Krótkie wprowadzenie do metody FDTD (cz. 2(2D))

Równania Maxwella

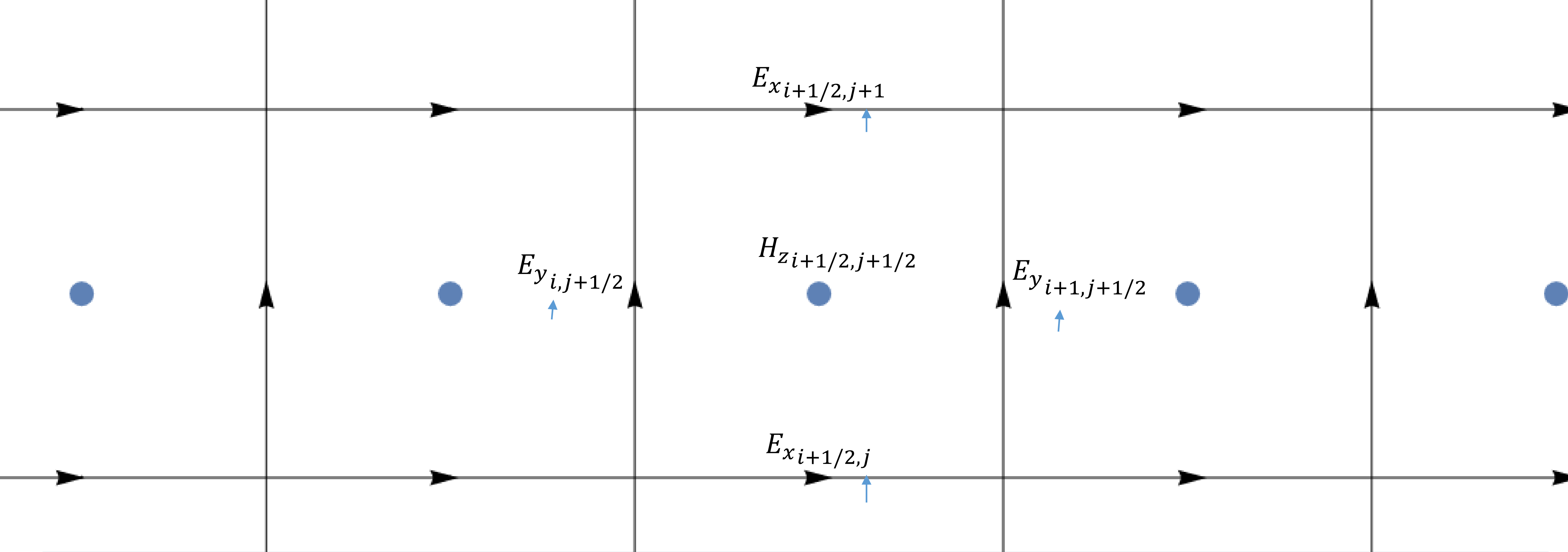
- $\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$
- $\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$

$$\boldsymbol{J} = \boldsymbol{J}_{source} + \sigma \boldsymbol{E}$$

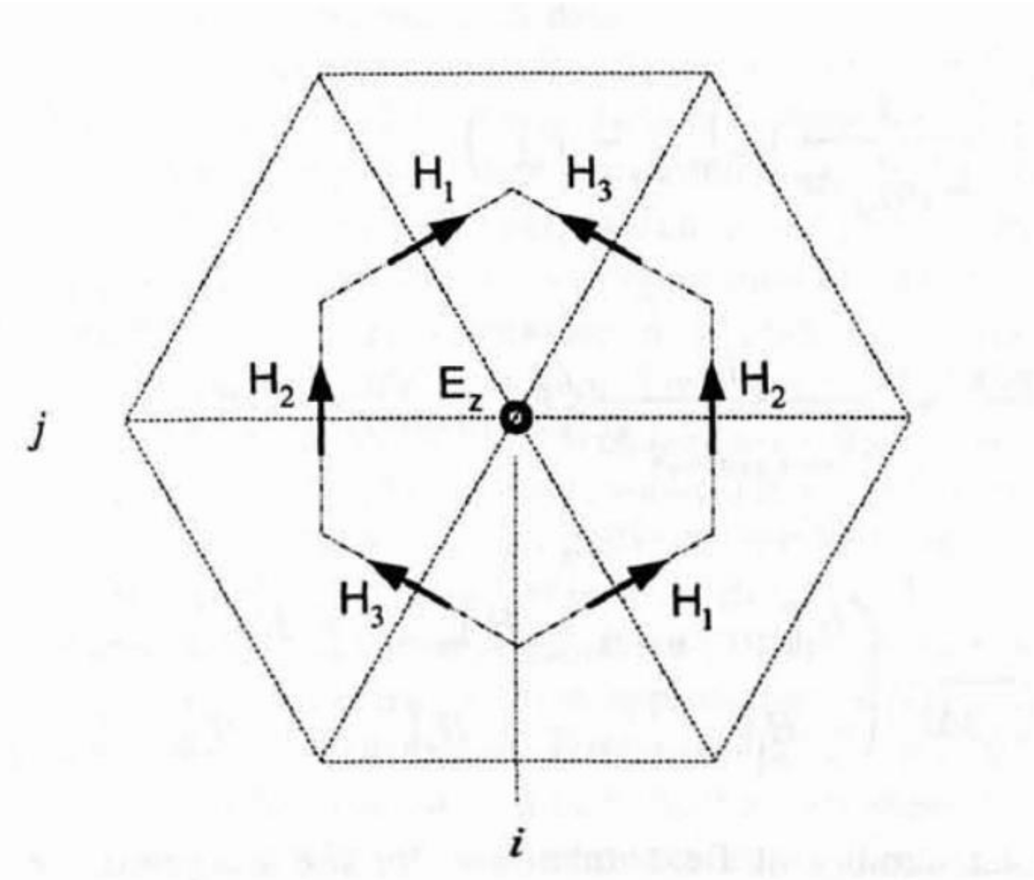
Źródło prądu

Mikroskopowe prawo Ohma

- $\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - J_x\right)$
- $\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(-\frac{\partial H_z}{\partial x} - J_y\right)$
- $\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$



$$\frac{H_{z_{i+1/2,j+1/2}}^{n+1} - H_{z_{i+1/2,j+1/2}}^{n}}{\Delta t} = \frac{1}{\mu}\left(\frac{E_{x_{i+1/2,j+1}}^{n+1/2} - E_{x_{i+1/2,j}}^{n+1/2}}{\Delta y} - \frac{E_{y_{i+1,j+1/2}}^{n+1/2} - E_{y_{i,j+1/2}}^{n+1/2}}{\Delta x}\right)$$



źr: Allen Taflove, Susan C. Hagness, *Computational electrodynamics – FDTD method*

Równania Maxwella

- $\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$
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$$\boldsymbol{J} = \boldsymbol{J}_{source} + \sigma \boldsymbol{E}$$

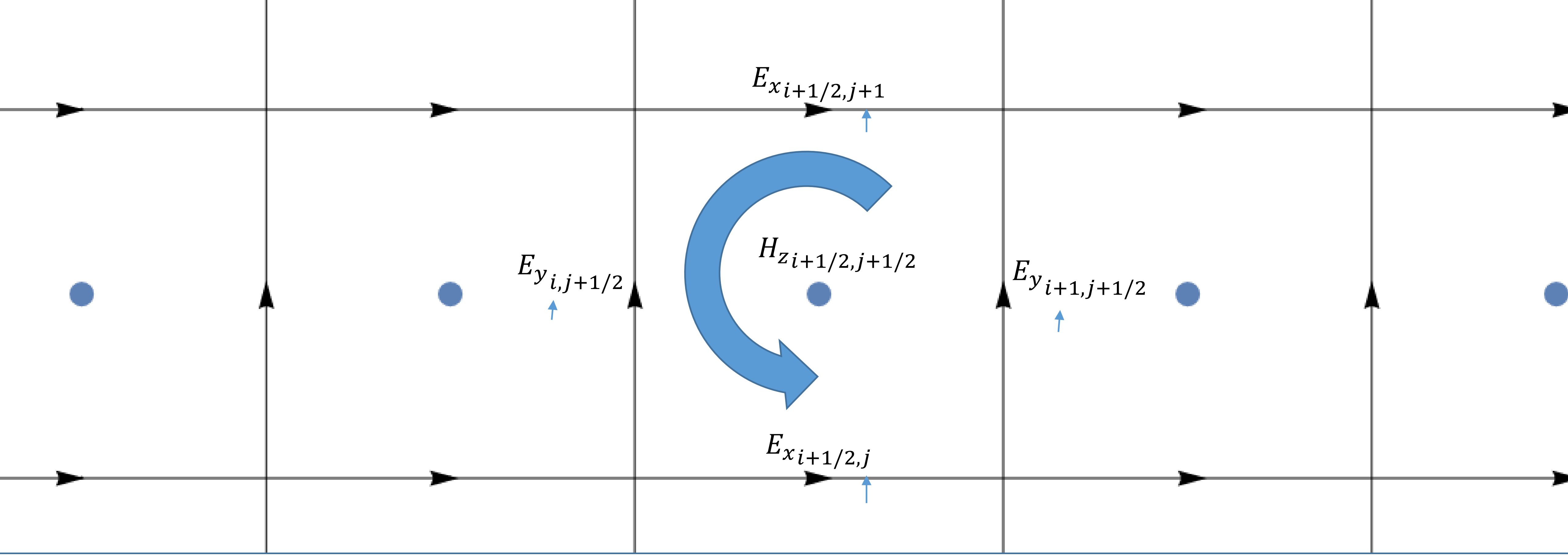
Źródło prądu

Mikroskopowe prawo Ohma

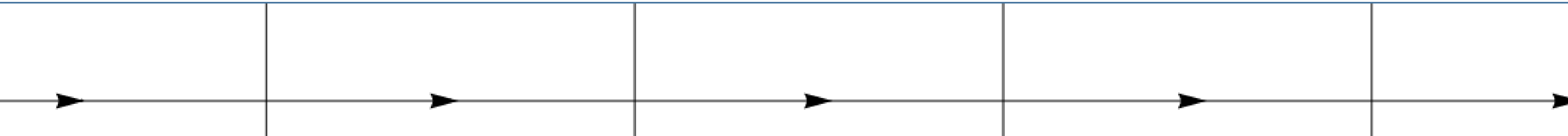
$\Updownarrow$

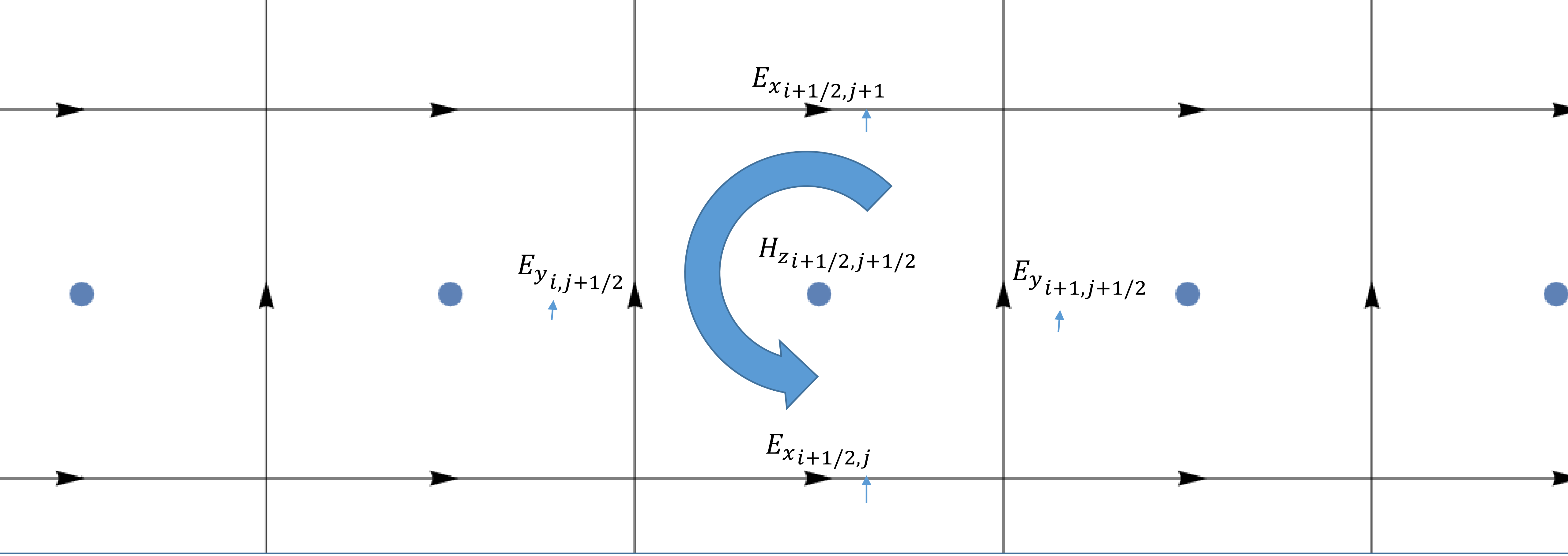
- $\frac{\partial}{\partial t} \int_S \boldsymbol{D} \cdot \boldsymbol{ds} = \int_C \boldsymbol{H} \cdot \boldsymbol{dl} - \boldsymbol{I}$
- $\frac{\partial}{\partial t} \int_S \boldsymbol{B} \cdot \boldsymbol{ds} = -\int_C \boldsymbol{E} \cdot \boldsymbol{dl}$

$$\boldsymbol{I} = \int_S \boldsymbol{J} \cdot \boldsymbol{ds}$$

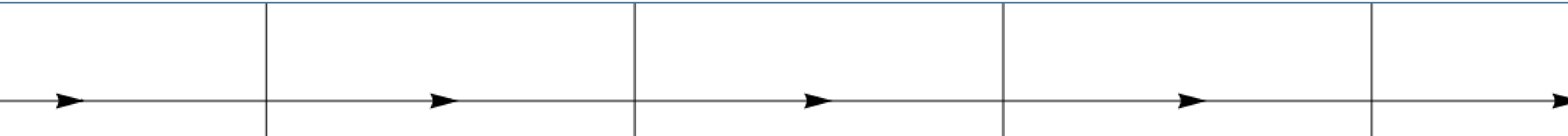


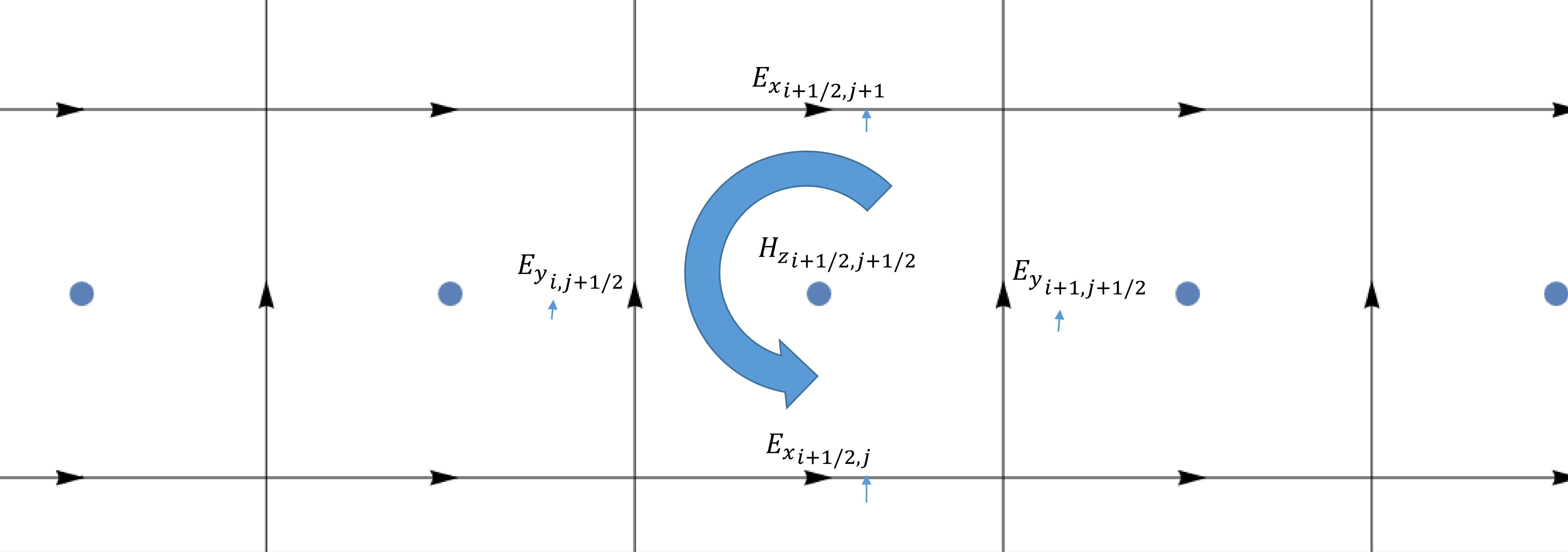
$$\frac{\partial}{\partial t}\int H_{z_{i+1/2,j+1/2}}^{n+1/2}\, ds = -\frac{1}{\mu}\int E_{i+1/2,j+1/2}^{n\,+1/2}\, dl$$



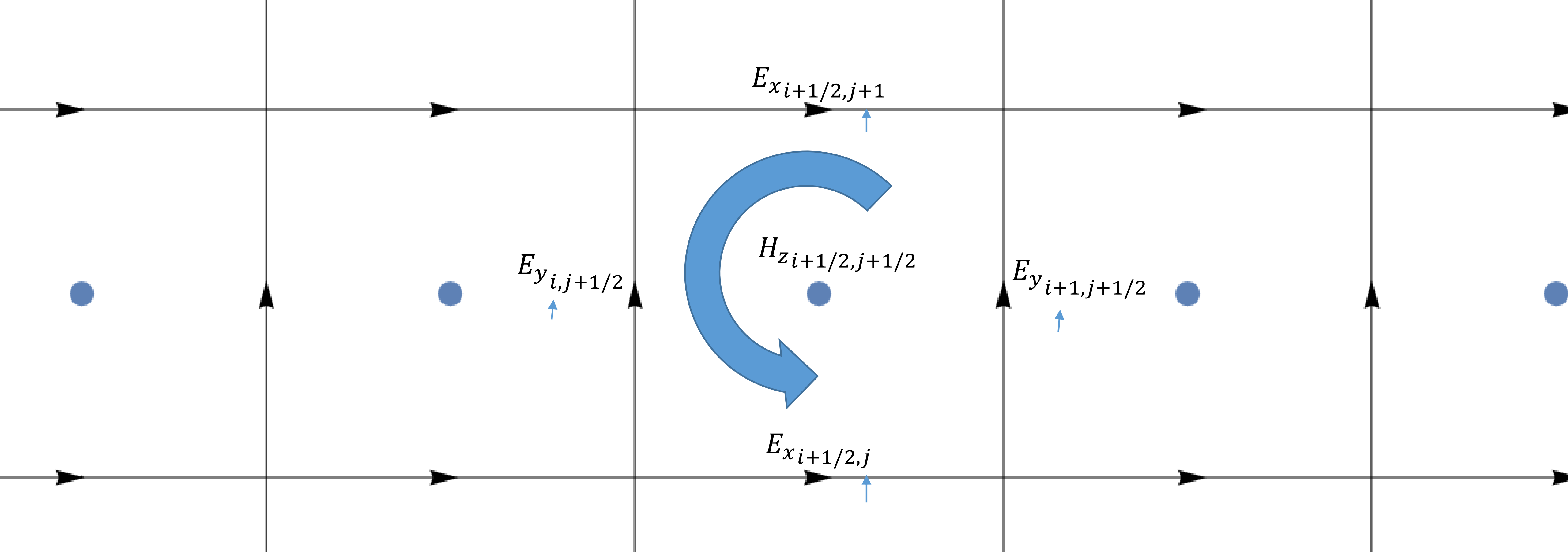


$$\frac{\partial}{\partial t}\Delta x \Delta y H_{z_{i+1/2,j+1/2}}^{n+1/2} = \frac{1}{\mu}\left(\Delta x\left(E_{x_{i+1/2,j+1}}^{n+1/2} - E_{x_{i+1/2,j}}^{n+1/2}\right) + \Delta y\left(E_{y_{i+1,j+1/2}}^{n+1/2} - E_{y_{i,j+1/2}}^{n+1/2}\right)\right)$$

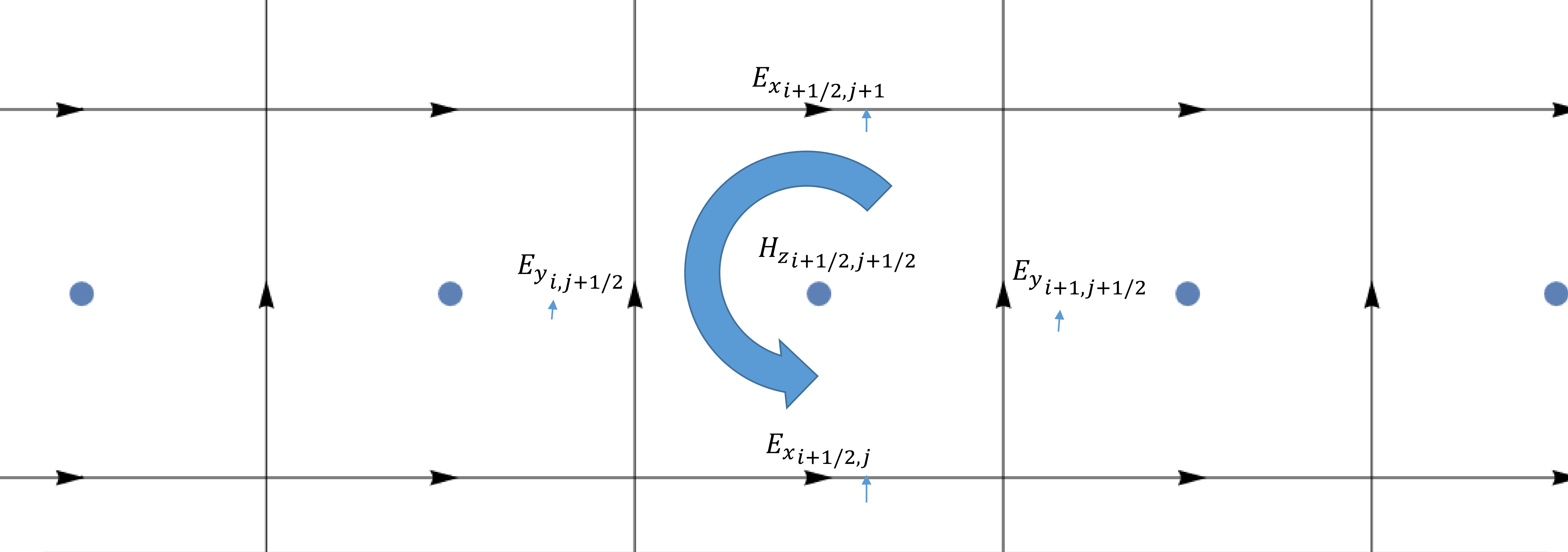




$$\Delta x \Delta y \frac{\partial}{\partial t} H_{z_{i+1/2,j+1/2}}^{n+1/2} = \frac{1}{\mu}\left(\Delta x \left(E_{x_{i+1/2,j+1}}^{n+1/2} - E_{x_{i+1/2,j}}^{n+1/2}\right) + \Delta y \left(E_{y_{i+1,j+1/2}}^{n+1/2} - E_{y_{i,j+1/2}}^{n+1/2}\right)\right)$$



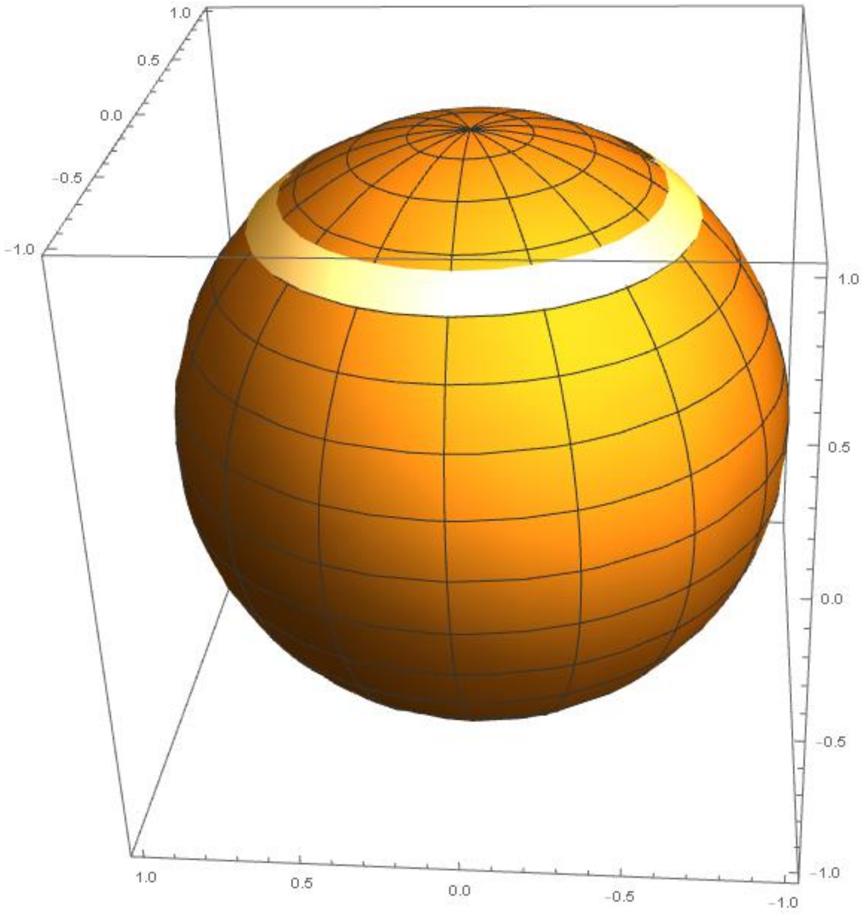
$$\Delta x \Delta y \frac{H_{z_{i+1/2,j+1/2}}^{n+1} - H_{z_{i+1/2,j+1/2}}^{n}}{\Delta t}$$
$$= \frac{1}{\mu}\left(\Delta x \left(E_{x_{i+1/2,j+1}}^{n+1/2} - E_{x_{i+1/2,j}}^{n+1/2}\right) + \Delta y \left(E_{y_{i+1,j+1/2}}^{n+1/2} - E_{y_{i,j+1/2}}^{n+1/2}\right)\right)$$

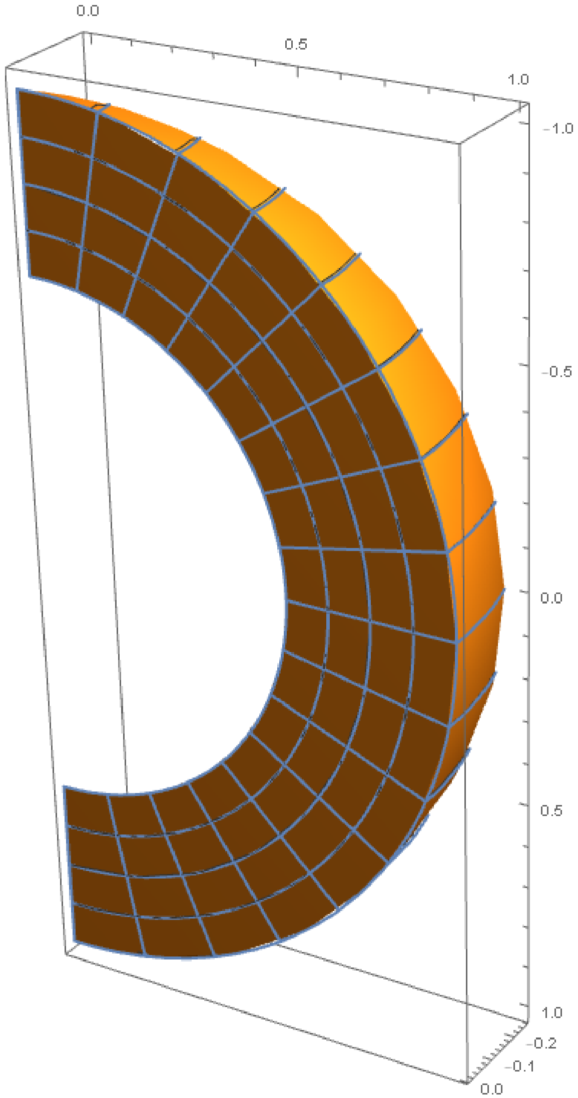


$$\frac{H_{z\,i+1/2,j+1/2}^{\,n+1} - H_{z\,i+1/2,j+1/2}^{\,n}}{\Delta t} = \frac{1}{\mu}\left(\frac{E_{x\,i+1/2,j+1}^{\,n+1/2} - E_{x\,i+1/2,j}^{\,n+1/2}}{\Delta y} + \frac{E_{y\,i+1,j+1/2}^{\,n+1/2} - E_{y\,i,j+1/2}^{\,n+1/2}}{\Delta x}\right)$$

Modelowanie wnęki sferycznej

Zakładamy symetrię ze względu na φ
=> brak gradientu ze względu na φ



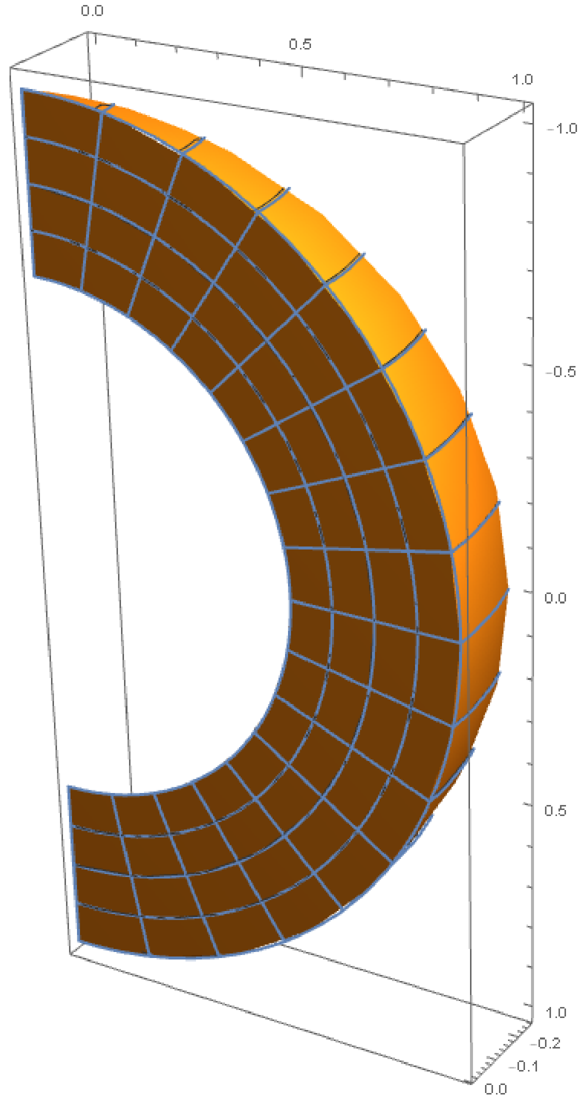


Modelowanie wnęki sferycznej

Zakładamy symetrię ze względu na φ
$\Rightarrow$ brak gradientu ze względu na φ

$$\varepsilon\frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta H_\varphi) - \frac{\partial H_\theta}{\partial\varphi}\right]$$
$$\varepsilon\frac{\partial E_\theta}{\partial t} = \frac{1}{r\sin\theta}\frac{\partial H_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi)$$
$$\varepsilon\frac{\partial E_\varphi}{\partial t} = \frac{1}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial H_r}{\partial\theta}\right]$$

$$\mu\frac{\partial H_r}{\partial t} = -\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\varphi) - \frac{\partial E_\theta}{\partial\varphi}\right]$$
$$\mu\frac{\partial H_\theta}{\partial t} = -\frac{1}{r\sin\theta}\frac{\partial E_r}{\partial\varphi} + \frac{1}{r}\frac{\partial}{\partial r}(rE_\varphi)$$
$$\mu\frac{\partial H_\varphi}{\partial t} = -\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial\theta}\right]$$



Modelowanie wnęki sferycznej

Zakładamy symetrię ze względu na φ
$\Rightarrow$ brak gradientu ze względu na φ

$$\varepsilon\frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta H_\varphi) - \cancel{\frac{\partial H_\theta}{\partial\varphi}}\right]$$

$$\varepsilon\frac{\partial E_\theta}{\partial t} = \cancel{\frac{1}{r\sin\theta}\frac{\partial H_r}{\partial\varphi}} - \frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi)$$

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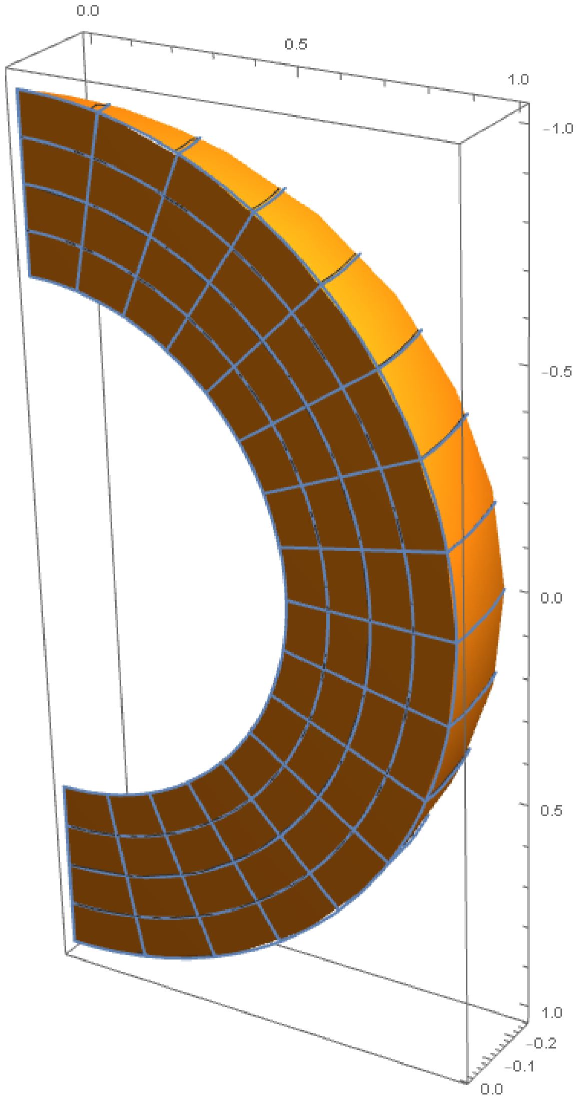
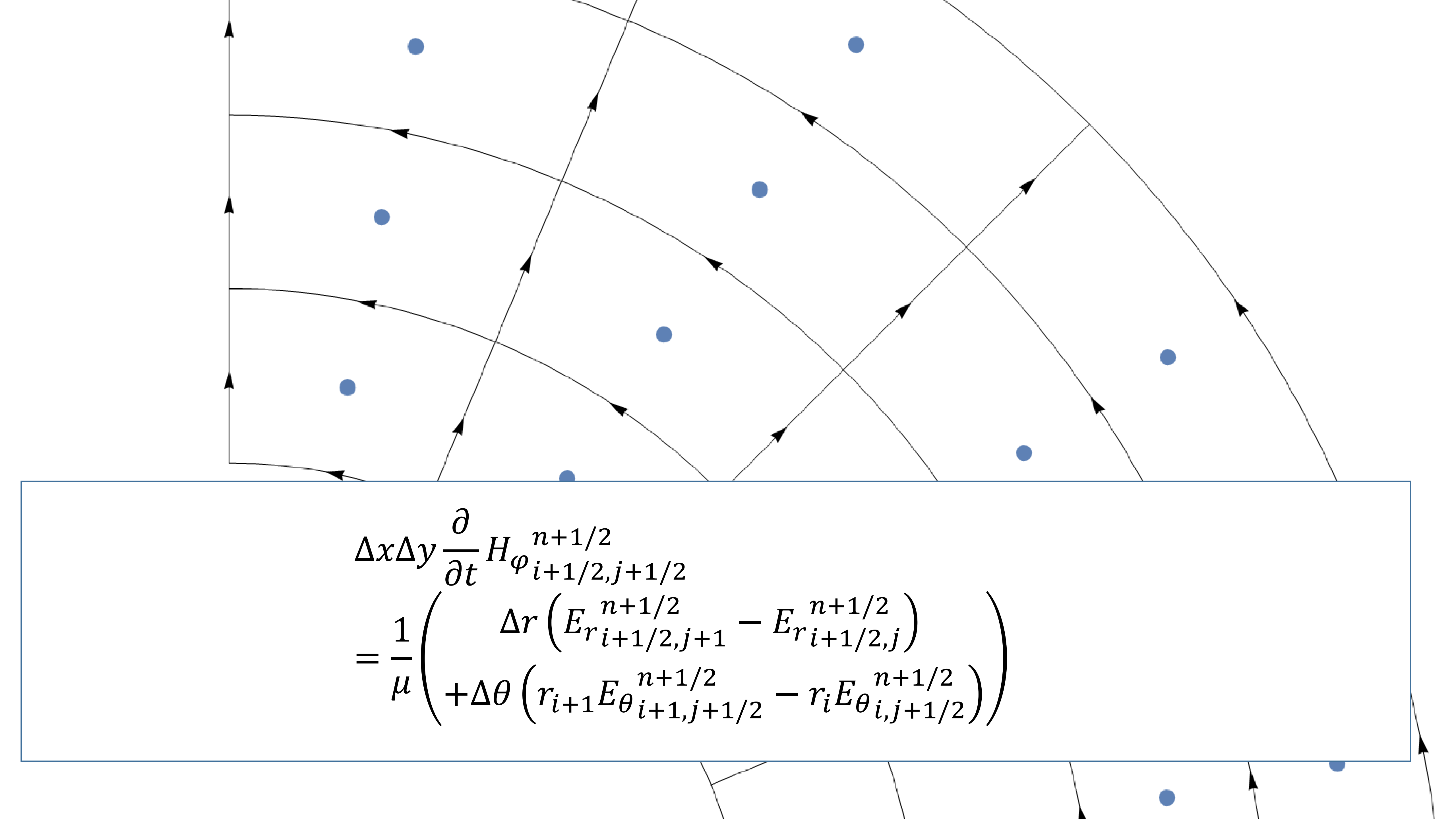
$$\mu\frac{\partial H_r}{\partial t} = -\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\varphi) - \cancel{\frac{\partial E_\theta}{\partial\varphi}}\right]$$

$$\mu\frac{\partial H_\theta}{\partial t} = \cancel{-\frac{1}{r\sin\theta}\frac{\partial E_r}{\partial\varphi}} + \frac{1}{r}\frac{\partial}{\partial r}(rE_\varphi)$$

$$\mu\frac{\partial H_\varphi}{\partial t} = -\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial\theta}\right]$$





$$\Delta x \Delta y \frac{\partial}{\partial t} H_{\varphi}{}_{i+1/2,j+1/2}^{n+1/2} = \frac{1}{\mu}\left(\begin{array}{c} \Delta r\left(E_{r}{}_{i+1/2,j+1}^{n+1/2} - E_{r}{}_{i+1/2,j}^{n+1/2}\right) \\ +\Delta\theta\left(r_{i+1} E_{\theta}{}_{i+1,j+1/2}^{n+1/2} - r_i E_{\theta}{}_{i,j+1/2}^{n+1/2}\right) \end{array}\right)$$