How to solve the Island Alleys puzzle?

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Introduction

The objective of the game is to connect horizontally and vertically adjacent dots by clicking between them so that the lines form a simple loop - with no loose ends - that goes through all the dots (technically speaking the line forms a Hamiltonian cycle on a grid).

The lines of the loop enclose an island. The island is exactly one square wide at all places, that's why and I call the paths "alleys". Where two or more alleys meet at a right angle, there is always a number indicating the total distance from that square to the shores in all directions: East, West, North and South. Use these numbers to reconstruct the shape of the entire island. No guessing is needed, only logic.
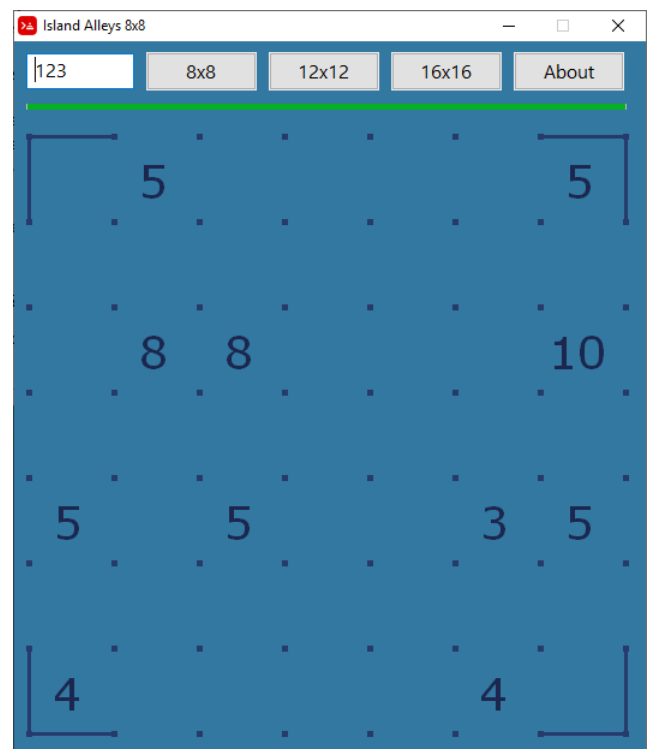
A random game is chosen if no number is entered in the field, otherwise the number indicates the game number.
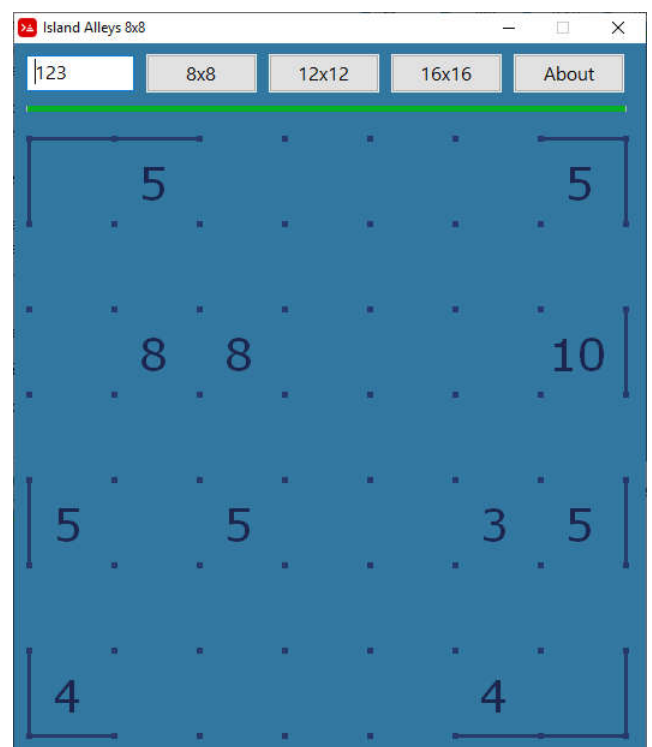
How to so solve it?

Here's what the 8x8 game number 123 looks like in the beginning:
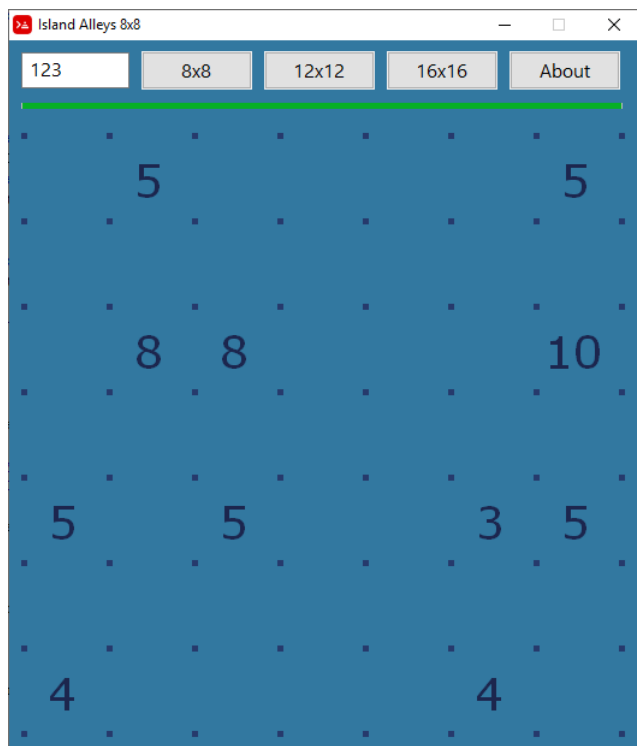
The final result – the loop outlining of the island – must go through all the dots, that's why the four corner dots always contribute their adjacent edges:


The numbers are always inside the island, so the squares with numbers at the border of the grid must have an edge on the outside:

Beside this, the numbers are always at a corner or a crossroad. Moreover, the island is exactly one square wide at all places. This means that the squares with numbers at the border (but not at the corners!) that have no numbered neighbors, contribute an alley towards the center to the island:
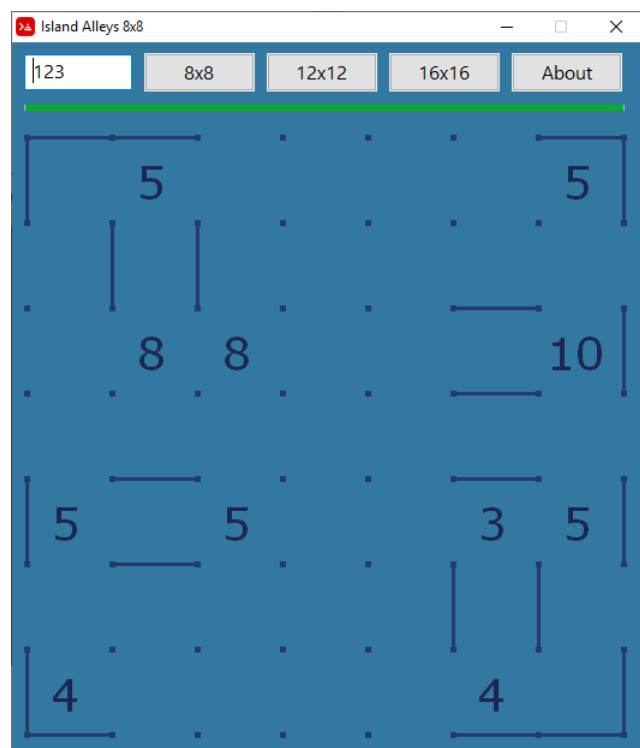

Those at the corners must be at a place where two alleys intersect, so they contribute two alleys. At this point we are not sure how long are the alleys, that's why we extend only one square away from the corners:
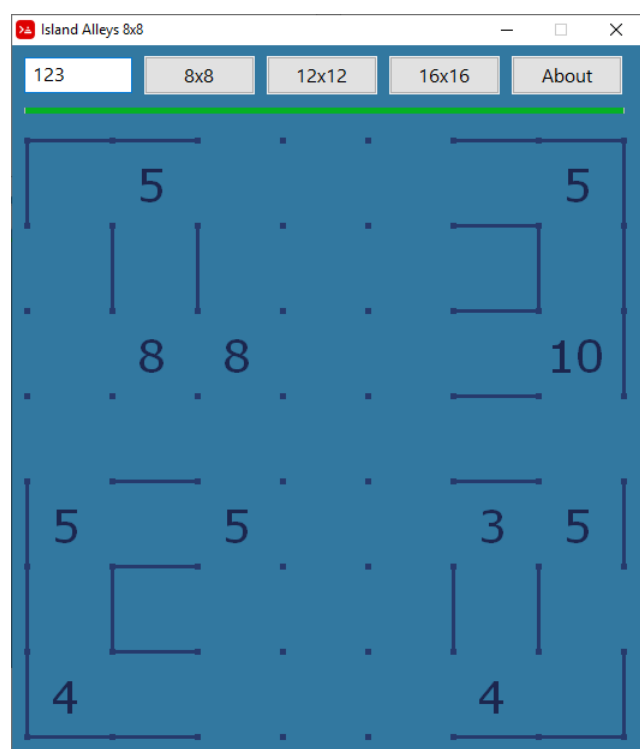

Now we can clean up the corner areas. Looking at the top-left corner with a 5 to the right, we can conclude that there must be a line at the bottom of the square and not on the left side of the square below it. Similarly, the square below must have its bottom edge connected, so that the dots on the left is part of the loop. We make the same for the bottom-right corner:
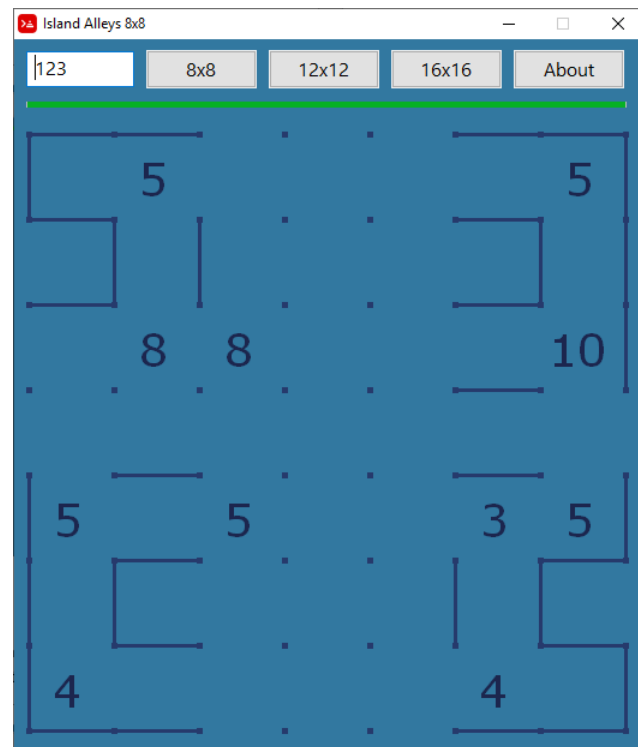

Now that the initial preparations are done, let's start counting. The 3 square already has exactly 3 squares attached to it, so the left edge must be drawn in order to prevent the island from going to the left:
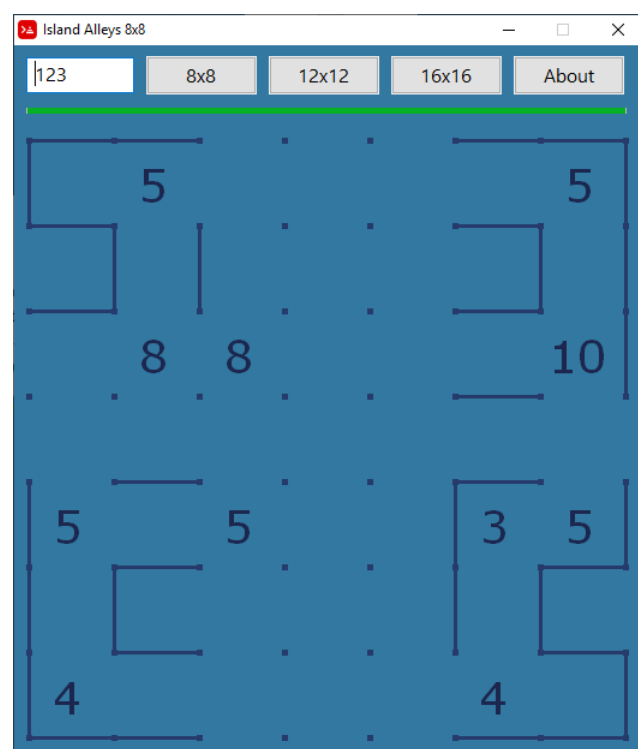
The 4 at the bottom-right has 3 squares attached and can only extend to the left, that's why we mark the alley one square to the left. Since it's an empty square, we must mark all its top, left and bottom edges:
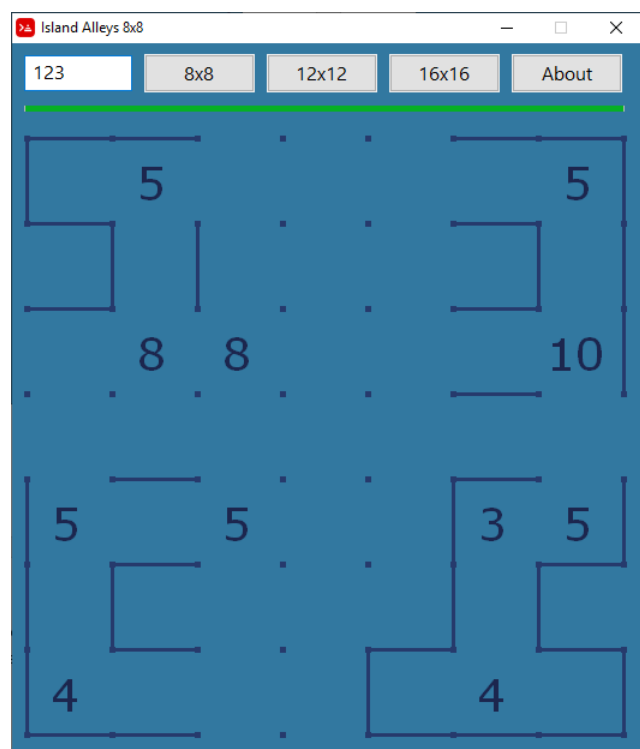

Now look at the 4 at the bottom-left. It has 3 squares attached and can seemingly extend one square to the top or to the right. In fact, it can't extend to the above, because the line will be disconnected from the line above:
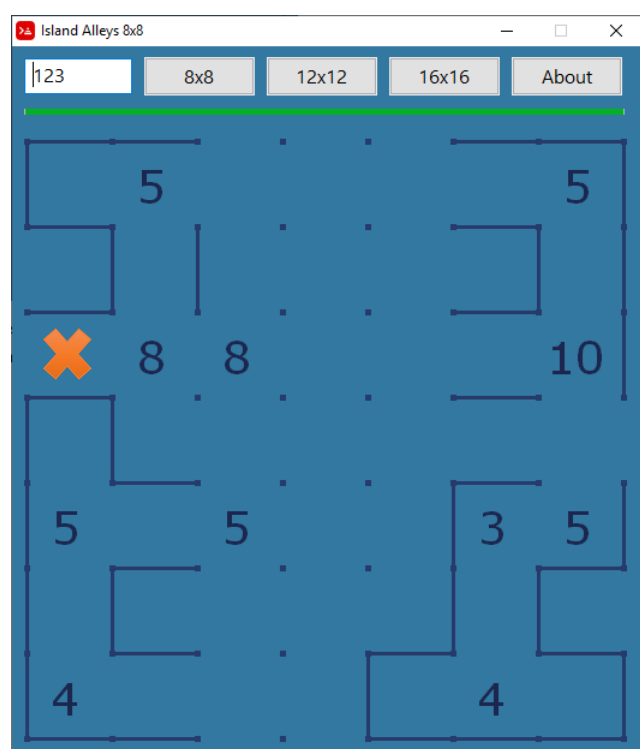

That's why the 5 above it must have its top edge drawn and the alley be extended one more square to the right of the 4:
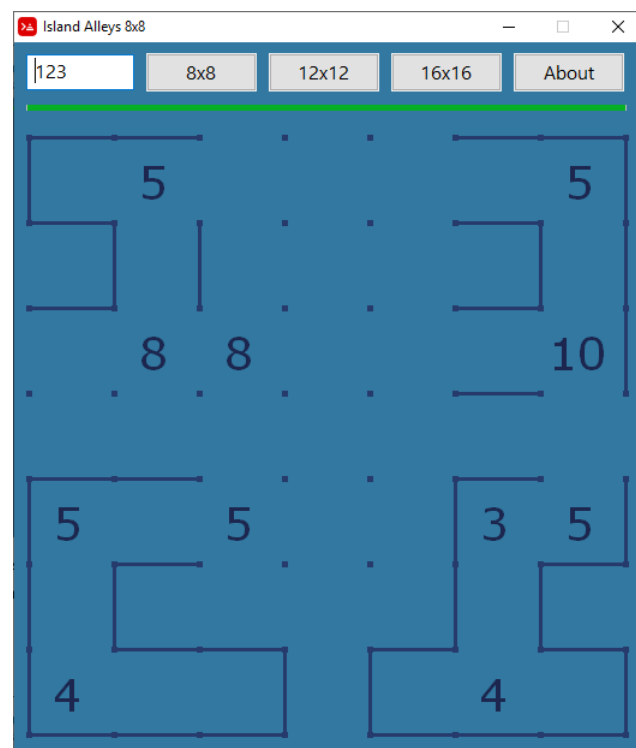
Let's look at the 5's at the right border of the grid. The bottom one already has 1 square and can only extend upward. This also means that the upper 5 is on the same alley and have 4 squares from it. It already has one to the left, so we mark its left edge:
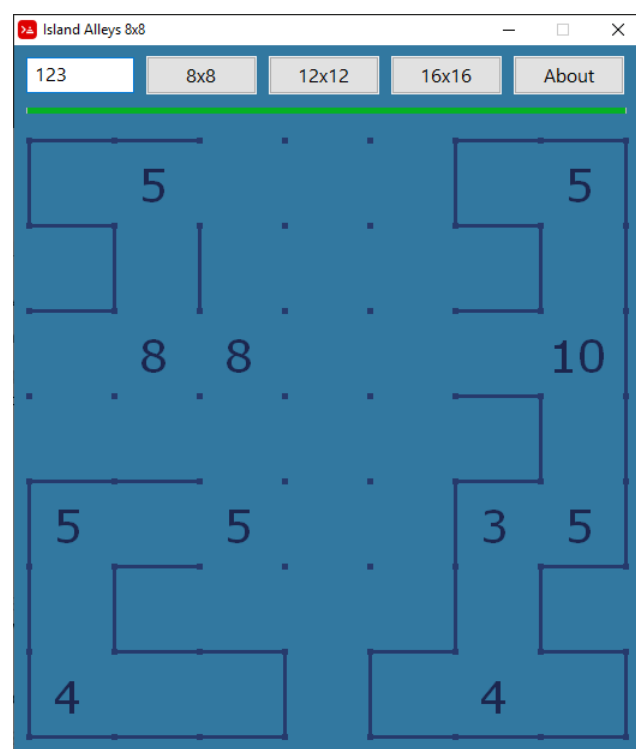
It's now clear where can the 10 extend to – to the left. It has 4 squares already, so the left alley must extend to the left border of the grid. We can mark all the empty squares, because there can't be any branches there, as well as the leftmost square's left edge:
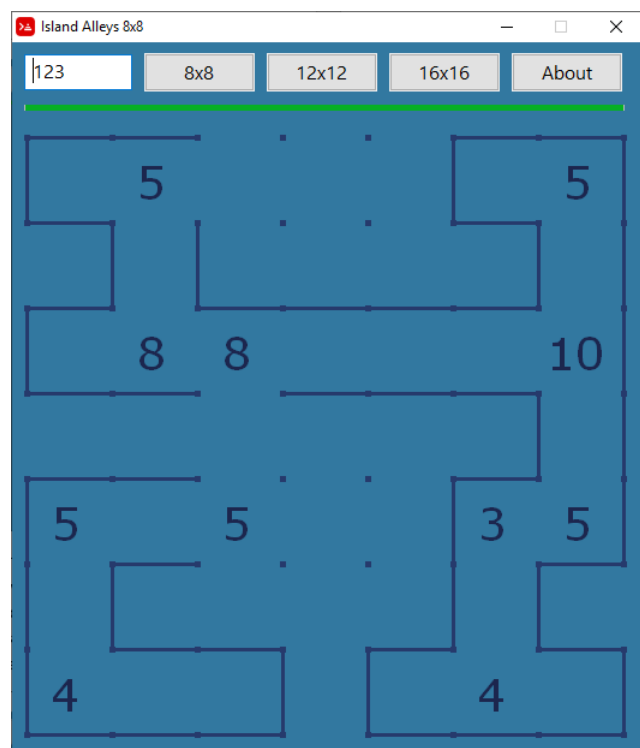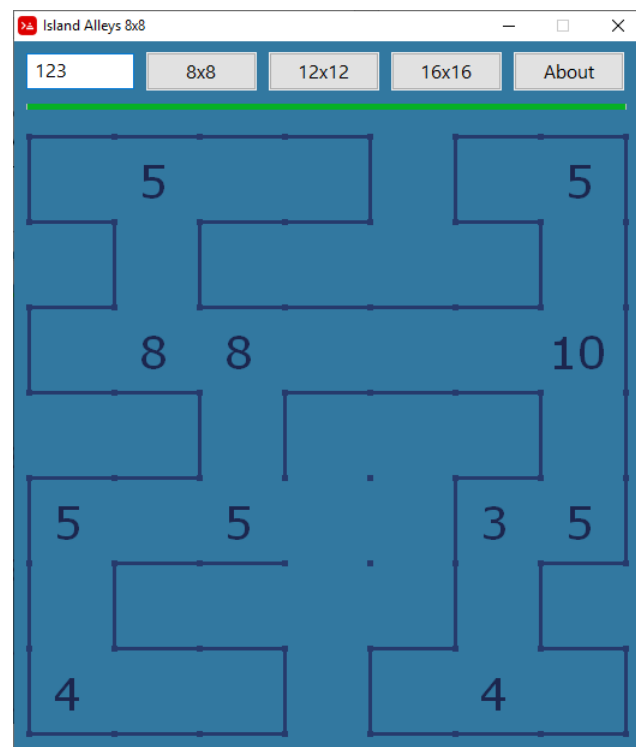

Please note that the left 8 can't extend downward, so we mark its bottom edge. Similarly, the right 8 can't extend upward, that's why we mark its top edge too.
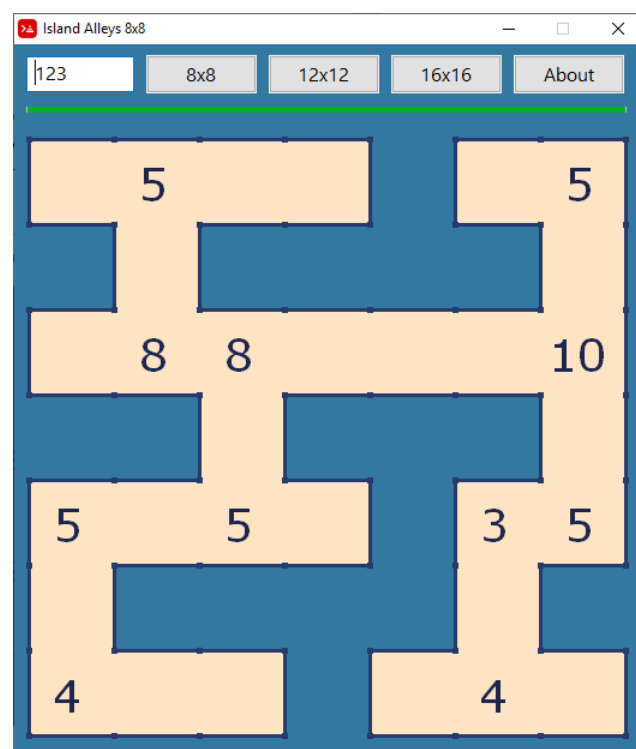
The top-left 5 already has 3 squares to it and can only extend exactly 2 squares to the right:
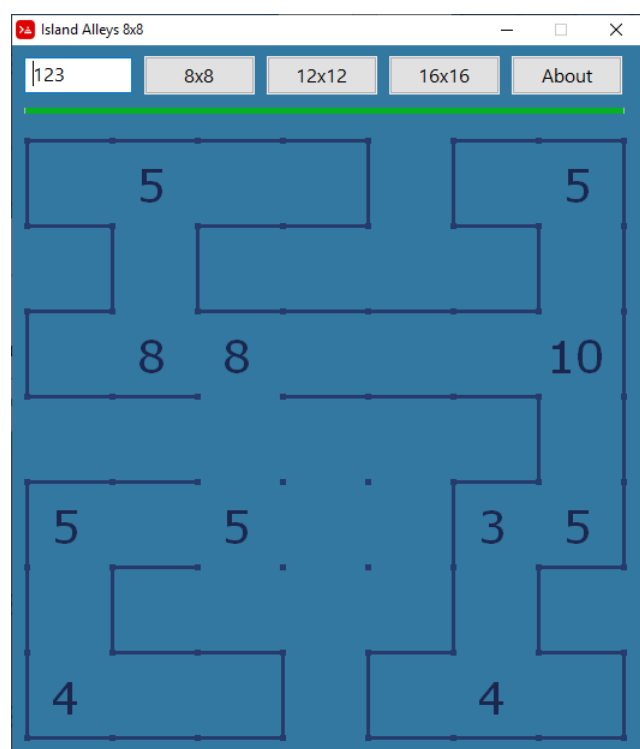
The 8 square has 6 horizontal squares already attached to it and can only extend 2 squares downwards:

We left with one square that has to be attached to the last 5:

And the 123rd 8x8 game is successfully solved!

Enjoy the "Island Alleys"!