Midterm Exam 3

December 23, 2022

Name: ____________ Student No.: ____________ Seat No.: ____________

1. The tangent. (10%)

   Find the equation of the tangent to the curve $\begin{cases} x(t) &= e^{\sqrt{t}} \\ y(t) &= t - \ln(t^2) \end{cases}$ through the point $(x(4), y(4))$.

2. The arc length.

   (a) (10 %) Find the length of the curve $x = 3\cos(t) - \cos(3t)$, $y = 3\sin(t) - \sin(3t)$, $0 \le t \le 2\pi$.

   (b) (10 %) Find the length of the curve $x = \cos(s)$, $y = s + \sin(s)$, $0 \le s \le 2\pi$.

   (c) (10 %) Find the length of the polar curve $r = 1 + \sin(\theta)$ for $0 \le \theta \le \pi$.

3. The area.

   (a) (10 %) Find the area of the surface obtained by rotating the infinite region $\Omega = \{(x, y)|x \ge 1, 0 \le y \le \frac{1}{x}\}$ about the x-axis.

   (b) (10 %) Let R be the region bounded below by the graph of $y = x^3 - x$ and bounded above by the graph of $y = \sin(\pi x)$. Find the area of R.

4. The volume.

   (a) (10 %) Find the volume of the solid obtained by rotating the infinite region $\Omega = \{(x, y)|x \ge 1, 0 \le y \le \frac{1}{x}\}$ about the x-axis.

   (b) (10 %) Find the volume of the solid obtained by rotating the region D about the x-axis, where D is bounded by the x-axis and the curve, $x = \theta - \sin(\theta)$ and $y = 1 - \cos(\theta)$ with $0 \le \theta \le \pi$.

   (c) (10 %) The base of a solid S is the region enclosed by curves $y = \sec x$, $y = \tan x$, $x = 0$ and $x = \pi/6$. The cross-sections perpendicular to the x-axis are squares. Find the volume of S.

   (d) (10 %) Let R be the region bounded below by the graph of $y = x^3 - x$ and bounded above by the graph of $y = \sin(\pi x)$. Find the volume of the solid obtained by rotating the region R about the line $x = -1$.

5. (20 points) Let $V(t)$ be the volume of the solid obtained by rotating the region $A(t)$ about the y-axis, where $A(t) = \{(x, y)|0 \le x \le t, 0 \le y \le \frac{1 + \sin^2(x)}{2}\}$. For what value of $r \le 2$ such that the limit $\lim_{t \to 0^+} \frac{V(t)}{t^r}$ exists.