t)]$

$$\phi_{AM}(t) = [A_C + m(t)] \cdot \cos(\omega_C t) = A_C \cdot \cos(\omega_C t) + m(t) \cdot \cos(\omega_C t)$$



We will generally use this format.

But other forms exist for this equation:

Some books write (such as in Haykin & Moher, 2009)

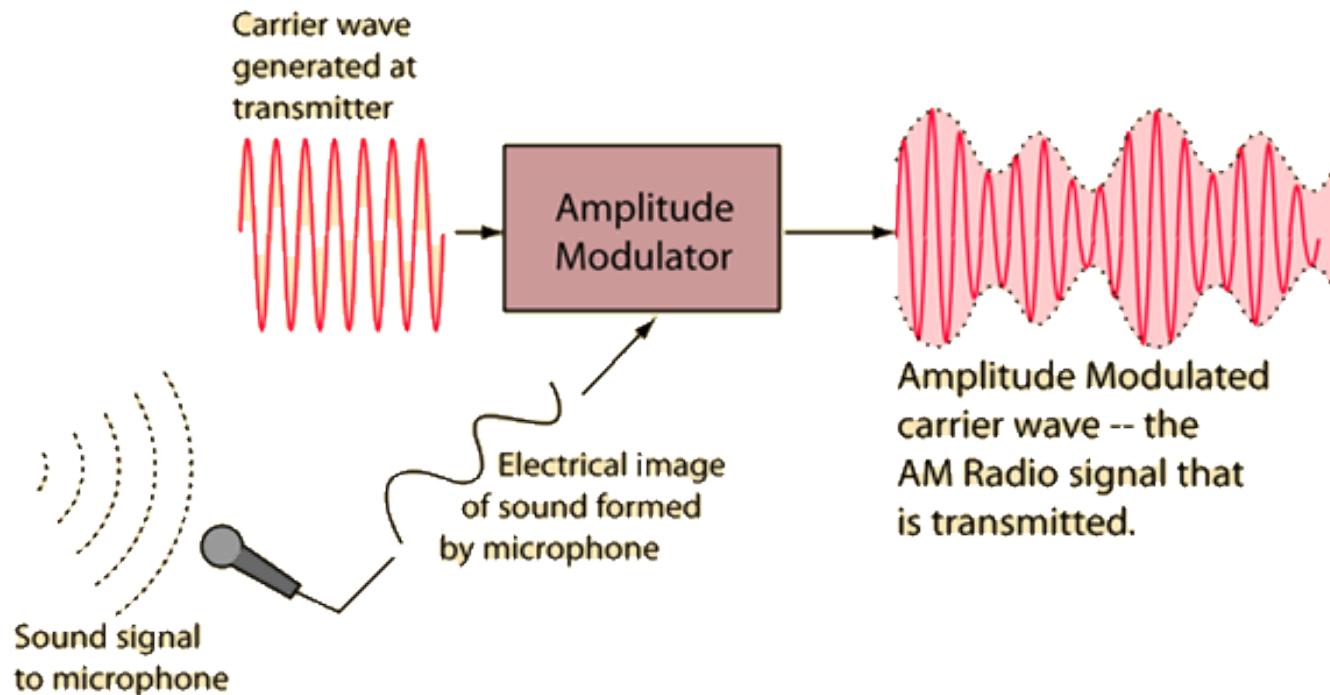
$$\phi_{AM}(t) = A_C [1 + k_{am} m(t)] \cdot \cos(\omega_C t) = A_C \cdot \cos(\omega_C t) + A_C k_{am} m(t) \cdot \cos(\omega_C t)$$

where  $k_{am}$  is the "amplitude sensitivity."

And other books write (Leon W. Couch, II, 2013)

$$\phi_{AM}(t) = A_C [1 + m(t)] \cdot \cos(\omega_C t) = A_C \cdot \cos(\omega_C t) + A_C m(t) \cdot \cos(\omega_C t)$$

# Amplitude Modulation (DSB with Carrier) Illustrated

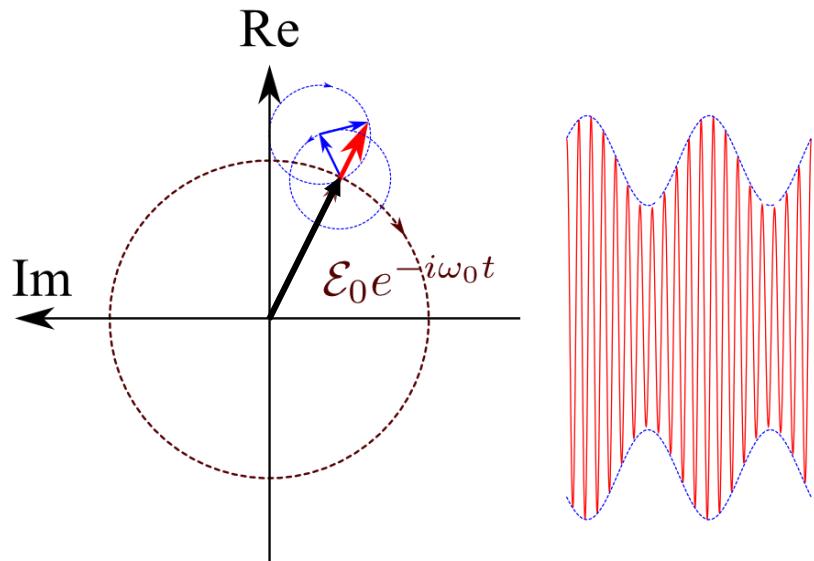


<http://hyperphysics.phy-astr.gsu.edu/hbase/Audio/bcast.html>

# Phasor View of Amplitude Modulation

Example showing tone modulation

a) Amplitude modulation phasor diagram



Black vector  $\Rightarrow$  Carrier signal

Red vector  $\Rightarrow$  AM modulated signal

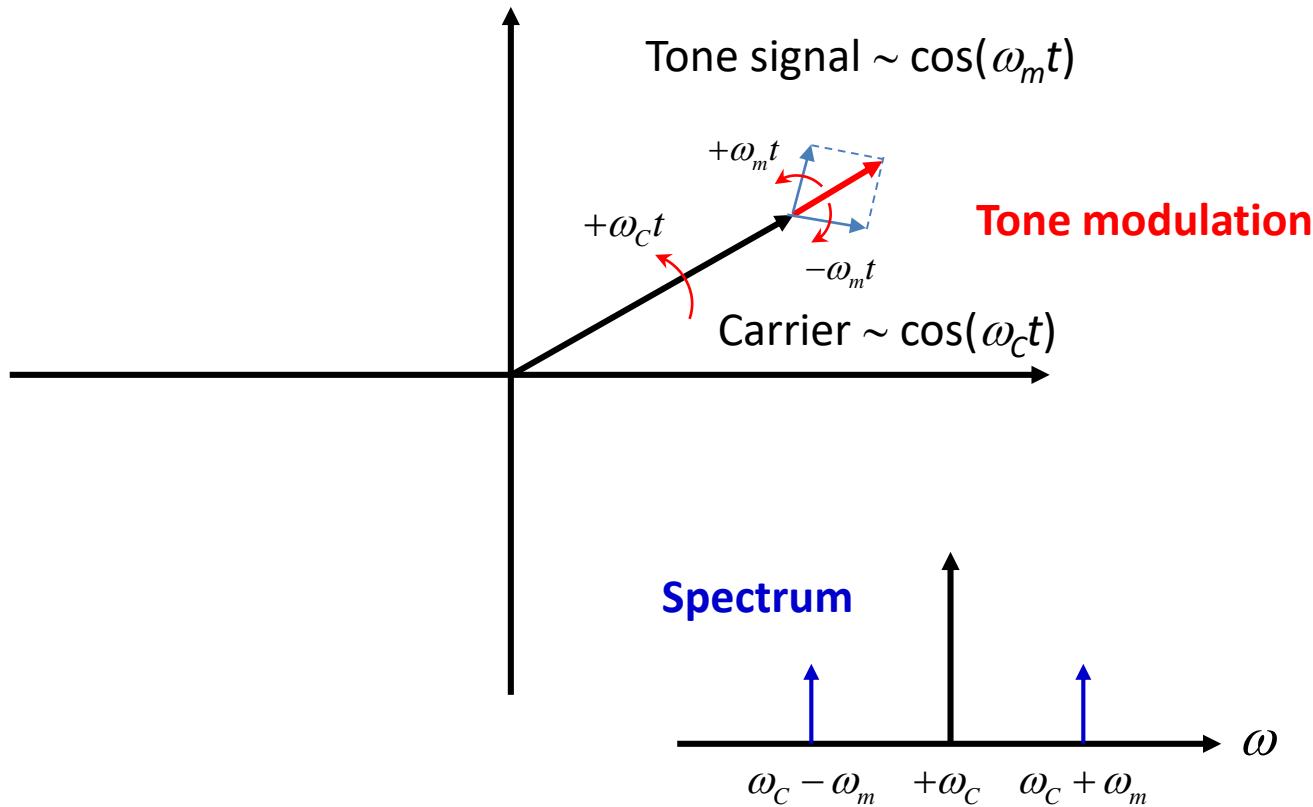
Modulated oscillation is a sum of these three vectors and is given by the red vector. In the case of amplitude modulation (AM), the modulated oscillation vector is always in phase with the carrier field while its length oscillates with the modulation frequency. The time dependence of its projection onto the real axis gives the signal strength as drawn to the right of the corresponding phasor diagram.

<https://inspirehep.net/record/1093258/plots>

# Phasor Expression of Amplitude Modulation

Complex  
Exponential  
Format

$$\phi_{AM}(t) = \operatorname{Re} \left[ e^{j\omega_C t} \left( 1 + \frac{e^{j\omega_m t}}{2} + \frac{e^{-j\omega_m t}}{2} \right) \right]$$



## Phasor Interpretation of AM DSB with Carrier (continued)

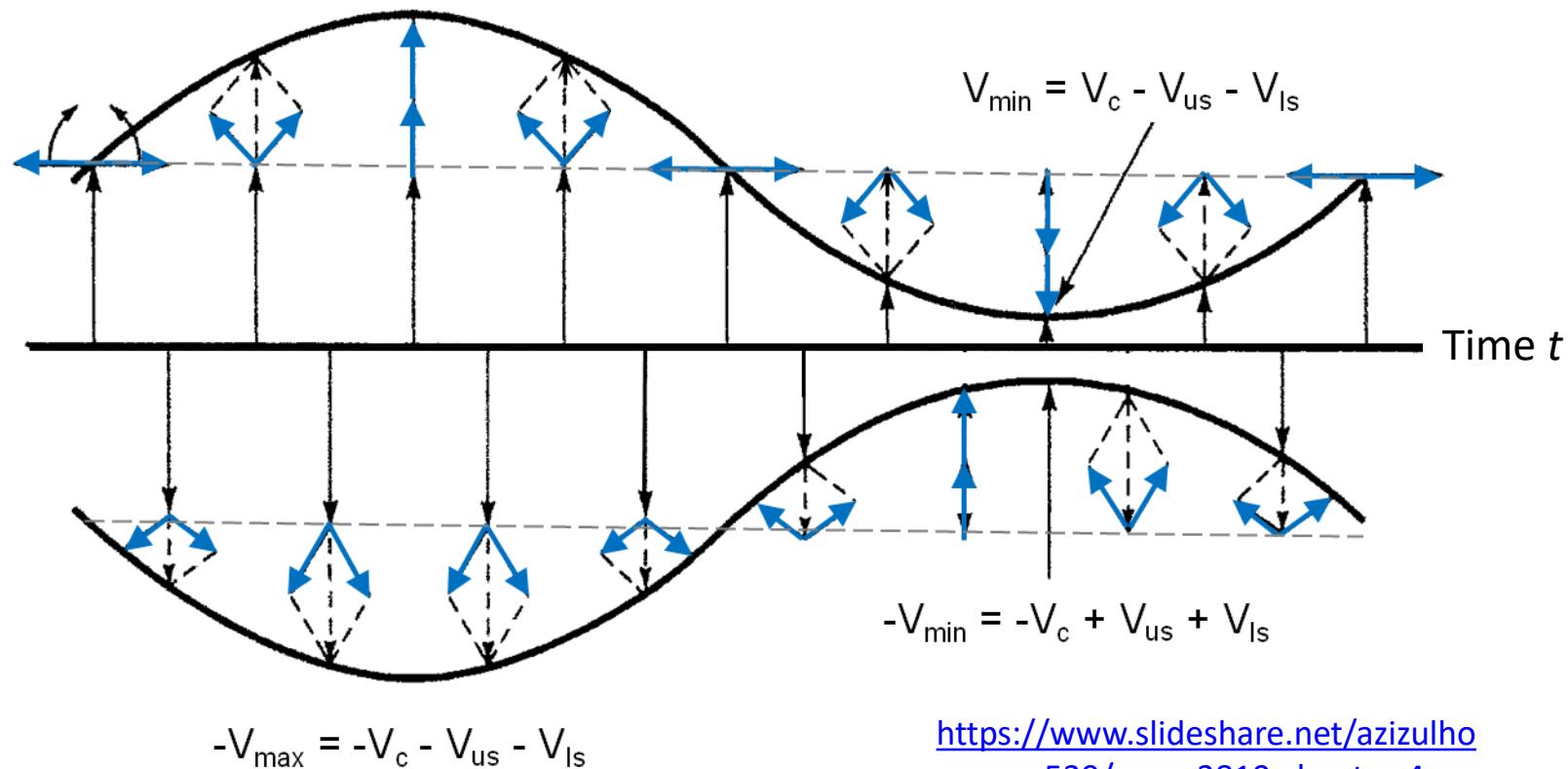
Tone modulation

$V_{us}$  = Voltage of upper sideband phasor

$V_{ls}$  = Voltage of lower sideband phasor

$V_c$  = Voltage of the carrier

$$V_{max} = V_c + V_{us} + V_{ls}$$



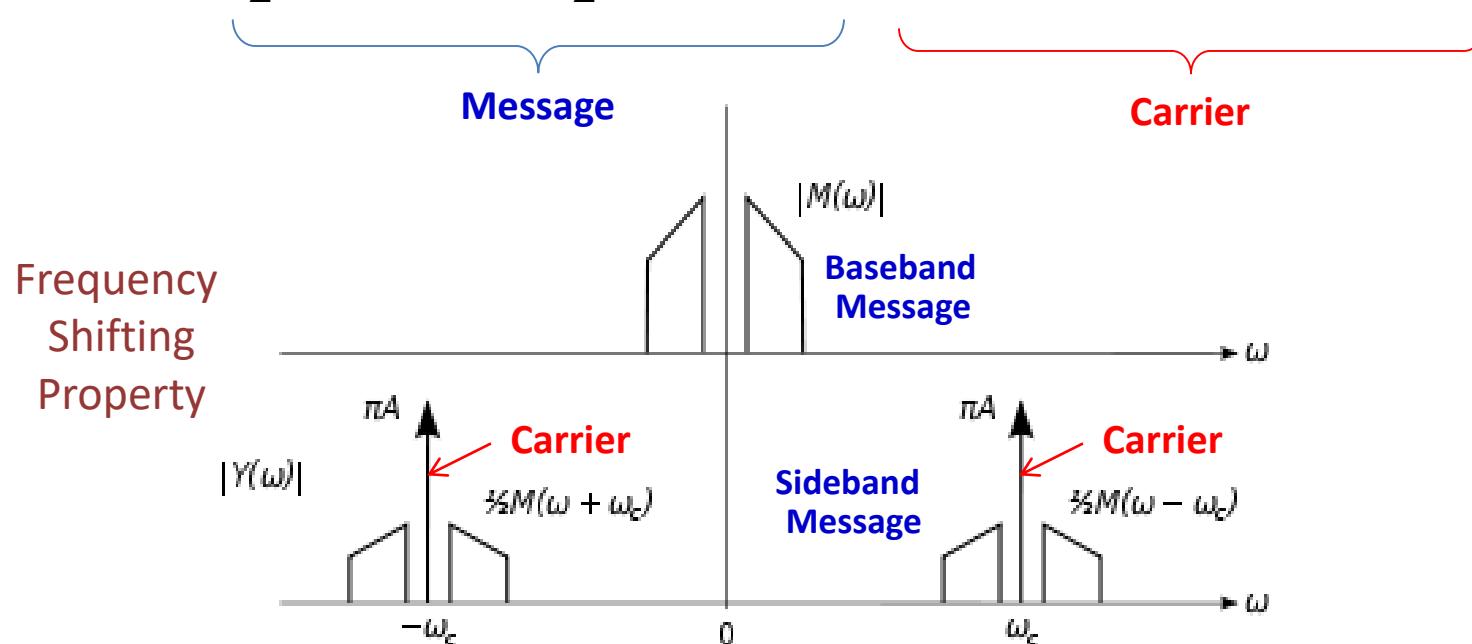
<https://www.slideshare.net/azizulhoque539/eeng-3810-chapter-4>

# Double-Sideband Amplitude Modulation Spectrum

$$\phi_{AM}(t) = m(t) \cdot \cos(\omega_c t) + A_C \cos(\omega_c t) = [A_C + m(t)] \cdot \cos(\omega_c t)$$

The spectrum  $\Phi_{AM}(\omega)$  is found from the Fourier transform of  $\phi_{AM}(t)$

$$\mathcal{F}[\phi_{AM}(t)] = \frac{1}{2} M(\omega - \omega_c) + \frac{1}{2} M(\omega + \omega_c) + \pi A [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$



[https://en.wikipedia.org/wiki/Amplitude\\_modulation](https://en.wikipedia.org/wiki/Amplitude_modulation)

Agbo & Sadiku; Section 3.2.1; pp. 89 to 91

## AM Modulation Index Basics – Definition

The amplitude modulation (AM) modulation index  $\mu$  can be defined as the ratio of the peak value of the message signal to the amplitude  $A_C$  of the carrier signal.

When expressed as a percentage it is the same as the depth of modulation. In other words it can be expressed as:

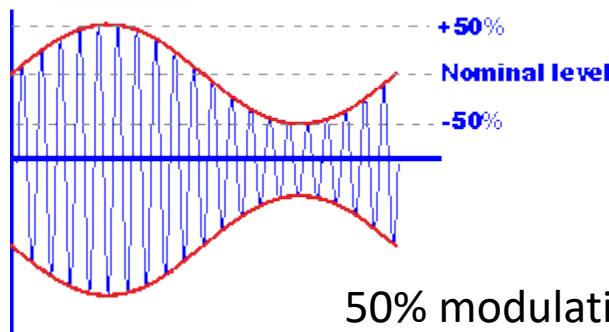
$$\text{Modulation Index } \mu = \frac{m_p}{A_C}$$

Agbo & Sadiku  
Page 85-86

where  $A_C$  is the carrier signal amplitude, and

$m_p$  is the peak modulation amplitude (maximum change in the RF amplitude relative to its un-modulated value).

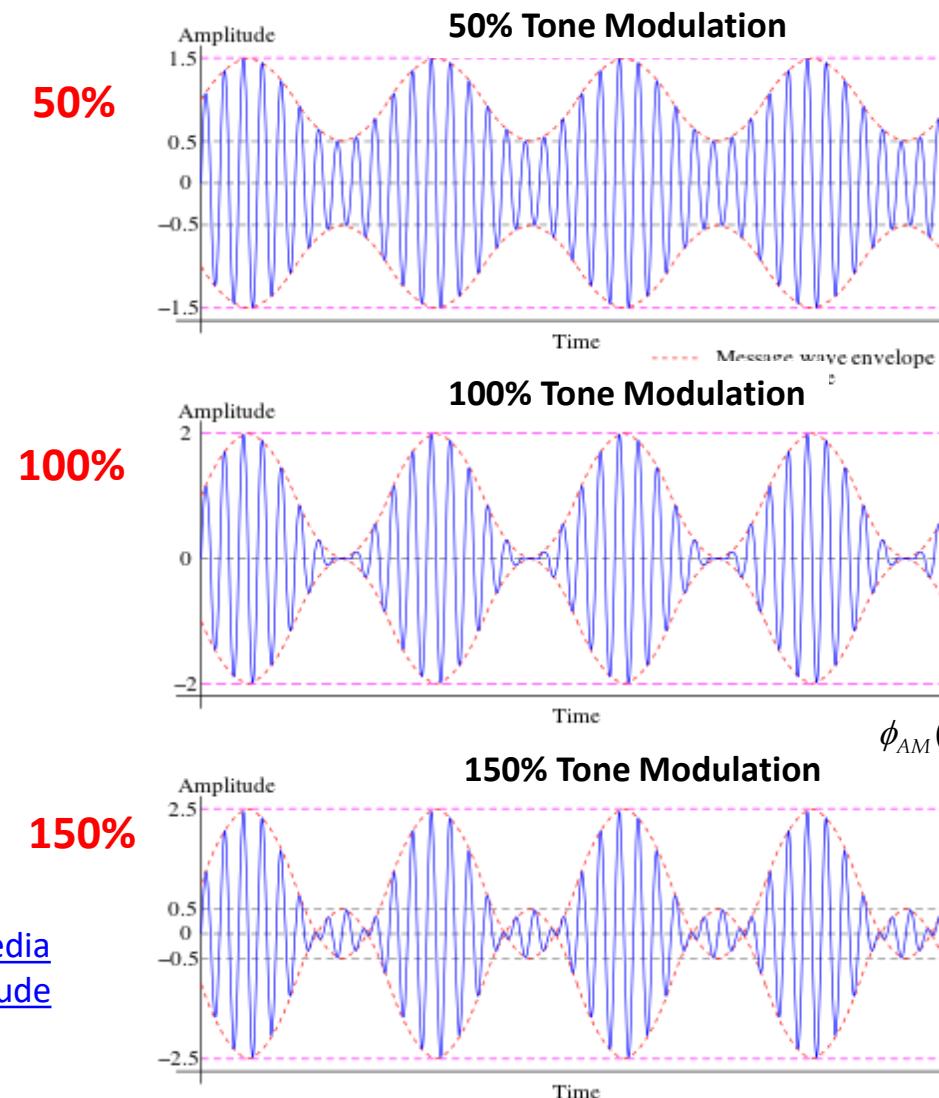
Example: An AM modulation index of 0.5 means the signal increases by a factor of 0.5, and decreases to 0.5, centered around its unmodulated level. See drawings below.



$$\phi_{AM}(t) = [A_C + m(t)] \cdot \cos(\omega_C t)$$

50% modulation illustrated

# AM Modulation Index Basics – Examples



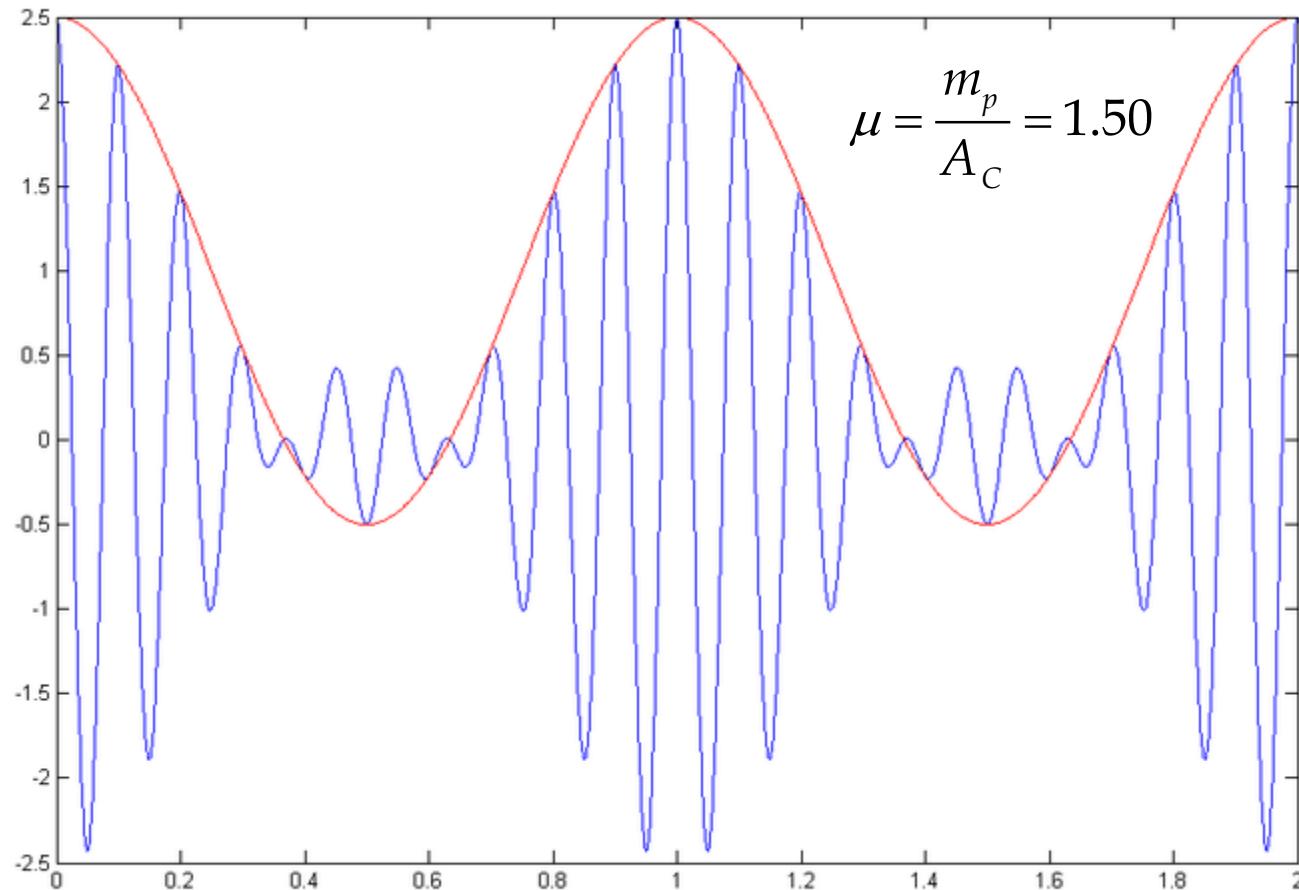
$$\mu = \frac{m_p}{A_C}$$

$$\phi_{AM}(t) = A_C [1 + \mu \cdot \cos(\omega_m t)] \cdot \cos(\omega_C t)$$

Overmodulation  
or  
Envelope Distortion

[https://en.wikipedia.org/wiki/Amplitude\\_modulation](https://en.wikipedia.org/wiki/Amplitude_modulation)

## AM Overmodulation → Envelope Distortion



[https://upload.wikimedia.org/wikipedia/commons/d/da/AM\\_150%25\\_modulation\\_depth.png](https://upload.wikimedia.org/wikipedia/commons/d/da/AM_150%25_modulation_depth.png)

## Power Efficiency of Amplitude Modulation

Given the AM signal:  $\phi_{AM}(t) = A_c \cos(\omega_c t) + m(t) \cos(\omega_c t)$

The power in the carrier is  $P_c = \frac{A_c^2}{2}$  {Let  $P_s \triangleq$  power in one sideband}

The power in the sidebands (modulated message) is

$$P_s = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) \cos^2(\omega_c t) dt = \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) [1 + \cos(2\omega_c t)] dt$$

But  $\int_{-T/2}^{T/2} m^2(t) [\cos(2\omega_c t)] dt = 0$ , and so

we are left with  $P_s = \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) dt \triangleq \frac{1}{2} P_m$

That is,  $P_s$  is one-half the total message power  $P_m$ .

In AM the power in the message (useful power) is the power in the two sidebands. Next, we define **power efficiency**  $\eta$ .

**Agbo & Sadiku; Section 3.2.2; pp. 91 to 92**

## Power Efficiency in Amplitude Modulation (continued)

The power efficiency of a modulated signal is the ratio of the power in the message part of the signal relative to the total power of the modulated signal.

$$\text{Power efficiency } \eta = \frac{\text{messsage power}}{\text{total power}} = \frac{\text{sideband power}}{\text{total power}}$$

$$\text{In symbols, } \eta = \frac{P_s}{P_c + P_s} = \frac{\frac{1}{2}P_m}{P_c + \frac{1}{2}P_m}, \quad \text{and} \quad P_c = \frac{A_c^2}{2}$$

$$\therefore \boxed{\eta = \frac{P_m}{A_c^2 + P_m}}$$

## Power Efficiency in Amplitude Modulation (continued)

In general, the form of  $P_m$  is complicated and not known precisely. However, we can study AM power efficiency  $\eta$  using a tone modulation message.

For tone modulation,  $m(t) = m_p \cos(\omega_C t) = \mu A_C \cos(\omega_C t)$ ,

$$\therefore P_m = \frac{m_p^2}{2} = \frac{(\mu A_C)^2}{2} \quad \text{and} \quad \eta = \frac{m_p^2}{2A_C^2 + m_p^2} = \frac{\mu^2}{2 + \mu^2}$$

Example: (For double-sideband with carrier is AM)

Modulation index = $\mu$	$\eta = \frac{\mu^2}{2 + \mu^2}$
0.25	0.0303 or 3.03 %
0.5	0.111 or 11.1 %
1.0	0.333 or 33.3 %

**Conclusion: AM power efficiency is very low (highly undesirable).**

## Power Efficiency in Amplitude Modulation (continued)

In general, the form of  $P_m$  is complicated and not known precisely. In practice, we find  $P_m$  by the integral,

$$P_m = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) \cdot \cos^2(\omega_C t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T/2}^{T/2} m^2(t) \cdot [1 + \cos(2\omega_C t)] dt$$
$$P_m = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T/2}^{T/2} m^2(t) dt.$$

If  $m(t)$  is a **single tone**, then we have

$$m(t) = m_p \cdot \cos(\omega_C t), \quad \text{then}$$

$$P_m = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m_p^2 \cdot \cos^2(\omega_C t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T/2}^{T/2} m_p^2 \cdot [1 + \cos(2\omega_C t)] dt$$

$$P_m = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T/2}^{T/2} m_p^2 dt = \frac{1}{2} (m_p^2) \left[ \frac{(T/2) - (-T/2)}{T} \right] = \frac{1}{2} (m_p^2)$$

$$\text{and } P_s = \frac{P_m}{2} = \frac{1}{4} (m_p^2)$$

## Can We Reduce Transmitted Power in AM?

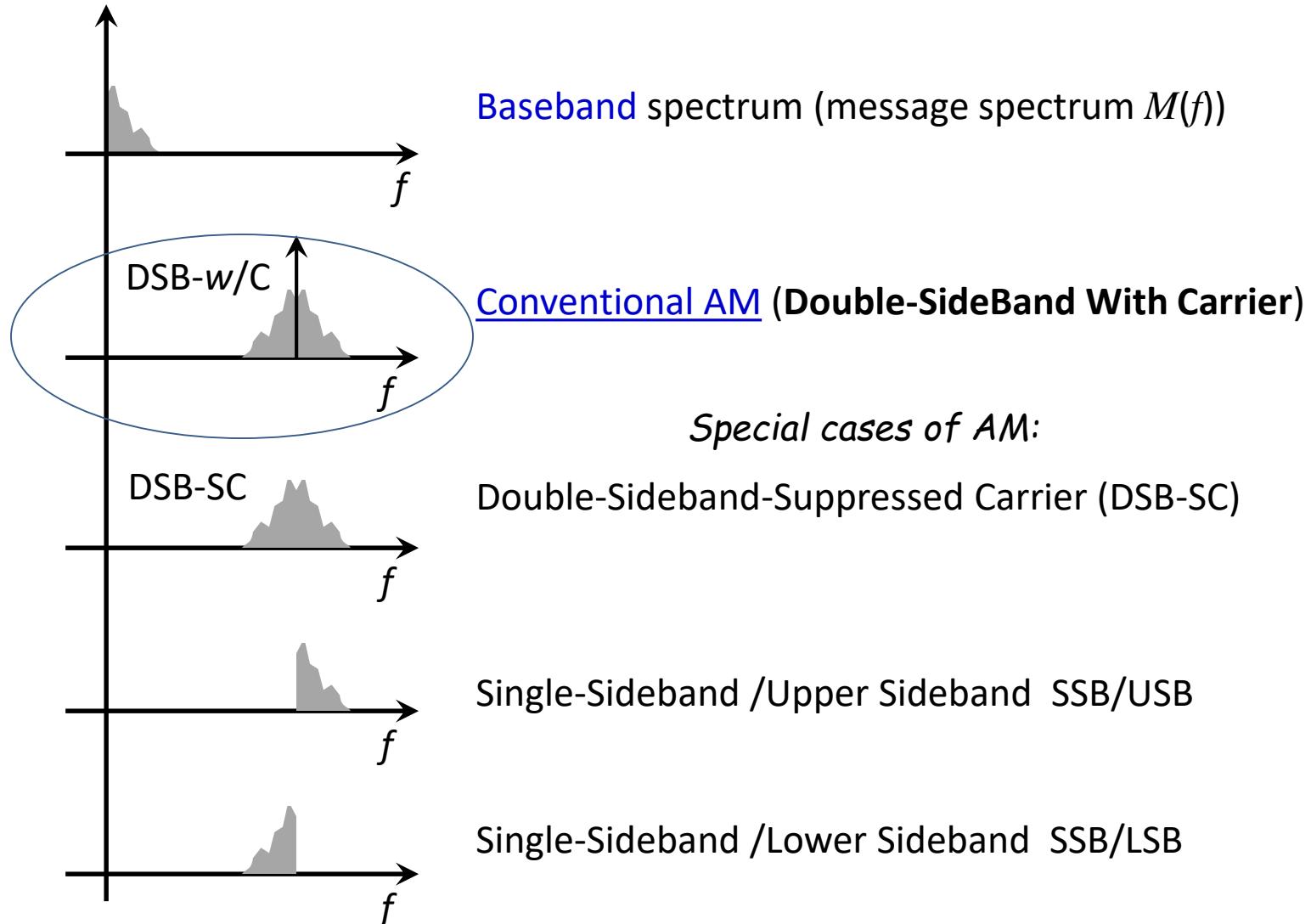
- 1. Do we need to transmit the carrier, which is the majority of the transmitted power, in AM?**

**No**, but we will need to have a more sophisticated demodulation scheme in the receiver. We will do this with “double-sideband suppressed carrier” modulation.

- 2. Do we have to transmit both sidebands in AM?**

**No**, because both sidebands contain identical information. We will do this by introducing “single-sideband modulation” (without carrier).

## Preview: Categories of Amplitude Modulation



# **Generation of Amplitude Modulated Signals**

Agbo & Sadiku present two methods for AM generation:

## **1. Nonlinear AM modulator**

Almost any nonlinearity will work, but a very inexpensive but strongly nonlinear device is the diode. Transistors are also nonlinear and work well as modulators (but more complicated).

## **2. Switching AM modulator**

Switching is an easily attained function with diodes and transistors in electronic circuits.

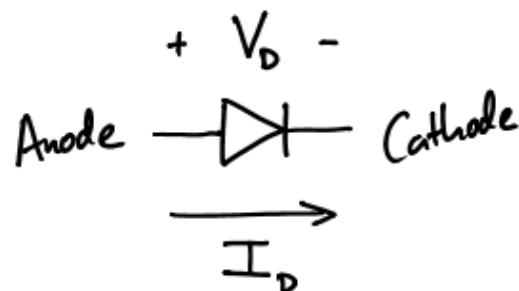
There is also a third method:

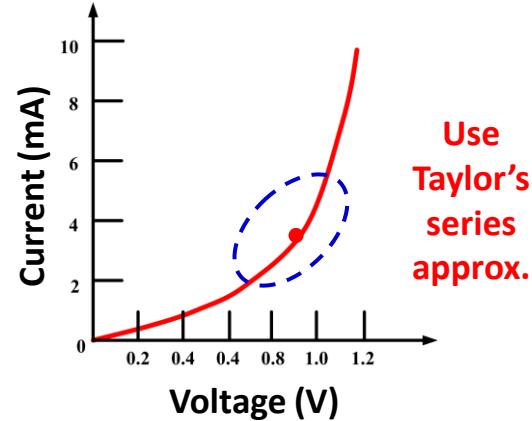
## **3. Electronic multipliers (such as Gilbert cells) can be used as modulators.**

**Agbo & Sadiku; Section 3.2.3; pp. 93 to 95**

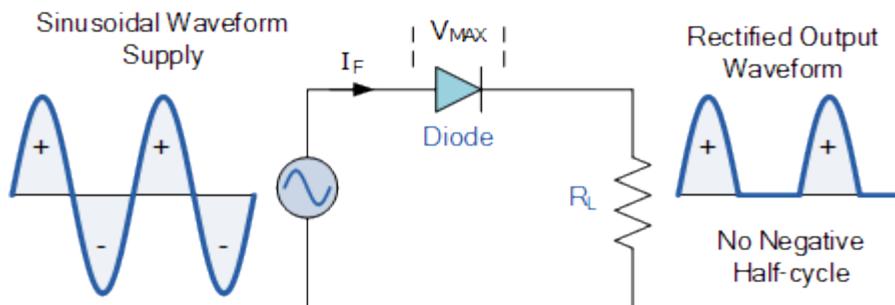
# Diode Operation Applied to AM Modulators & Demodulators

1. As **nonlinear circuit components** (primarily the “square law” part )

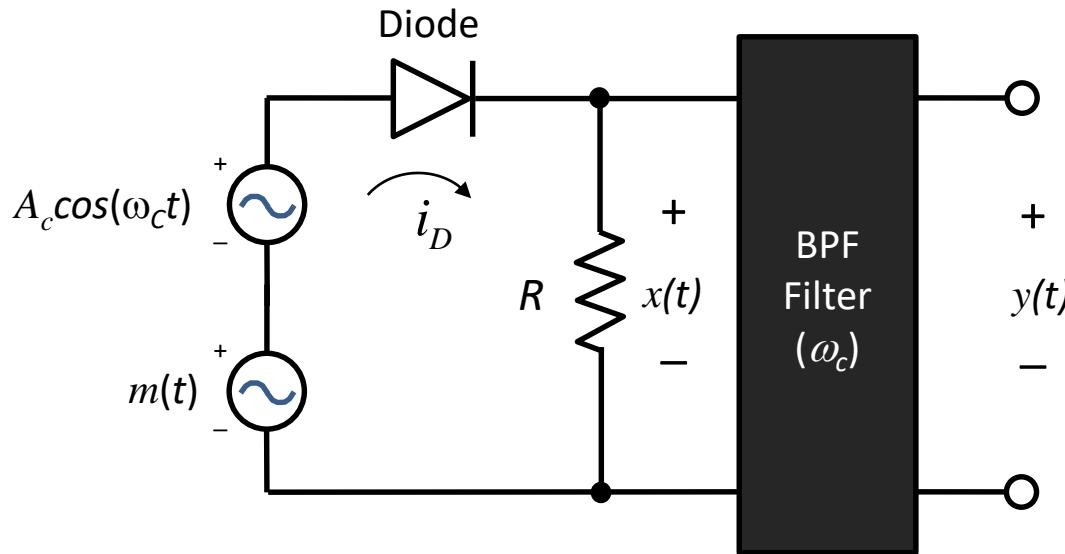
  
 $I_D = I_{sat} \left( e^{qV_D/kT} - 1 \right)$



2. As “on-off” switches (they have to be driven hard to do this)



# Using Nonlinearity For Modulation (i.e., AM Generation)



The diode is the nonlinear component (it has an exponential characteristic). Using a **Taylor's series** we can express the diode current  $i_D$  as (with only first two terms of the Taylor's series),

$$i_D(t) = b_1 v_D(t) + b_2 v_D^2(t); \quad v_D(t) \text{ is diode voltage.}$$

The voltage across resistor  $R$  is given by

$$x(t) = i_D(t)R = b_1 R v_D(t) + b_2 R v_D^2(t) = a_1 v_D(t) + a_2 v_D^2(t)$$

"Square Law" behavior

## Using Nonlinearity For Modulation (continued)

We now can evaluate voltage  $x(t)$

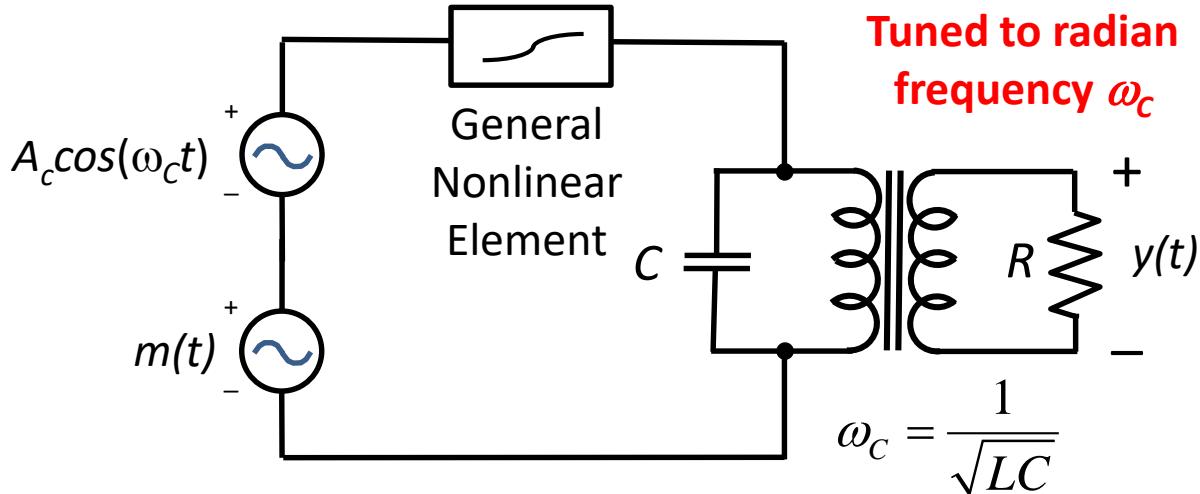
$$\begin{aligned}x(t) &= a_1 [m(t) + A_C \cos(\omega_C t)] + a_2 [m(t) + A_C \cos(\omega_C t)]^2 \\x(t) &= a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_C^2}{2} [1 + \cos(2\omega_C t)] \\&\quad + a_1 A_C \left[ 1 + \frac{2a_2 m(t)}{a_1} \right] \cos(\omega_C t)\end{aligned}\tag{Eq. 3.23}$$

Applying the bandpass filter about  $\omega_C$ , the output voltage  $y(t)$  is

$$y(t) = \phi_{AM}(t) = a_1 A_C \left[ 1 + \frac{2a_2 m(t)}{a_1} \right] \cos(\omega_C t) \iff$$

Note: For  $\mu$  to be less than unity, we require  $\frac{2a_2 m(t)}{a_1} < 1$ .

## Using Nonlinearity For Modulation (continued)



### Comments:

1. Can use a general nonlinear element (not just a “square law” device)
2. The filter can be as simple as a LC resonator
3. This is about the simplest of all modulators (it is unbalanced)

## Using General Nonlinearity For Modulation

$$v_{out} = v_{DC} + Gv_{in} + Av_{in}^2 + Bv_{in}^3 + \dots$$

Taylor's series

Input signals:  $v_{in} = A_{RF} \cos(\omega_{RF}t) + B_{LO} \cos(\omega_{LO}t)$

$v_{out} \Rightarrow \omega_{RF}$  and  $\omega_{LO}$

*tone*                            *carrier*

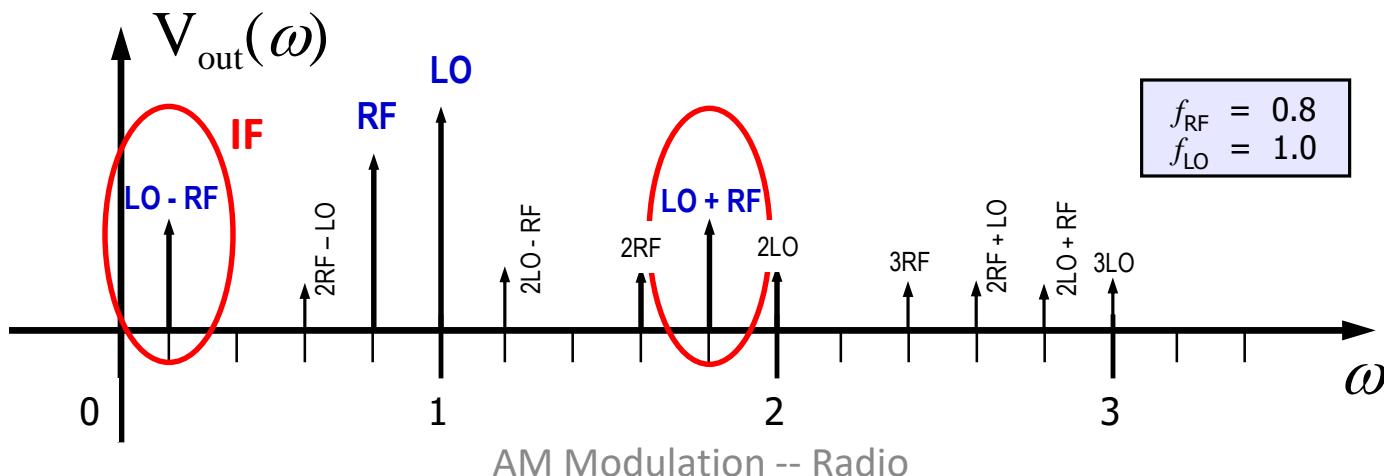
(linear)

$v_{out}^2 \Rightarrow (\omega_{RF} + \omega_{LO}), (\omega_{LO} - \omega_{RF}), 2\omega_{RF}$  and  $2\omega_{LO}$

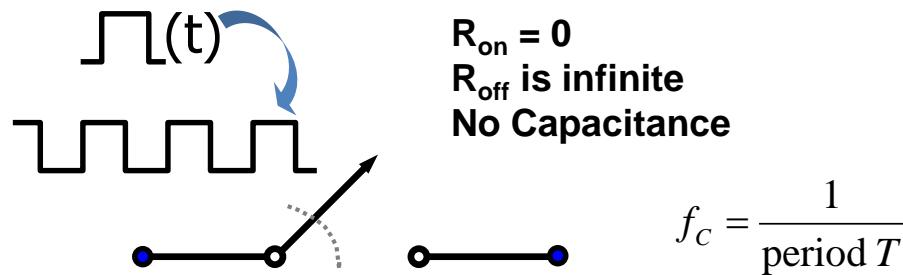
(square law)

$v_{out}^3 \Rightarrow (2\omega_{RF} + \omega_{LO}), (2\omega_{RF} - \omega_{LO}), (2\omega_{LO} + \omega_{RF}), (2\omega_{LO} - \omega_{RF}), 3\omega_{RF}$  &  $3\omega_{LO}$

**Conclusion: Nonlinearity generates new frequencies.**



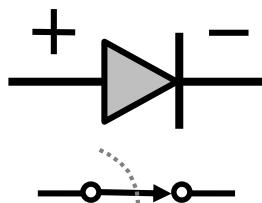
# Switching Amplitude Modulator – The Switch



$$p(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \dots \right]$$

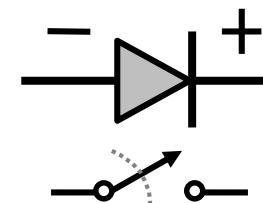
By driving a diode with sufficient AC voltage it acts like a switch:

**Forward bias**



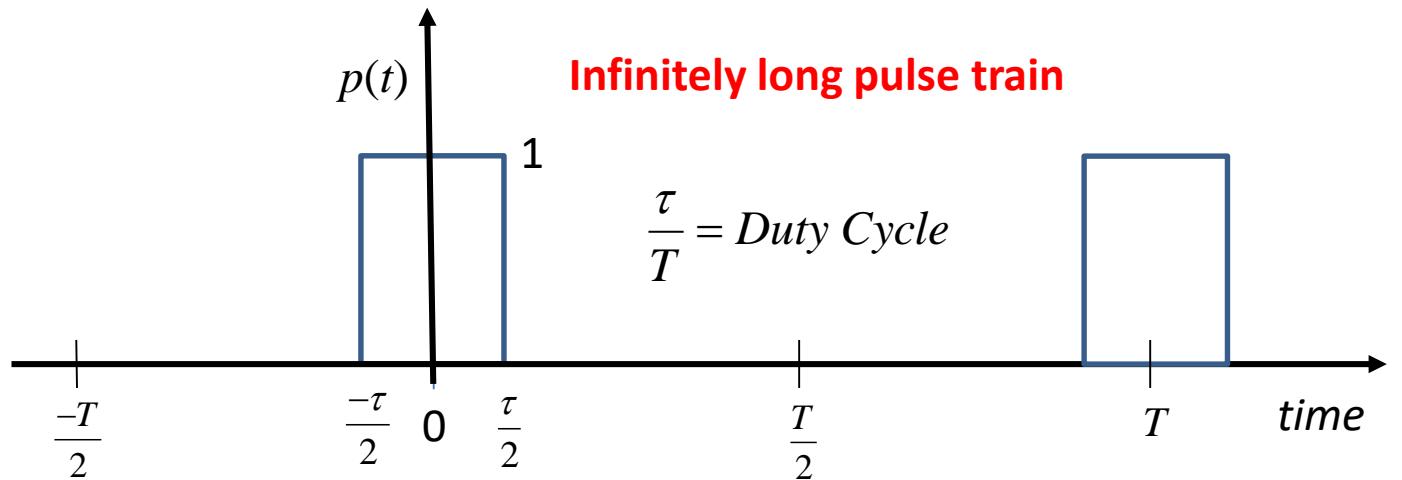
Current flow  
Switch closed

**Reverse bias**



No current flow  
Switch open

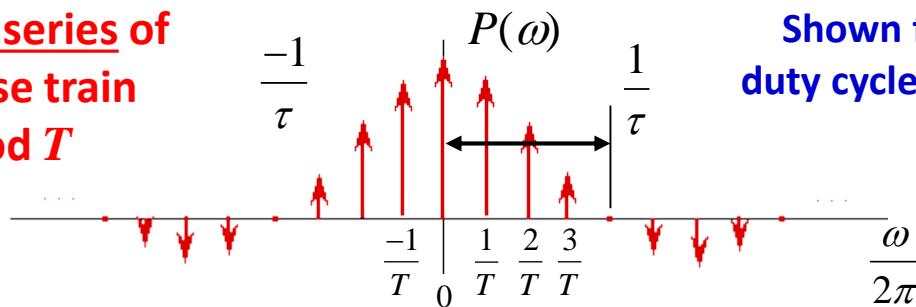
# Switching Amplitude Modulator – Pulse Spectrum Generated



$$p(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega t) - \frac{1}{3} \cos(3\omega t) + \frac{1}{5} \cos(5\omega t) - \dots \right]$$

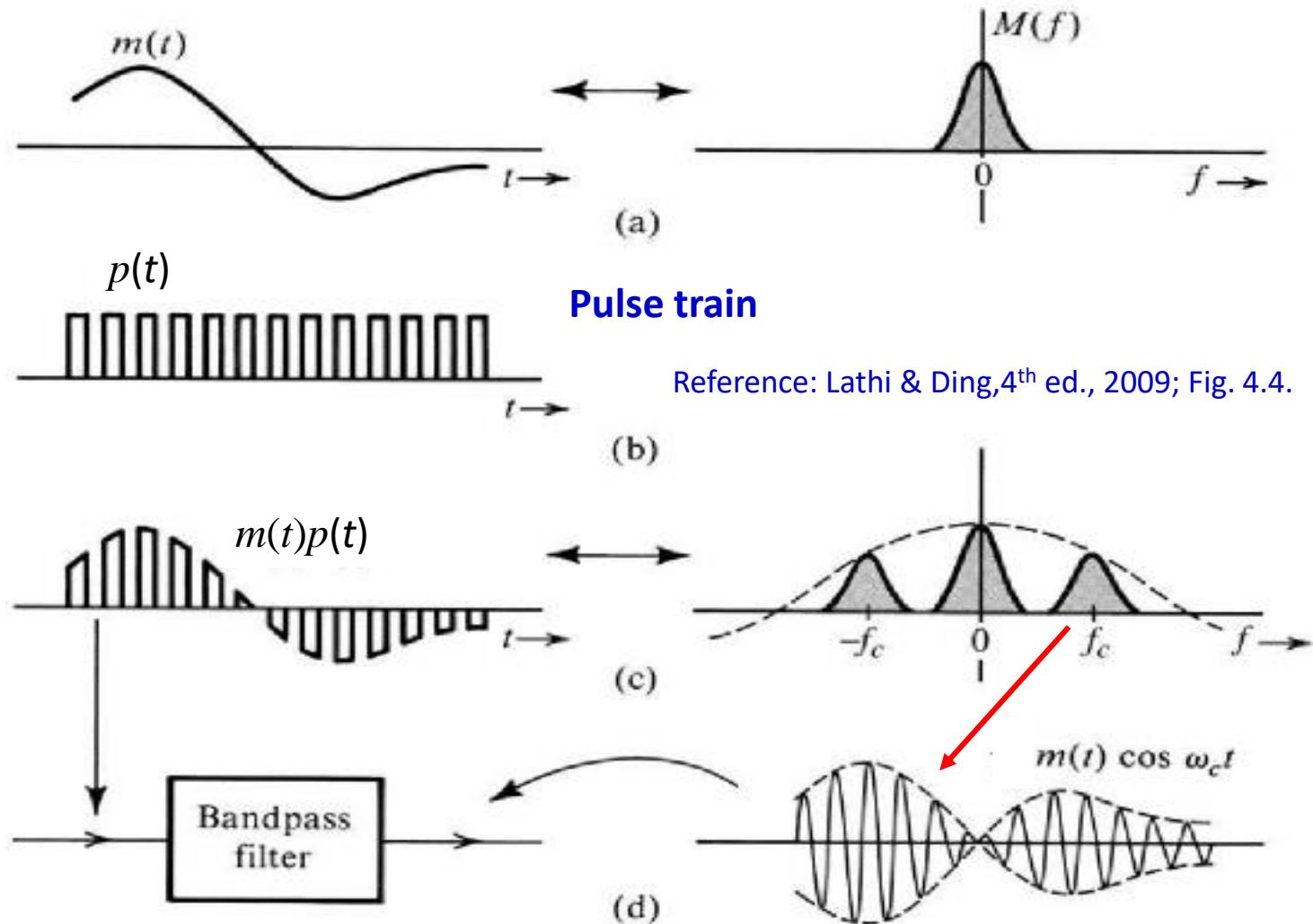
Fourier Series representation

Fourier series of  
the pulse train  
of period  $T$



Uses the “Frequency Shifting” Property of the Fourier Transform.

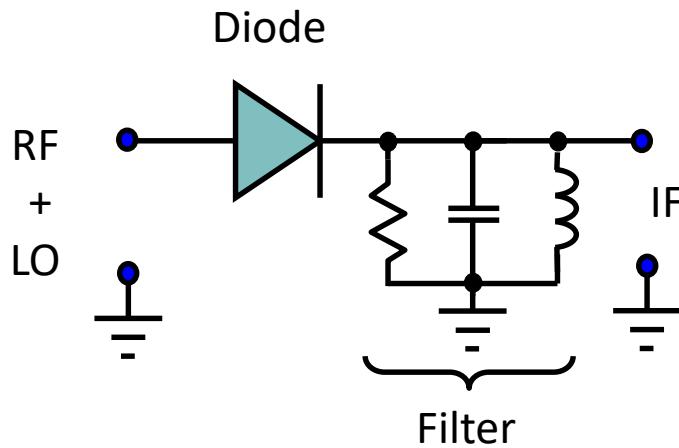
# Switching Modulator – Generating $m(t)\cos(\omega_c t)$



# Diode Mixer For Modulation and Demodulation

A “hopelessly unsophisticated” mixer.

— Tom Lee (Stanford University)



This is the same circuit used in the Foxhole radio for demodulation.

The unbalanced single-diode mixer has no isolation and no conversion gain.

Single-diode mixers have been used in many applications --

- (1) Detectors for radar in WW II
- (2) Early UHF Television tuners
- (3) Crystal radio detectors
- (4) mm-wave & sub-mm-wave receivers

# AM Demodulation

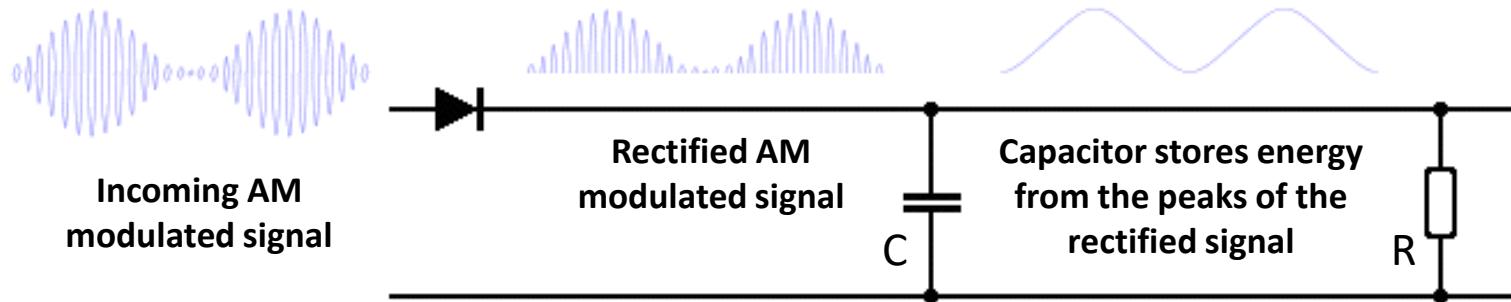
Section 3.2.4 (pp. 95 to 99)

**Coherent (*i.e.*, synchronous) demodulation** (or **detection**) is a method to recover the message signal from the received modulated signal that requires a carrier at the receiver. This carrier signal must match in frequency and phase the received signal.

But . . . Amplitude Modulation has the advantage of not requiring coherent detection methods. Non-coherent methods can be used which are much simpler to implement.

- 1. AM Envelope Detector**
  
- 2. AM Rectifier Detector**

# AM Envelope Detector Circuit



**Key idea: Capacitor captures the voltage peaks of rectified waveform**

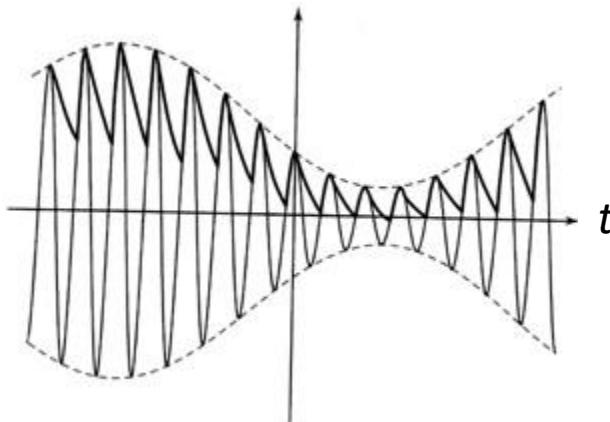
Envelope Detection requires the an RC network with time constant  $\tau = RC$

Two conditions must be met for an envelope detector to work:

- (1) Narrowband [meaning  $f_c \gg$  bandwidth of  $m(t)$ ]
- (2)  $A_C + m(t) \geq 0$

# Choosing the RC Time Constant in Envelope Detector

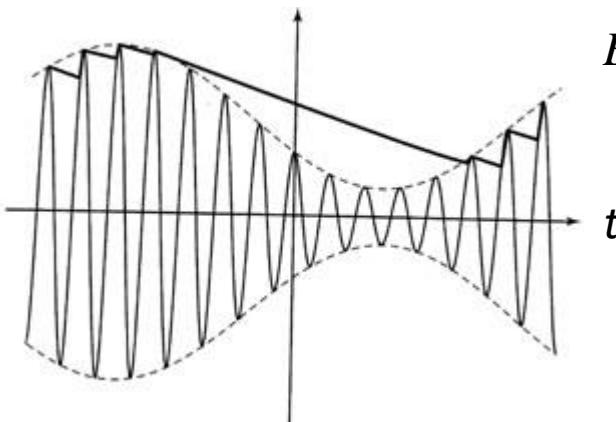
How the envelope is captured



Time constant  $\tau = RC$  too short.

Design criteria is  $2\pi B < \frac{1}{RC} \ll 2\pi f_C$

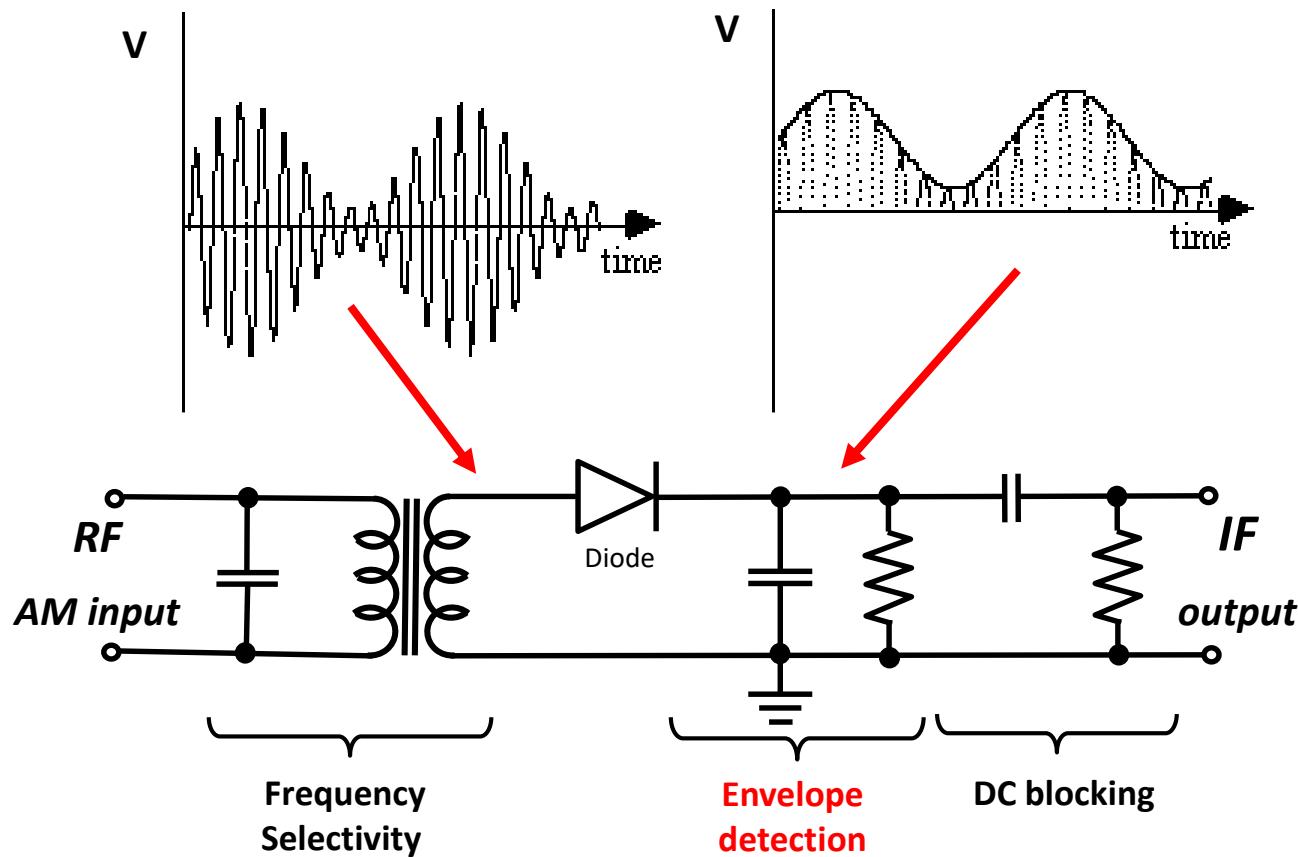
$B$  Hz is bandwidth of message signal.



Time constant  $\tau = RC$  too long.

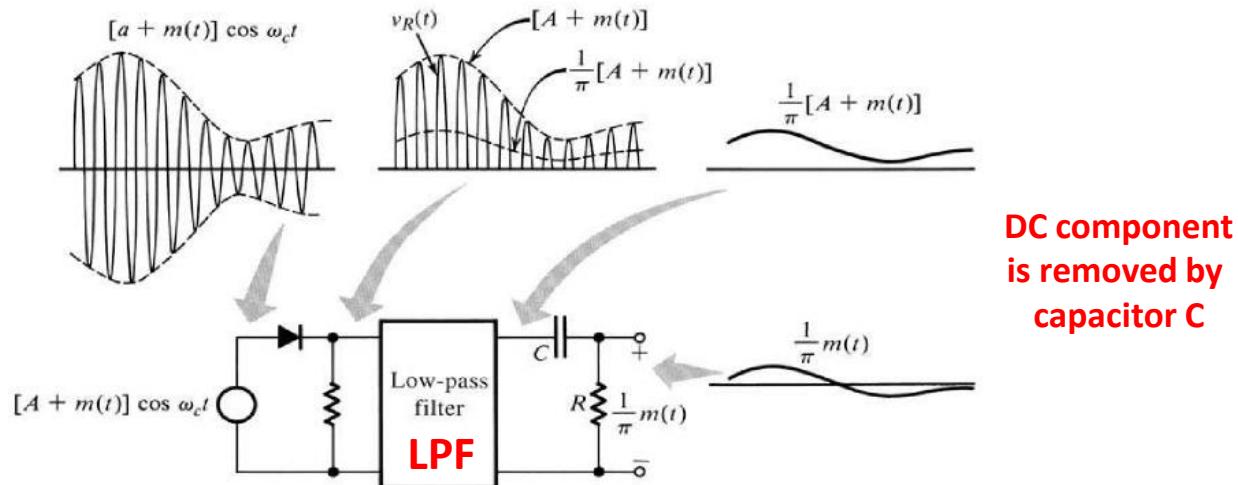
Reference: Lathi & Ding, 4<sup>th</sup> ed., 2009; Fig. 4.11.

# Practical Demodulation of an AM Signal



The user has a choice of changing the filter to meet their needs.

# AM Rectifier Detection



$$V_{rect}(t) = [(A_C + m(t)) \cos(\omega_C t)] \cdot p(t)$$

Reference: Lathi & Ding, 4<sup>th</sup> ed., 2009; Fig. 4.10.

$$= [(A_C + m(t)) \cos(\omega_C t)] \left\{ \frac{1}{2} + \frac{2}{\pi} \left[ (\cos(\omega_C t) - \frac{1}{3}(\cos(3\omega_C t) + \frac{1}{5}(\cos(5\omega_C t) - \dots) \right] \right\}$$

$$= \frac{1}{\pi} (A_C + m(t)) + \text{other terms.}$$

= **dc term + baseband term**

Note: Multiplication with  $p(t)$  allows rectifier detection to act essentially as a synchronous detection without a carrier being generated at the receiver.

## Double-Sideband Suppressed Carrier AM

Conventional AM transmits both the message and carrier signals. Hence, the its power efficiency  $\eta$  is low,

$$\eta = \frac{P_m}{A_C^2 + P_m} < 100\%$$

If  $(A_C)^2$  approaches zero, then  $\eta$  approaches 100%.

$$\phi_{AM}(t) = [A_C + m(t)] \cdot \cos(\omega_C t)$$

For DSB-SC (double sideband -- suppressed carrier) we have

$$\phi_{DSB-SC}(t) = m(t) \cdot \cos(\omega_C t) \quad \text{with a FT pair} \quad m(t) \Leftrightarrow M(\omega)$$

$$\mathcal{F}[m(t) \cos(\omega_C t)] = \Phi_{DSB-SC}(\omega) = \frac{1}{2} [M(\omega - \omega_C) + M(\omega + \omega_C)]$$

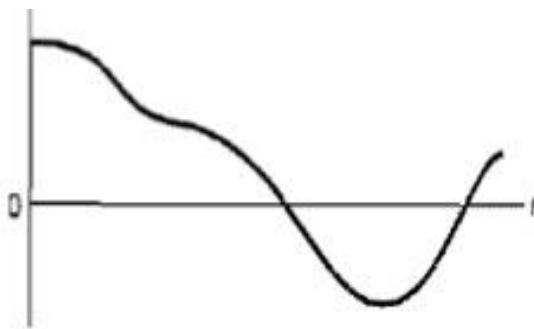
## Double-Sideband Suppressed Carrier (continued)

- The standard form of a DSB-SC wave is defined by

$$\phi_{DSB-SC}(t) = m(t) \cos(\omega_C t)$$

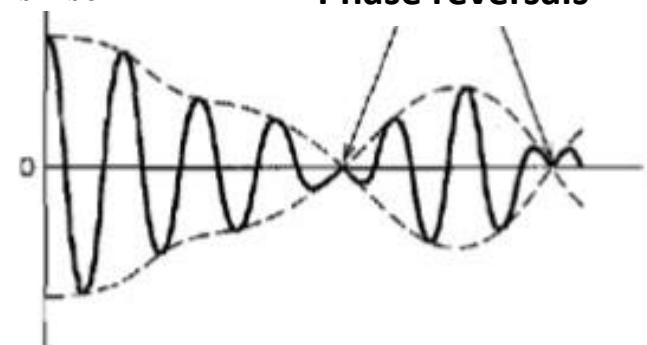
- This modulated wave undergoes a phase reversal whenever the message signal  $m(t)$  crosses zero, as illustrated in figure below

$m(t)$



(a) Baseband signal

$\phi_{DSB-SC}(t)$



(b) DSB-SC modulated wave

The penalty for using DSB-SC is a complex detection scheme is needed.  
We can't use simple envelope detection.

# Double-Sideband Suppressed Carrier Generation

## Modulation:

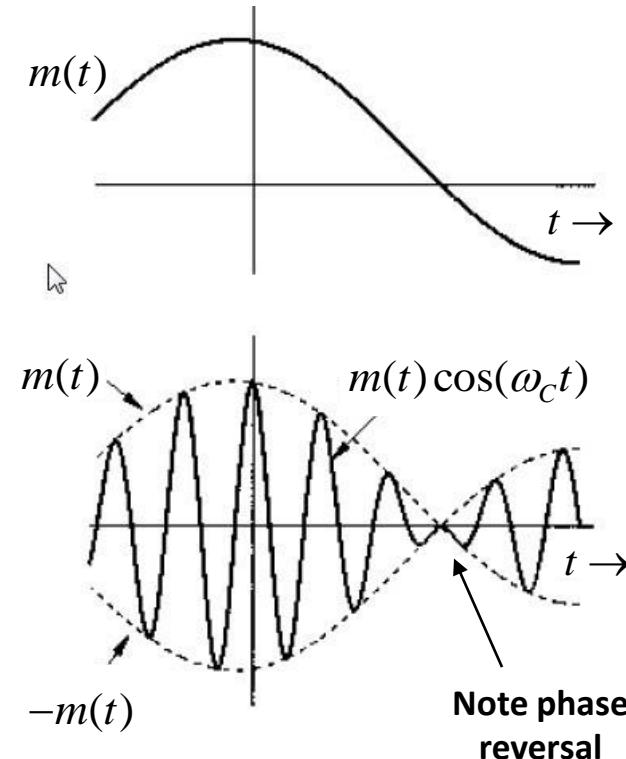
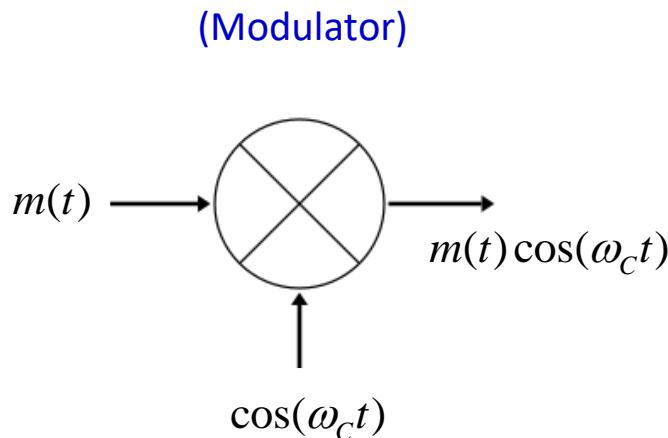
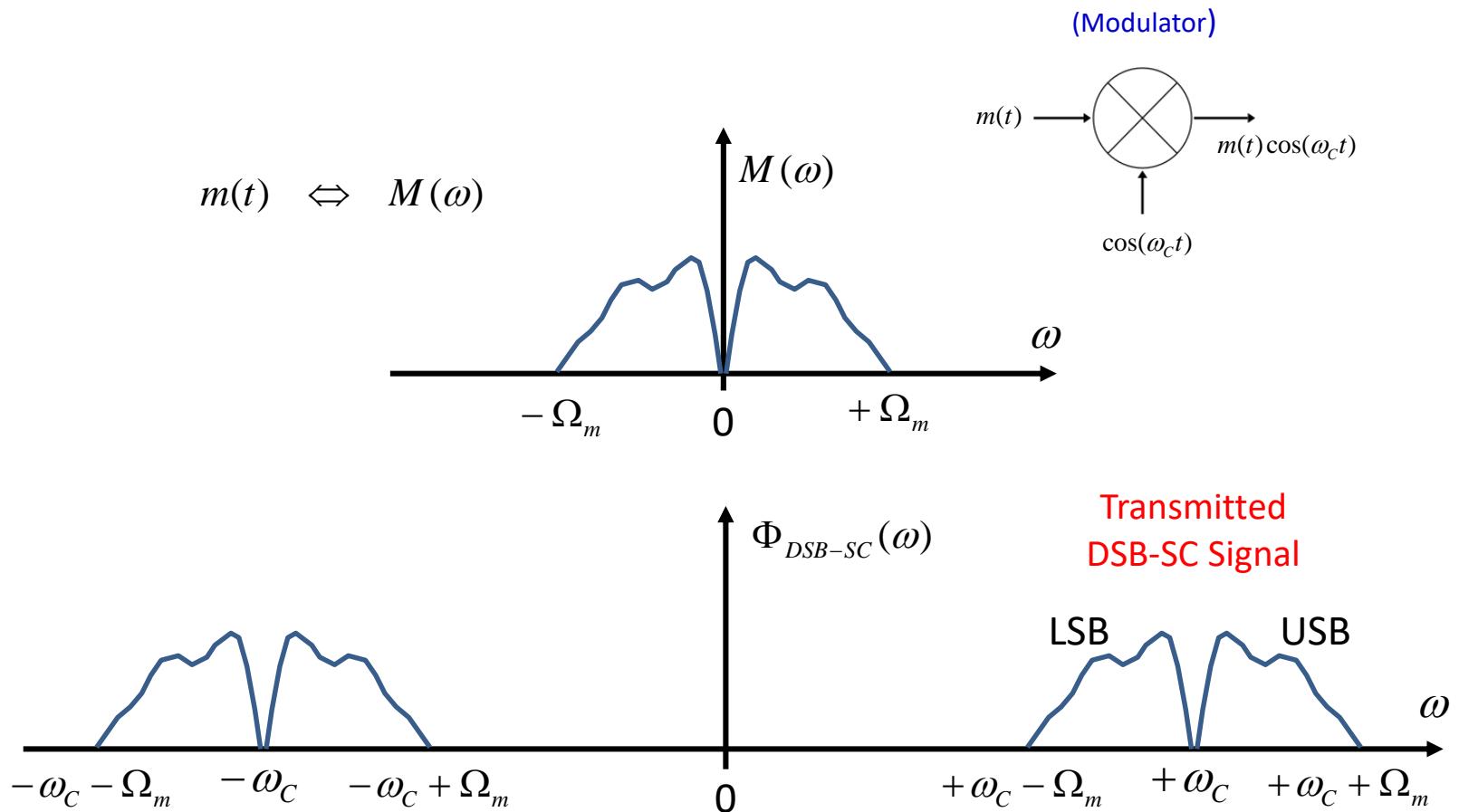


Figure 3.11 in Agbo & Sadiku

**DSB-SC Output**

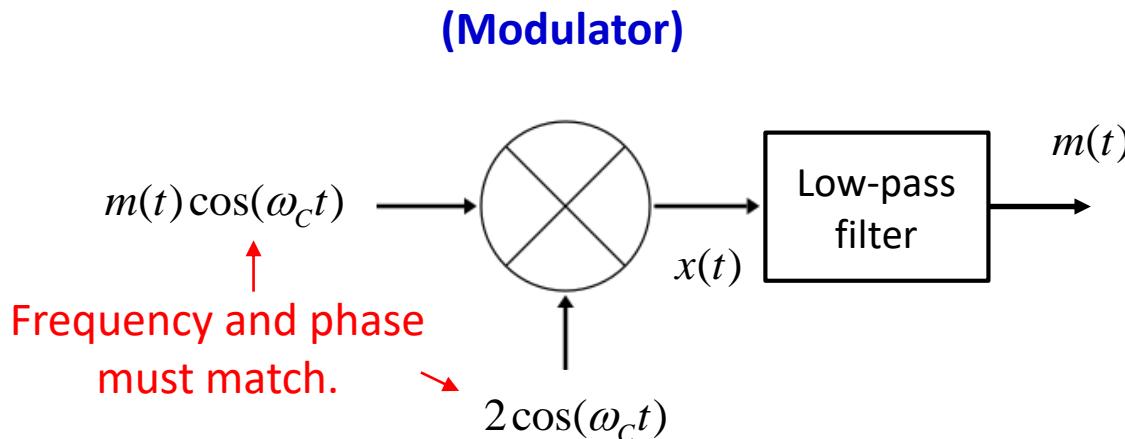
# Double-Sideband Suppressed Carrier Spectrum



**DSB-SC has USB and LSB spectra but no carrier impulses at  $\pm \omega_c$ .**

## Double-Sideband Suppressed Carrier (continued)

### Demodulation:



$$x(t) = 2m(t) \cdot \cos^2(\omega_c t) = m(t) + m(t) \cdot \cos(2\omega_c t)$$

Used the identity:  $\cos^2 \varphi = \frac{1}{2} [1 + \cos(2\varphi)]$

The Fourier transform of  $x(t)$  is

$$X(\omega) = M(\omega) + \frac{1}{2} [M(\omega - 2\omega_c) + M(\omega + 2\omega_c)]$$

# DSB-SC Synchronous Demodulation

April 11, 1961

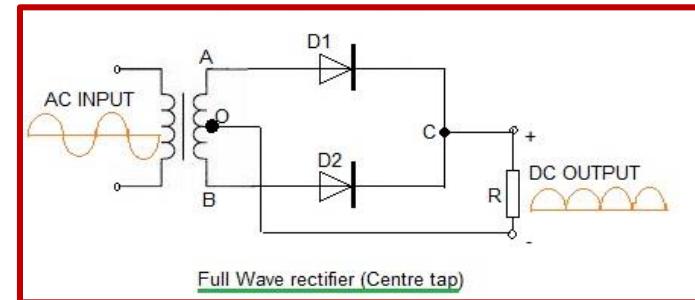
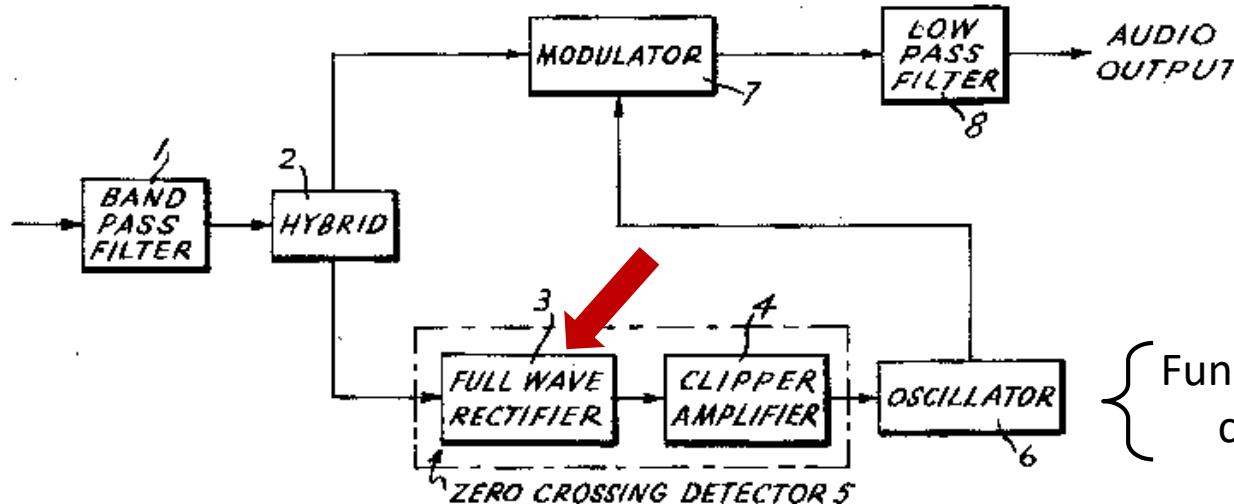
J. W. HALINA

2,979,611

SYNCHRONOUS DEMODULATION SYSTEM

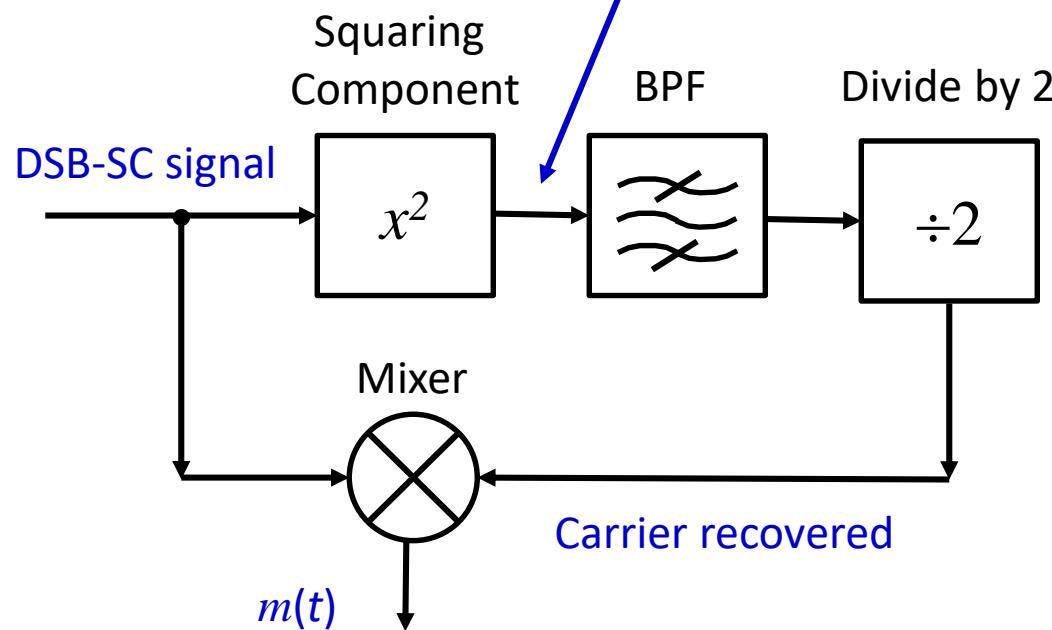
Filed Dec. 16, 1957

Fig. 1



## DSB-SC Synchronous Demodulation (continued)

$$\cos^2 \omega t = \frac{1}{2} [1 + \cos(2\omega t)]$$

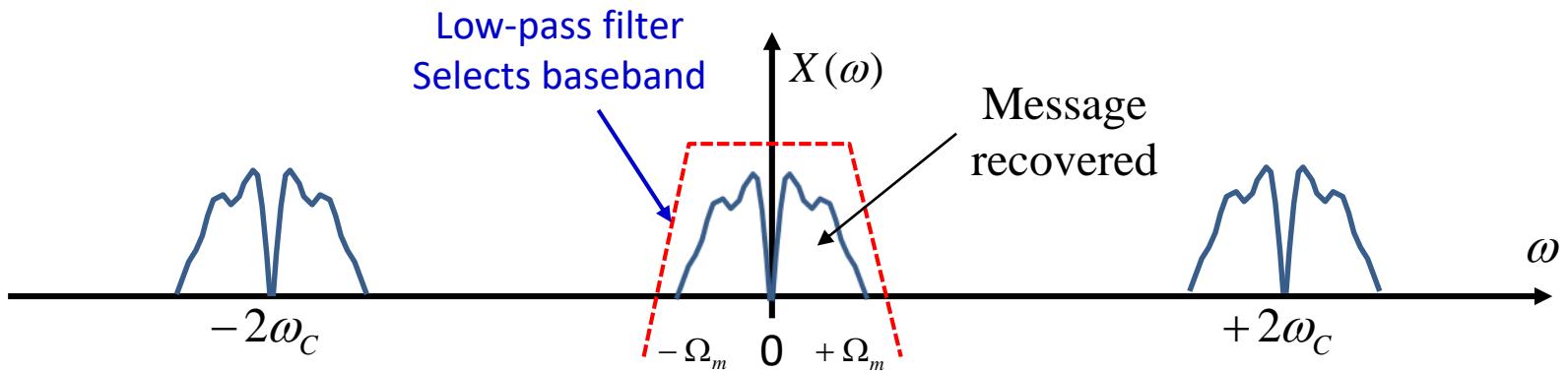


## Double-Sideband Suppressed Carrier (continued)

Let us examine the spectrum of the demodulated DSB-SC signal.

The message sidebands are shifted from being centered at  $\pm \omega_c$  back to the about the origin ( $\omega = 0$ ) and  $\pm 2\omega_c$ . This is illustrated below.

Note: We now need coherent (*i.e.*, synchronous) detection!



We filter out the signals centered at  $\pm 2\omega_c$ .

## Double-Sideband Suppressed Carrier (continued)

Example 3.5 (from page 101 of Agbo & Sadiku):

We are given a carrier signal of  $A_c \cos(\omega_C t)$  and a tone message signal of  $m(t) = A_m \cos(\omega_m t)$

Therefore, the AM signal is

$$\phi_{DSB-SC}(t) = A_c A_m \cdot \cos(\omega_C t) \cdot \cos(\omega_m t) = \frac{A_c A_m}{2} [\cos(\omega_C - \omega_m)t + \cos(\omega_C + \omega_m)t]$$

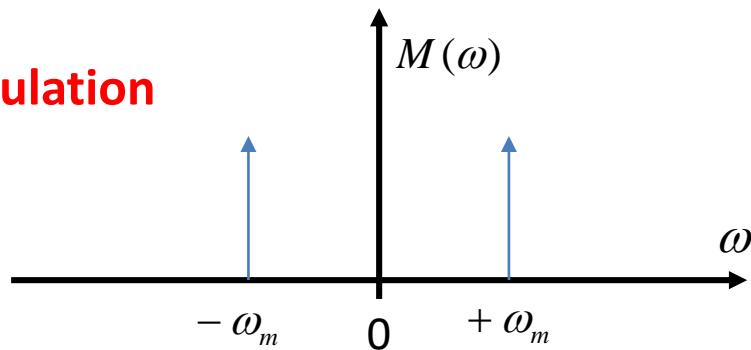
This comes from the identity:  $\cos(\theta) \cdot \cos(\phi) = \frac{1}{2} [\cos(\theta - \phi) + \cos(\theta + \phi)]$

Next, we take the Fourier transform to get

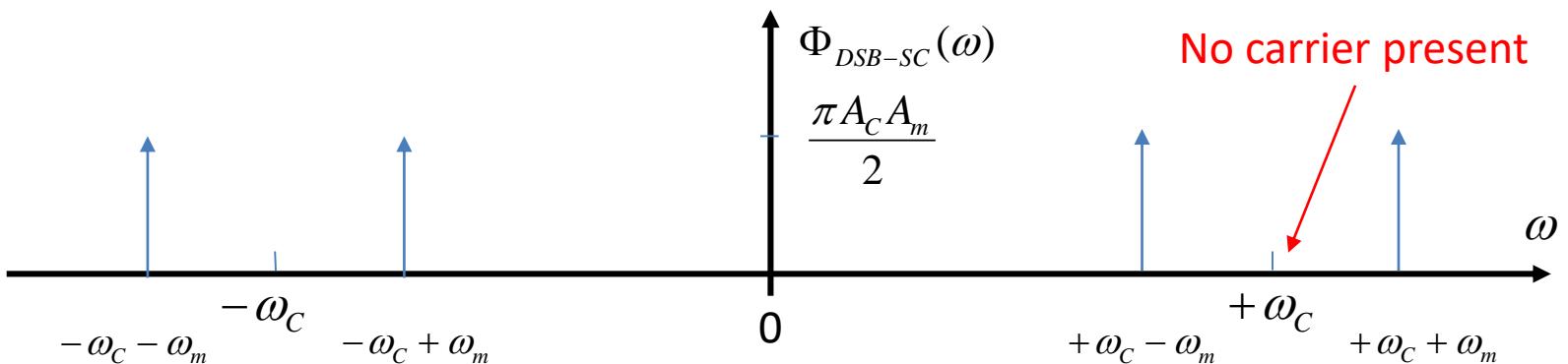
$$\Phi_{DSB-SC}(\omega) = \frac{\pi A_c A_m}{2} \left\{ [\delta(\omega - \omega_C + \omega_m) + \delta(\omega + \omega_C - \omega_m)] + [\delta(\omega - \omega_C - \omega_m) + \delta(\omega + \omega_C + \omega_m)] \right\}$$

## Double-Sideband Suppressed Carrier (continued)

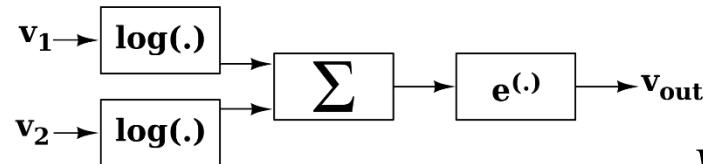
Tone Modulation



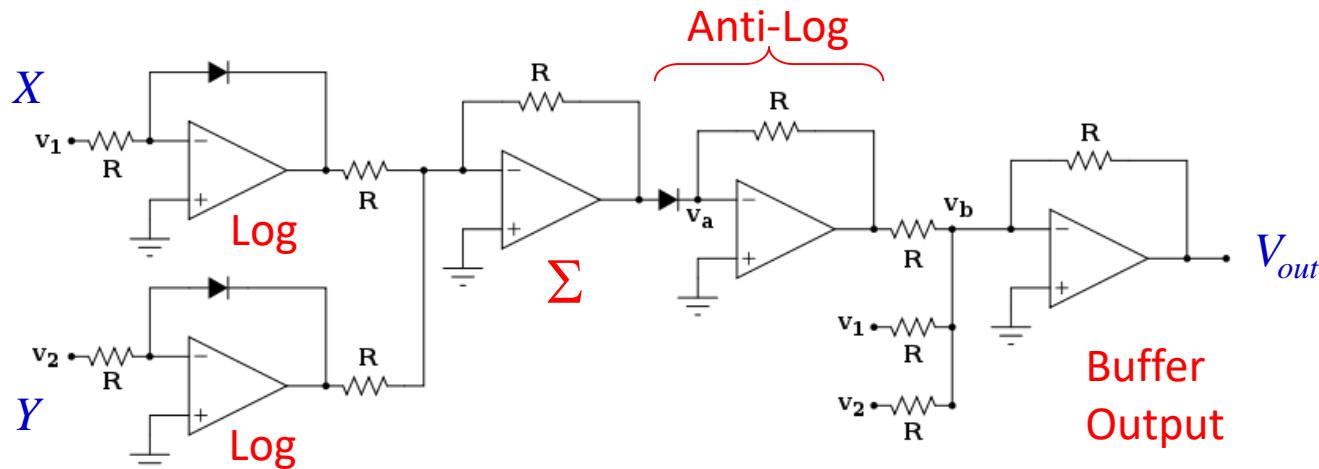
No carrier present



# Analog Product Modulator



$$V_{out} = kX \cdot Y$$

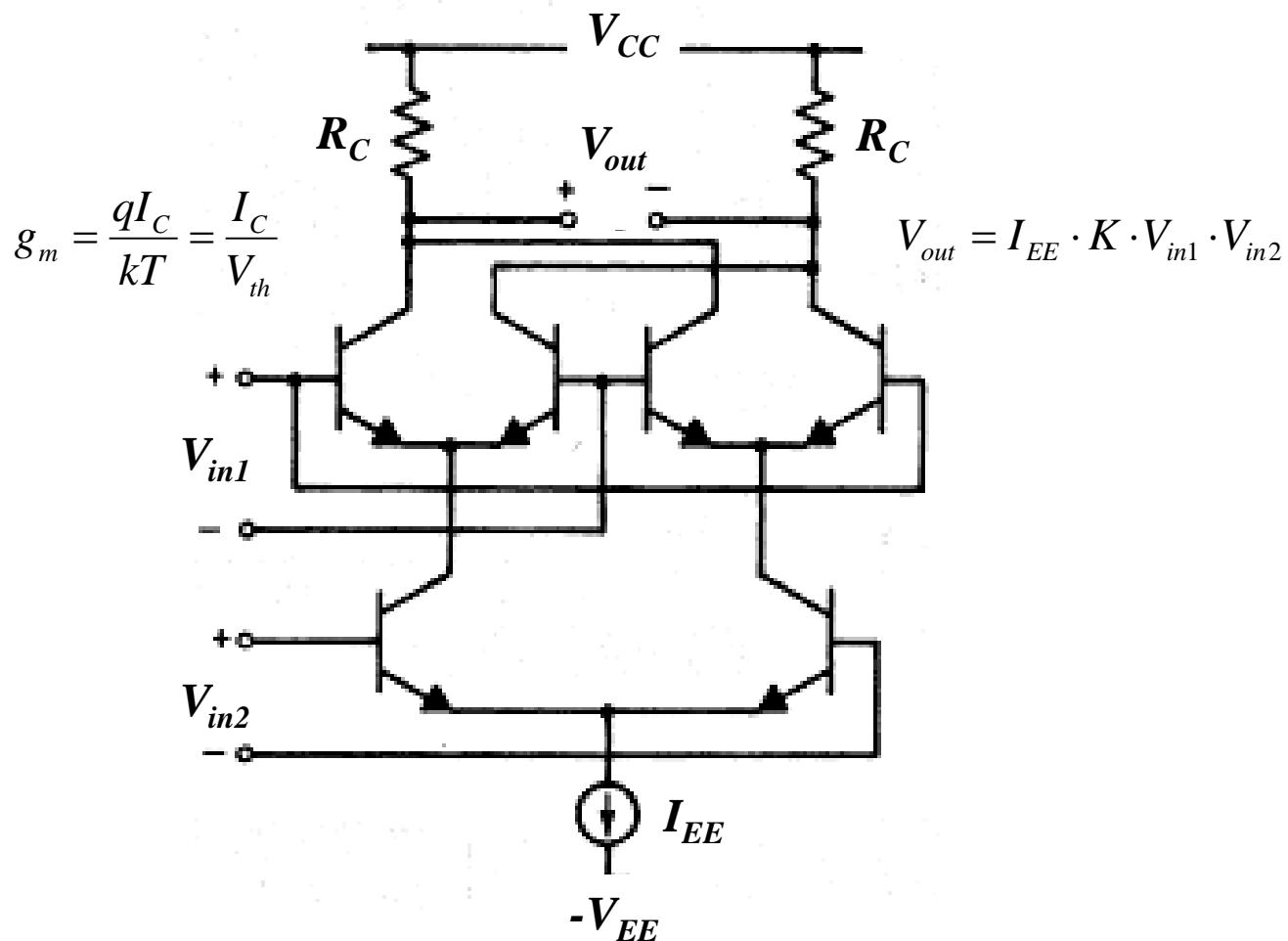


DSB-SC is used primarily today for point-to-point communications where a small number of receivers are involved.

One can buy commercial ICs that perform this function.

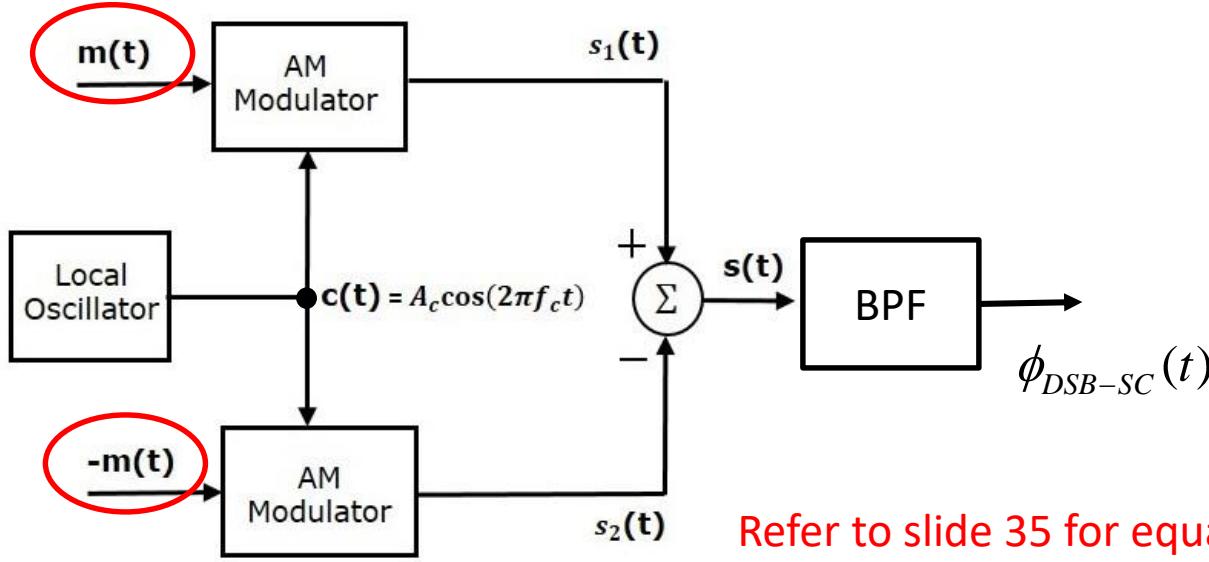
[https://en.wikibooks.org/wiki/Electronics/Analog\\_multipliers](https://en.wikibooks.org/wiki/Electronics/Analog_multipliers)

# Gilbert Cell Multiplier



<http://www.iitk.ac.in/eclub/ee381/AnalogMultipliers.pdf>

## Non-Linear DSB-SC Modulator



Refer to slide 35 for equations.

$$s_1(t) = a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_C^2}{2} [1 + \cos(2\omega_C t)] + a_1 A_C \left[ 1 + \frac{2a_2 m(t)}{a_1} \right] \cos(\omega_C t)$$

$$s_2(t) = -a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_C^2}{2} [1 + \cos(2\omega_C t)] + a_1 A_C \left[ 1 - \frac{2a_2 m(t)}{a_1} \right] \cos(\omega_C t)$$

$$s(t) = s_1(t) - s_2(t) = 2a_1 m(t) + 4a_2 A_C \cdot m(t) \cdot \cos(\omega_C t)$$

$$\therefore \phi_{DSB-SC}(t) = 4a_2 A_C \cdot m(t) \cdot \cos(\omega_C t) \Leftarrow \text{BPF selected this}$$

## Non-Linear DSB-SC Modulator (continued)

$$\phi_{DSB-SC}(t) = 4a_2 A_C \cdot m(t) \cdot \cos(\omega_C t)$$

Note that this expression contains no carrier signal. Why?

Answer: The modulator is a **balanced configuration** and this results in the carrier signal being cancelled (that assumes perfect balance, of course).

Definition: A **balanced modulator** does not output either the carrier component or the message component. When both are missing, we say it is **double-balanced**.

## Switching DSB-SC Modulators

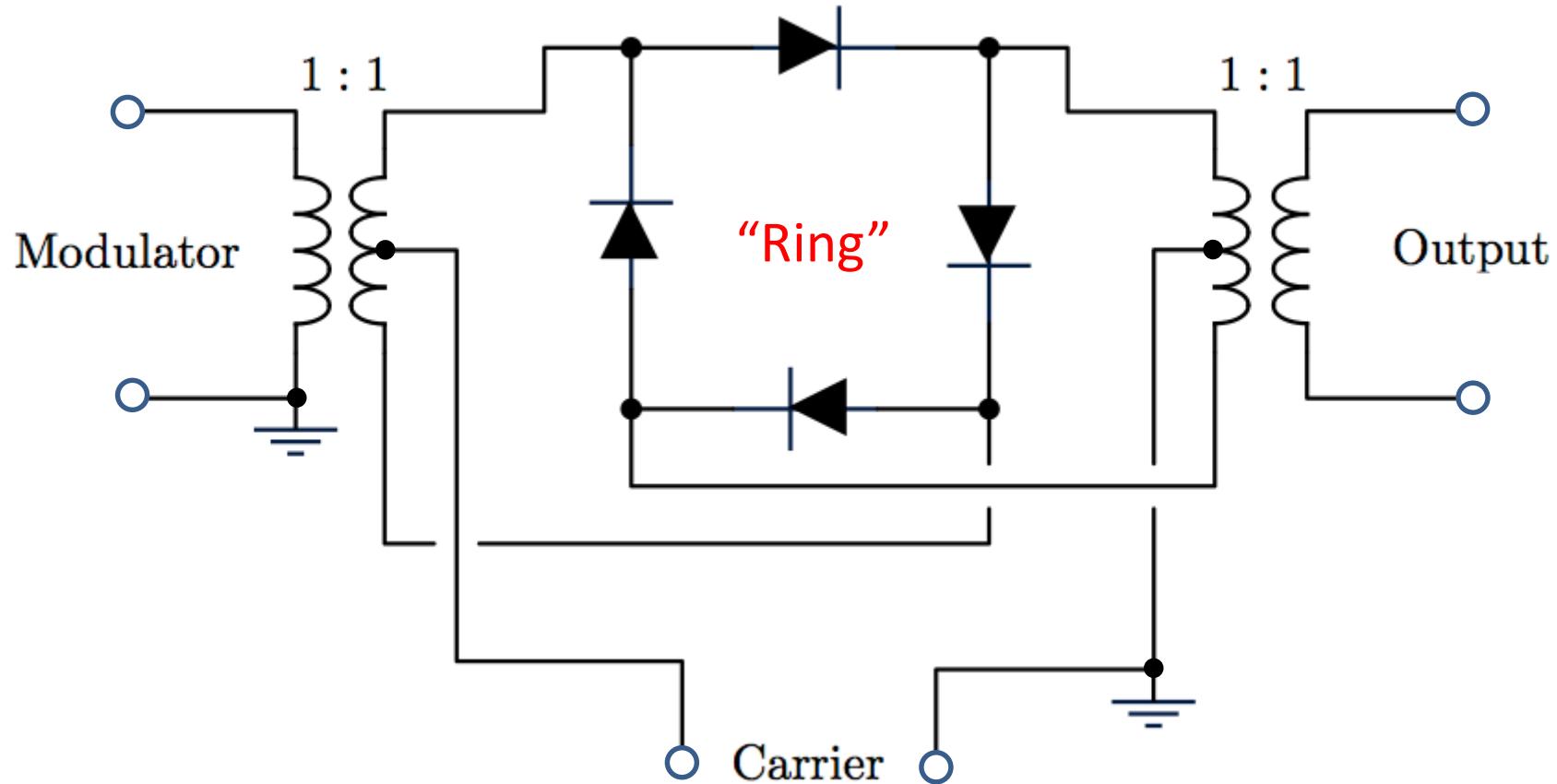
Agbo & Sadiku present three switching modulators:

1. Series-bridge modulator
2. Shunt-bridge modulator, and
3. **Ring modulator**

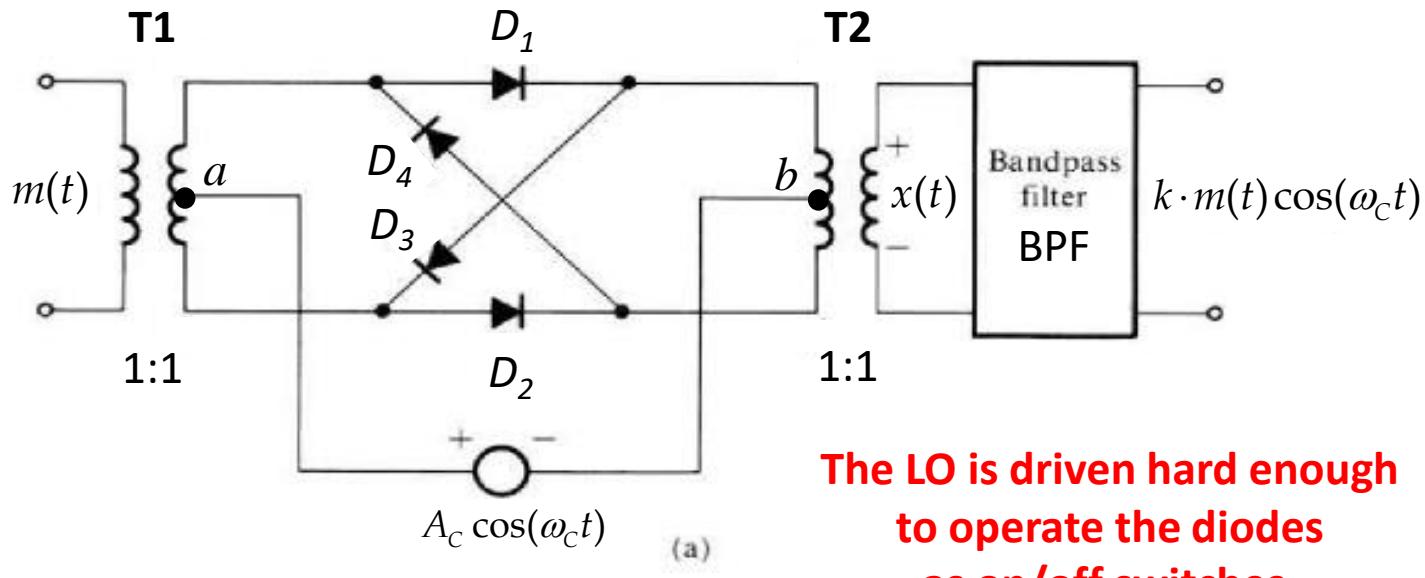
We are only going to discuss the ring modulator because it is the most widely used and contains the fundamental principle of operation for all of them. It is important you understand how it works.

**Note: Examples of series-bridge and shunt-bridge modulators are shown in Figure 3.16 (a) and (b), page 106 of Agbo and Sadiku.**

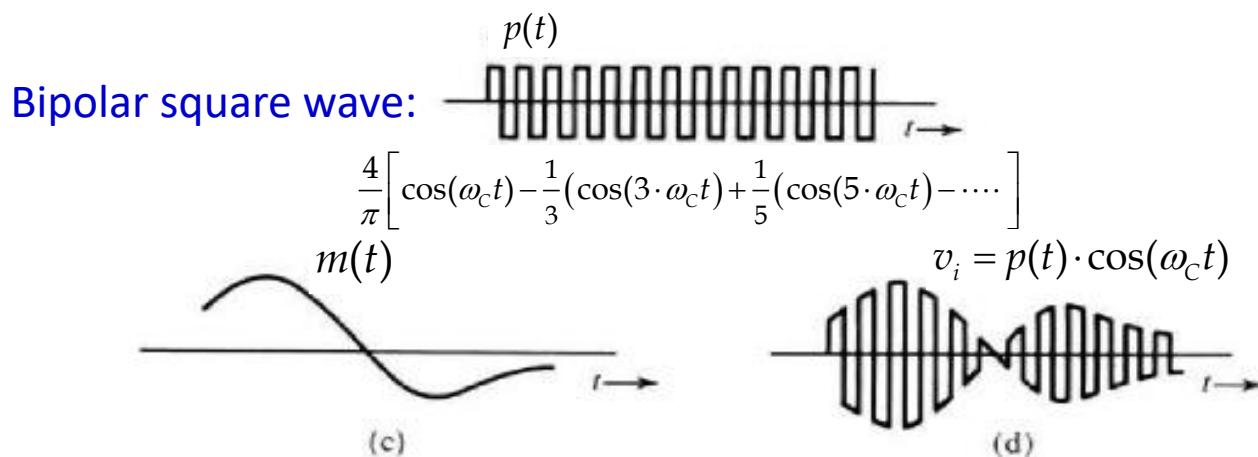
## Ring Modulator for DSB-SC Generation



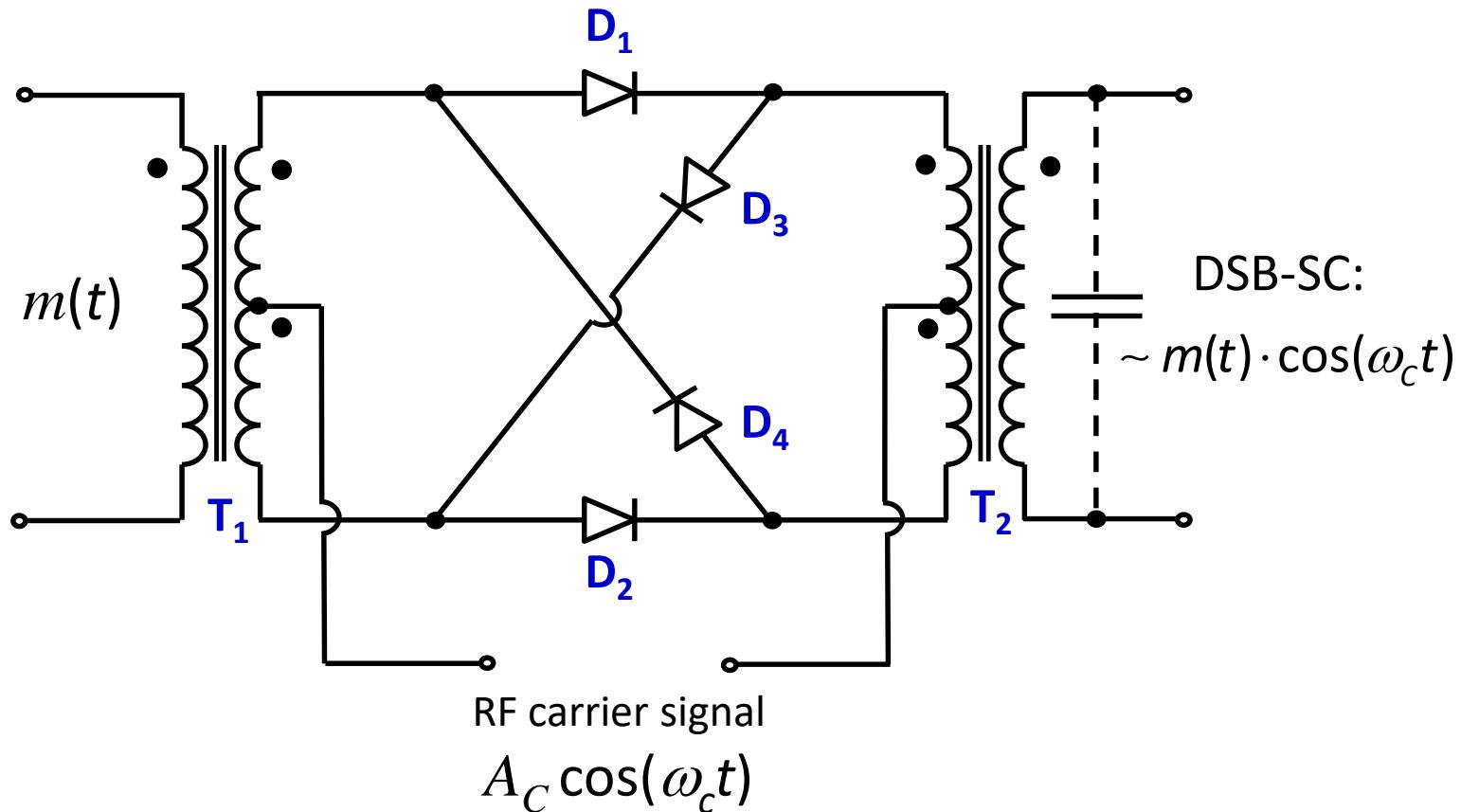
# Double-Balanced Diode Ring Modulator



The LO is driven hard enough  
to operate the diodes  
as on/off switches.



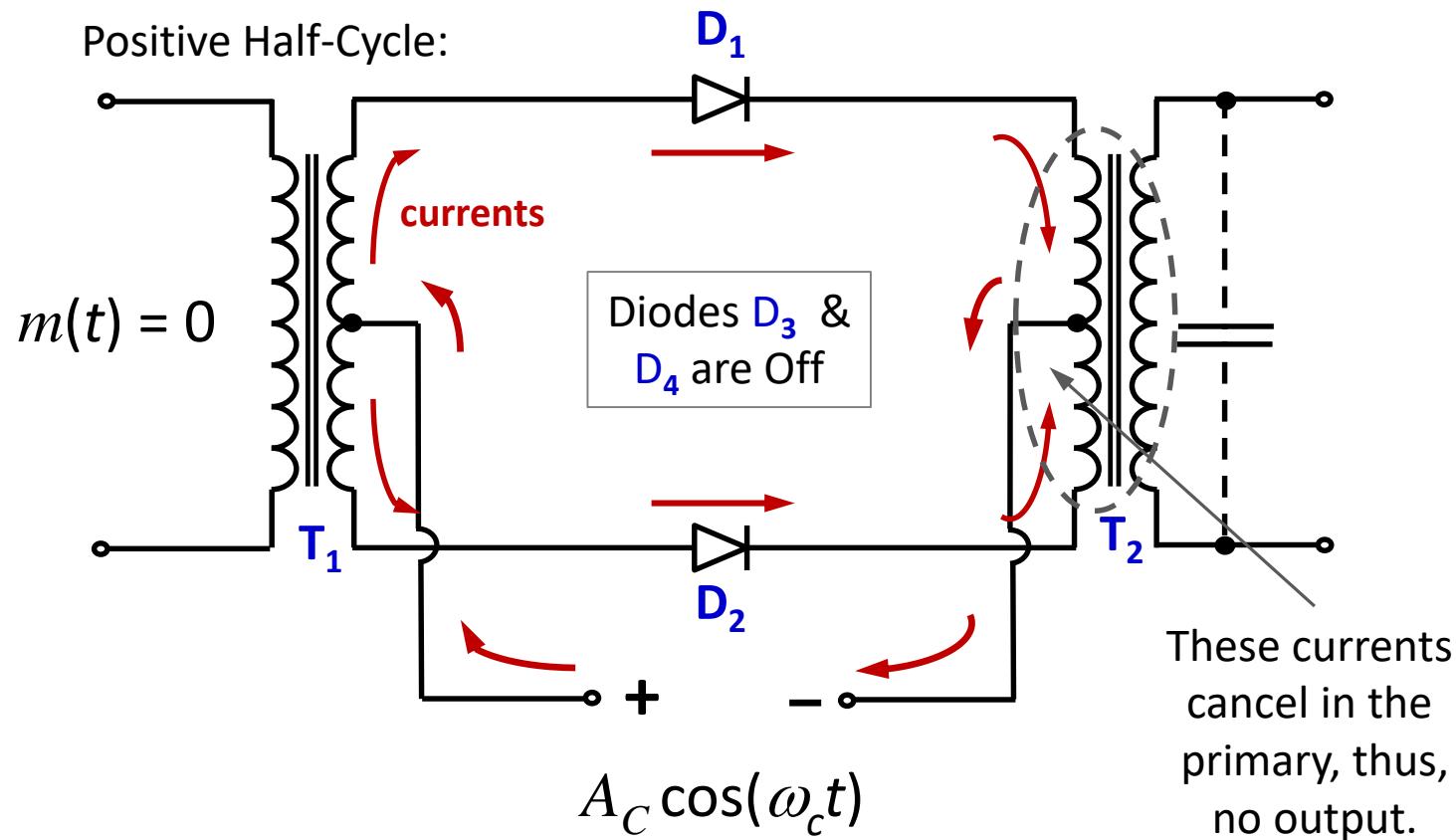
## Double-Balanced Diode Ring Modulator (continued)



Assume the diodes act as perfect switches (either “on” or “off”) and are controlled by the RF carrier signal (requires large amplitude).

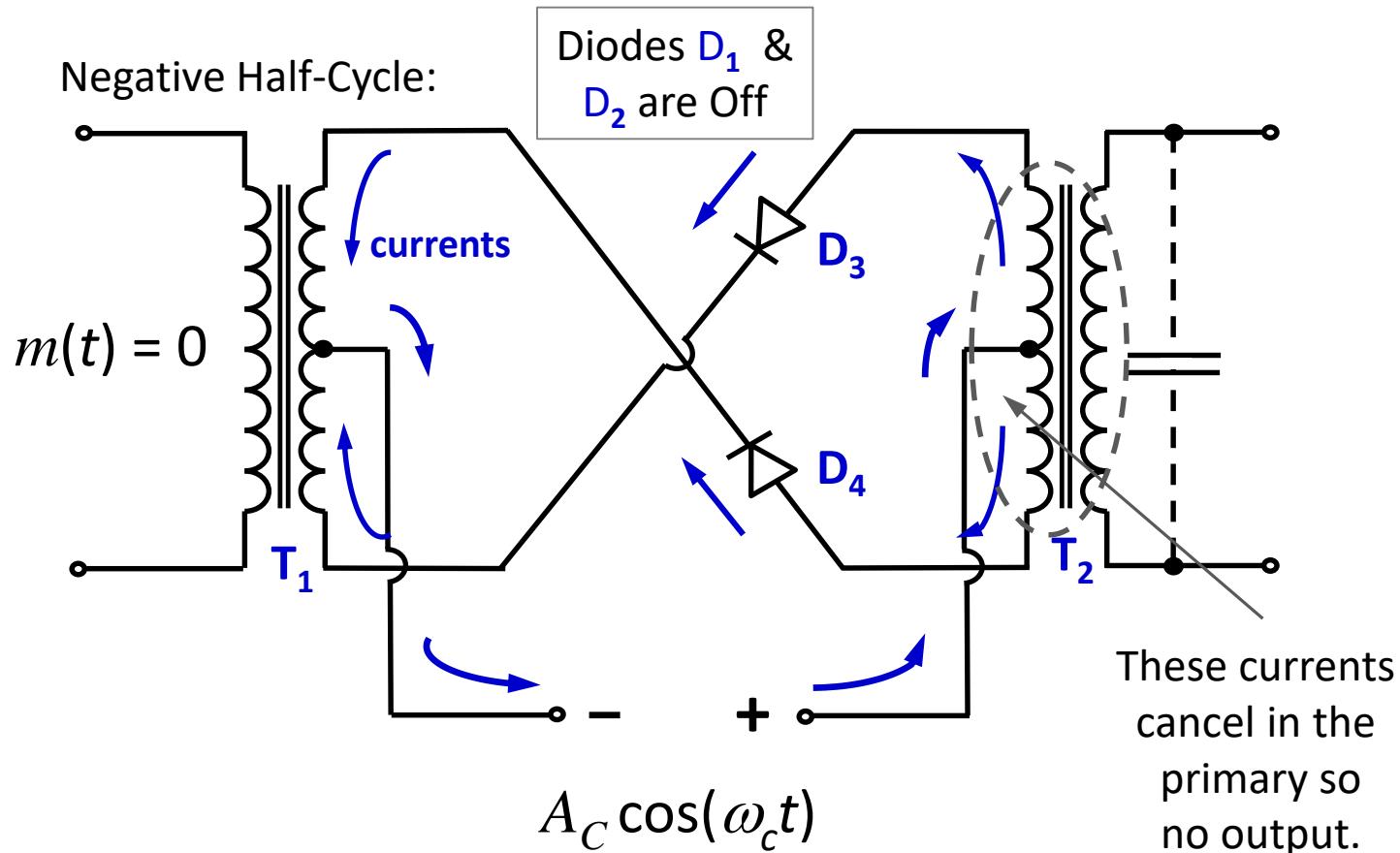
## Double-Balanced Diode Ring Modulator (continued)

Operation in the positive half-cycle of the carrier signal



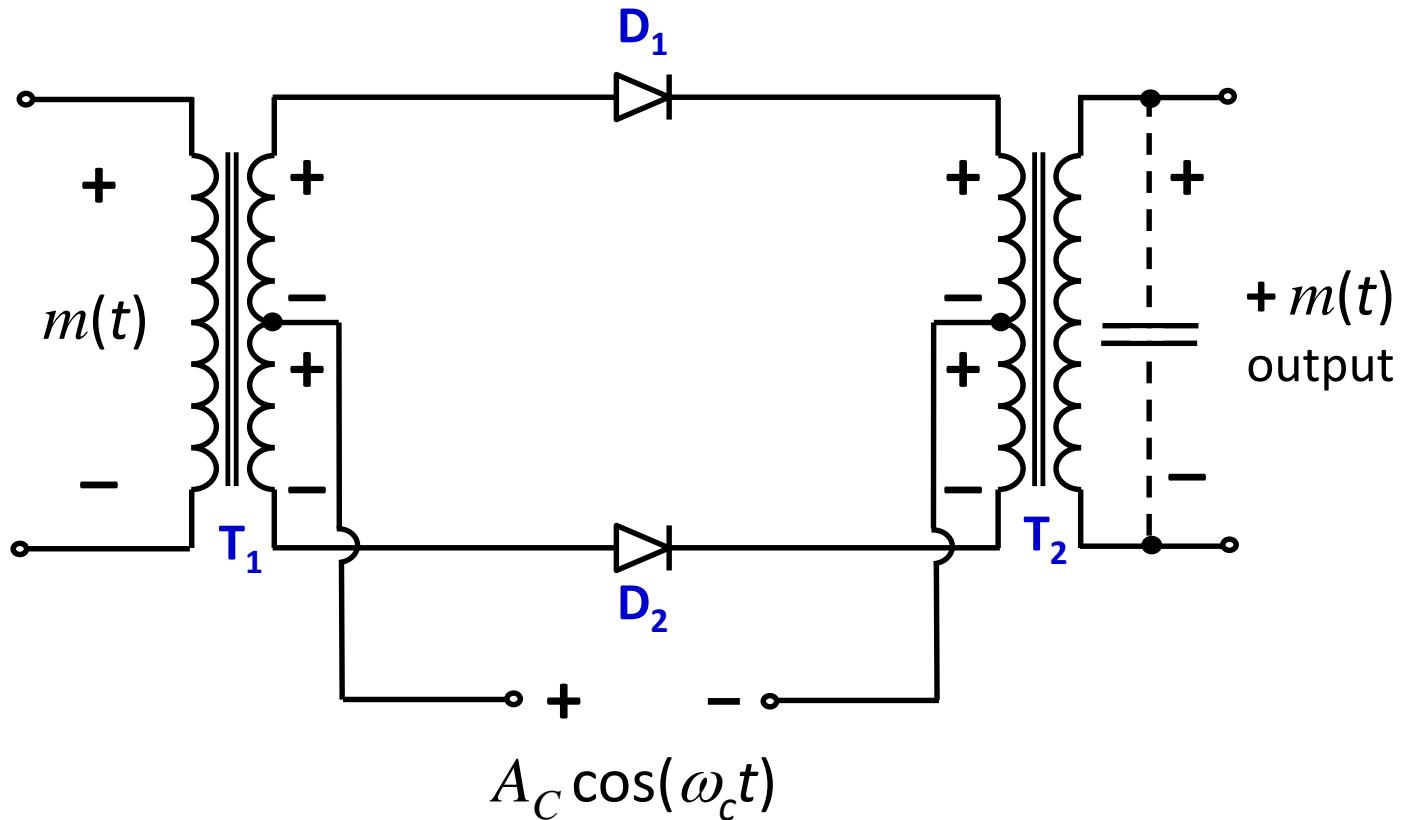
## Double-Balanced Diode Ring Modulator (continued)

Operation in the negative half-cycle of the carrier signal



## Double-Balanced Diode Ring Modulator (continued)

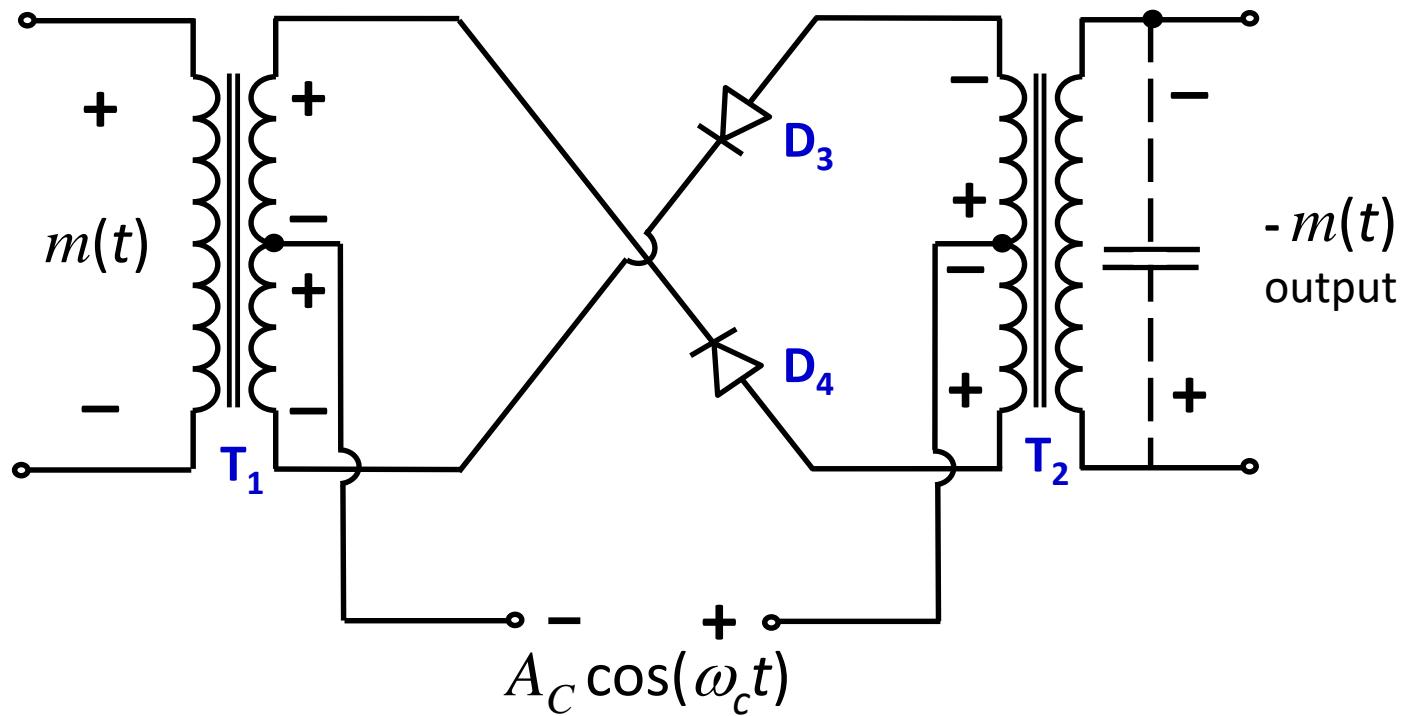
Operation in the positive half-cycle of the carrier signal passes message signal  $m(t)$  to output.



Diodes D<sub>1</sub> and D<sub>2</sub> are “on” and the secondary of T<sub>1</sub> is applied directly to T<sub>2</sub>.

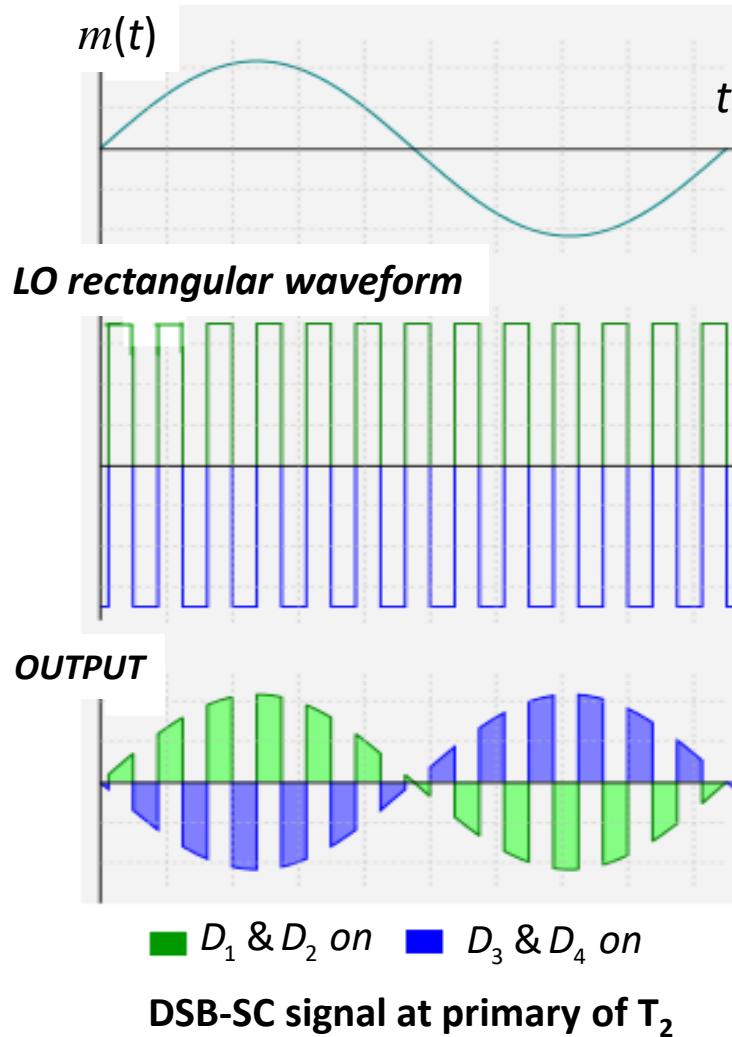
## Double-Balanced Diode Ring Modulator (continued)

Operation in the negative half-cycle of the carrier signal  
inverts message signal  $m(t)$  at the output.



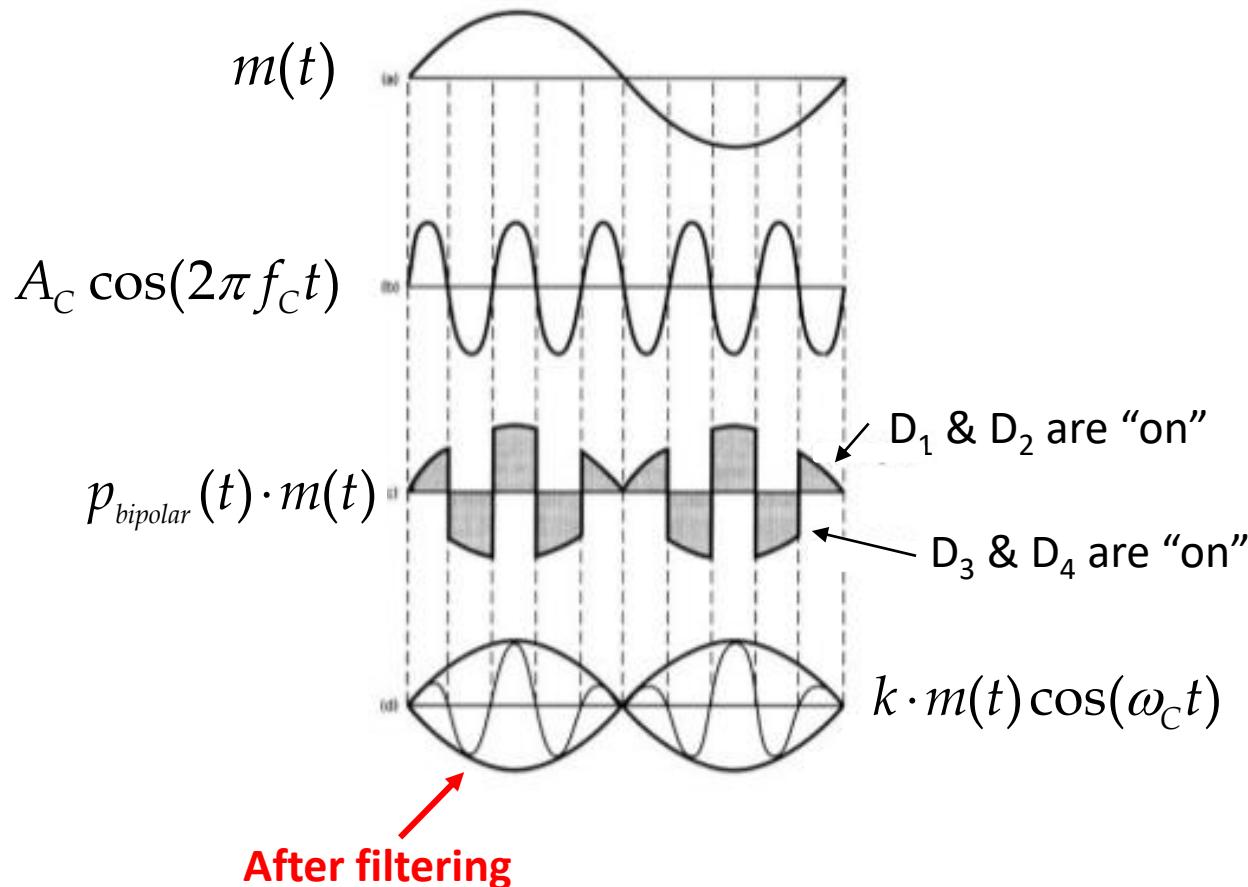
Diodes  $D_3$  and  $D_4$  are “on” and the secondary of  $T_1$  is applied directly to  $T_2$ .

# Double-Balanced Diode Ring Modulator (continued)



<https://electronicspost.com/ring-modulator-for-the-double-sideband-suppressed-carrier-generation/>

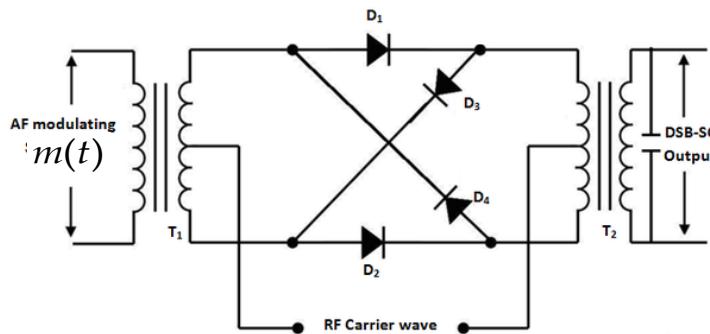
# Double-Balanced Diode Ring Modulator Waveforms



# Double-Balanced Diode Ring Modulator

## Requirements:

1. The carrier signal is higher in amplitude than the modulating signal  $m(t)$ .
2. The carrier signal must be of sufficient amplitude to fully switch the diodes between “on” and “off” states.
3. The carrier signal switches the diodes on and off at a rate higher than the highest frequency contained in  $m(t)$ .
4. The message signal  $m(t)$  is chopped into segments, alternating between two amplitudes;  $+m(t)$  and  $-m(t)$ .



## Double-Balanced Diode Ring Modulator

The mathematics behind the Diode Ring Modulator:

$$p_{bipolar}(t) = 2 \left( p(t) - \frac{1}{2} \right) = 2p(t) - 1 \quad \Leftarrow \quad \text{This is a bipolar square wave train.}$$

$$p_{bipolar}(t) = 2 \left\{ \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \dots \right] \right\}$$

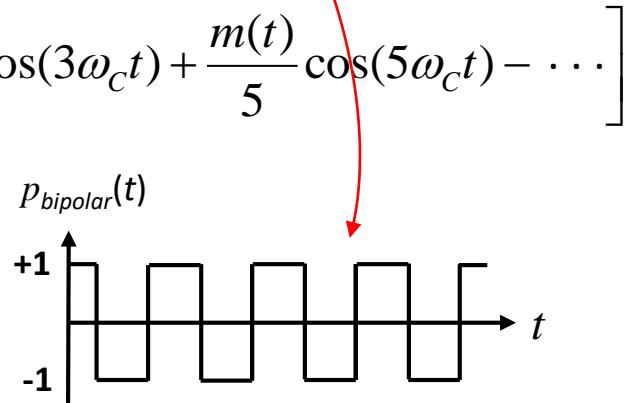
$$p_{bipolar}(t) = \frac{4}{\pi} \left[ \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \dots \right]$$

Therefore, the output is found by the product,

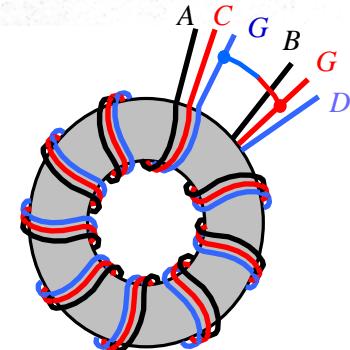
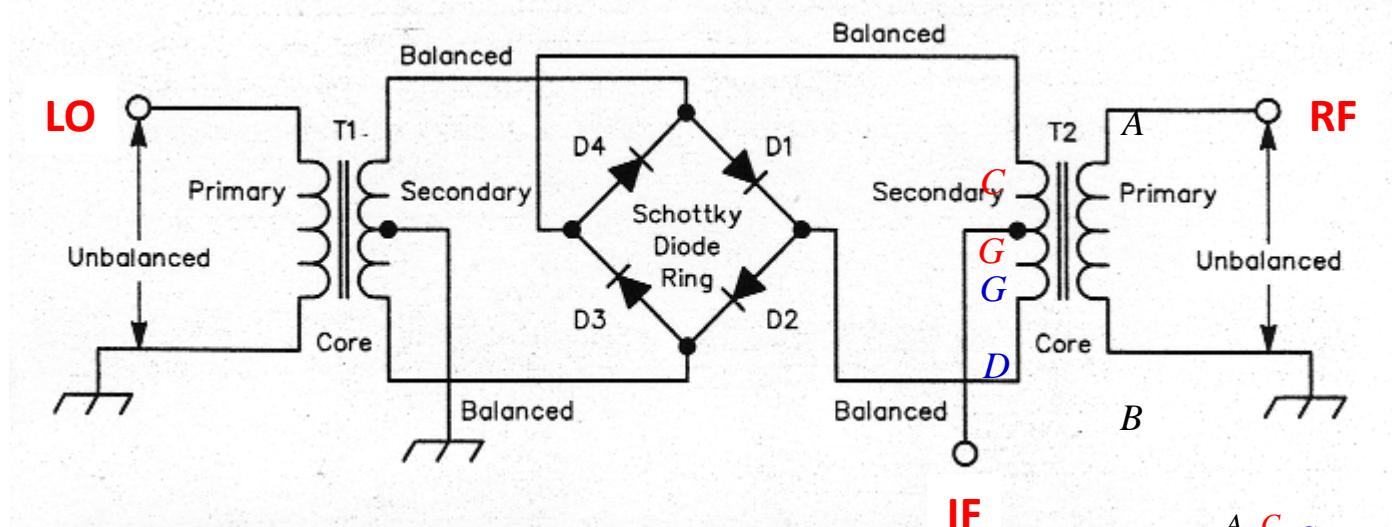
$$x(t) = m(t)p_{bipolar}(t) = \frac{4}{\pi} \left[ m(t) \cdot \cos(\omega_c t) - \frac{m(t)}{3} \cos(3\omega_c t) + \frac{m(t)}{5} \cos(5\omega_c t) - \dots \right]$$

Upon passing through the BPF,

$$\phi_{DSB-SC}(t) = \frac{4}{\pi} \left[ m(t) \cdot \cos(\omega_c t) \right]$$



# Double-Balanced Diode Ring Modulator/Mixer



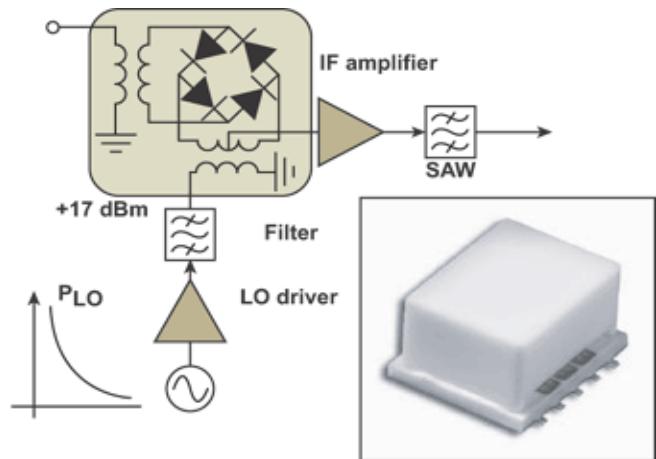
Trifilar-Wound Toroid

<http://kambing.ui.ac.id/onnopurbo/orari-diklat/teknik/qrp/Broadband%20Transformers.htm>

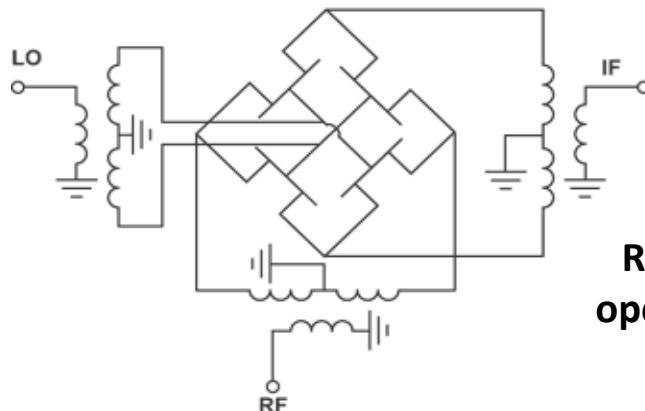
It is inexpensive and easy to build a ring mixer.

# Commercial Diode Ring Mixer (Mini-Circuits)

It is even easier  
to buy a ring mixer  
component.



Mixer in surface-mount package



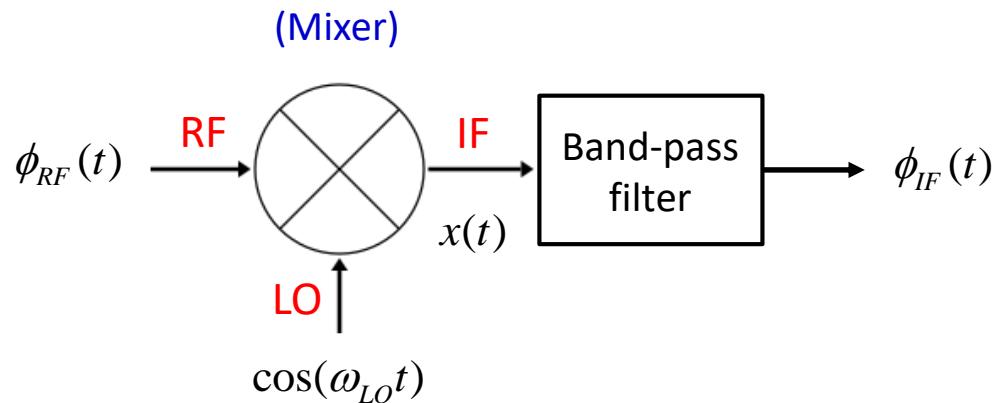
Ring of FET devices  
operated as nonlinear  
resistances

1. These diagrams show a typical diode ring quad (top) or FET passive mixer (bottom); the inset photograph shows is a surface-mount-packaged mixer from Mini-Circuits (Brooklyn, NY).

# Mixers

Frequency mixing → frequency conversion → heterodyning

A mixer translates the modulation around one carrier frequency to another frequency. In a receiver, this is usually from a higher RF frequency to a lower IF frequency. In a transmitter, it's the inverse.



We know that a LTI circuit can't perform frequency translation. Mixers can be realized with either time-varying circuits or non-linear circuits.

## Digression: What is Heterodyning?

**Heterodyning** is a signal processing technique invented in 1901 by Canadian inventor-engineer Reginald Fessenden that creates new frequencies by combining or mixing two frequencies using a nonlinear device. Edwin Armstrong invented the superheterodyne receiver in 1918.

Using an electronic circuit to combine an input radio frequency signal (RF) with another signal that is locally generated (LO) to produce new frequencies (IF): one being the sum of the two frequencies and the other being the difference of the two frequencies.

<http://www.yourdictionary.com/heterodyning#SgeI5zTvo7VV96o7.99>

### Applications of heterodyning:

1. Used in communications to generate new frequencies.
2. Move modulated signals from one frequency channel to another.
3. Used in the superheterodyne radio receivers able to select from multiple communication channels.

## Frequency Translation By Mixing

Let  $\phi_{RF}(t) = m(t) \cdot \cos(\omega_C t)$  and  $\phi_{IF}(t) = m(t) \cdot \cos(\omega_{IF} t)$

The local oscillator (LO) is proportional to  $\cos(\omega_{LO} t)$

The mixer (or multiplier) output  $x(t)$  is given by

$$x(t) = 2m(t) \cdot \cos(\omega_C t) \cdot \cos(\omega_{LO} t)$$

(A.) Choosing  $\omega_{LO} = \omega_C - \omega_{IF}$ , we have

$$x(t) = m(t) \left[ \cos((\omega_C - \omega_{IF} - \omega_C)t) + \cos((\omega_C - \omega_{IF} + \omega_C)t) \right]$$

$$x(t) = m(t) \cdot \cos(\omega_{IF} t) + m(t) \cdot \cos((2\omega_C - \omega_{IF})t)$$

Note: Used even property of cosines [i.e.,  $\cos(-\theta) = \cos(\theta)$ ]

(B.) Choosing  $\omega_{LO} = \omega_C + \omega_{IF}$ , then we have

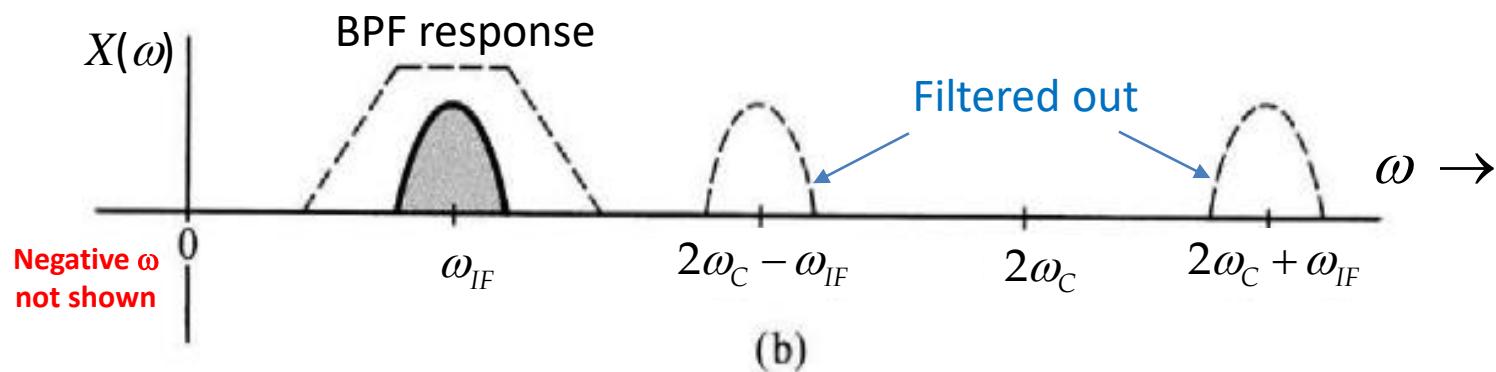
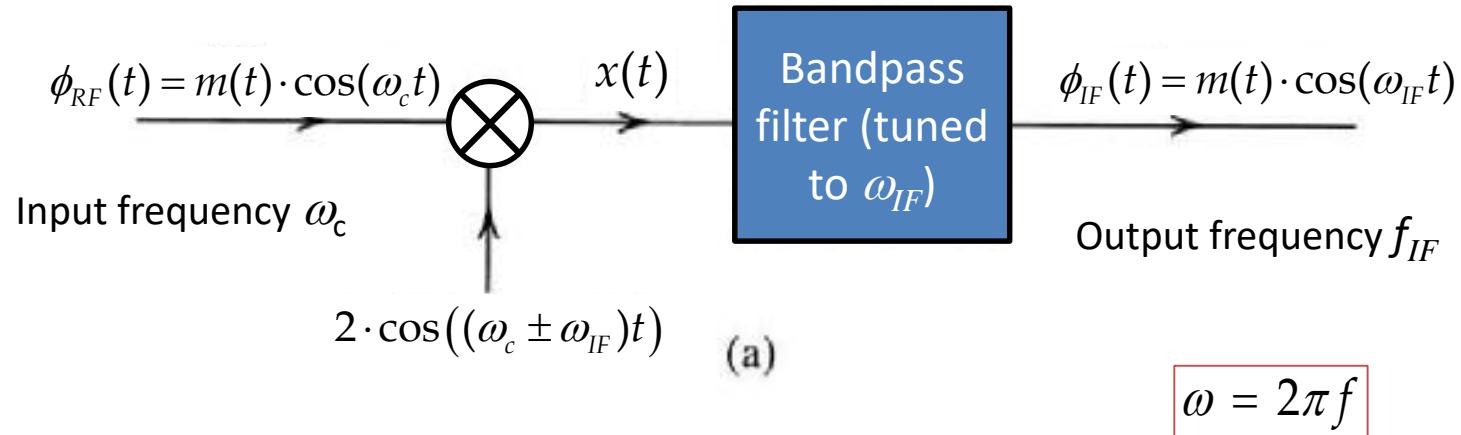
$$x(t) = m(t) \left[ \cos((\omega_C + \omega_{IF} - \omega_C)t) + \cos((\omega_C + \omega_{IF} + \omega_C)t) \right]$$

$$x(t) = m(t) \cdot \cos(\omega_{IF} t) + m(t) \cdot \cos((2\omega_C + \omega_{IF})t)$$

## Frequency Conversion From $\omega_c$ to $\omega_{IF}$ With a Mixer

Multiplying a modulated signal by a sinusoidal moves the frequency band to sum and difference frequencies.

Example: We want to convert from frequency  $\omega_c$  to frequency  $\omega_{IF}$ .



Note: Super-heterodyning:  $\omega_c + \omega_{IF}$ ; Sub-heterodyning:  $\omega_c - \omega_{IF}$

## Mixer Example (Example 3.7 on Page 110)

Example: Derive the relationship between  $\omega_{LO}$  and  $\omega_C$  so that centering the bandpass filter of the mixer is at  $\omega_{LO} - \omega_C$  and also ensure that  $\omega_{IF}$  is less than (*i.e.*, below)  $\omega_C$ .

**Answer:**

We know that  $\omega_{LO} = \omega_C \pm \omega_{IF}$  in general. We must meet two conditions:

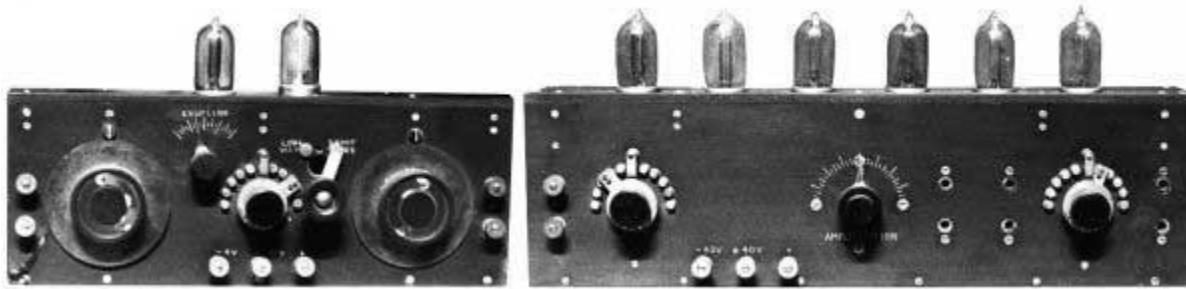
$$(1) \quad \omega_{IF} = \omega_{LO} - \omega_C \quad \text{and} \quad (2) \quad \omega_{IF} < \omega_C$$

Start by assuming  $\omega_{LO} = \omega_C + \omega_{IF}$  that meets the first condition; then the second condition,  $\omega_{IF} < \omega_C$ , implies that

$$\omega_{LO} - \omega_C < \omega_C \rightarrow \boxed{\omega_{LO} < 2\omega_C}$$

## Digression

### Superheterodyne Receiver



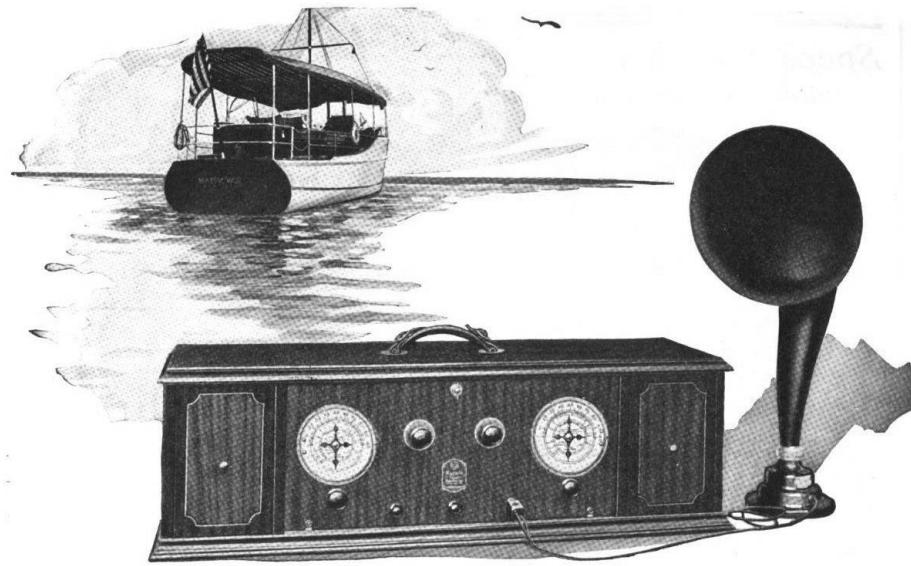
The superheterodyne, the circuit used in virtually all modern radios, was invented by Edwin Armstrong in 1918 while he worked in a US Army Signal Corps laboratory in Paris during World War I. This is one of the receivers constructed at that laboratory, shown in a 1920 article in an amateur radio magazine. It is constructed in two parts. The righthand section consists of the mixer and local oscillator. Now on display at the Smithsonian.

[https://commons.wikimedia.org/wiki/File:Prototype\\_Armstrong\\_superheterodyne\\_receiver\\_1920.jpg](https://commons.wikimedia.org/wiki/File:Prototype_Armstrong_superheterodyne_receiver_1920.jpg)



## First Commercial Superheterodyne Receiver

An advertisement for the **RCA Radiola AR-812** radio – it was the first commercially produced superheterodyne radio receiver. The superheterodyne receiver circuit was invented by US engineer Edwin Armstrong in 1918 during World War I. The rights were purchased by RCA, and the AR-812 was released March 4, 1924. It used 6 UV-199 triodes: a mixer, a local oscillator, two IF and two audio amplifier stages, with an IF of 45 kHz, and was priced at \$289. It was semi-portable, with compartments for the batteries in back and a handle on top, and it weighed 30 pounds.



[https://en.wikipedia.org/wiki/Superheterodyne\\_receiver](https://en.wikipedia.org/wiki/Superheterodyne_receiver)

**Take It Aboard**

# Advantages of the Superheterodyne Receiver

## Superior selectivity in signal reception

Uses fixed frequency filters allowing for excellent adjacent channel signal rejection and also helps with image rejection.

## Able to receive multiple modulation schemes

Ability to incorporate a wide variety of different types of demodulators.

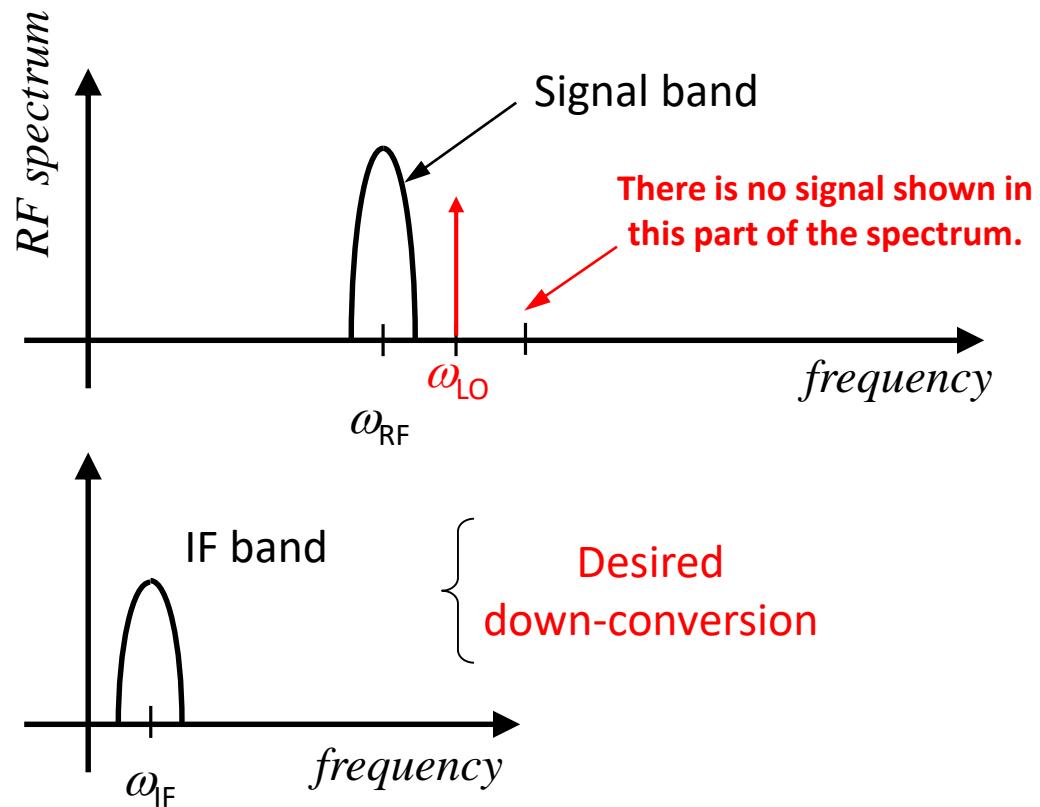
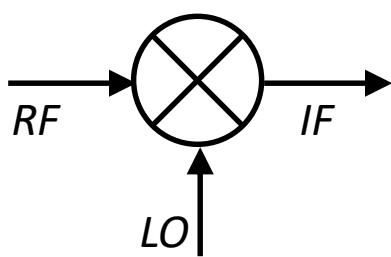
## Able to receive very high frequency signals

Most of the receiver's signal processing is done at lower frequencies using mixers to down convert higher frequency incoming signals (easier to filter at lower frequencies, lower cost components, etc.).

The word **heterodyne** is derived from the Greek roots *hetero* -"different", and *- dyne* "power".

# Image Signals in Mixers (1)

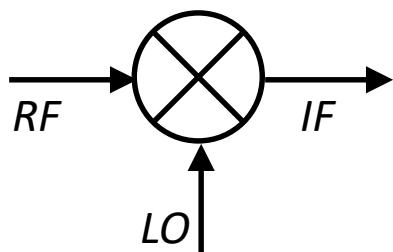
This converts the spectrum at the RF carrier frequency down to the spectrum centered at the IF frequency.



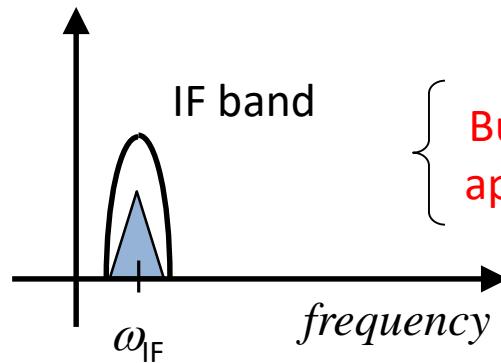
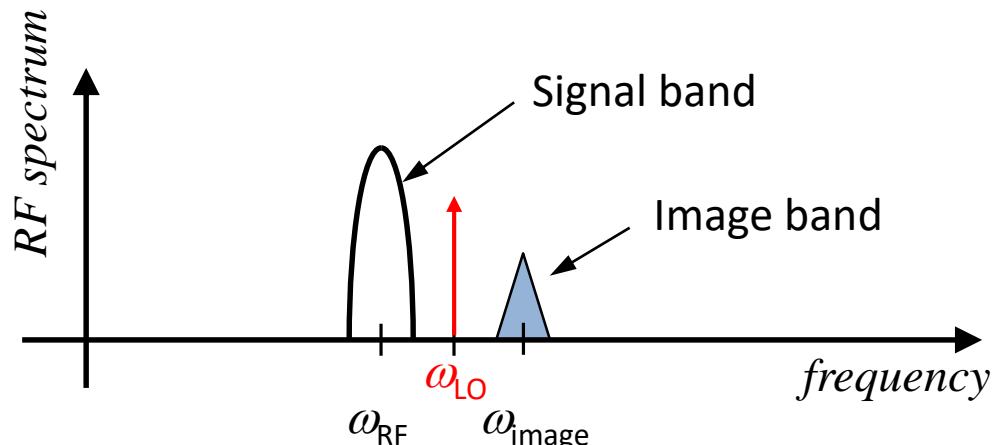
$$\omega_{IF} = \omega_{LO} - \omega_{RF}$$

## Image Signals in Mixers (2) – Now an Image Signal Appears

Now both the spectrum at the RF carrier frequency and the undesired image spectrum are down converted to the spectrum centered at the IF frequency.



The image spectrum is not wanted.

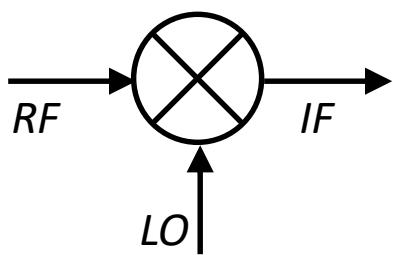


But, now both signals appear in the IF band.

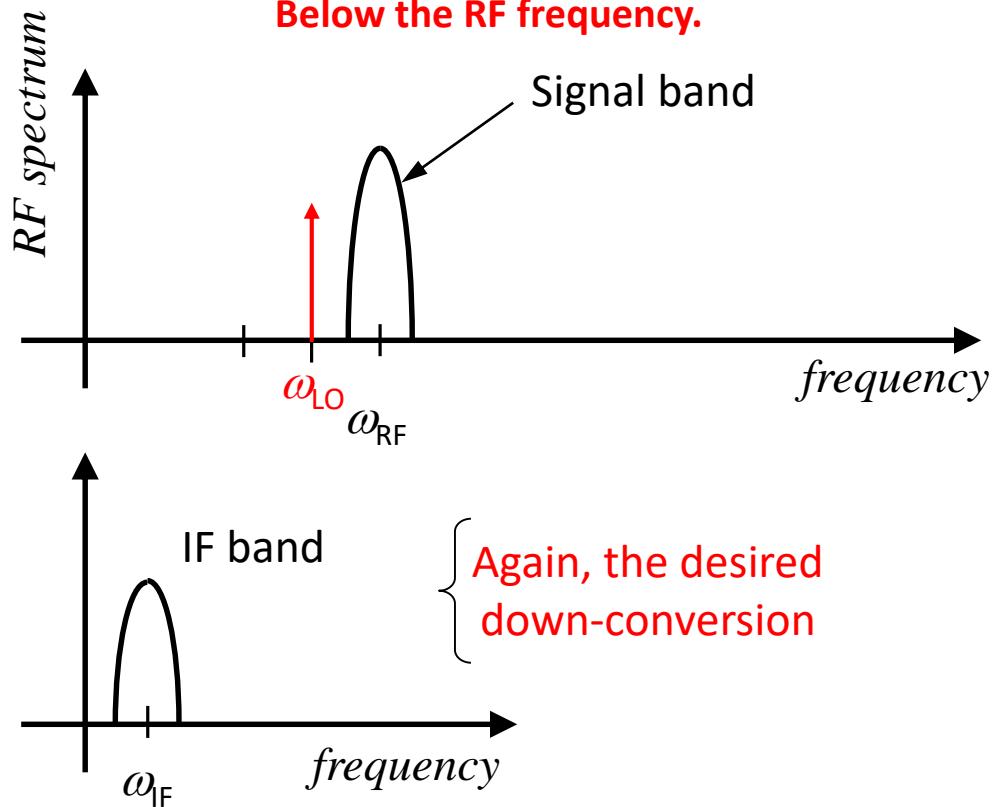
$$\begin{aligned}\omega_{IF} &= \omega_{LO} - \omega_{RF} \\ \text{and} \\ \omega_{IF} &= \omega_{image} - \omega_{LO}\end{aligned}$$

## Image Signals in Mixers (3)

This converts the spectrum at the RF carrier frequency down to the spectrum centered at the IF frequency.



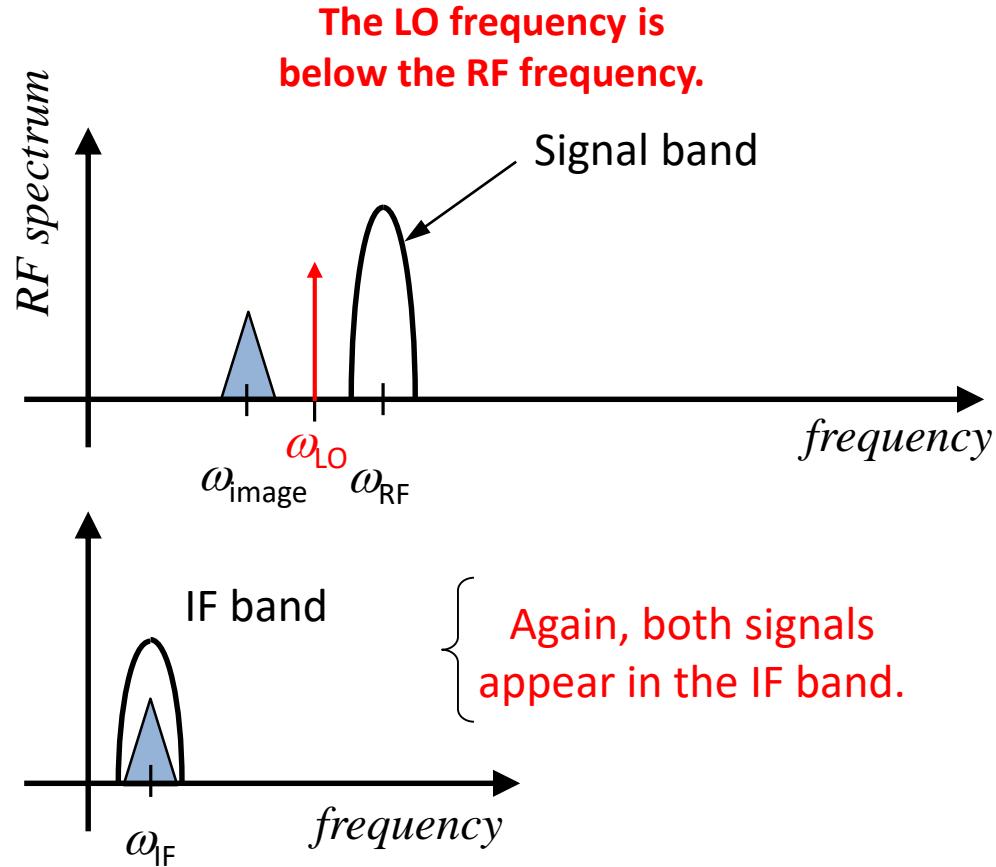
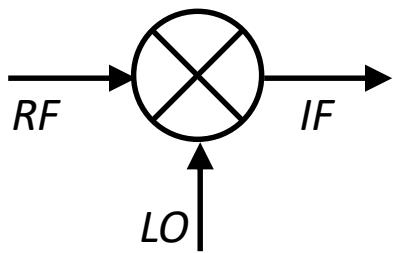
Suppose the LO frequency is Below the RF frequency.



$$\omega_{IF} = \omega_{RF} - \omega_{LO}$$

## Image Signals in Mixers (4) – With Image Signal

As before both the spectrum at the RF carrier frequency and the undesired image spectrum are down converted to the spectrum centered at the IF frequency.

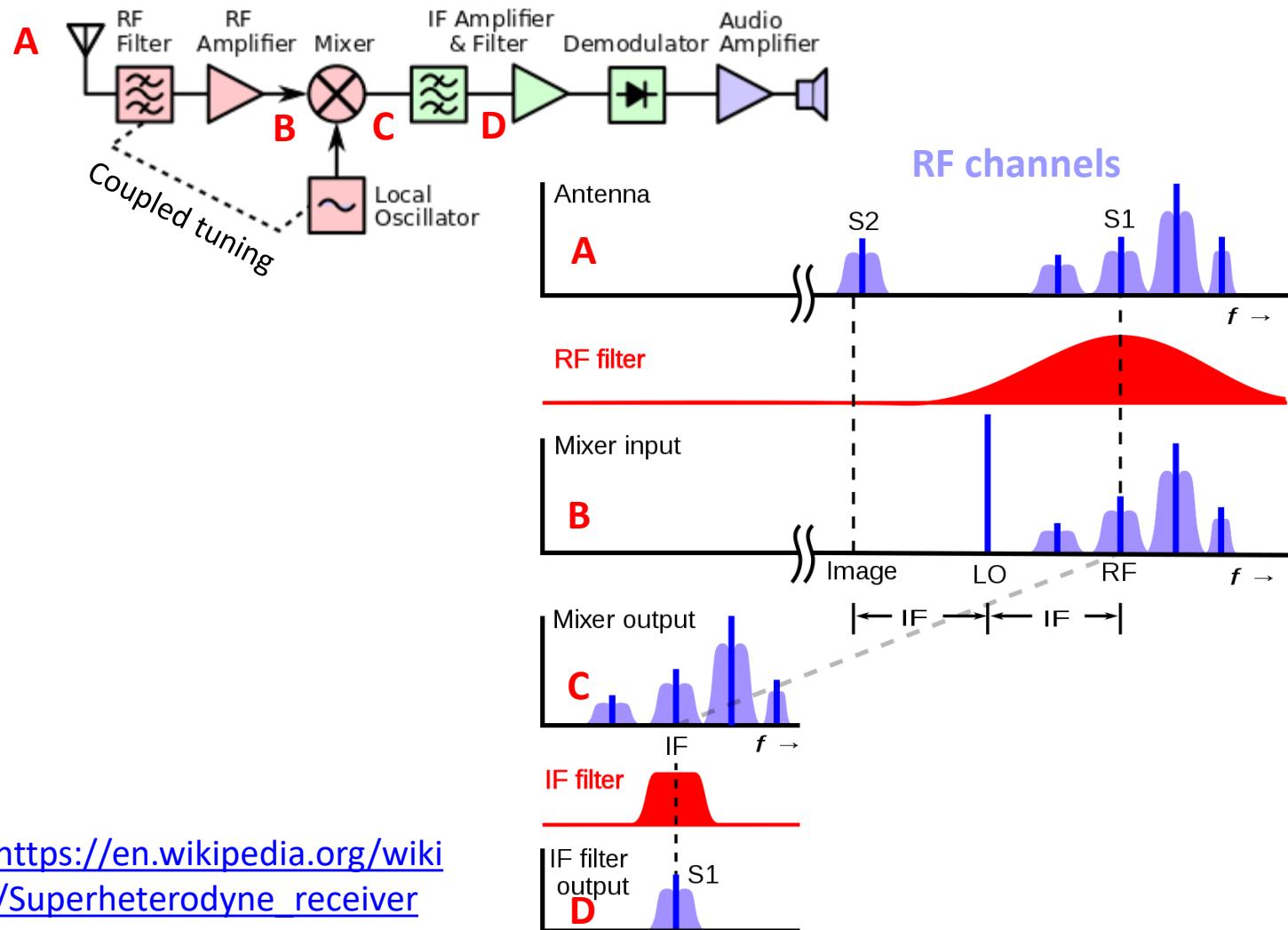


$$\omega_{\text{IF}} = \omega_{\text{RF}} - \omega_{\text{LO}}$$

and

$$\omega_{\text{IF}} = \omega_{\text{LO}} - \omega_{\text{image}}$$

# Image Rejection in a Single Mixer Heterodyne Receiver



[https://en.wikipedia.org/wiki/Superheterodyne\\_receiver](https://en.wikipedia.org/wiki/Superheterodyne_receiver)

# **Simultaneous Selectively and Image Rejection**

## **High Frequency IF**

Making the IF frequency higher enables image signals to be far from the wanted signal; thus, RF filtering is relatively easier and increases the rejection level of image signals.

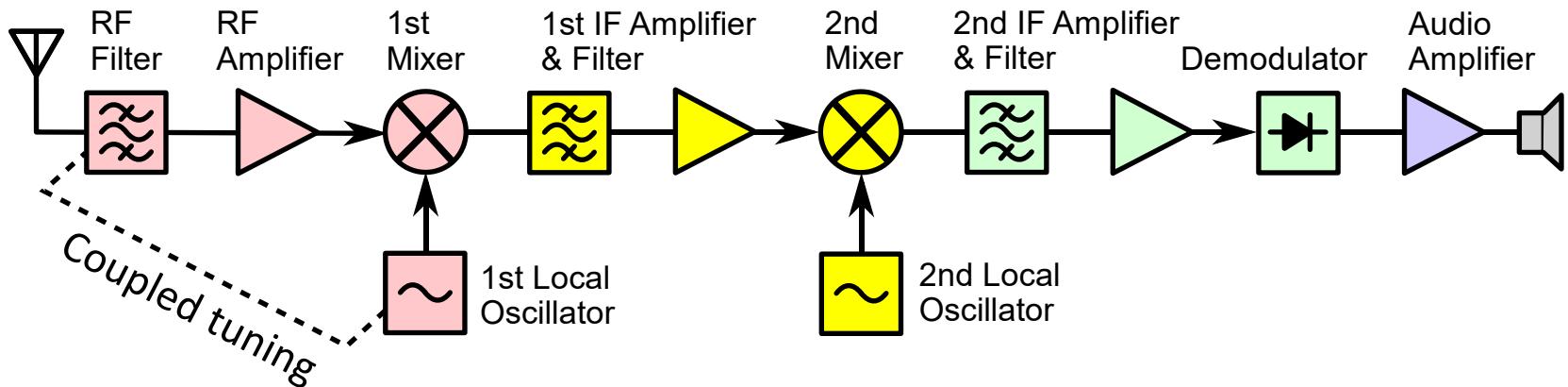
## **Low Frequency IF**

The advantage of a low frequency IF allows for filters providing much better adjacent channel rejection by filtering – this improves the selectivity of the receiver. And low frequency filters are less expensive also.

## **Solution:**

These two conflicting requirements can not be simultaneously satisfied by a single IF frequency. A solution is to use two IF frequencies via a double-conversion superheterodyne architecture.

# A Double-Conversion Superheterodyne Receiver



Many high performance spectrum analyzer receivers use triple-conversion or quadruple-conversion superheterodyne architectures.

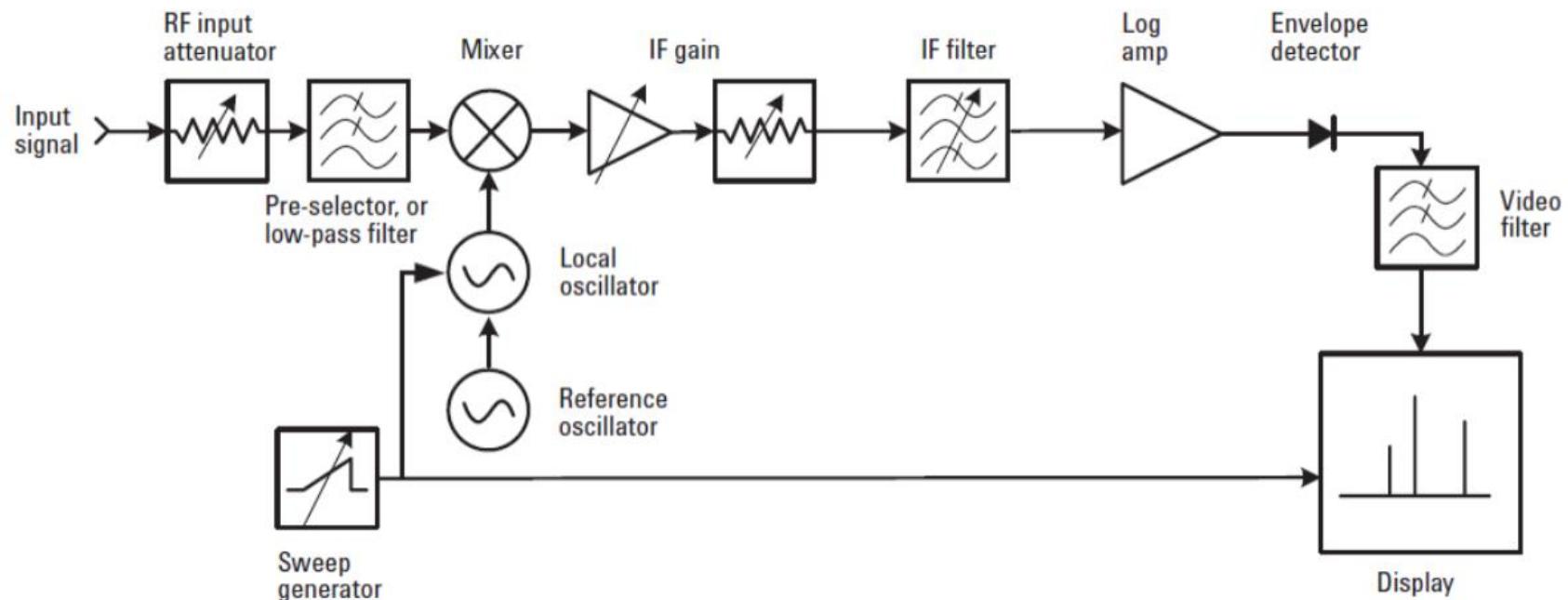
A good reference is

[https://www.keysight.com/upload/cmc\\_upload/All/5952-0292EN.pdf](https://www.keysight.com/upload/cmc_upload/All/5952-0292EN.pdf)

[https://commons.wikimedia.org/wiki/File:Double-conversion\\_superheterodyne\\_receiver\\_block\\_diagram.svg](https://commons.wikimedia.org/wiki/File:Double-conversion_superheterodyne_receiver_block_diagram.svg)

# Simplified Swept-Tuned Spectrum Analyzer Block Diagram

Keysight Application Note  
September 15, 2016

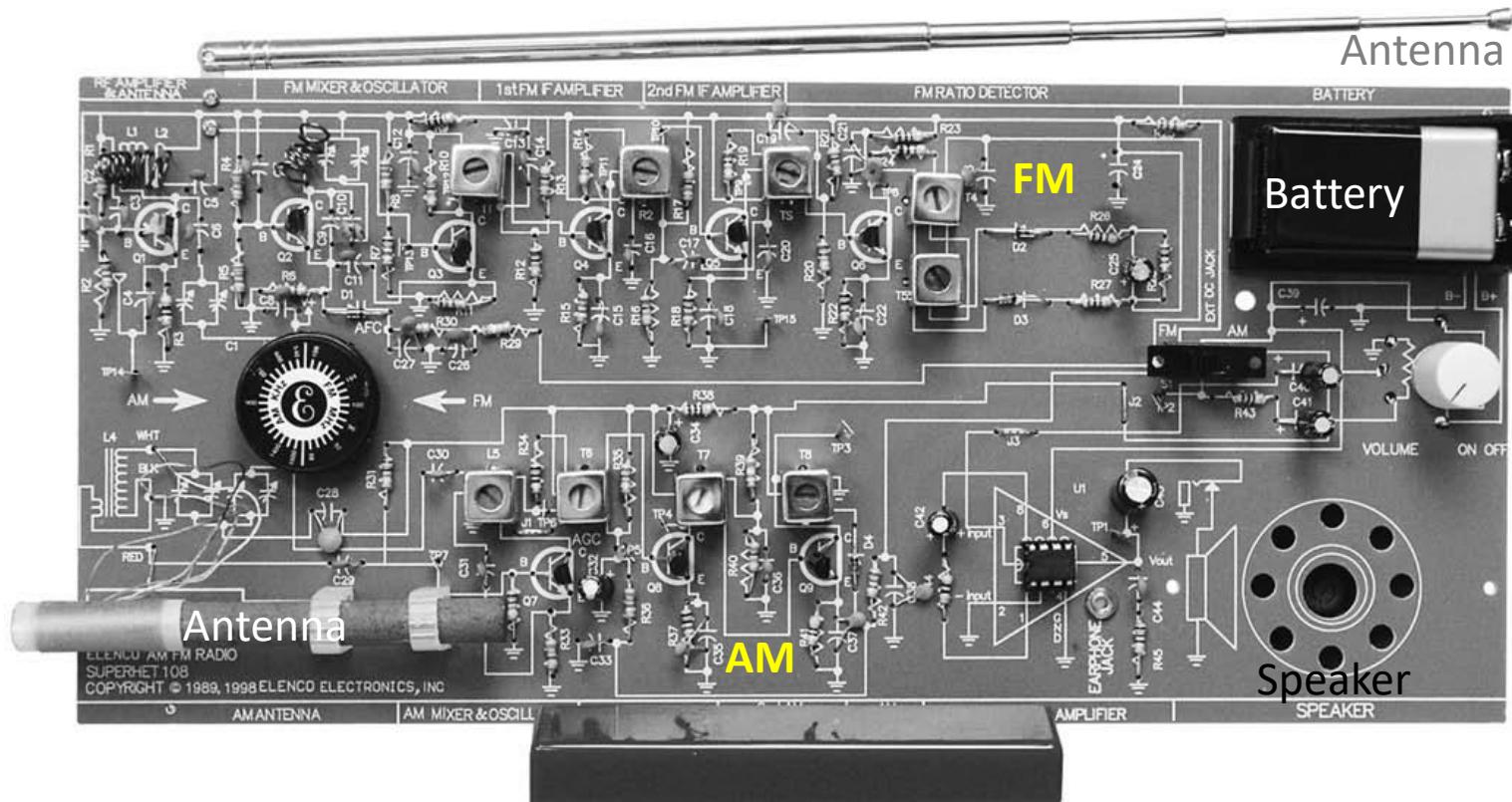


[https://www.keysight.com/upload/cmc\\_upload/All/5952-0292EN.pdf](https://www.keysight.com/upload/cmc_upload/All/5952-0292EN.pdf)

<http://www.microwavejournal.com/articles/print/27120-part-3-overcoming-rfmicrowave-interference-challenges-in-the-field-using-rtsa>

# Elenco AM/FM Dual-Radio Receiver Kit

Model AM/FM-108CK Superhet Radio



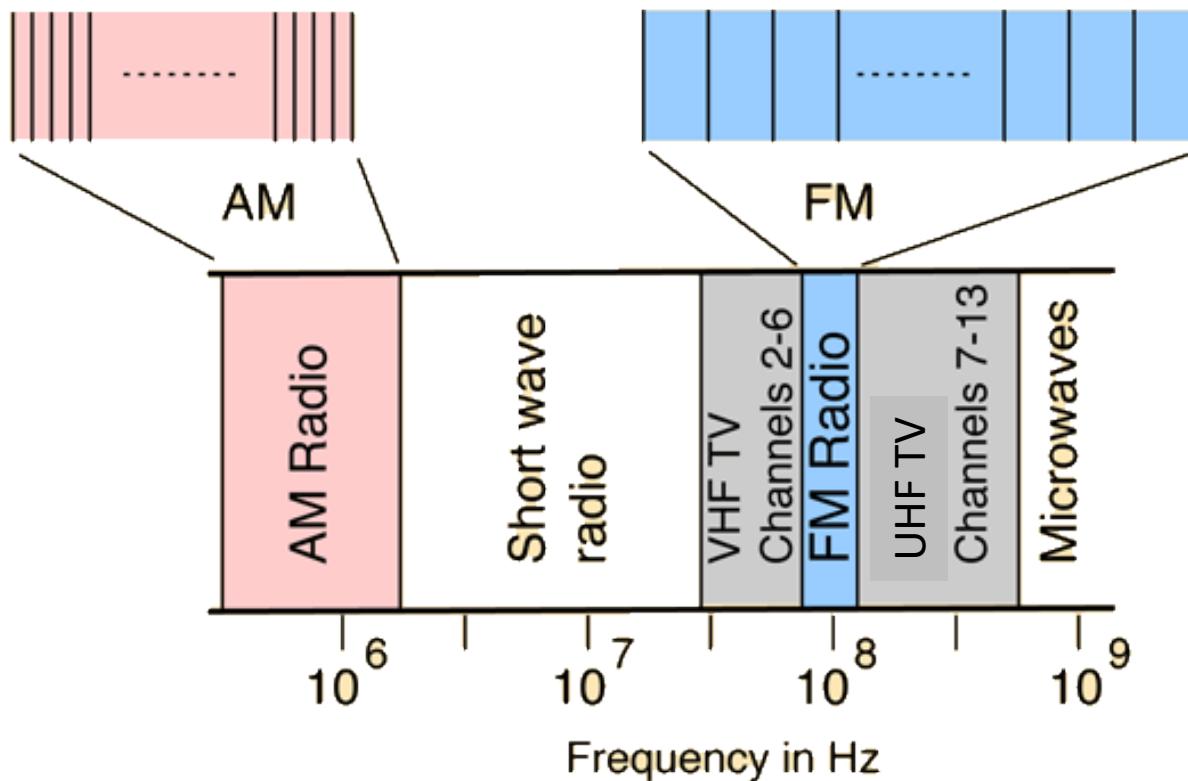
<https://www.elenco.com/product/amfm-radio-kitcombo-ic-transistor/>

AM Modulation -- Radio

## AM and FM Broadcast Bands

10 kHz bandwidth from  
**540 kHz to 1720 kHz**  
(118 possible bands)

200 kHz bandwidth from  
**88.1 MHz to 108.1 MHz**  
(100 possible bands)



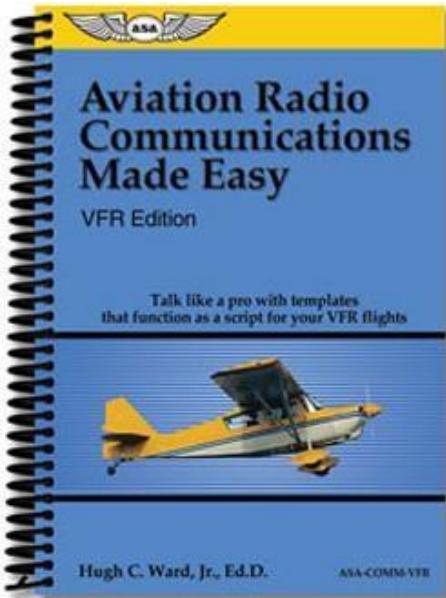
<http://hyperphysics.phy-astr.gsu.edu/hbase/Audio/radio.html>

## Example: AM Broadcast Stations



<http://ypropubmedia.com/online-radio-advertising-services-delhi/>

## Example: Aircraft Communication (Airband)



<https://aviatorsattic.com/product-category/pilot-amt-library/pilots-library/communication/>

Airband – 108 MHz to 137 MHz

108 to 117.95 MHz has 200 channels of 50 kHz each for navigational aids such as VOR beacons and precision approach systems (ILS localizer).

VHF Omni-Directional Range  
Instrument Landing System localizer

**118 to 136.975 MHz is divided into 2,280 channels (each 8.33 kHz of bandwidth) for AM voice transmissions.**



AM Modulation -- Radio

## Another Mixer Example (Practice Problem 3.8, Page 110)

For a frequency converter the carrier frequency of the output signal is 425 kHz and the carrier frequency of the AM input signal ranges from 500 kHz to 1500 kHz. Find the tuning ratio of the local oscillator

$$\frac{\omega_{LO,\max}}{\omega_{LO,\min}},$$

If the frequency of the local oscillator is given by (a)  $\omega_{IF} = \omega_{LO} - \omega_C$  and (b)  $\omega_{IF} = \omega_C + \omega_{LO}$ .

**Answer:**

(a)  $\omega_{IF} = \omega_{LO} - \omega_C \rightarrow \omega_{LO} = \omega_C + \omega_{IF} \rightarrow$  superheterodyning

$$\frac{\omega_{LO,\max}}{\omega_{LO,\min}} = \frac{\omega_{C,\max} + \omega_{IF}}{\omega_{C,\min} + \omega_{IF}} = \frac{1500 + 425}{500 + 425} = 2.081$$

(b)  $\omega_{IF} = \omega_C + \omega_{LO} \rightarrow \omega_{LO} = \omega_C - \omega_{IF} \rightarrow$  sub-heterodyning

$$\frac{\omega_{LO,\max}}{\omega_{LO,\min}} = \frac{\omega_{C,\max} - \omega_{IF}}{\omega_{C,\min} - \omega_{IF}} = \frac{1500 - 425}{500 - 425} = 14.33$$

# Quadrature Amplitude Modulation (QAM)

Fact: Both conventional AM and DSB-SC AM are wasteful of bandwidth.

One way to improve of bandwidth efficiency is with quadrature amplitude modulation (QAM). It involves two data streams: the I-channel and the Q-channel.

Bandwidth efficiency is improved by allowing two signals to share the same bandwidth of a channel. But this can only be done if the two modulated signals are orthogonal to each other. We now see how this can be accomplished.

A better name for QAM might be **quadrature-carrier multiplexing**.



<https://www.rcrwireless.com/20160901/test-and-measurement/what-is-64-qam-tag6-tag99>

## Quadrature-Carrier Multiplexing (QCM)

Quadrature-carrier multiplexing allows for transmitting two message signals on the same carrier frequency.

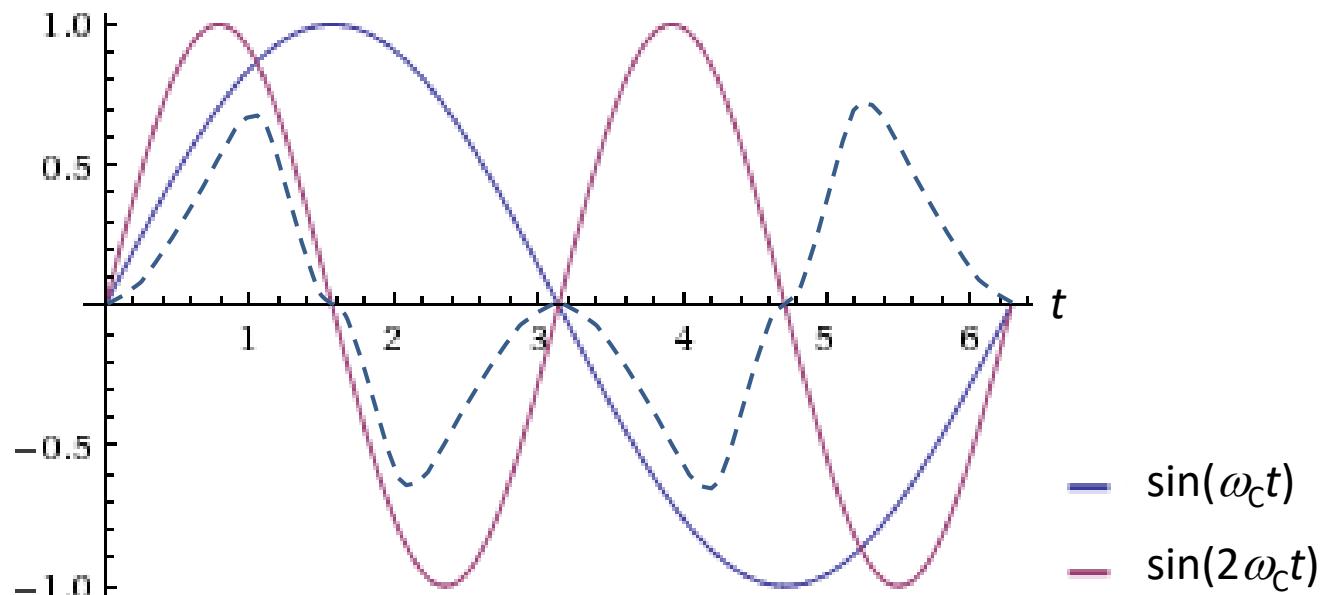
- (1) Two quadrature carriers are multiplexed together,
- (2) Signal  $m_I(t)$  modulates the carrier  $\cos(\omega_C t)$ , and  
Signal  $m_Q(t)$  modulates the carrier  $\sin(\omega_C t)$ .
- (3) The two modulated signals are added together & transmitted over the channel as

$$\varphi_{QAM}(t) = m_I(t) \cdot \cos(\omega_C t) + m_Q(t) \cdot \sin(\omega_C t)$$

## Quadrature Amplitude Modulation Features

1. QAM transmits two DSB-SC signals in the bandwidth of one DSB-SC signal.
2. Interference between the two modulated signals of the same frequency is prevented by using two carriers in phase quadrature. This is because they are **orthogonal** to each other.
3. The In-phase (I-phase) channel modulates the  $\cos(\omega_c t)$  signal and the Quadrature-phase (Q-phase) channel modulates the  $\sin(\omega_c t)$  signal.
4. The carriers used in the transmitter and receiver are synchronous with each other. In fact, they must be almost exactly in quadrature with each other, otherwise they experience cochannel interference.
5. Low-pass filters are used to extract the baseband signals  $m_I(t)$  and  $m_Q(t)$  in the receiver.

## Illustrating Orthogonality With Sinusoidal Signals - I

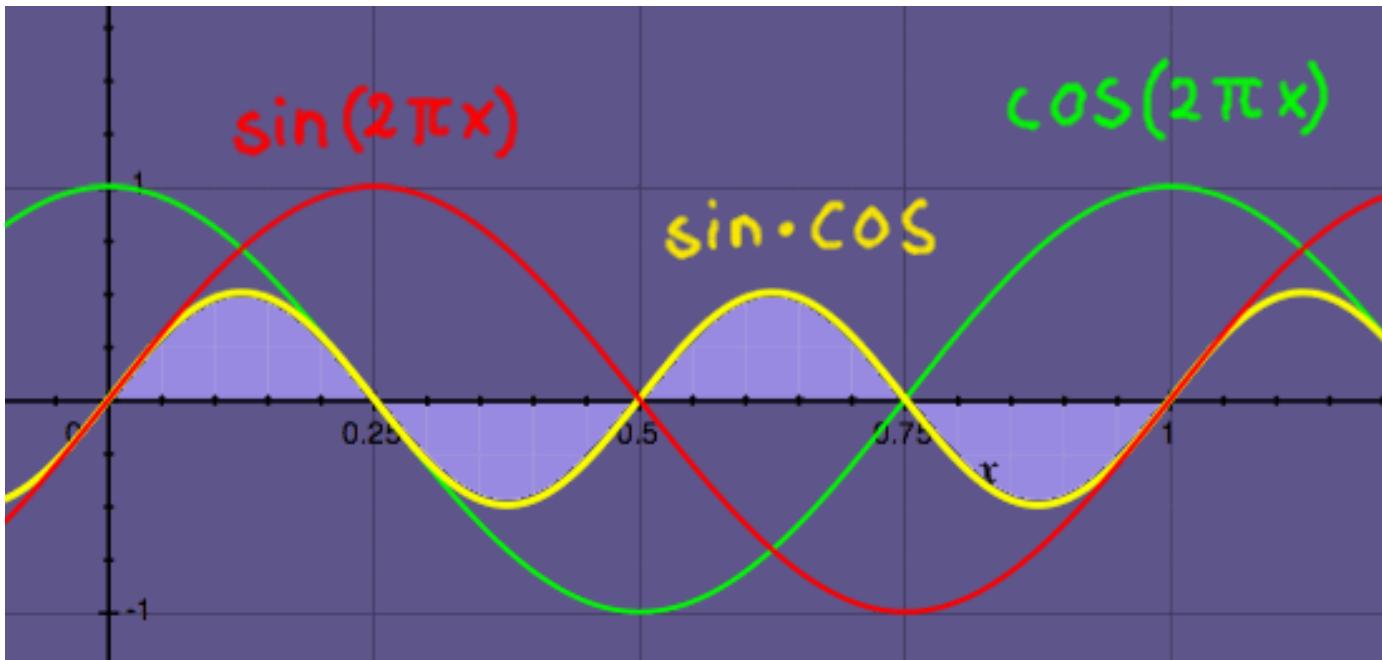


Signals are harmonically related.

$$\int_0^T \sin(\omega_C t) \cdot \sin(2\omega_C t) dt = 0 \quad \text{and} \quad \int_0^T \cos(\omega_C t) \cdot \cos(2\omega_C t) dt = 0$$

<https://math.stackexchange.com/questions/474398/waves-of-differing-frequency-are-orthogonal-help-me-understand>

## Illustrating Orthogonality With Sinusoidal Signals - II



Area of product over one period is equal to zero.  
Therefore, the signals are orthogonal.

$$\int_0^T \sin(\omega_c t) \cdot \cos(\omega_c t) dt = 0$$

<http://www.katjaas.nl/sinusoids2/sinusoids2.html>

# Quadrature Amplitude Modulation (QAM)

## QAM Can Be Viewed As Combination of ASK and PSK

- Quadrature Amplitude Modulation uses the phase and amplitude of the carrier signal to encode data. QAM finds widespread use in current and emerging wireless standards, including Wi-Fi, Digital Video Broadcast (DVB), WiMAX, IEEE 802.11n, and HSDPA/HSUPA.
- The QAM modulation scheme encodes data by varying both amplitude and phase of the carrier signal. Thus, it is sometimes viewed as a combination of ASK and PSK modulation.

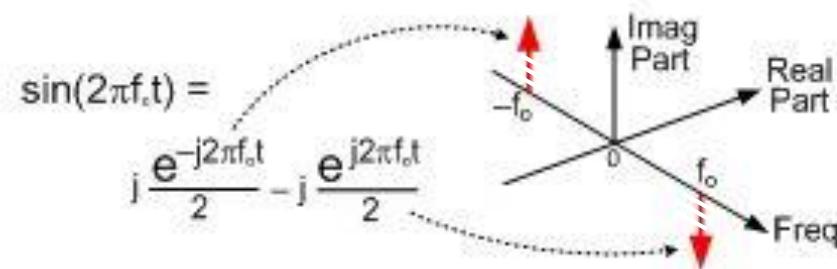
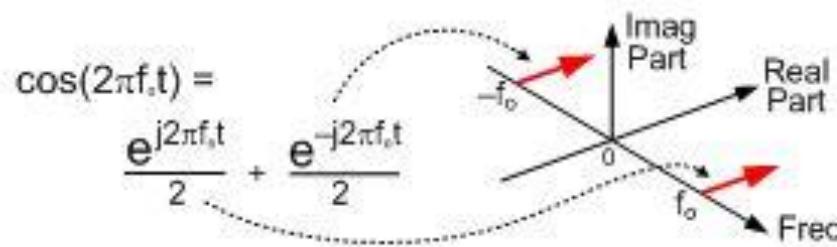
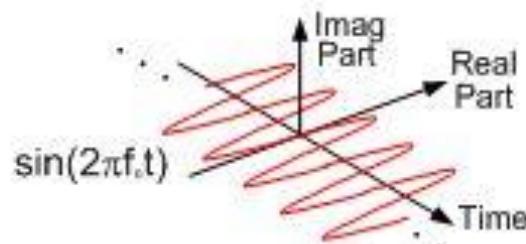
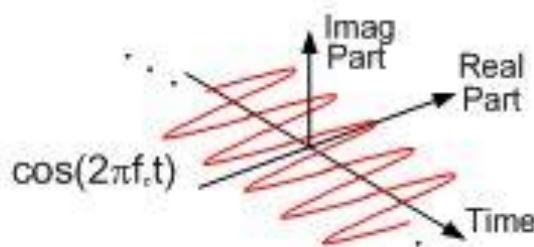
$$A_C \cos(\omega_C t + \theta) = I(t) \cos(\omega_C t) + Q(t) \sin(\omega_C t)$$

where  $I(t) = A_C \cos(\theta)$  and  $Q(t) = A_C \sin(\theta)$

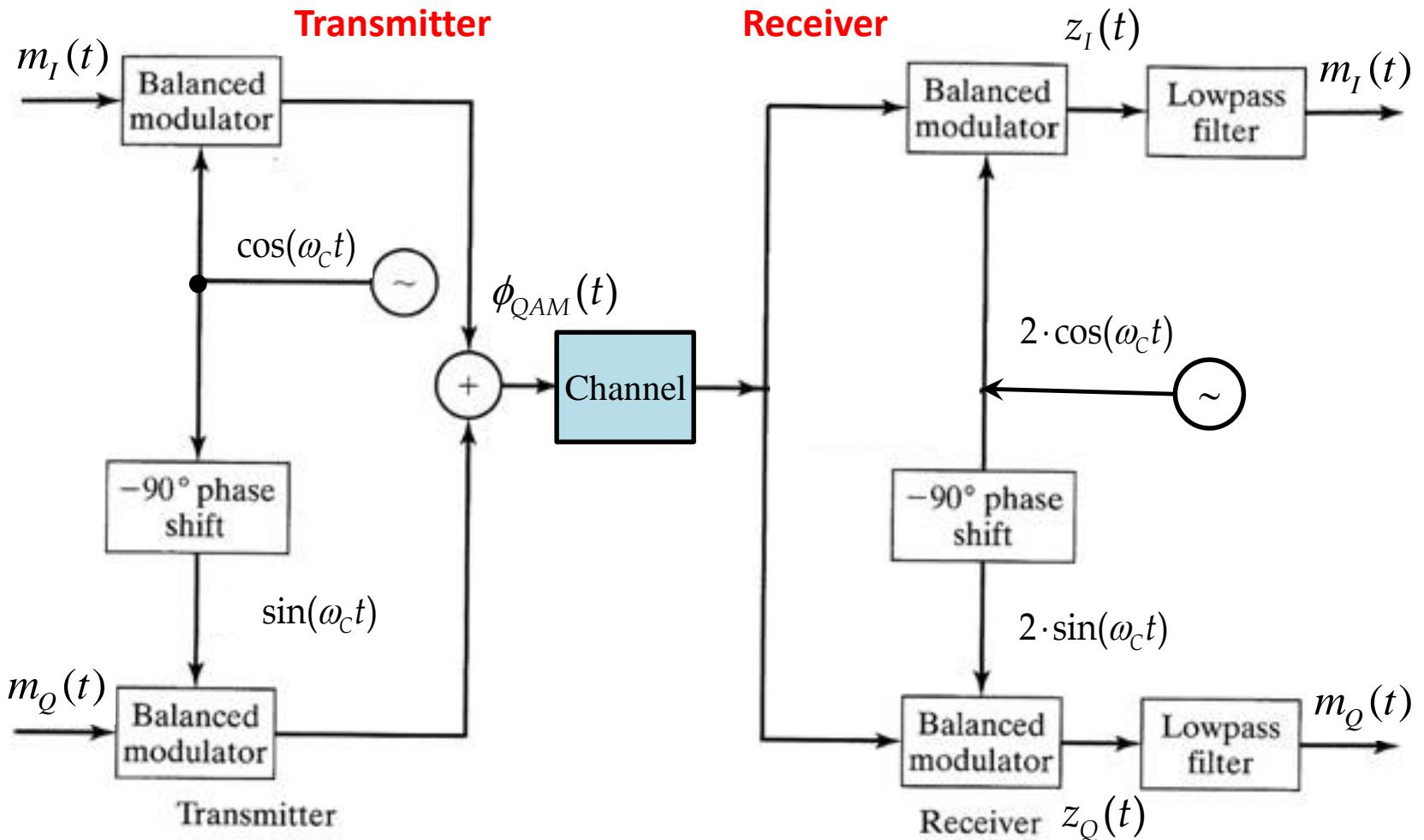
**HSDPA** (High-Speed Downlink Packet Access) is a packet-based mobile telephony protocol used in 3G UMTS radio networks.

<https://www.slideshare.net/ahsanhalini/quadrature-amplitude-modulation-54999195>

## Review: Spectral Lines for Sine and Cosine Signals



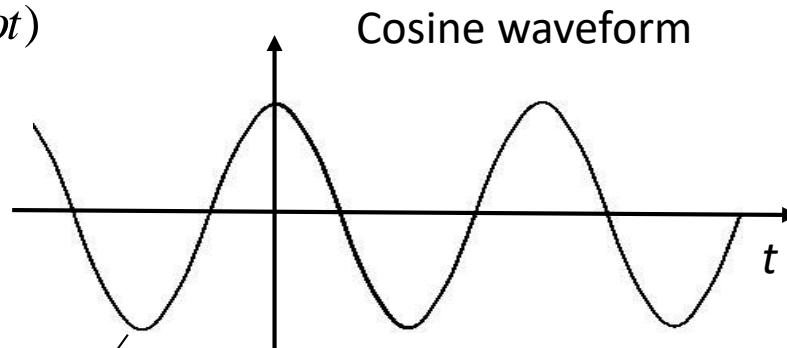
# Quadrature Amplitude Modulation and Demodulation



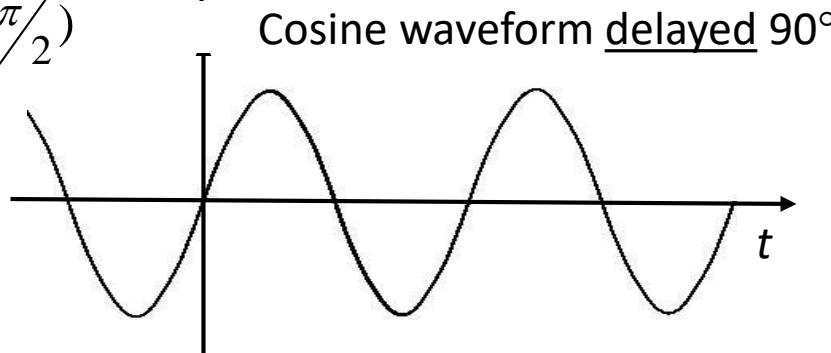
Note:  $\cos(\omega_C t - 90^\circ) = \sin(\omega_C t)$

## Illustrating $\pm 90$ Degree ( $\pm \pi/2$ ) Phase Shift of a Cosine Wave

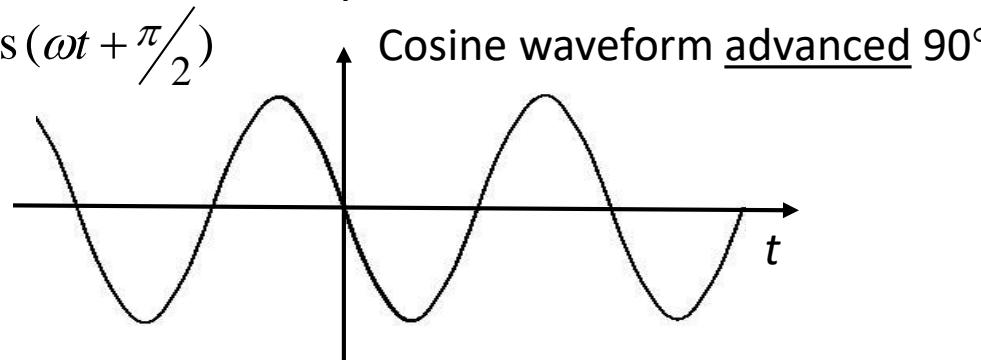
$$x(t) = \cos(\omega t)$$



$$x(t) = \cos(\omega t - \pi/2)$$



$$x(t) = \cos(\omega t + \pi/2)$$



## Quadrature Amplitude Demodulation (QAM)

$$\varphi_{QAM}(t) = m_I(t)\cos(\omega_C t) + m_Q(t)\sin(\omega_C t)$$

$$z_I(t) = 2 \cdot \cos(\omega_C t) [\varphi_{QAM}(t)] = 2 \cdot \cos(\omega_C t) [m_I(t)\cos(\omega_C t) + m_Q(t)\sin(\omega_C t)]$$

$$z_I(t) = 2m_I(t) \cdot \cos^2(\omega_C t) + 2m_Q(t) \cdot \cos(\omega_C t) \cdot \sin(\omega_C t)$$

$$z_I(t) = m_I(t) + m_I(t) \cdot \cos(2\omega_C t) + m_Q(t) \sin(2\omega_C t)$$

We recover  $m_I(t)$  by passing  $z_I(t)$  through a LPF.

and

$$z_Q(t) = 2 \cdot \sin(\omega_C t) [\varphi_{QAM}(t)] = 2 \cdot \sin(\omega_C t) [m_I(t)\cos(\omega_C t) + m_Q(t)\sin(\omega_C t)]$$

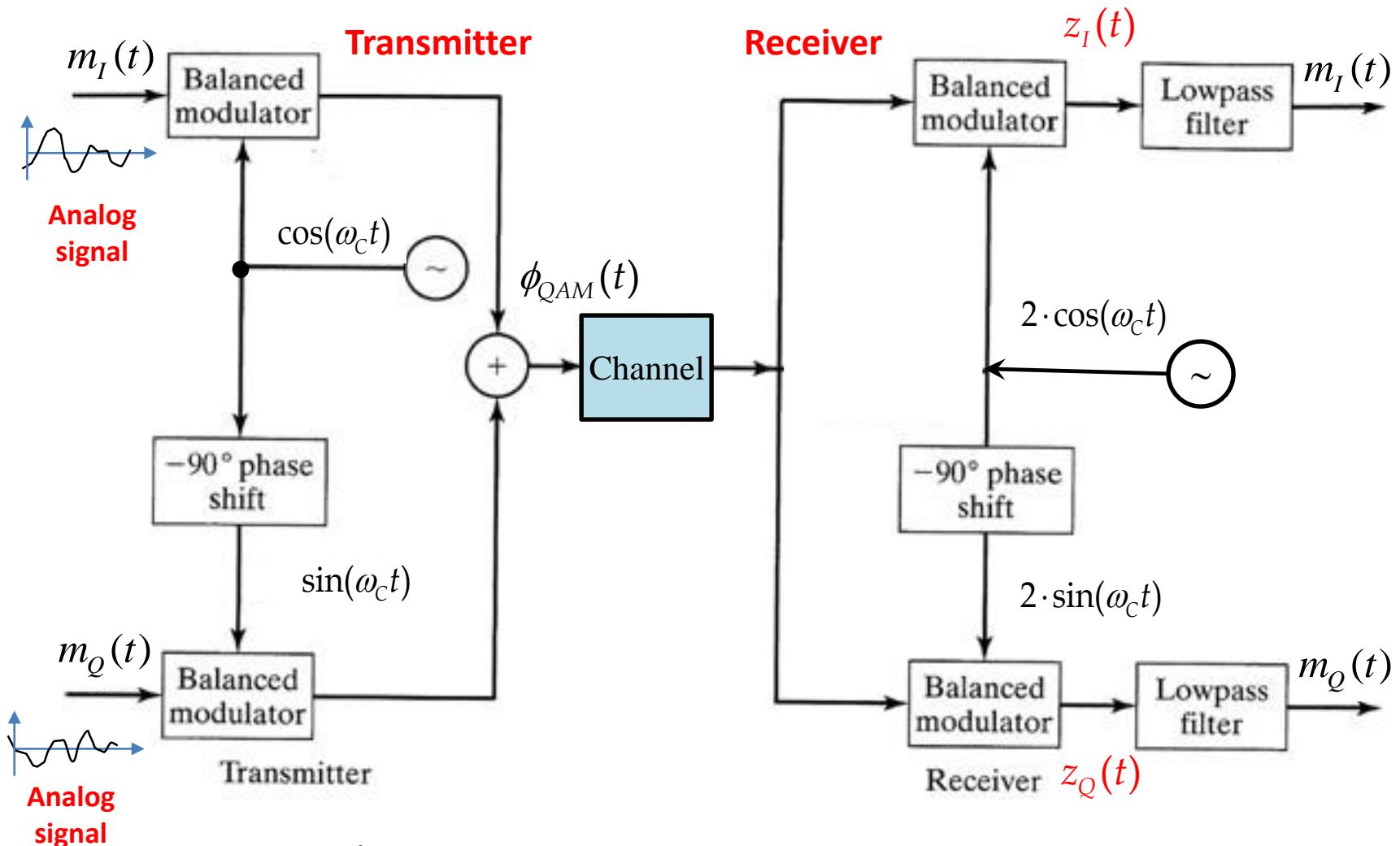
$$z_Q(t) = 2m_Q(t) \cdot \sin^2(\omega_C t) + 2m_I(t) \cdot \sin(\omega_C t) \cdot \sin(\omega_C t)$$

$$z_Q(t) = m_Q(t) - m_Q(t) \cdot \cos(2\omega_C t) + m_I(t) \sin(2\omega_C t)$$

We recover  $m_Q(t)$  by passing  $z_Q(t)$  through a LPF.

**Page 111  
Agbo & Sadiku**

# Quadrature-Amplitude Modulation & Demodulation

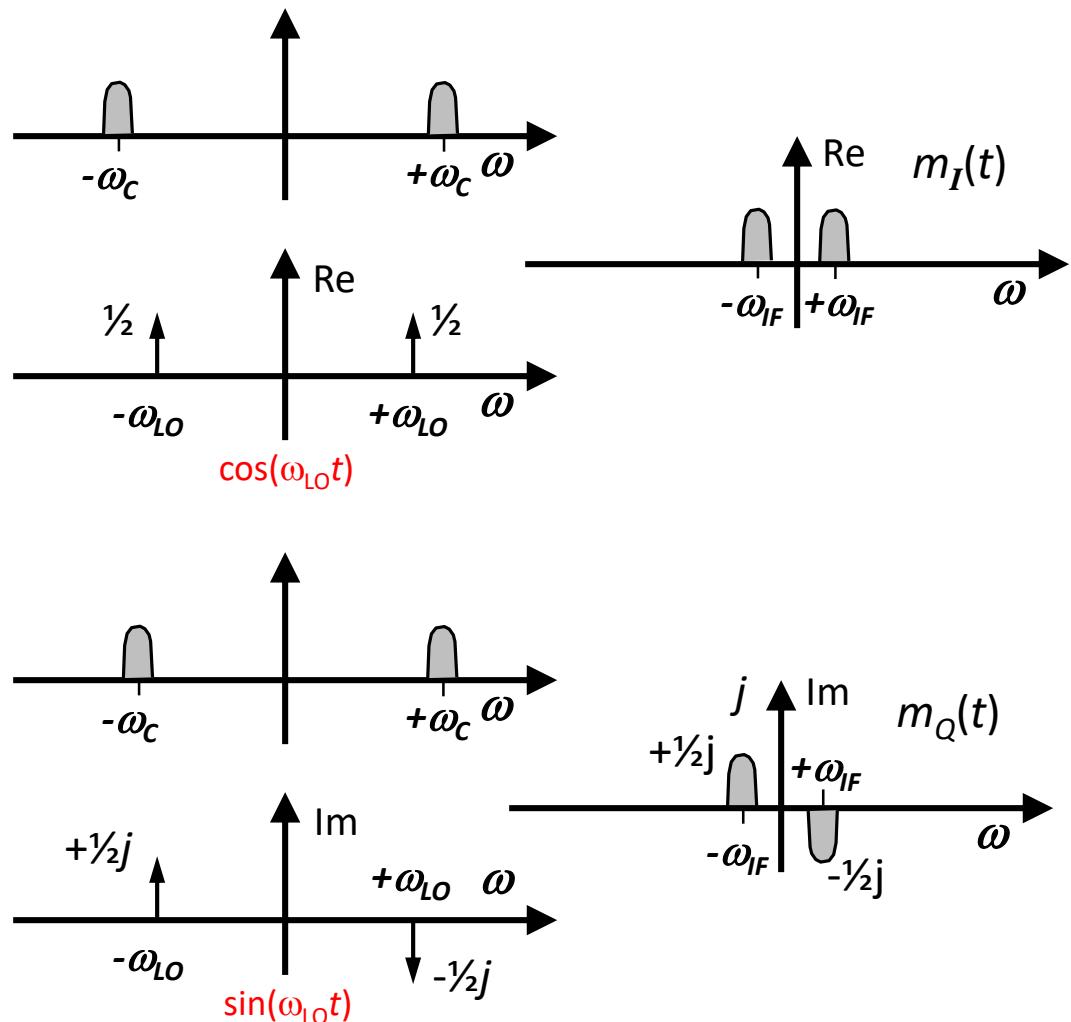
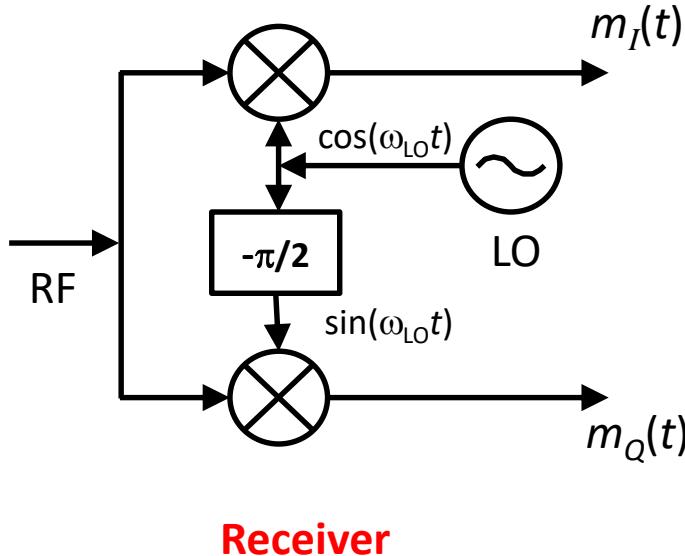


$$\phi_{QAM}(t) = m_I(t) \cos(\omega_C t) + m_Q(t) \sin(\omega_C t)$$

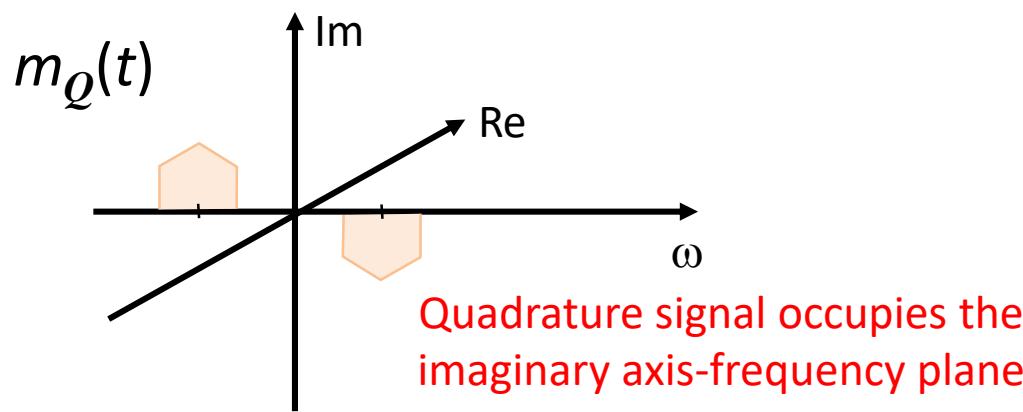
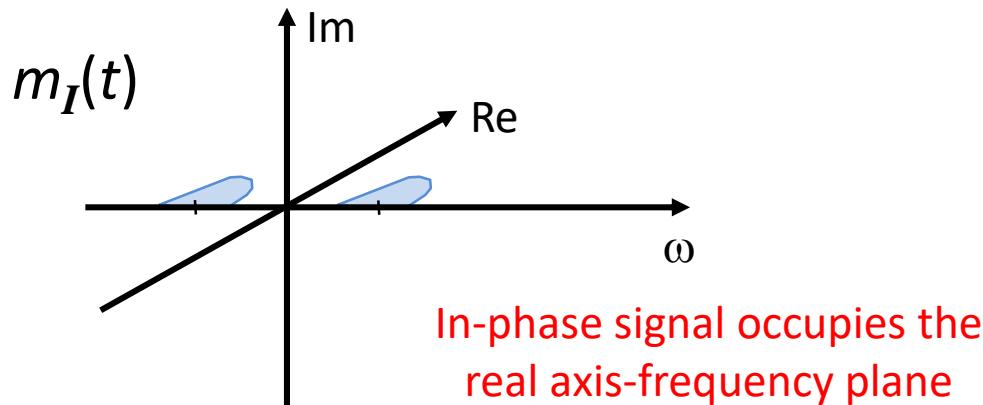
# Quadrature-Amplitude Demodulation

$$\omega_{IF} = \omega_C - \omega_{LO}$$

## Quadrature Downconverter



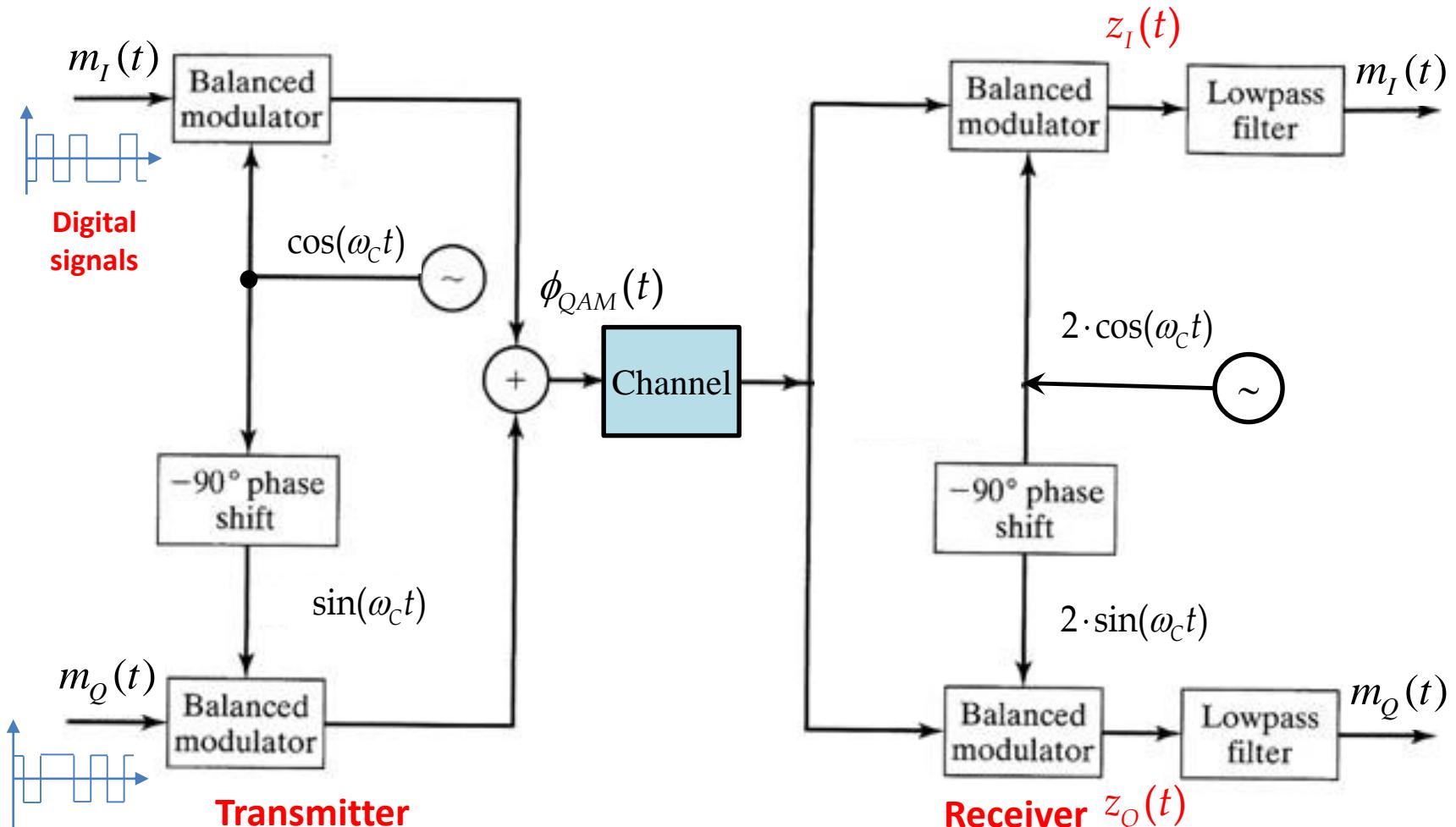
## Quadrature-Amplitude Demodulation (continued)



This illustrates the orthogonality of the two modulated signals.

# Quadrature-Amplitude Modulation & Demodulation

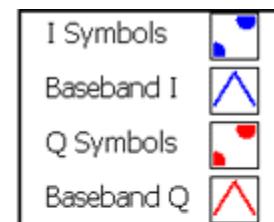
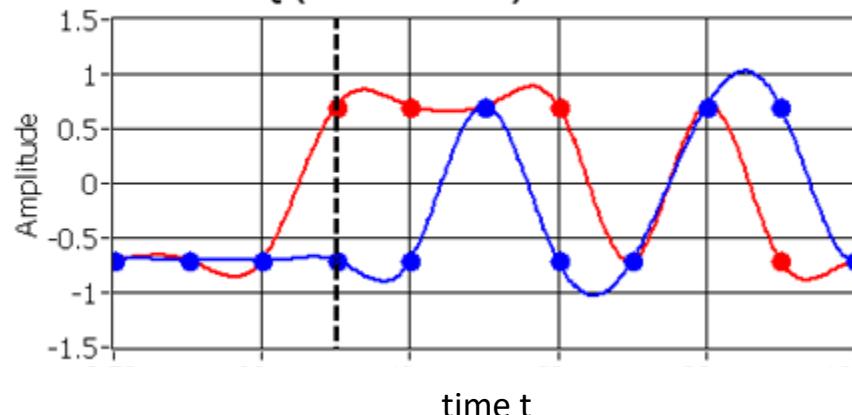
Now it becomes a **digital communication system**



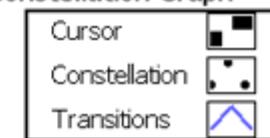
$$\phi_{QAM}(t) = m_I(t) \cos(\omega_C t) + m_Q(t) \sin(\omega_C t)$$

# Quadrature-Amplitude Modulation (4-QAM)

Baseband I and Q (Time Domain)

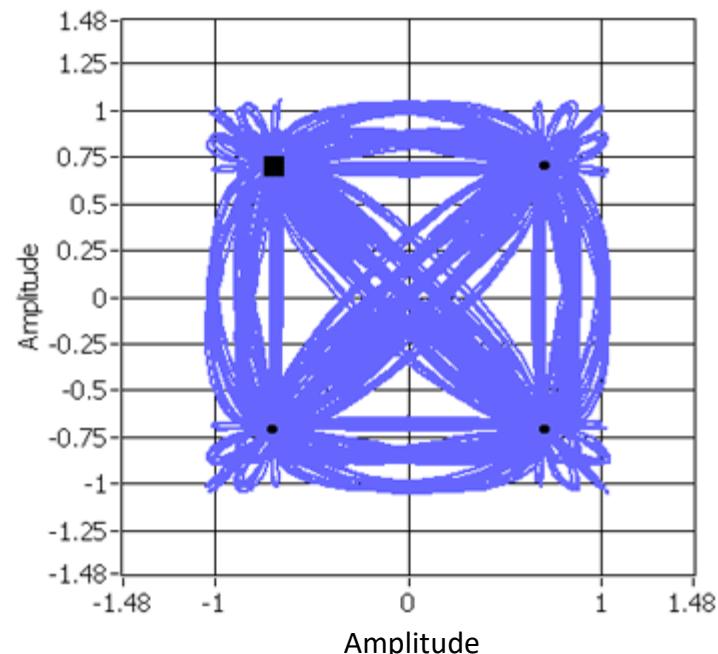


Constellation Graph



<http://www.ni.com/white-paper/3896/en/>

Constellation  
Diagram



# **Effect of frequency error and phase error in synchronous detection**

## QAM: Phase Error in Synchronous Detection

The local carrier in a DSB-SC receiver and a QAM receiver is  $2\cos(\omega_c t + \alpha)$ , while the signal carrier at the input of each receiver is  $\cos(\omega_c t)$ . That means the signal carrier and local carrier in the receivers are phase shifted relative to each other. Derive expressions for the demodulated output signals for both receivers. Compare your results for DSB-SC and QAM.

Solution: **(Example 3.9 on pp. 112-113)**

For the DSB-SC receiver (NOT QAM):

Phase error =  $\alpha$

$$x(t) = 2\cos(\omega_c t + \alpha) \cdot \phi_{DSB-SC}(t) = 2m(t) \cdot \cos(\omega_c t) \cdot \cos(\omega_c t + \alpha)$$

$$x(t) = m(t) \cdot \cos(\alpha) + m(t) \cdot \cos(\omega_c t + \alpha)$$

Therefore, low-pass filtering gives  $y(t) = m(t) \cdot \cos(\alpha)$

## QAM: Phase Error in Synchronous Detection (continued)

For the QAM receiver:

$$z_I(t) = 2 \cos(\omega_C t + \alpha) \cdot \phi_{QAM}(t) = 2 \cos(\omega_C t + \alpha) [m_I(t) \cdot \cos(\omega_C t) + m_Q(t) \cdot \sin(\omega_C t)]$$

$$z_I(t) = 2m_I(t) \cdot \cos(\omega_C t) \cdot \cos(\omega_C t + \alpha) + 2m_Q(t) \cdot \sin(\omega_C t) \cdot \cos(\omega_C t + \alpha)$$

$$z_I(t) = m_I(t) \cdot \cos(\alpha) + m_I(t) \cdot \cos(2\omega_C t + \alpha) - m_Q(t) \cdot \sin(\alpha) + m_Q(t) \cdot \sin(2\omega_C t + \alpha)$$

and

$$z_Q(t) = 2 \sin(\omega_C t + \alpha) \cdot \phi_{QAM}(t) = 2 \sin(\omega_C t + \alpha) [m_I(t) \cdot \cos(\omega_C t) + m_Q(t) \cdot \sin(\omega_C t)]$$

$$z_Q(t) = 2m_I(t) \cdot \cos(\omega_C t) \cdot \sin(\omega_C t + \alpha) + 2m_Q(t) \cdot \sin(\omega_C t) \cdot \sin(\omega_C t + \alpha)$$

$$z_Q(t) = m_I(t) \cdot \sin(\alpha) + m_I(t) \cdot \sin(2\omega_C t + \alpha) + m_Q(t) \cdot \cos(\alpha) - m_Q(t) \cdot \cos(2\omega_C t + \alpha)$$

The low-pass filters suppress the terms centered at  $2\omega_C$

Therefore,

$$\left. \begin{aligned} y_I(t) &= m_I(t) \cdot \cos(\alpha) - m_Q(t) \cdot \sin(\alpha) \\ y_Q(t) &= m_Q(t) \cdot \cos(\alpha) + m_I(t) \cdot \sin(\alpha) \end{aligned} \right\} \text{Co-channel interference}$$

## QAM: Frequency Error in Synchronous Detection

Compare the effect of a small frequency error in the local carrier for a DSB-SC receiver and a QAM receiver. The carrier at the transmitter is  $\cos(\omega_C t)$  and the carrier at the receiver is  $2\cos(\omega_C + \Delta\omega)t$ .

Answer: (Example 3.10 on pp. 113-114)

$$y_I(t) = m_I(t)\cos(\Delta\omega t) - m_Q(t)\sin(\Delta\omega t)$$

$$y_Q(t) = m_Q(t)\cos(\Delta\omega t) + m_I(t)\sin(\Delta\omega t)$$

Note the similarity to the answer to Example 3.9.  
You should now be able to guess the answer to a question involving both phase error  $\alpha$  and frequency error  $\Delta\omega$ . (Practice Problem 3.10 on page 114)

# Single Sideband (SSB) AM Modulation and Demodulation

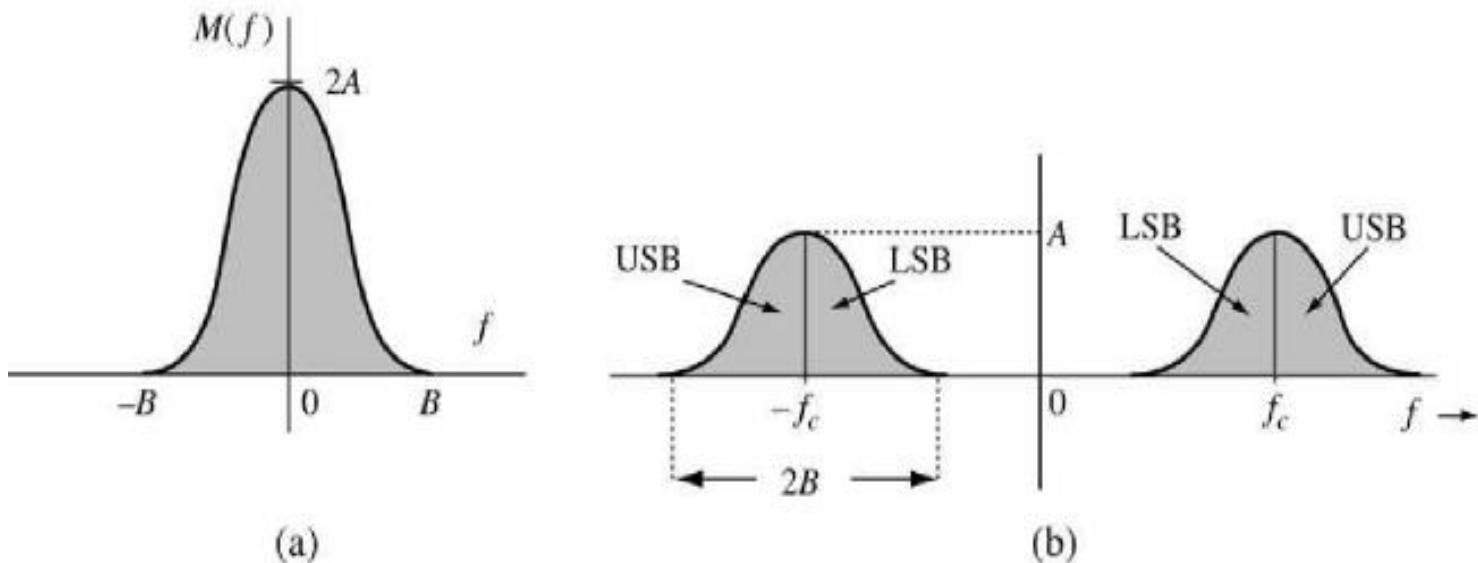


Filter method; Phasing Method; Weaver Method

<https://hamradioschool.com/hey-why-is-there-no-transmit-power-when-i-key-the-mic/>

## Single Sideband (SSB) AM

Why single sideband? DSB-SC is spectrally inefficient because it uses twice the bandwidth of the message. SSB addresses that issue.

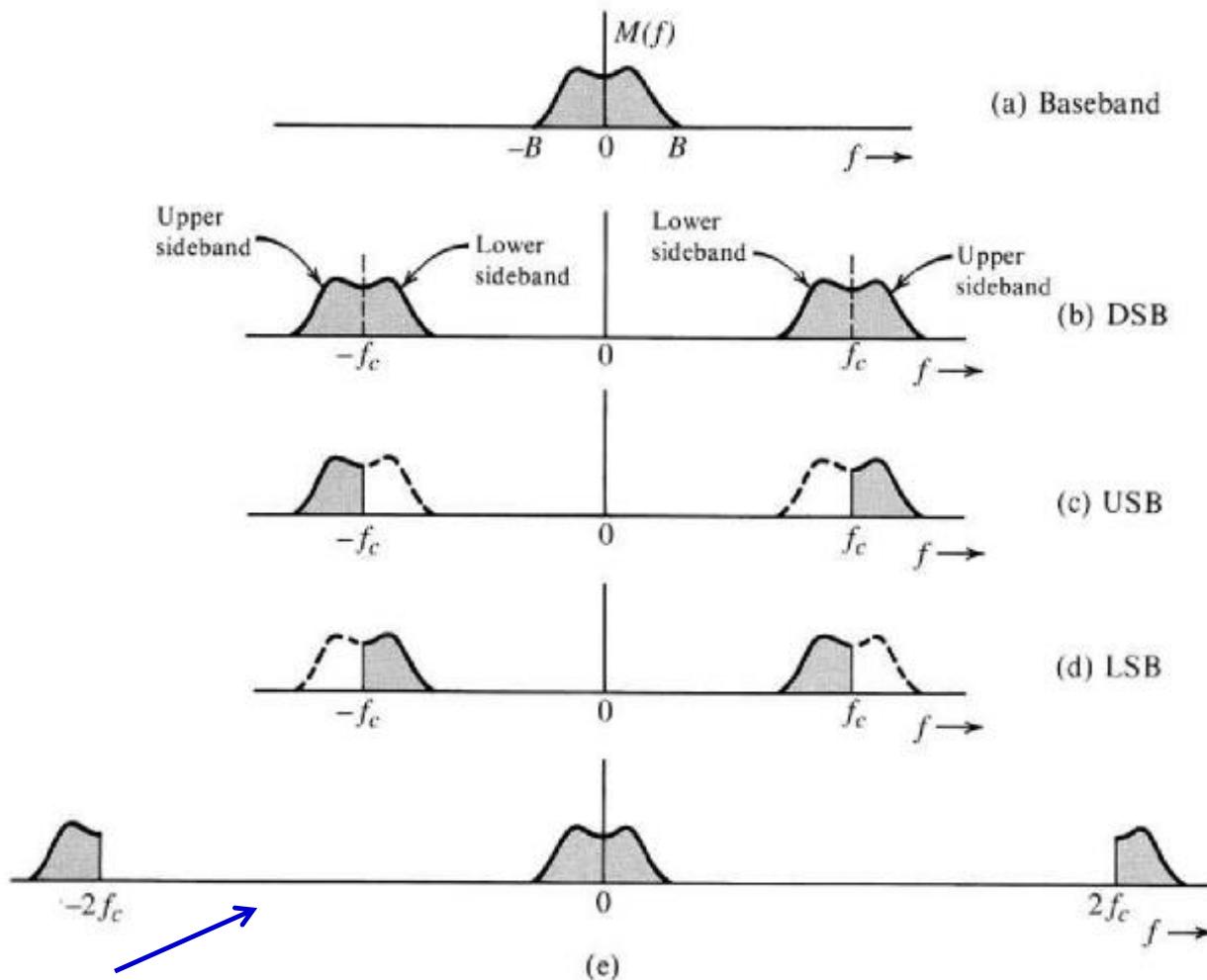


The signal can be reconstructed from either the upper sideband (USB) or the lower sideband (LSB).

SSB transmits a bandpass filtered version of the modulated signal.

Reference: Lathi & Ding, 4<sup>th</sup> ed., 2009; Fig. 4.12.

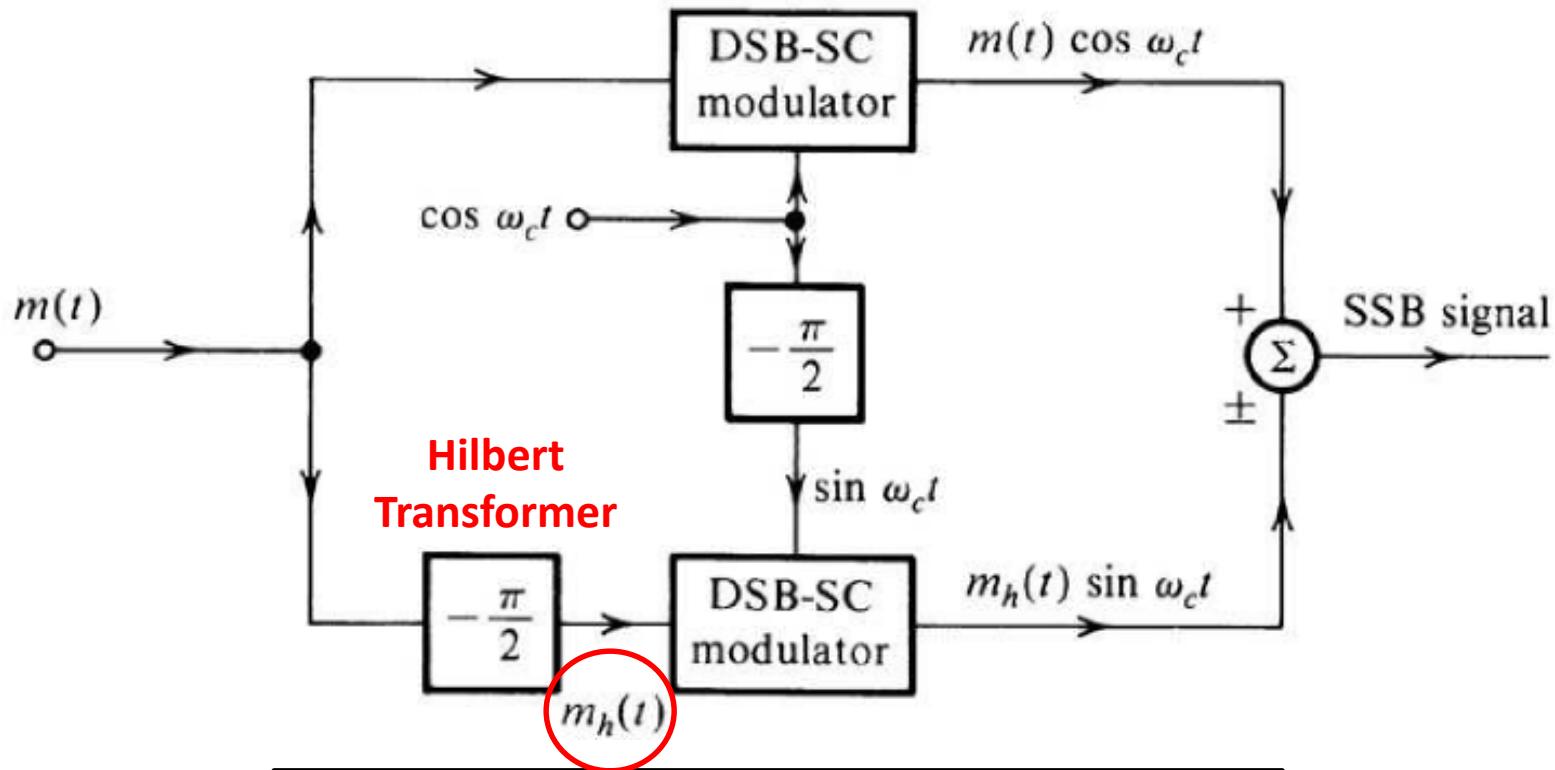
# Single Sideband (SSB) AM



Multiplication of a USB signal by  $\cos(\omega_C t)$  shifts the spectrum to left and right.

Reference: Lathi & Ding, 4<sup>th</sup> ed., 2009; Fig. 4.13.

## Phase-Shift Method to Generate SSB AM



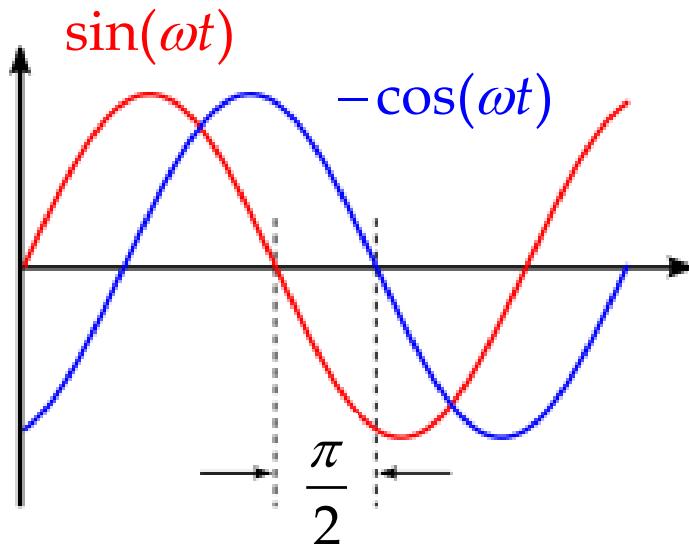
where minus sign applies to USB  
and plus sign applies to the LSB.

$m_h(t)$  is  $m(t)$  phase delayed by  $-\pi/2$

Reference: Lathi & Ding, 4<sup>th</sup> ed., 2009; Fig. 4.17.

AM Modulation -- Radio

## Reference Note: Quadrature Phase-Shifts & Hilbert Transform



For a  $+90^\circ$  (or  $\pi/2$ ) phase shift:

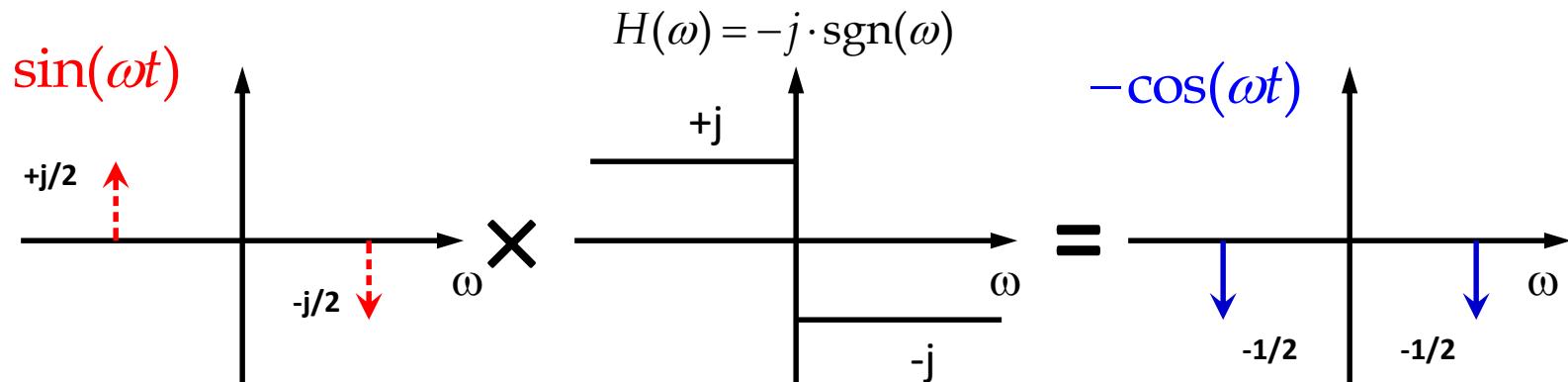
$$\sin(\omega t) \Rightarrow \cos(\omega t)$$

$$\cos(\omega t) \Rightarrow -\sin(\omega t)$$

For a  $-90^\circ$  (or  $-\pi/2$ ) phase shift:

$$\sin(\omega t) \Rightarrow -\cos(\omega t)$$

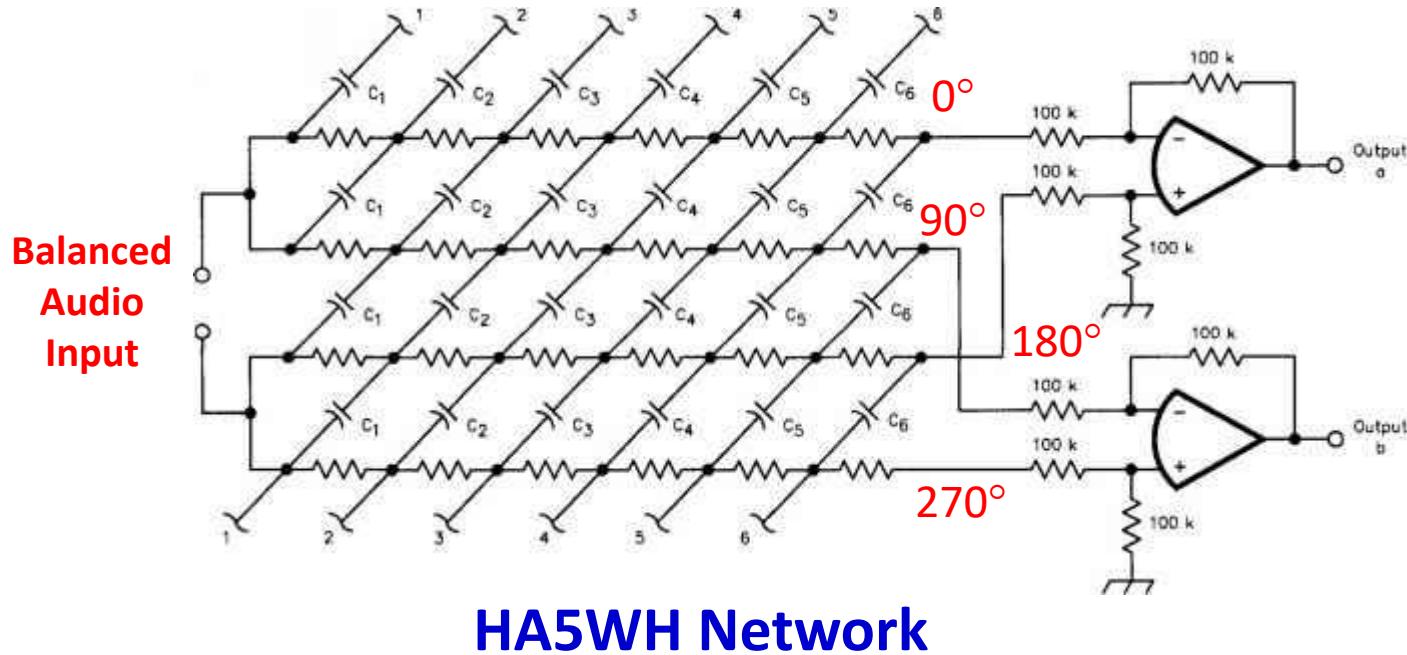
$$\cos(\omega t) \Rightarrow \sin(\omega t)$$



Hilbert transform

# Polyphase Filter – HA5WH Network (Gingell)

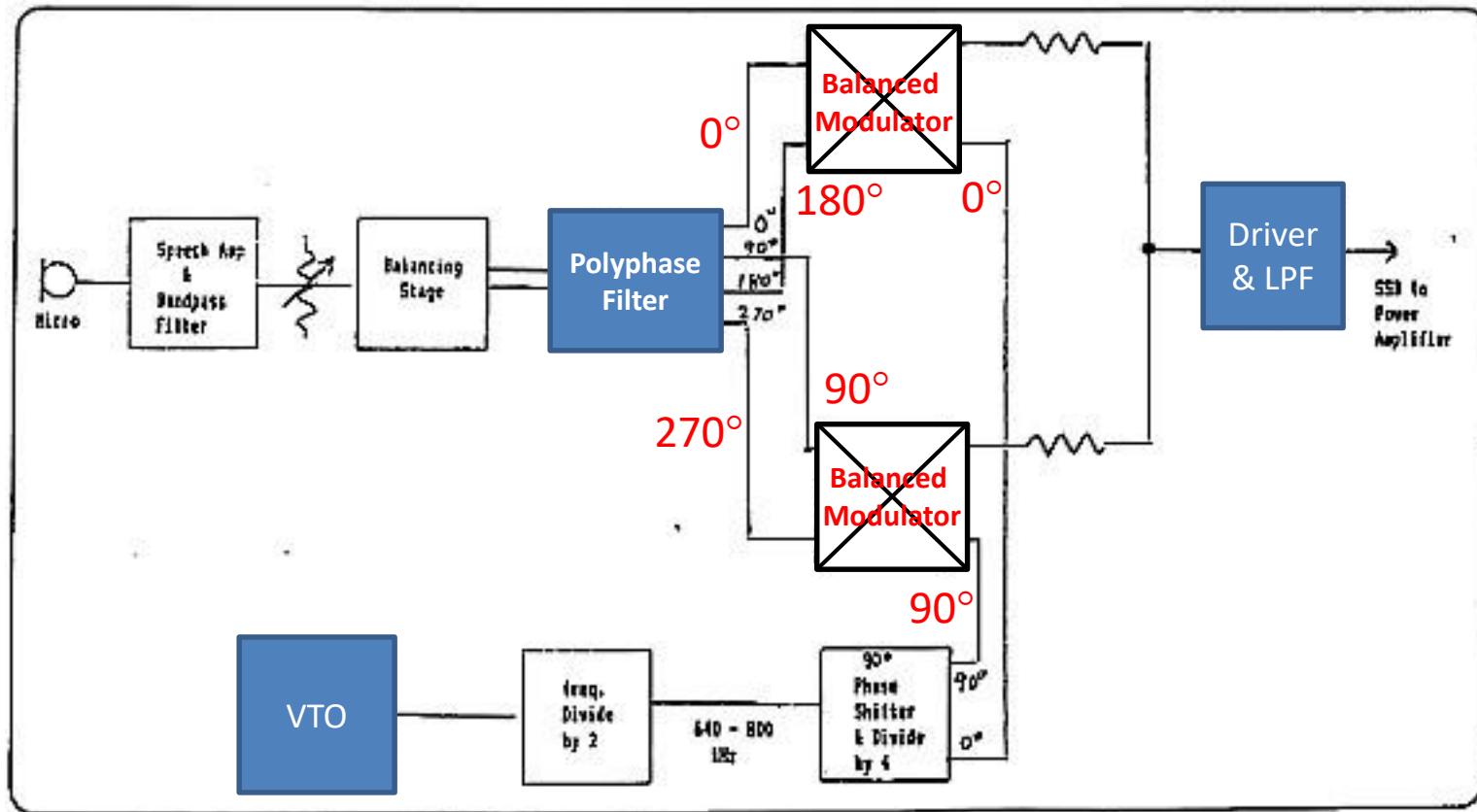
Polyphase filters are symmetric RC structures with inputs and outputs symmetrically arranged in relative phases.



Achieves constant phase shifts over 300 Hz to about 3,000 Hz with a 60 dB rejection of the other sideband.

# Using a Polyphase Filter in a SSB Generator

The ARRL Handbook for the Radio Amateur, (American Radio Relay League, Newington, 1993), and many previous editions.

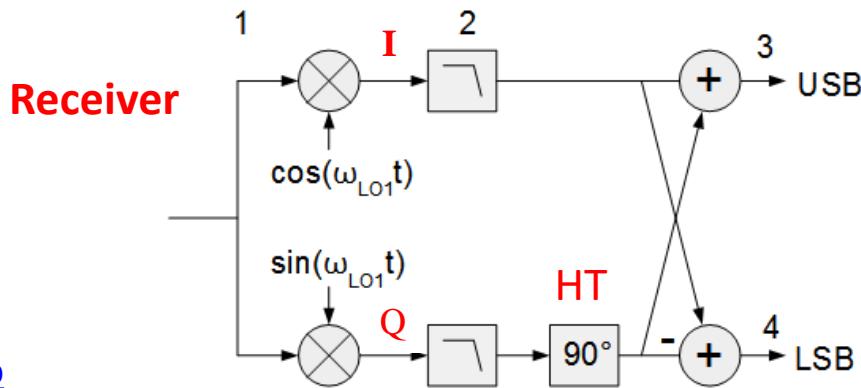


<https://www.radioexperimenter.us/re-04-1994/phaseshifi-network-analysis-and-optimization.html>

## Phase-Shift Method to Generate SSB AM

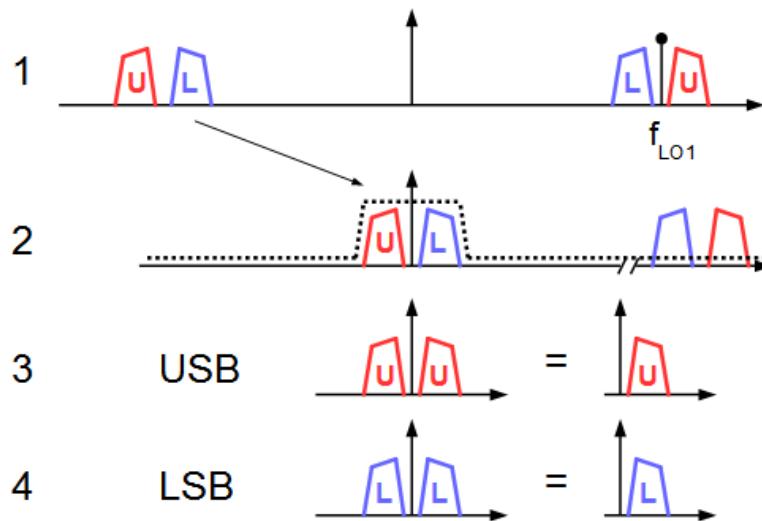
- The phasing method uses two balanced mixers to eliminate the carrier.
- The phasing method for SSB generation uses a phase-shift to cancel one of the sidebands.
- The carrier oscillator is applied to the upper balanced modulator along with the modulating signal.
- The carrier and modulating signals are both shifted by 90 degrees and applied to another modulator.
- Phase-shifting causes one sideband to cancel when the two modulator outputs are summed together.

# Phase-Shift Method for Receiving SSB Signals



Reference:

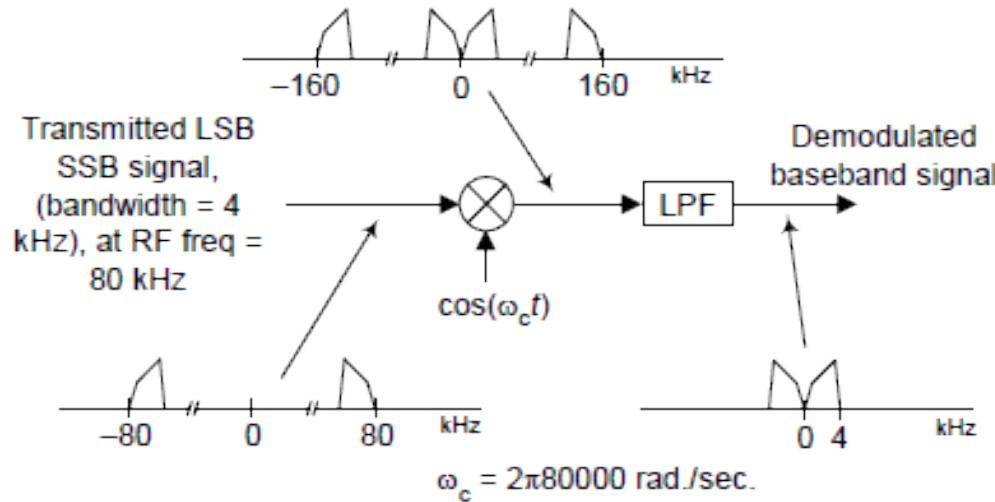
[https://www.dsp  
related.com/sho  
warticle/176.php](https://www.dsprelated.com/showarticle/176.php)



<http://www.panoradio-sdr.de/ssb-demodulation/>

# Synchronous Demodulation of SSB AM

Analog LSB demodulation (synchronous detection)



<https://www.dsprelated.com/showarticle/176.php>

$$x(t) = 2\phi_{SSB}(t) \cdot \cos(\omega_C t) = [m(t) \cdot \cos(\omega_C t) \mp m_h(t) \cdot \sin(\omega_C t)] 2\cos(\omega_C t)$$

$$x(t) = m(t) + [m(t) \cdot \cos(2\omega_C t) \mp m_h(t) \cdot \sin(2\omega_C t)]$$

and upon low-pass filtering we have

$$x(t) = m(t)$$

# SSB Generation: Weaver's Method

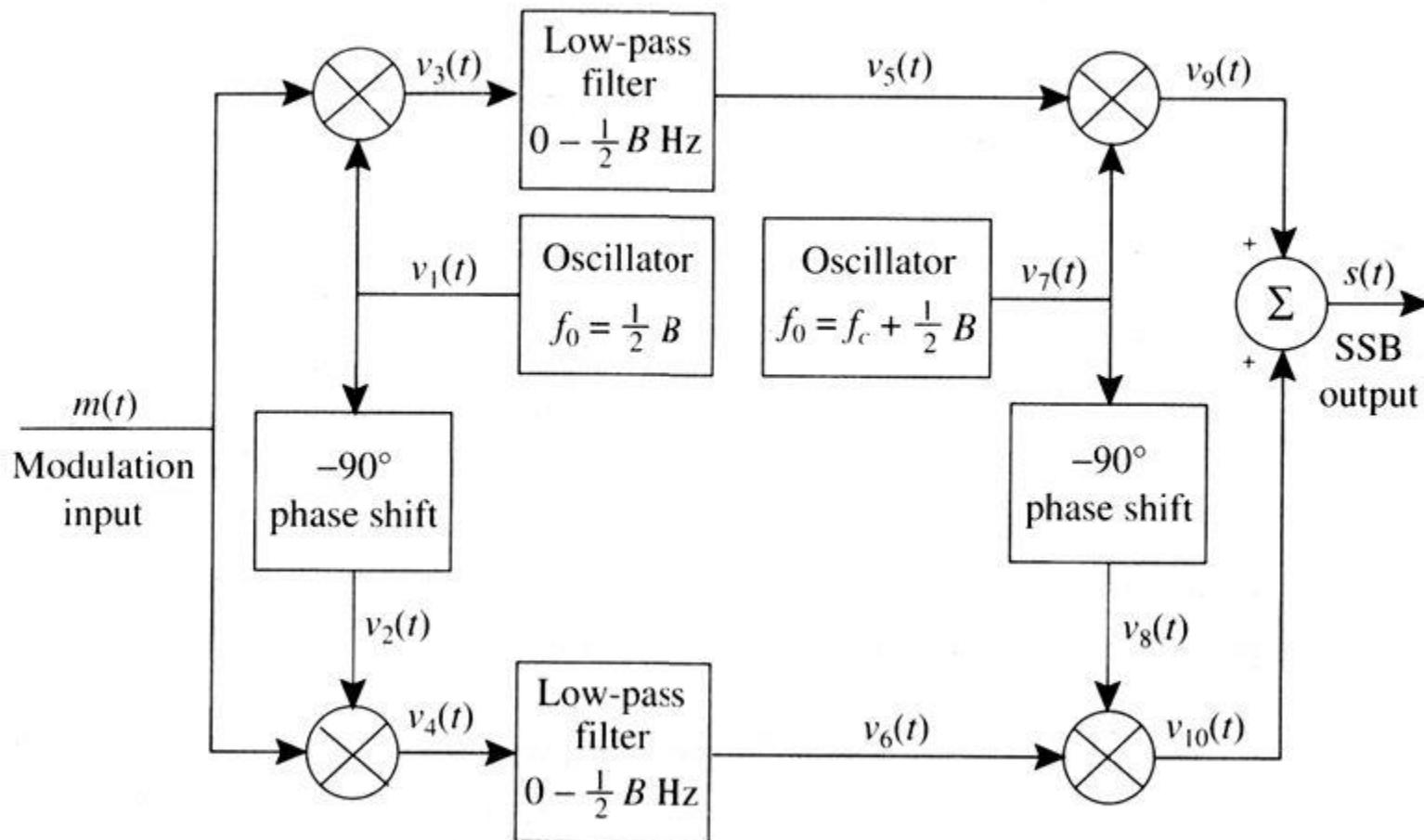
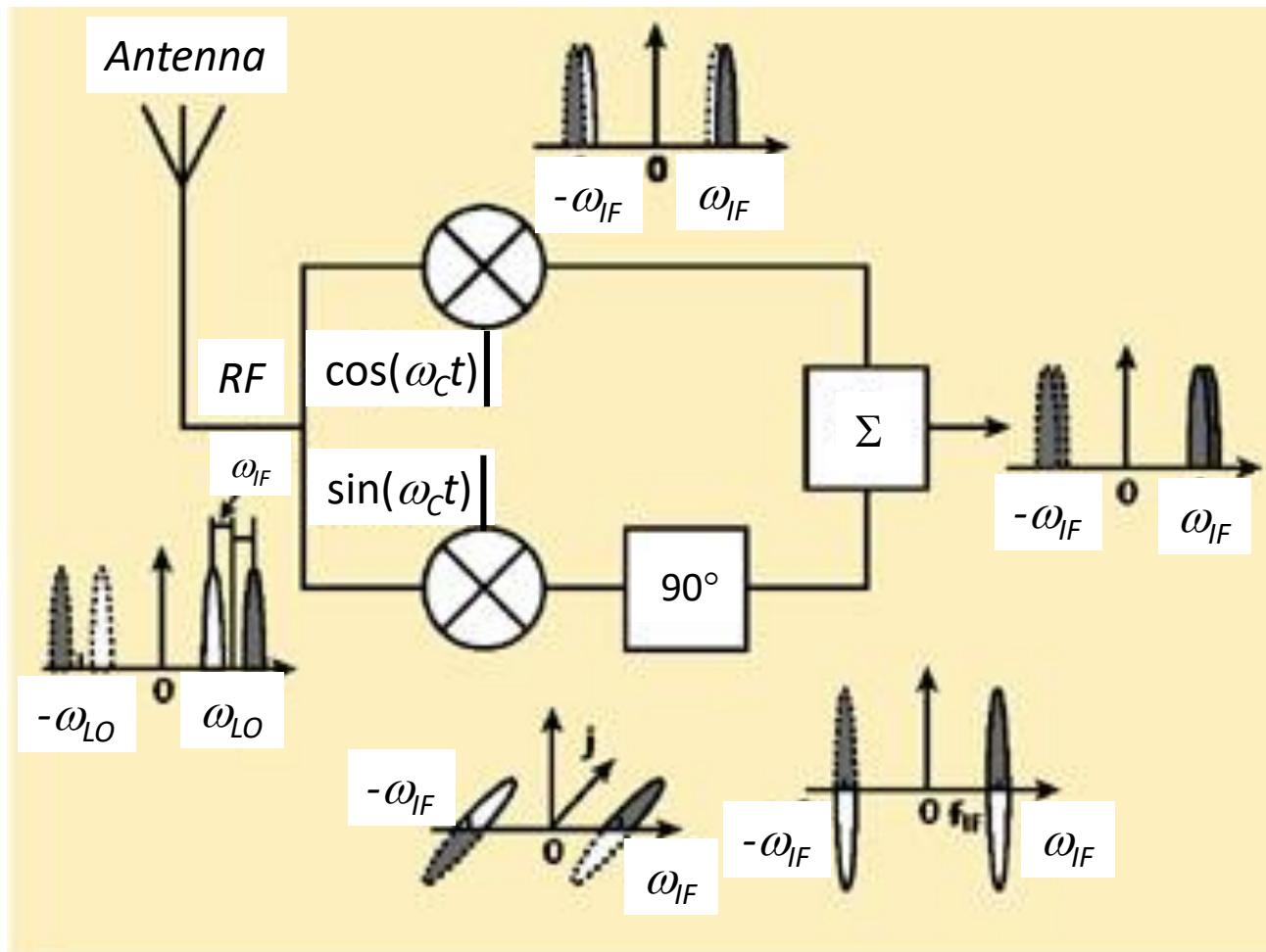


Figure: Weaver's method for generating SSB Signals

# Hartley Image-Rejection Architecture



<http://www.microwavejournal.com/articles/3226-on-the-direct-conversion-receiver-a-tutorial>

# SSB Mixer and Image Rejection Mixer Comparison

Single Sideband Mixer

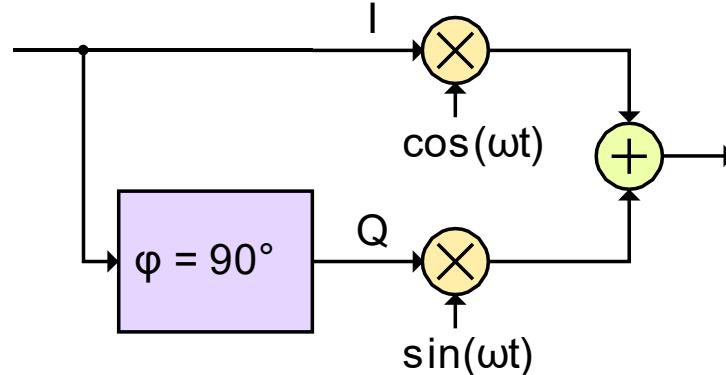
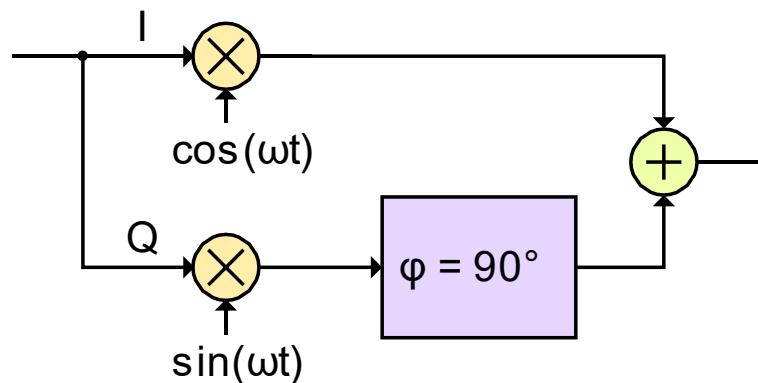
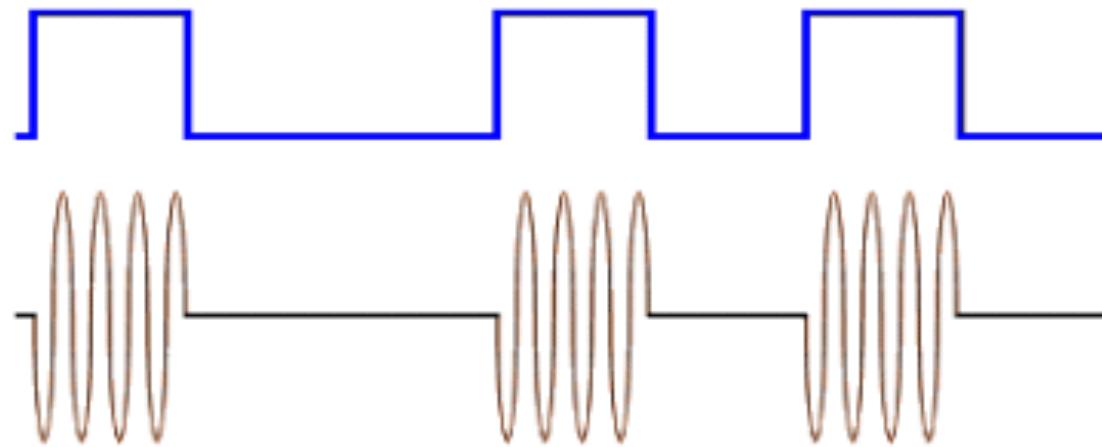


Image Rejection Mixer



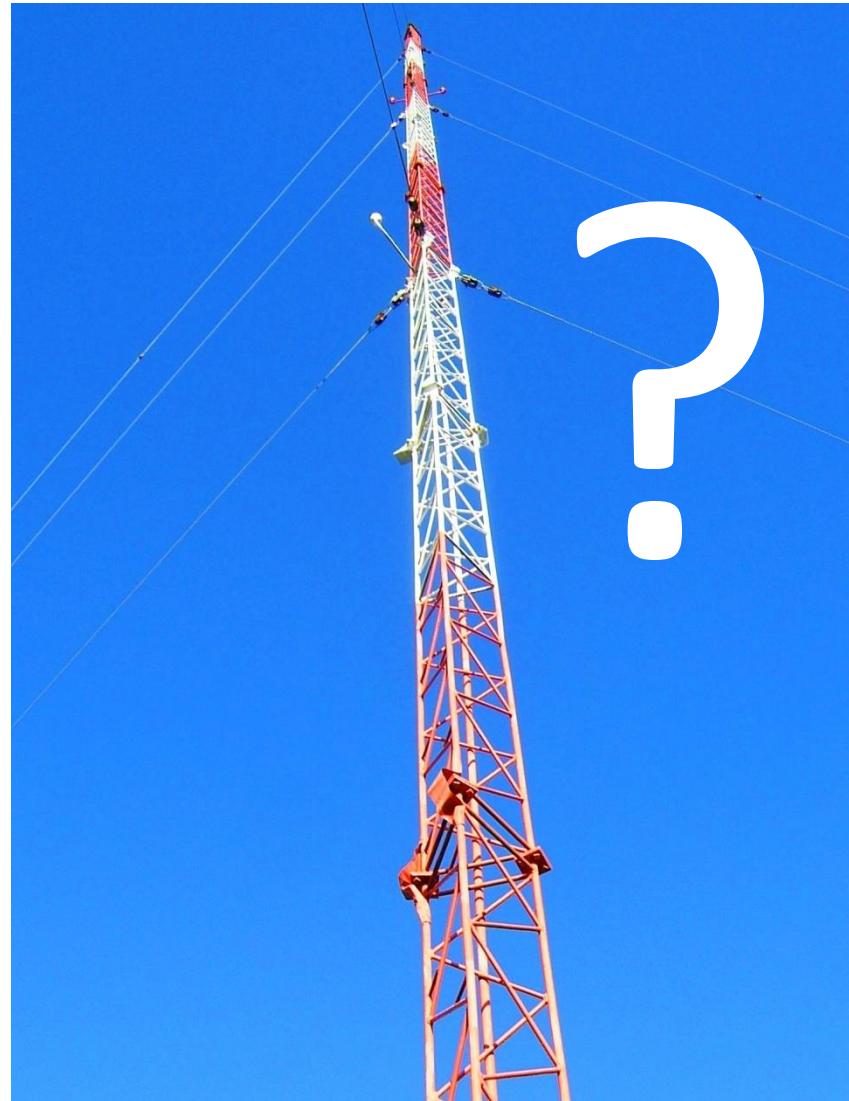
[https://commons.wikimedia.org/wiki/File:SSB\\_and\\_Image\\_Rejection\\_Mixer.svg](https://commons.wikimedia.org/wiki/File:SSB_and_Image_Rejection_Mixer.svg)

## Pulse Amplitude Modulation (PAM) → Digital Signal



How is PAM in digital communication similar to AM in analog communication?

# Questions



# Questions

## From Spring 2018 EE 442 Class

- 1. What is the point of creating a rectified output when using a diode for AM modulation?**

It combines the carrier signal with the message signal  $m(t)$ . See slides 34, 35 & 36 for illustration of this.

- 2. More about how mixers work.** (Three questions asked about mixers.)

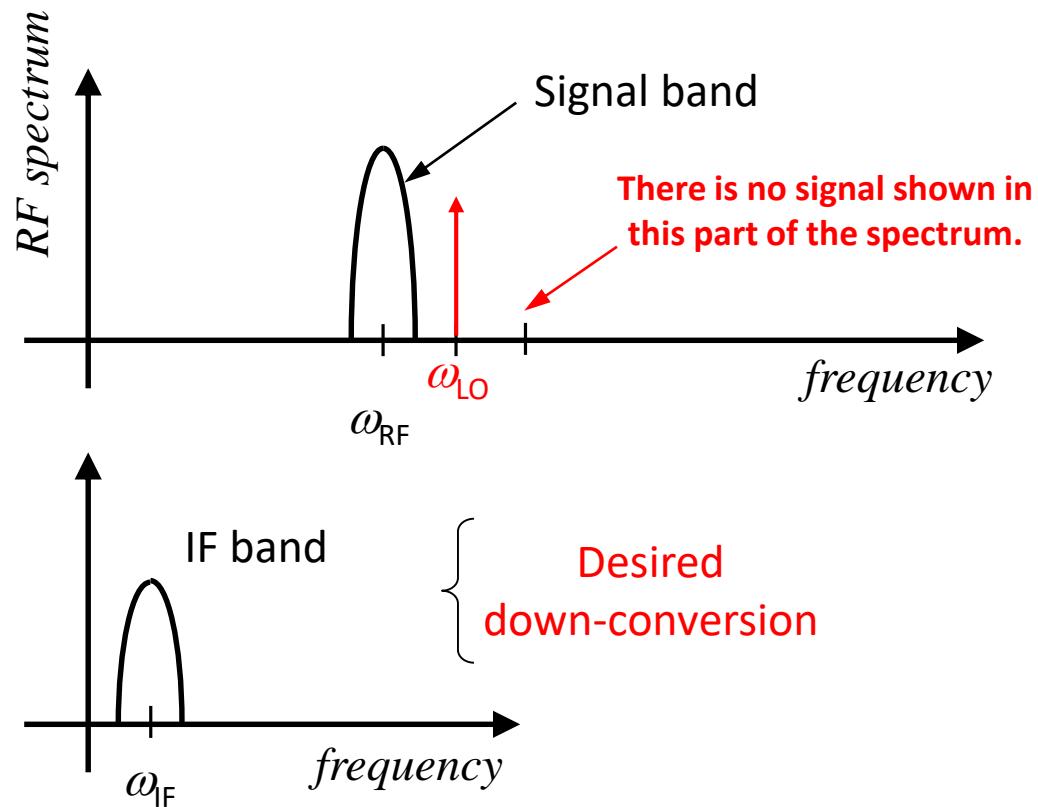
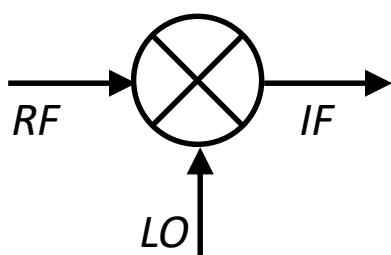
Two principles are used in mixers to create new frequencies: (1) Nonlinearity the I-V characteristics of a nonlinear device do this, and (2) time-varying switching will create new frequencies.

- 3. Explain the idea of images in mixers again.**

See the next four slides.

# Image Signals in Mixers (1)

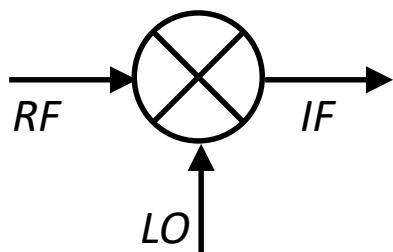
This converts the spectrum at the RF carrier frequency down to the spectrum centered at the IF frequency.



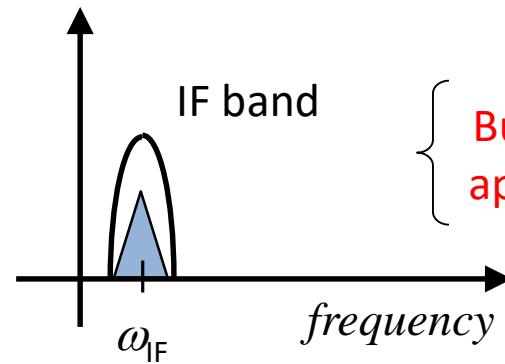
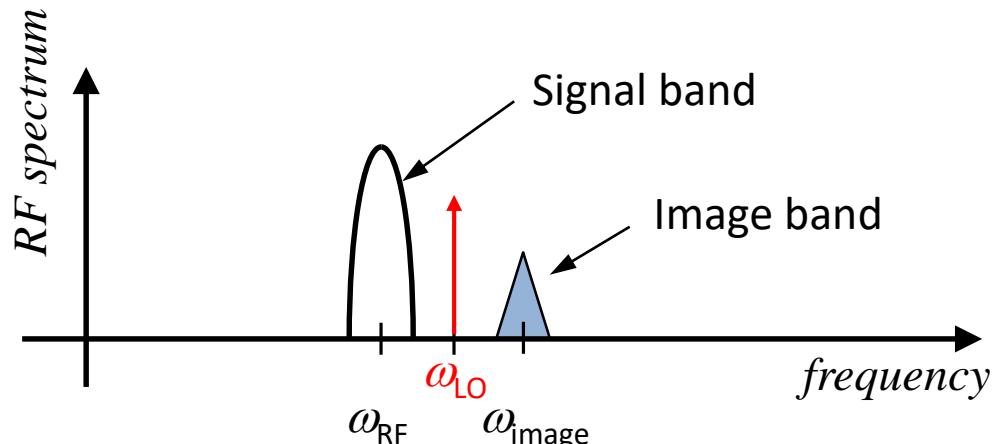
$$\omega_{IF} = \omega_{LO} - \omega_{RF}$$

## Image Signals in Mixers (2) – Now an Image Signal Appears

Now both the spectrum at the RF carrier frequency and the undesired image spectrum are down converted to the spectrum centered at the IF frequency.



The image spectrum is not wanted.

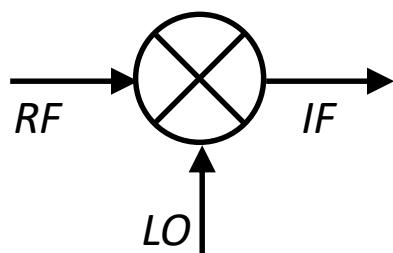


But, now both signals appear in the IF band.

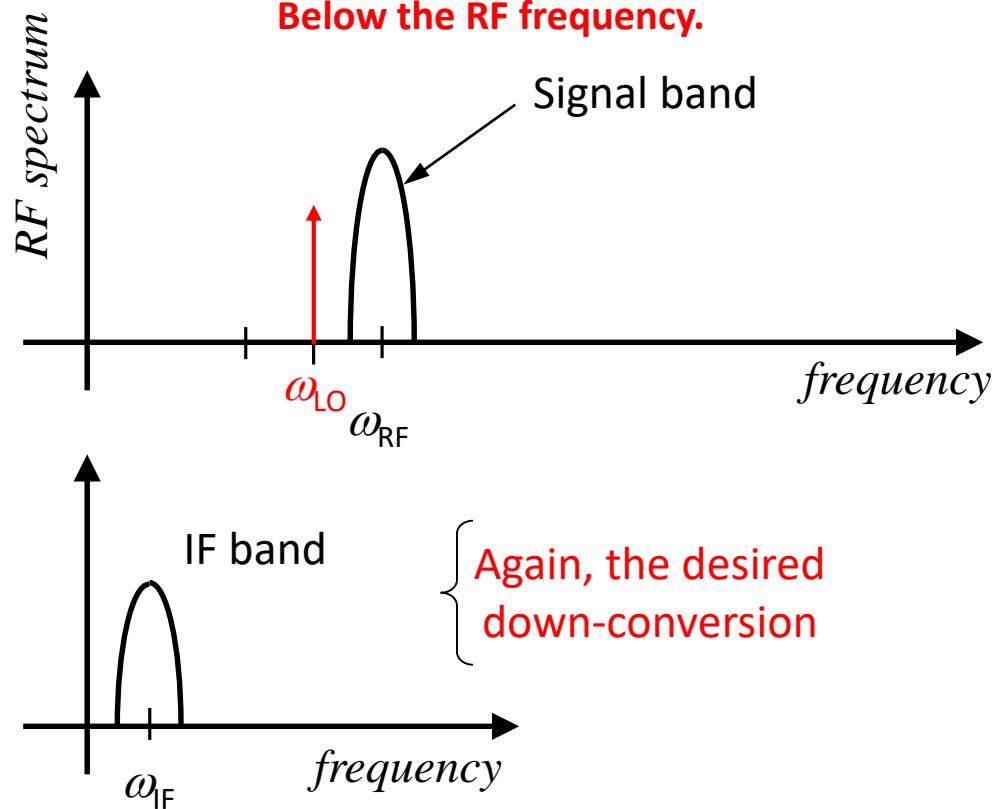
$$\begin{aligned}\omega_{IF} &= \omega_{LO} - \omega_{RF} \\ \text{and} \\ \omega_{IF} &= \omega_{image} - \omega_{LO}\end{aligned}$$

## Image Signals in Mixers (3)

This converts the spectrum at the RF carrier frequency down to the spectrum centered at the IF frequency.



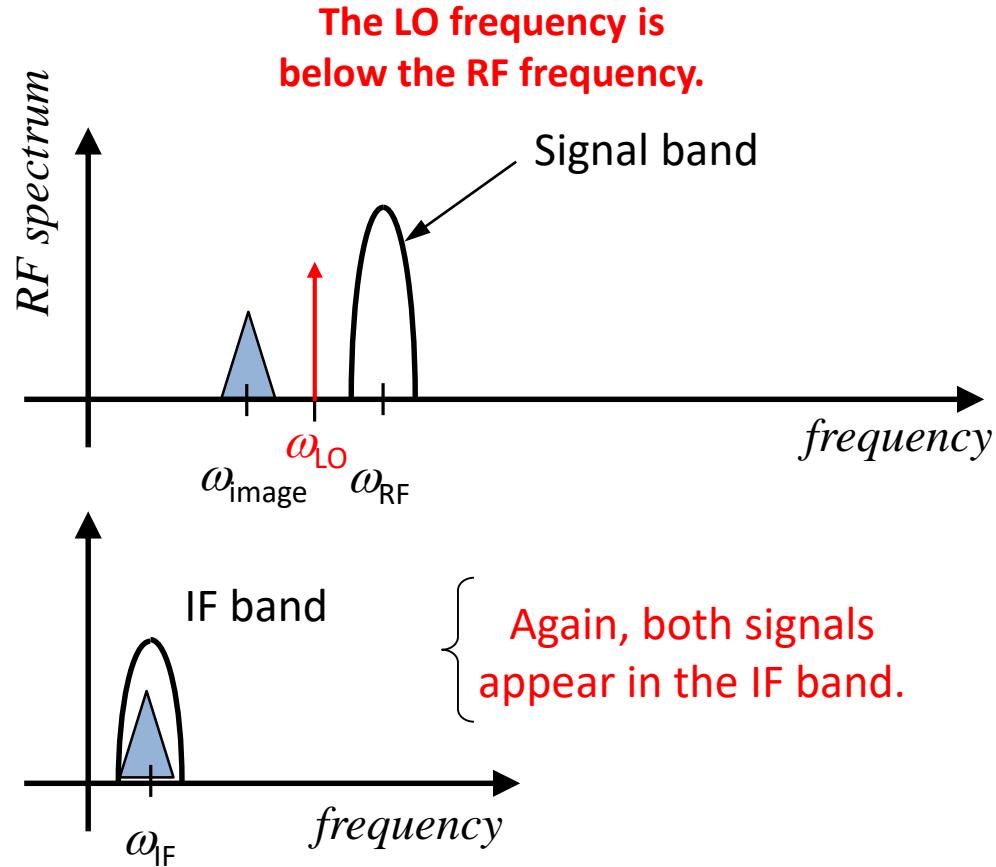
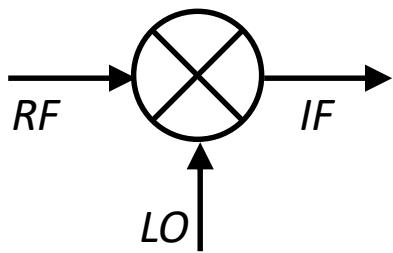
Suppose the LO frequency is Below the RF frequency.



$$\omega_{IF} = \omega_{RF} - \omega_{LO}$$

## Image Signals in Mixers (4) – With Image Signal

As before both the spectrum at the RF carrier frequency and the undesired image spectrum are down converted to the spectrum centered at the IF frequency.



$$\omega_{\text{IF}} = \omega_{\text{RF}} - \omega_{\text{LO}}$$

and

$$\omega_{\text{IF}} = \omega_{\text{LO}} - \omega_{\text{image}}$$

## More Questions

### 4. Why does FM take so much more bandwidth than AM?

It is because we force FM (and PM) signals to have constant amplitude. We show this in detail when we cover Angle Modulation.

### 5. Is there a point where having too many mixers can impact your frequency negatively?

Issues with using multiple mixers:

- a. Adds complexity (such as many mixing products come into play)
- b. Most mixers are lossy and need power gain to continue to process signals (and mixers add noise of their own to signals)
- c. Cost (not only of mixers but for LO oscillators and amplifiers)

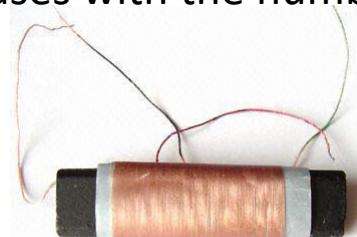
## More Questions

### 6. In an antenna why does the length have to be $\lambda/4$ ?

It actually does not have to be precisely  $\lambda/4$ , but making it longer does not have an advantage in maximizing its efficiency. Making it shorter does decrease the signal strength received. Also, we may want the antenna to resonate at the operating frequency to increase the efficiency of the antenna.

### 7. Also, if an AM signal has a wavelength of hundreds of feet, how can its antenna be so small?

The coil of wire picks up the time-variation of the electromagnetic wave's magnetic field and induces a current in the coil which becomes the signal at the input of the AM receiver. The current increases with the number of turns of the coil.



## More Questions

- 8. When you have no carrier signal, are you sending the message signal? If so, is there even any modulation of the amplitude or just the original signal?**

In the absence of a carrier signal, then only the message signal can be sent at the frequency band of the message signal. We say the message signal “modulates” the carrier signal.

- 9. Does the local oscillator change its frequency depending upon the RF frequency you want to select?**

I assume you are referring to the superheterodyne receiver.

Yes, the local oscillator frequency is changed to select the RF frequency you want to select. We require that frequency difference between the RF and LO frequencies is a constant.

## More Questions

### **10. How is a mixer similar/different from amplitude modulation?**

The mathematics of the AM says we use a multiplier to multiply the carrier signal with the message signal. A mixer performs signal multiplication as required in amplitude modulation.

### **11. Why do we use sub-heterodyning? What are the common applications of sub-heterodyning?**

When analyzing a system we focus upon the RF frequencies involved and Possible local oscillator frequencies. The focus is upon frequency conversion. Considerations: Do we want to work with higher or lower frequencies? What frequency bands are to be avoided?

### **12. Which modulation is the most useful today? AM, FM or PM Why?**

FM is the most widely used. It has better noise immunity.

## More Questions

### **13. How do you build a mixer?**

Example are on slides 62 and 73 illustrating the diode ring mixer.

### **14. How would you handle having your oscillator being 1% off?**

A 1% offset in frequency is very large. You might lock it to a reference frequency (such as a precision crystal oscillator). You might use a phase-lock loop to slave the local oscillator to the correct frequency. You might use a pilot signal broadcast with the modulated carrier. There are many other possible solutions.

### **15. From our homework assignments what would you consider the most important problems?**

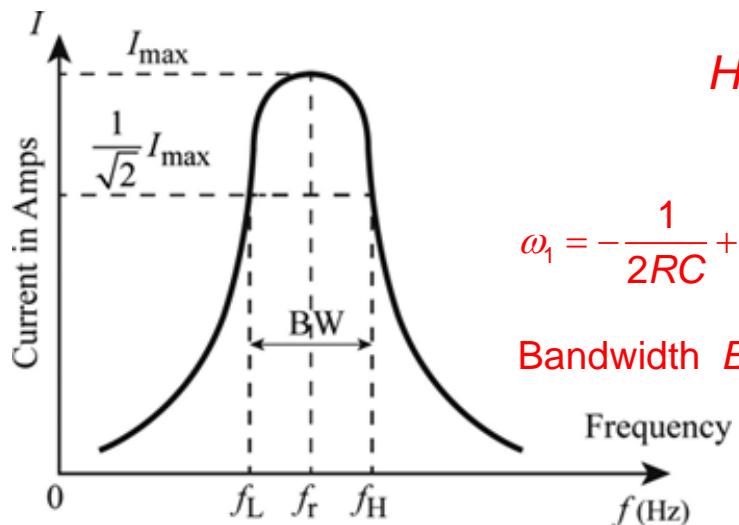
All of them. During the review session before the first midterm I will give a more detailed and specific answer to this question.

## More Questions

### 16. What happens to an over-modulated signal? Can the signal still be used?

Over-modulation leads to distortion in the message. It can still be used in voice communication if the voice over-modulation is not too severe. The point is that voice can still be understood even with moderate distortion.

### 17. Why do RLC resonator circuits have a -3 dB frequency corresponding to a bandwidth of $B = 1/RC$ ?



$$H(\omega) = \frac{i_R}{i(t)} = \frac{j\omega L}{R + j\omega L + (j\omega)^2 RCL}$$

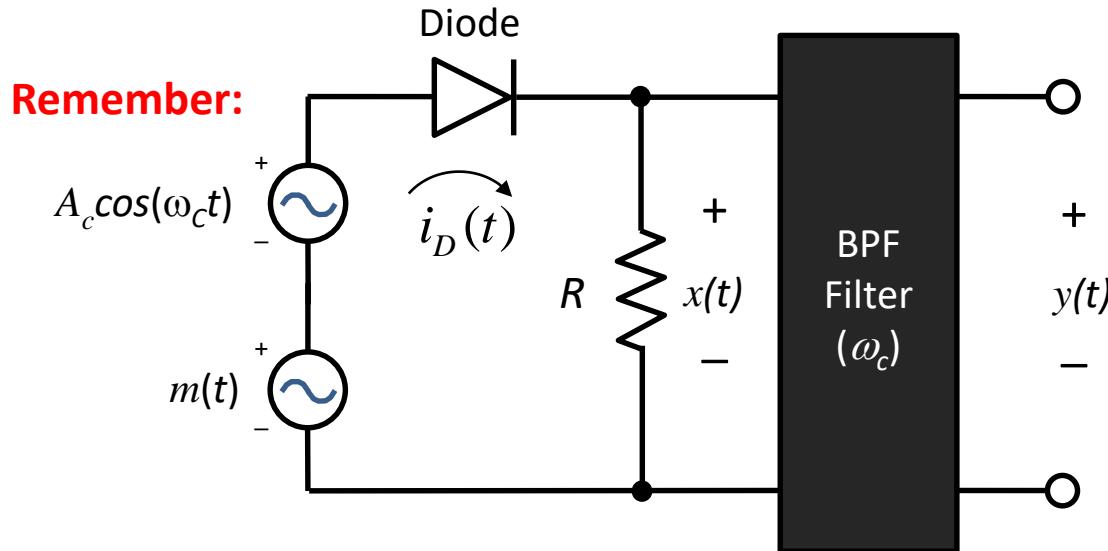
$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + (LC)^2} \quad \& \quad \omega_2 = +\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + (LC)^2}$$

$$\text{Bandwidth } B = \omega_2 - \omega_1 = \frac{1}{RC}$$

Figure 1

## More Questions

18. How does  $x(t) = i_D(t)R$  come about? Why is it not KVL?



The diode is the nonlinear component (it has an exponential characteristic). Using a Taylor's series we can express the diode current  $i_D$  as (only first two terms of Taylor's series),

$$i_D(t) = b_1 v_D(t) + b_2 v_D^2(t); \quad v_D(t) \text{ is diode voltage.}$$

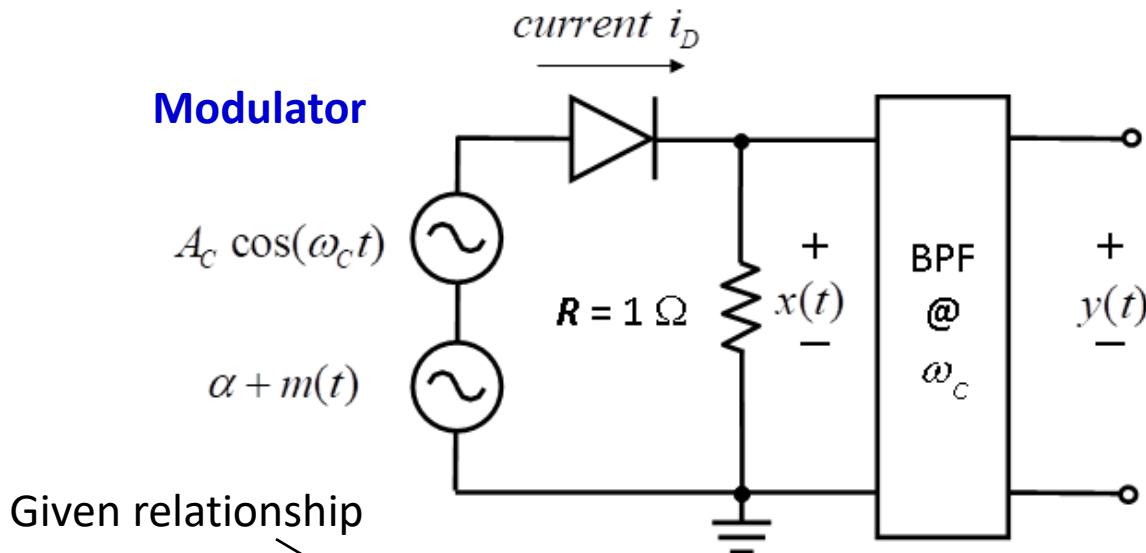
The voltage across resistor R is given by

$$x(t) = i_D(t)R = b_1 R v_D(t) + b_2 R v_D^2(t) = a_1 v_D(t) + a_2 v_D^2(t)$$

"Square Law" behavior

## More Questions (2018)

19. Can you explain using nonlinearity for modulation much like Problem 3 in Homework #3?



$$x(t) = i_D R = \beta(4v_D + v_D^2) \cdot R, \text{ but } R = 1\Omega$$

Substituting for  $v_D$  ( $= \alpha + m(t) + A_C \cos(\omega_c t)$ ),

$$x(t) = 4\beta[(\alpha + m(t) + A_C \cos(\omega_c t))] + \beta[(\alpha + m(t) + A_C \cos(\omega_c t))^2]$$

$$x(t) = 4\beta[(\alpha + m(t) + A_C \cos(\omega_c t))] + \beta[(\alpha + m(t))^2 + 2A_C(\alpha + m(t)) \cdot \cos(\omega_c t) + A_C^2 \cos^2(\omega_c t)]$$

$$\text{But we know } A_C^2 \cos^2(\omega_c t) = \frac{A_C^2}{2}(1 + \cos(2\omega_c t))$$

Continued next slide →

## More Questions

Problem 3 in Homework #3 continued:

$$x(t) = 4\beta[(\alpha + m(t) + A_c \cos(\omega_c t))] + \beta[(\alpha + m(t))^2 + 2A_c(\alpha + m(t)) \cdot \cos(\omega_c t) + A_c^2 \cos^2(\omega_c t)]$$

$$x(t) = \left( 4\beta(\alpha + m(t)) + \beta(\alpha + m(t)^2) + \frac{\beta A_c^2}{2} \right) + \left( 2\beta A_c(\alpha + m(t)) \cdot \cos(\omega_c t) \right) + \frac{\beta A_c^2}{2} \cos(2\omega_c t)$$

The band-pass filter (BPF) passes only terms of  $\cos(\omega_c t)$ , thus  $y(t)$  is

$$y(t) = 4\beta A_c \cdot \cos(\omega_c t) + 2\beta A_c(\alpha + m(t)) \cdot \cos(\omega_c t)$$

$$y(t) = (4\beta A_c + 2\beta A_c \alpha) \cdot \cos(\omega_c t) + m(t) \cdot \cos(\omega_c t)$$

$$y(t) = (K) \cdot \cos(\omega_c t) + m(t) \cdot \cos(\omega_c t)$$

## More Questions

### **20. What is the primary form of noise production in AM systems?**

The greatest noise problem in AM channels is interference, noise pickup, & fading in wireless transmission, all of which distort the amplitude of the transmitted signal.

### **21. QAM (Several asked about QAM so we need to cover it again)**

I will review QAM again after questions are answered.

# More Questions

## 22. Go over Problem 2 in Homework #2

### Problem 2 Square Law Device (20 points)

You are given a square-law component with an input to output relationship of

$$y(t) = A + B(g(t))^2$$

- (a) To explore the behavior of this device we let the input signal  $g(t)$  be a sinusoidal tone, that is,  $g(t) = \cos(\omega t)$ .

**Answer:** The square-law device generates a frequency that is the double of the single tone frequency  $f$ . To show this we make use of the trigonometric identity:

$$y(t) = A + B(g(t))^2 = A + B[\cos(\omega t)]^2 = A + \frac{B}{2}[1 + \cos(2\cdot\omega t)]$$

$$y(t) = \left(A + \frac{B}{2}\right) + \frac{B}{2}\cos(2\cdot\omega t)$$

- (b) What frequencies does the cubic term (that is,  $D[g(t)]^3$ ) generate when driven by  $g(t) = \cos(\omega t)$ ?

$$y(t) = A + Bg(t) + C(g(t))^2 + D(g(t))^3 + \text{other terms.}$$

**Answer:** The cubic term in the series generates a frequency that is triple of the frequency of  $g(t)$ , that is, frequency  $3f$ . This comes from using the trigonometric identity of  $\cos^3(x) = \frac{1}{4}[3\cos(x) + \cos(3x)]$ . Thus, the  $\cos^3(2\pi f t)$  term gives us both a  $\cos(2\pi f t)$  term (not so interesting) and a  $\cos(3 \cdot 2\pi f t)$  term (which is a new frequency being introduced).

## More Questions

### 23. Why does milliwatts relate to dBm rather than just use milliwatts?

We express milliwatts (mW) in decibels by

$$\text{Power in dBm} = 10 \cdot \log_{10} \left( \frac{P_{mW}}{1 \text{ mW}} \right) \text{ dBm} \quad (\text{Note: logarithm of a ratio})$$

We use this because logarithms add rather than multiply in calculations.

#### Example:

Suppose a signal of 3 dBm power drives an amplifier of gain = 13 dB. What is the output power of the amplifier.

Answer: Output power (in dB) = 3 dBm + 13 dB = 16 dBm,

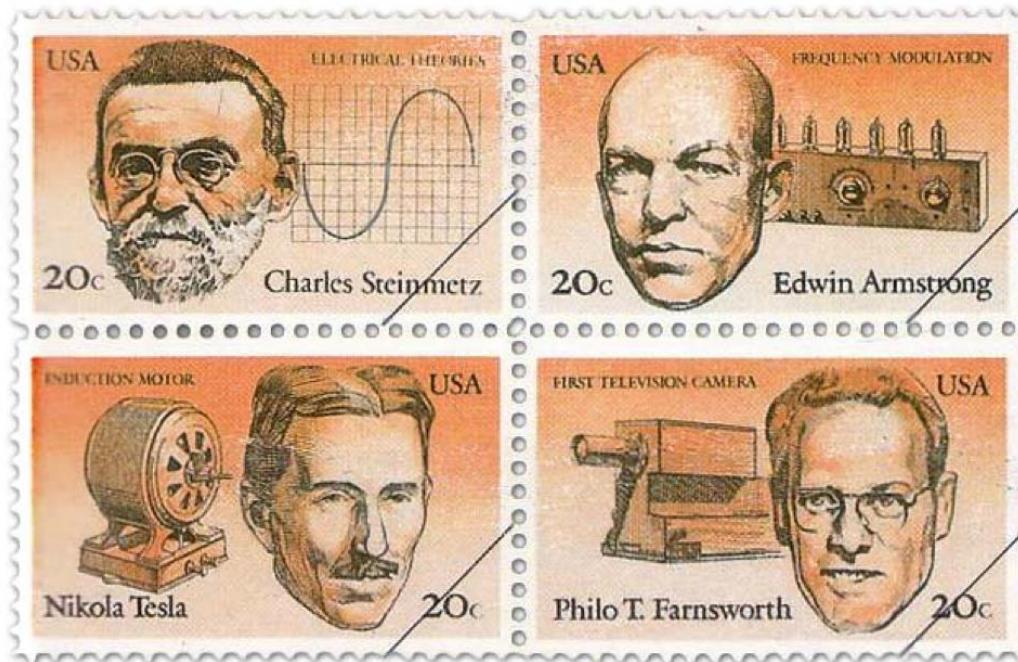
compared to 2 mW (= 3 dBm) multiplied by gain of 20 (= 13 dB) = 40 mW

## **More Questions**

**24. What is the one question you think we should have asked?**

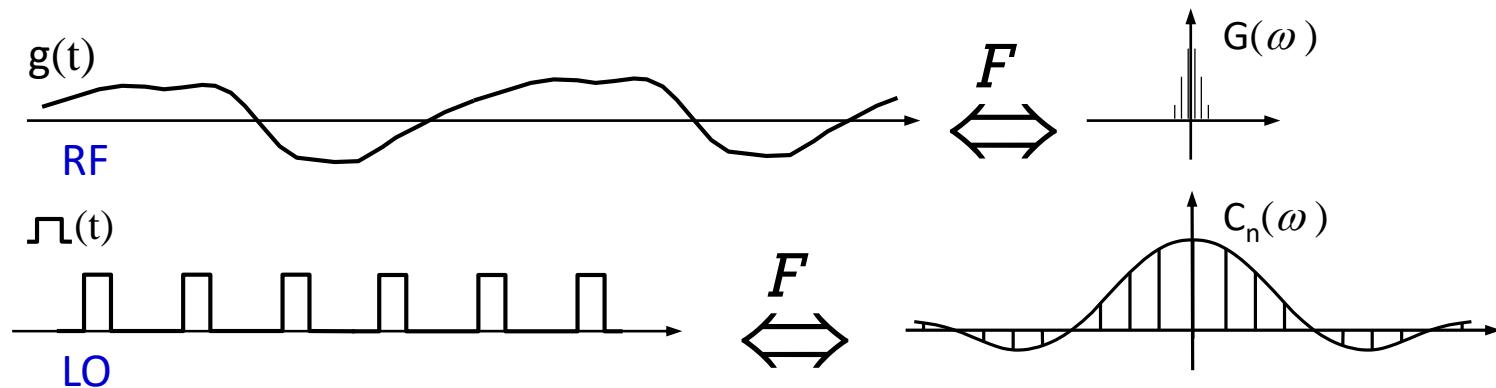
**Answer: The one for the topic that is troubling you.**

# Additional Slides & Illustrations

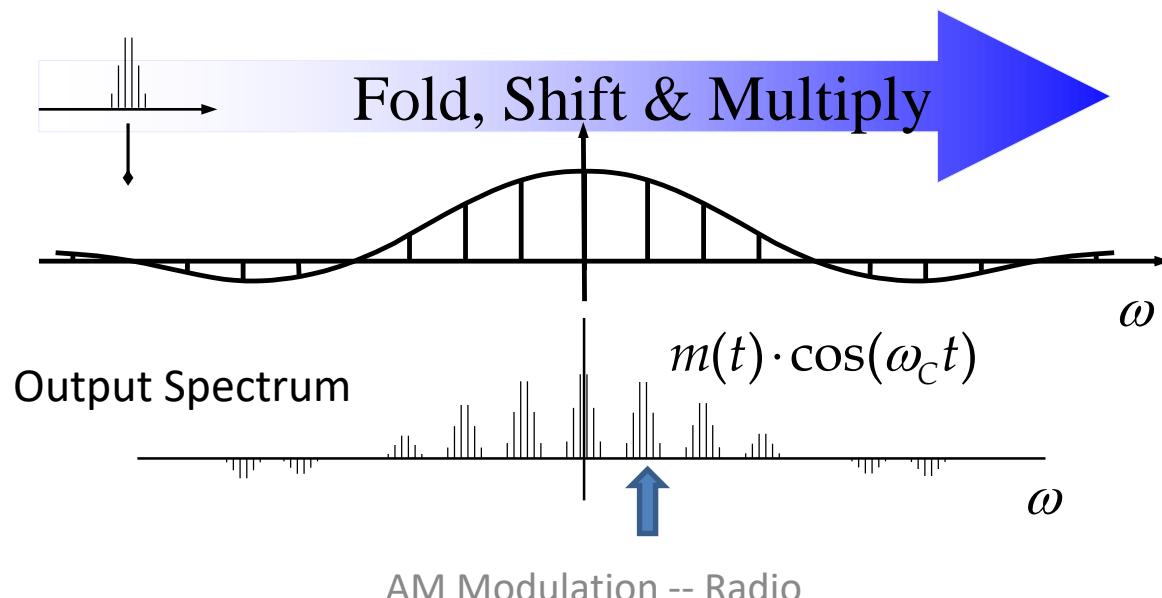


[https://www.stonybrook.edu/commcms/electrical/news/2017/armstrong\\_award.php](https://www.stonybrook.edu/commcms/electrical/news/2017/armstrong_award.php)

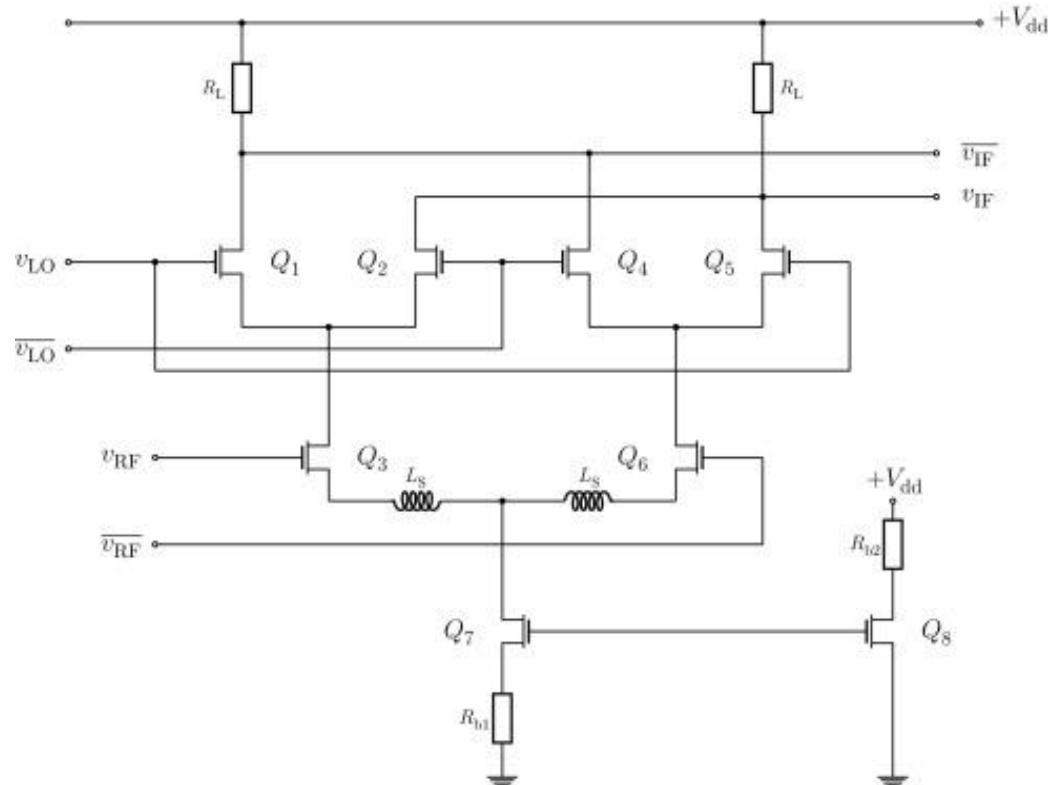
## Generating $m(t) \cdot \cos(\omega_C t)$ using Convolution



From Convolution Theorem:  $\Pi(t) g(t) \Leftrightarrow C_n(\omega) * G(\omega)$



# Gilbert Cell Multiplier Using FET Devices



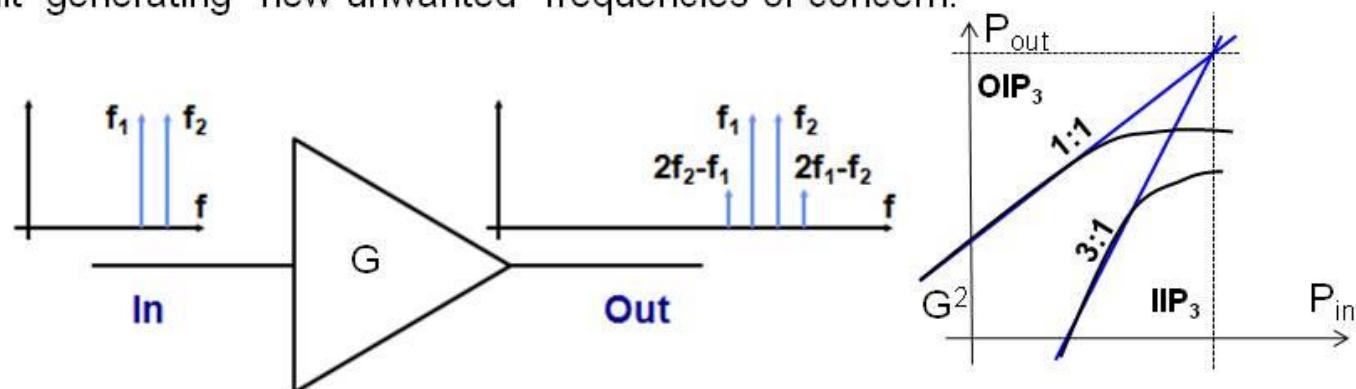
<https://www.sciencedirect.com/topics/engineering/gilbert-cell>

# LNA IIP3 Linearity

IIP3 = Third-order Input Intercept Point

IIP3 – “Intercept Point 3” is a common linearity measure usually employed in receivers for amplifiers and other active circuits.

2 signals closely spaced in frequency and equal in amplitude drive a “non linear circuit” generating “new unwanted” frequencies of concern.



$$v_{in} = A[\cos(\omega_1 t) + \cos(\omega_2 t)]$$

$$\begin{aligned} v_{out} \approx & G \cdot A [\cos(\omega_1 t) + \cos(\omega_2 t)] + \\ & K \cdot A^3 [\cos(2\omega_1 - \omega_2)t + \cos(2\omega_2 - \omega_1)t] \\ & + \dots \text{other freqs} \end{aligned}$$

[https://www.exporegate.com/video.aspx?video\\_id=5](https://www.exporegate.com/video.aspx?video_id=5)

# Modern Mechanix (December 1952)

Today it is reality!

A vintage advertisement for the Dick Tracy Wrist Radio. At the top left, a small illustration shows a man in a yellow trench coat and hat looking at his wristwatch, which has '2-WAY WRIST RADIO' printed on its face. The main illustration features a large, detailed drawing of the wristwatch with a circular face and a leather strap. To the left of the watch, the text reads: 'One of the most compact sets you've ever seen!' Below the watch, three diagonal lines point towards it with the text: 'No Batteries', 'No Electricity', and 'No Tubes'. On the right side of the watch, the words 'DICK TRACY' are embossed on the strap. The central text of the ad reads: 'DICK TRACY WRIST RADIO . . . for Kids from 6 to 60 WEAR IT LIKE A WATCH USE IT AS A RADIO! SHOCKPROOF—SAFE!' Below this, a detailed description of the product is provided: 'It *really* works. You've seen it in comic strips—now it's available for gift giving. Uses Radar Crystal Detector as developed by U. S. Air Forces. Receives regular AM radio broadcasts. Can be connected with wire for use as telephone system or as extra personal speaker for your home radio. Nothing to wear out or replace. Aerial and ground required for better reception.' At the bottom right, the price '\$2.98 P.P.' is listed. The bottom of the ad contains the company information: 'Leotone Radio Corp., 65 Dey St., Dept. SM, New York, N. Y.'

<http://blog.modernmechanix.com/dick-tracy-wrist-radio/>