Krótkie wprowadzenie do metody FDTD

Równania Maxwella

- $\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$
- $\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$

$$\boldsymbol{J} = \boldsymbol{J}_{source} + \sigma \boldsymbol{E}$$

Mikroskopowe
prawo Ohma

Źródło prądu

Schemat numeryczny

Przykład – cienki przewód (1D)

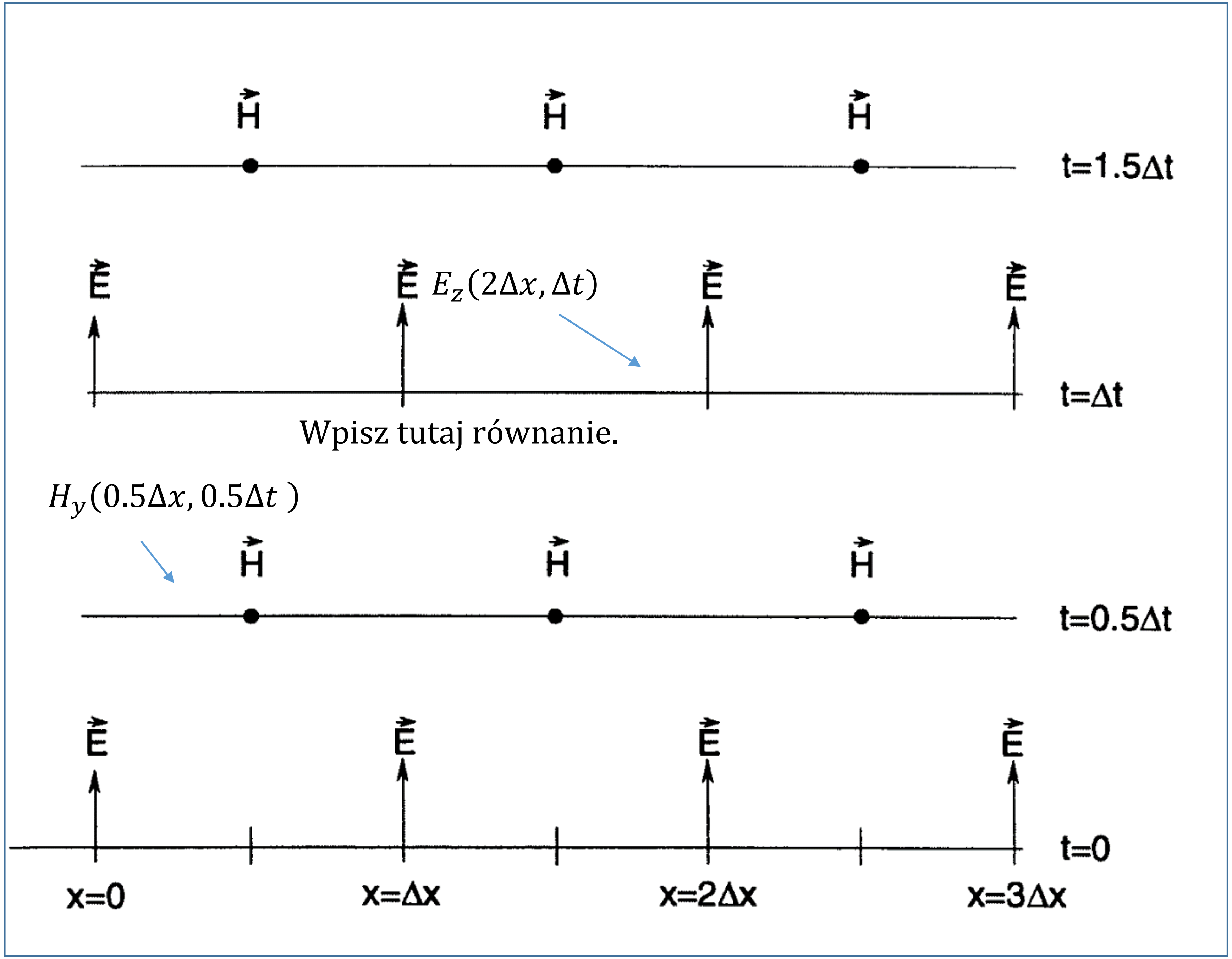
- $\boldsymbol{E} = (0, 0, E_z)$
- $\boldsymbol{H} = (0, H_y, 0)$
- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - J_z - \sigma E_z\right)$
- $\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x}$

$\Rightarrow$ **Schemat numeryczny**

- Dla prostoty $\mu, \varepsilon = const$

$$H_y(x,t) = H_y(i\Delta x, n\Delta t) \rightarrow {H_y}_i^n$$

$$E_z(x,t\ ) = E_z(i\Delta x, n\Delta t) \rightarrow {E_z}_i^n$$



Pochodna centralna

$$f(x+dx) = f(x) + f'(x)dx + f''(x)\frac{dx^2}{2} + f'''(x)\frac{dx^3}{6} + O(dx^4)$$
$$f(x-dx) = f(x) - f'(x)dx + f''(x)\frac{dx^2}{2} - f'''(x)\frac{dx^3}{6} + O(dx^4)$$

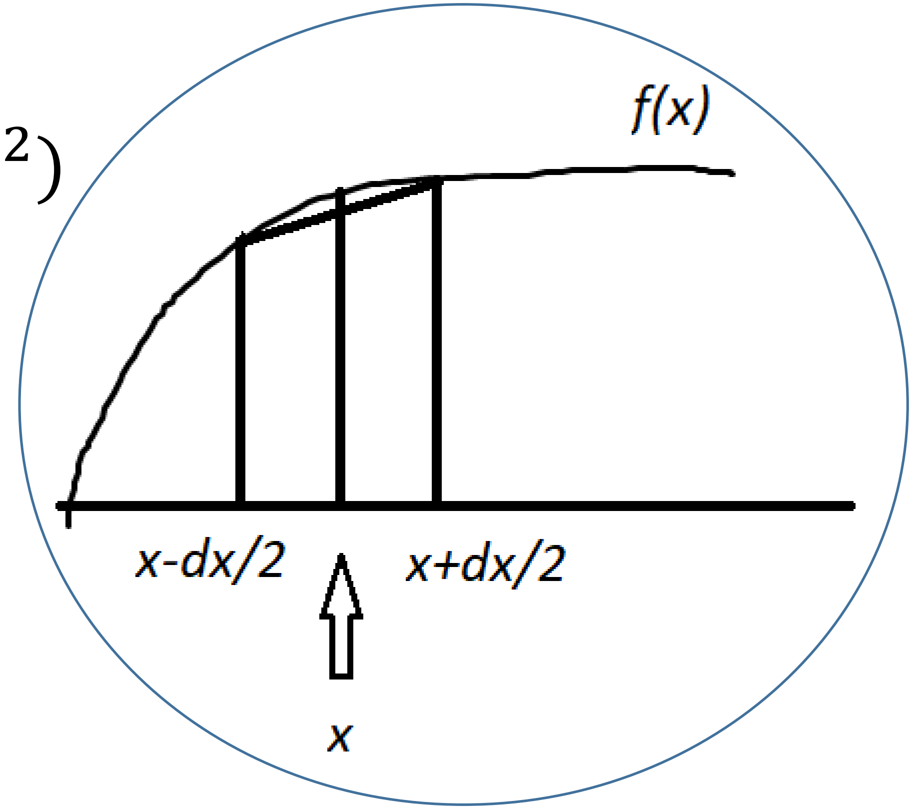
$$f(x+dx) - f(x-dx) = 2f'(x)dx + f'''(x)\frac{dx^3}{3} + O(dx^4)$$
$$\frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = f'(x) + f'''(x)\frac{\Delta x^2}{6} + O(\Delta x^3)$$

$$\boxed{f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O(\Delta x^2)}$$

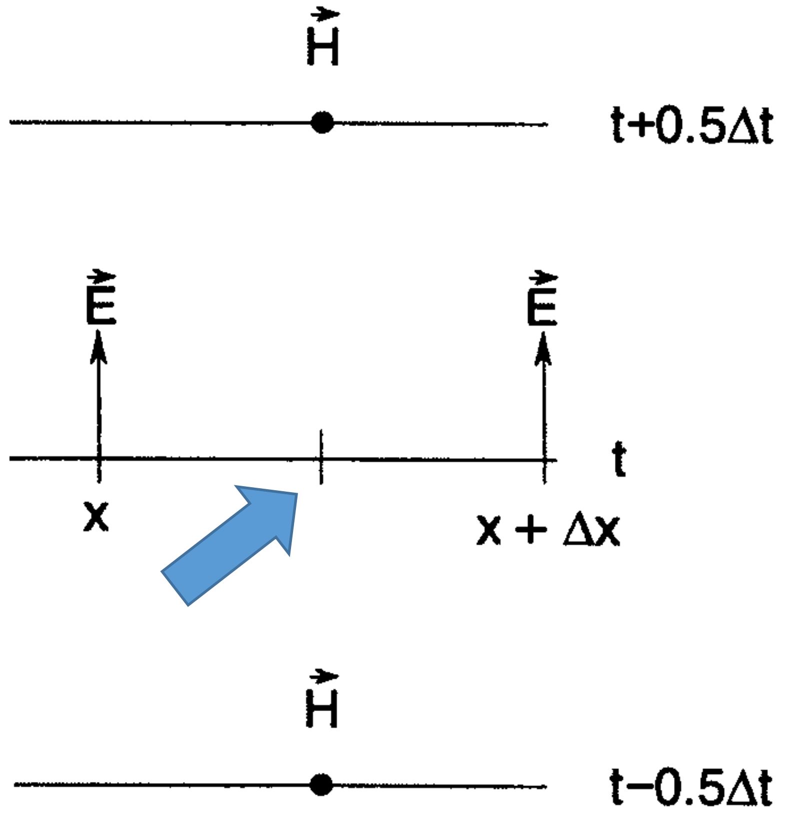
Pochodna centralna

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O(\Delta x^2)$$

$$f'(x) = \frac{f(x+\Delta x/2) - f(x-\Delta x/2)}{\Delta x} + O(\Delta x^2)$$

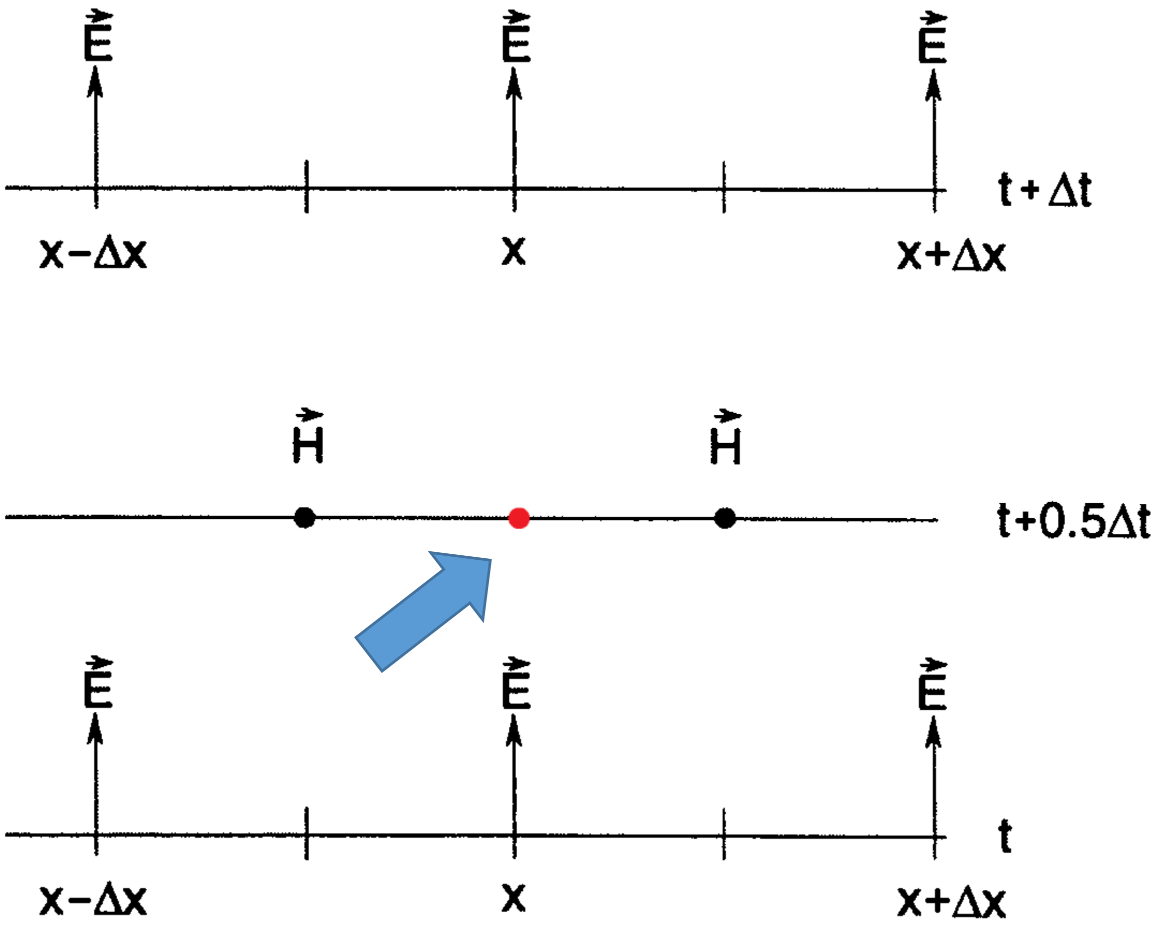


- $\dfrac{\partial H_y}{\partial t} = \dfrac{1}{\mu}\dfrac{\partial E_z}{\partial x}$
- $\dfrac{{H_y}_{i+1/2}^{n+1/2} - {H_y}_{i+1/2}^{n-1/2}}{\Delta t} = \dfrac{1}{\mu}\left(\dfrac{{E_z}_{i+1}^{n} - {E_z}_{i}^{n}}{\Delta x}\right)$
- ${H_y}_{i+1/2}^{n+1/2} = {H_y}_{i+1/2}^{n-1/2} + \dfrac{\Delta t}{\mu \Delta x}\left({E_z}_{i+1}^{n} - {E_z}_{i}^{n}\right)$



- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - J_z - \sigma E_z\right)$
- $\frac{{E_z}_i^{n+1} - {E_z}_i^{n}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{{H_y}_{i+1/2}^{n+1/2} - {H_y}_{i-1/2}^{n+1/2}}{\Delta x} - J_i - \sigma {E_z}_i^{n+1/2}\right)$

$$ {E_z}_i^{n+1/2} = \frac{{E_z}_i^{n+1} + {E_z}_i^{n}}{2} $$



- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - J_x - \sigma E_z\right)$
- $\frac{{E_z}_i^{n+1} - {E_z}_i^{n}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{{H_y}_{i+1/2}^{n+1/2} - {H_y}_{i-1/2}^{n+1/2}}{\Delta x} - J_i^{n+1/2} - \sigma_i {E_z}_i^{n+1/2}\right)$

- ${E_z}_i^{n+1} = \frac{\varepsilon - \sigma_i \Delta t/2}{\varepsilon + \sigma_i \Delta t/2} {E_z}_i^{n} + \frac{\Delta t}{\varepsilon + \sigma_i \Delta t/2}\left(\frac{{H_y}_{i+1/2}^{n+1/2} - {H_y}_{i-1/2}^{n+1/2}}{\Delta x} - J_i^{n+1/2}\right)$
- ${H_y}_{i+1/2}^{n+1/2} = {H_y}_{i+1/2}^{n-1/2} + \frac{\Delta t}{\mu \Delta x}\left({E_z}_{i+1}^{n} - {E_z}_i^{n}\right)$

Przykład programu