Midterm Exam 2

November 15, 2022

Name: _____ Student No.: _____ Seat No.: _____

1. The limit. (30%)
   (a) $\lim_{x\to 0^+}(1+2x)^{3/x}$.
   (b) $\lim_{x\to\infty}(1-\frac{2}{x}+\frac{3}{x^2})^{4x}$.
   (c) $\lim_{x\to 0^+}(\sin x+\sin^{-1}x)^{\tan^{-1}x}$.
2. The differentiation. (20%)
   (a) Find the slope of the tangent to $x^3+x^2y-\cos(xy)=0$ at the point $(1,0)$.
   (b) Let $f(x)=\dfrac{g(h(x))}{h(x)}$ with $g(3)=3$, $g'(3)=7$, $h(6)=3$, and $h'(6)=-2$. Find $f'(6)$.
3. (10%) Find the slant asymptote of $u(x)=11x+2x\sin\dfrac{3}{x}$.
4. (10%) Suppose that $s(x)$ and $t(x)$ are tangent to each other at $x=-1$, and only intersect at there. If $s(x)$ and $t(x)$ are polynomials of the third degree with $s(x)-t(x)=x^3+ax^2+bx+c$, where a,b,c are real numbers, then find a, b, and c.
5. (10%) Let $r(x)=\dfrac{ax}{x^2+b^2}$ with a local minimum at $x=-3$ and $r'(0)=2$. Find a.
6. (20%) Suppose that $v(x)$ is defined on the interval $[a,\infty)$ with $a>0$ and differentiable on (a,∞).
   (a) Prove or disprove that if $v(x)$ has a limit as $x\to\infty$, then $\lim_{x\to\infty}v'(x)=0$.
   (b) Prove or disprove that if $\lim_{x\to\infty}\dfrac{v(x)}{x}=1$, then $\lim_{x\to\infty}v'(x)=1$.