Quiz 1

March 7, 2023

Name: _____ Student No.: _____

1. (20 %) At what points does the helix $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?
2. (20 %) Given a vector function $\mathbf{r}$ with $\mathbf{r}''$ exists. Show that $\dfrac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t)$.
3. (60 %) Given a curve $\mathbf{r}(t) = e^t \sin t\, \mathbf{i} + e^t \cos t\, \mathbf{j} + \sqrt{2} e^t\, \mathbf{k}$.
   (a) Find the arc length function for the curve measured from the point $P(0, 1, \sqrt{2})$ in the direction of increasing t.
   (b) Reparametrize the curve with respect to are length starting from P.
   (c) Find the point 2 units along the curve (in the direction of increasing t) from P.