Quiz 2

March 21, 2023

Name: ______________________ Student No.: ______________________

1. (40 %) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \cos\frac{\pi}{n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty}(\tan^{-1} n)^n$.

2. (40 %) Determine positive integers k such that the series is convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2}{(kn)!}$.

   (b) $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n\ln n[\ln(\ln n)]^k}$.

3. (20 %) Prove that if $a_n \geq 0$ and $\sum a_n$ converges, then $\sum a_n^2$ also converges.

4. (20 points) Given an alternating series $\sum(-1)^{n-1}b_n$, where $b_n > 0$ that satisfies (i) $b_{n+1} \leq b_n$ and (ii) $\lim\limits_{n\to\infty} b_n = 0$. Show that the alternating series is convergent (that is, to prove the Alternating Series Test).