Midterm Exam 2

May 5, 2023

Name: ____________________ Student No.: ____________________ Seat No.: ____________________

1. (20%) Find the limit.
   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x\sin(y)}{x^2+y^2}$.
   (b) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{\ln(x)\ln(y)}{(\ln(x))^2+(\ln(y))^2}$.
2. (10%) Let S be the surface obtained by rotating the curve $y = x^2 + 2$ on the xy-plane about the x-axis. Find the tangent plane to S at $(1, -2, \sqrt{5})$.
3. (20%) Prove or disprove.
   (a) If the partial derivatives f_x and f_y exist, then the directional derivative $D_u f(x, y) = \nabla f(x, y) \cdot u$, where $\nabla f(x, y)$ is the gradient of f at (x, y).
   (b) If the directional derivative $D_u f(0, 0)$ exists for any unit vector u, then $f(x, y)$ is continuous at $(0, 0)$.
4. (20%) Assume that $f(x, y)$ is differentiable and set $g(t) = f(at, bt)$, where a and b are constants.
   (a) Prove or disprove that if, for any $a, b \in \mathbb{R}$, g has a local maximum at 0, then f has a local maximum at $(0, 0)$.
   (b) Prove or disprove that if $\nabla f(0, 0) = \langle 0, 0\rangle$, then g has a local maximum at 0 for any $a, b \in \mathbb{R}$.
5. (10%) Find the absolute maximum value of $f(x, y, z) = xy - yz$ with the constraint $x^2 + y^2 + \dfrac{z^2}{2} = 24$.
6. (20%) Let $f(x, y) = \dfrac{1}{x} + \dfrac{1}{y}$ for $x \neq 0$ and $y \neq 0$.
   (a) Assume that f attains its maximum at P with the constraint $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$. Find P.
   (b) Let P be the point in part (a). Assume that, on the parametric curve $\langle t,\, at^2 + bt\rangle$, f has a local maximum at P. Find the values of a and b.