

# Krótkie wprowadzenie do metody FDTD

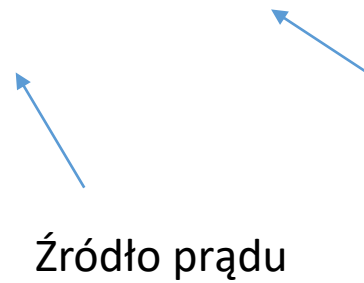
# Równania Maxwella

- $\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}$
- $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$



**Schemat numeryczny**

$$\mathbf{J} = \mathbf{J}_{source} + \sigma \mathbf{E}$$



Źródło prądu

Mikroskopowe  
prawo Ohma

# Przykład – cienki przewód (1D)

- $\mathbf{E} = (0, 0, E_z)$
- $\mathbf{H} = (0, H_y, 0)$
- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} - J_z - \sigma E_z \right)$
- $\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x}$



**Schemat numeryczny**

- Dla prostoty  $\mu, \varepsilon = \text{const}$

# Pochodna centralna

$$\begin{aligned}f(x + dx) &= f(x) + f'(x)dx + f''(x)\frac{dx^2}{2} + f'''(x)\frac{dx^3}{6} + O(dx^4) \\f(x - dx) &= f(x) - f'(x)dx + f''(x)\frac{dx^2}{2} - f'''(x)\frac{dx^3}{6} + O(dx^4)\end{aligned}$$

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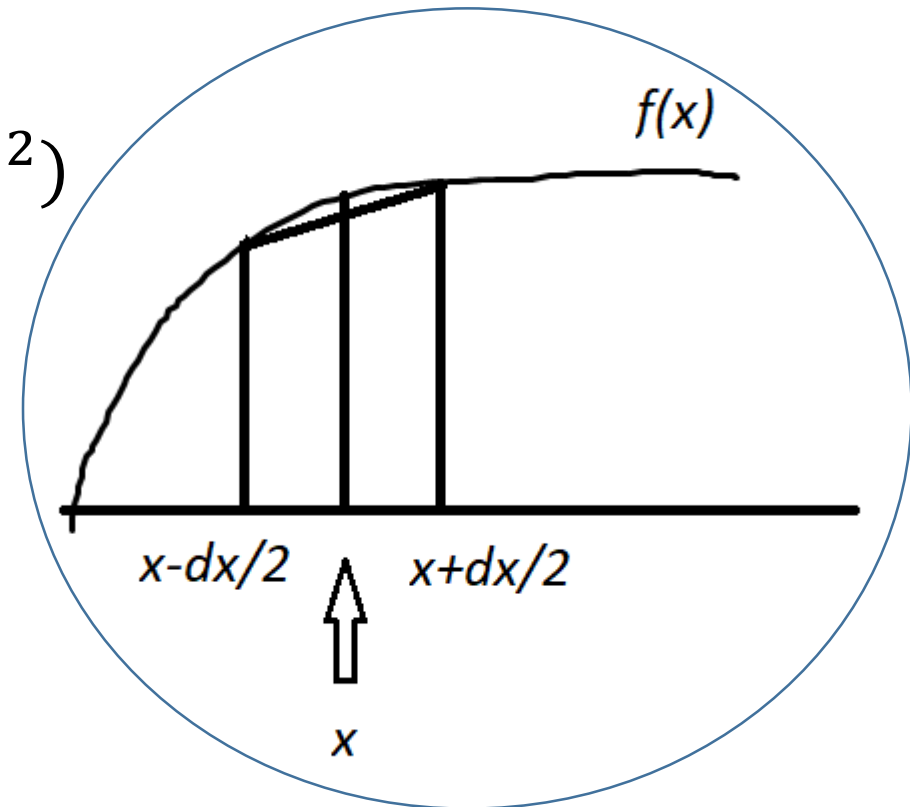
$$\begin{aligned}f(x + dx) - f(x - dx) &= 2f'(x)dx + f'''(x)\frac{dx^3}{3} + O(dx^4) \\ \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} &= f'(x) + f'''(x)\frac{\Delta x^2}{6} + O(\Delta x^3)\end{aligned}$$

$$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} + O(\Delta x^2)$$

# Pochodna centralna

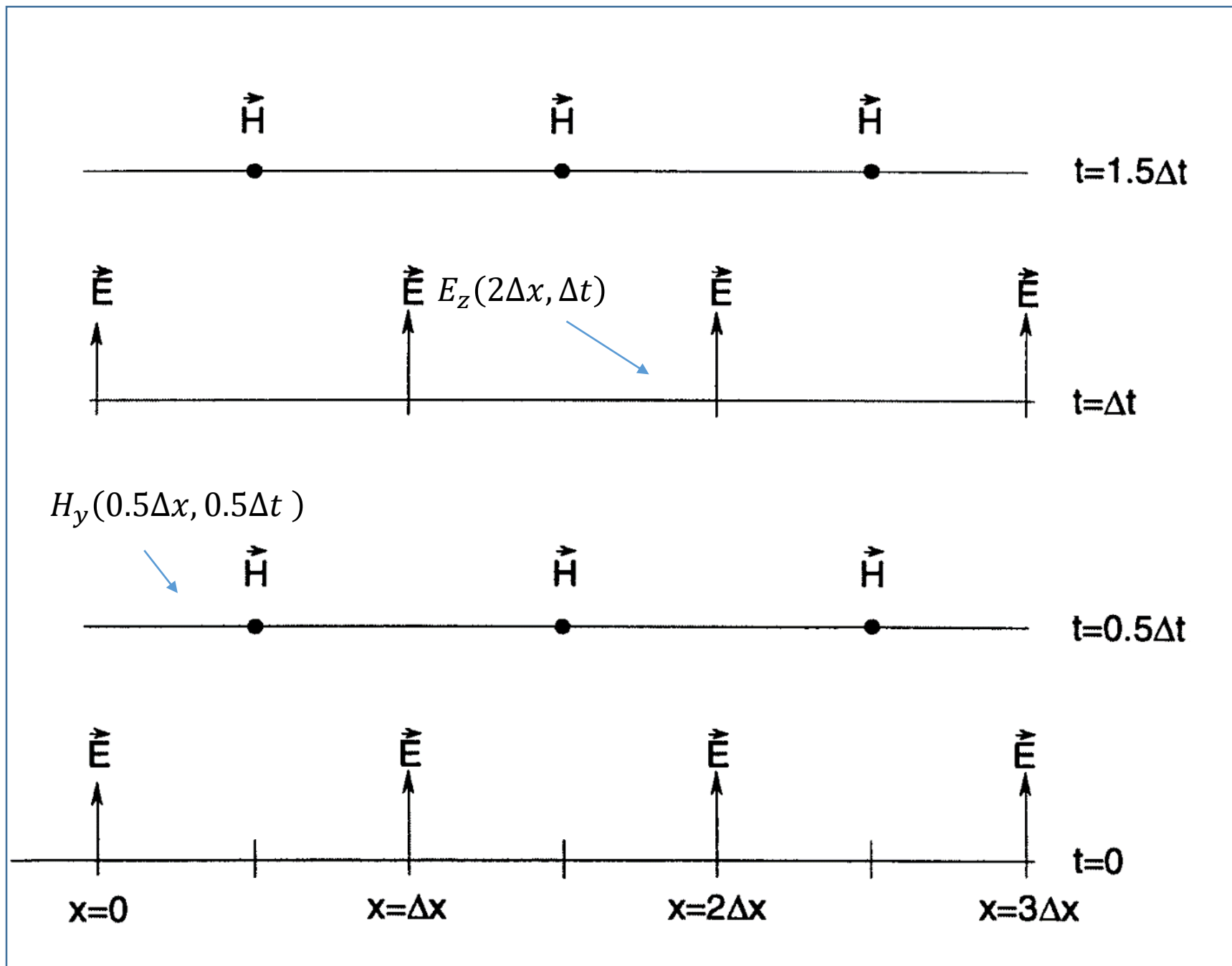
$$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} + O(\Delta x^2)$$

$$f'(x) = \frac{f(x + \Delta x/2) - f(x - \Delta x/2)}{\Delta x} + O(\Delta x^2)$$



$$H_y(x, t) = H_y(i\Delta x, n\Delta t) \rightarrow H_{y_i}^n$$

$$E_z(x, t) = E_z(i\Delta x, n\Delta t) \rightarrow E_{z_i}^n$$

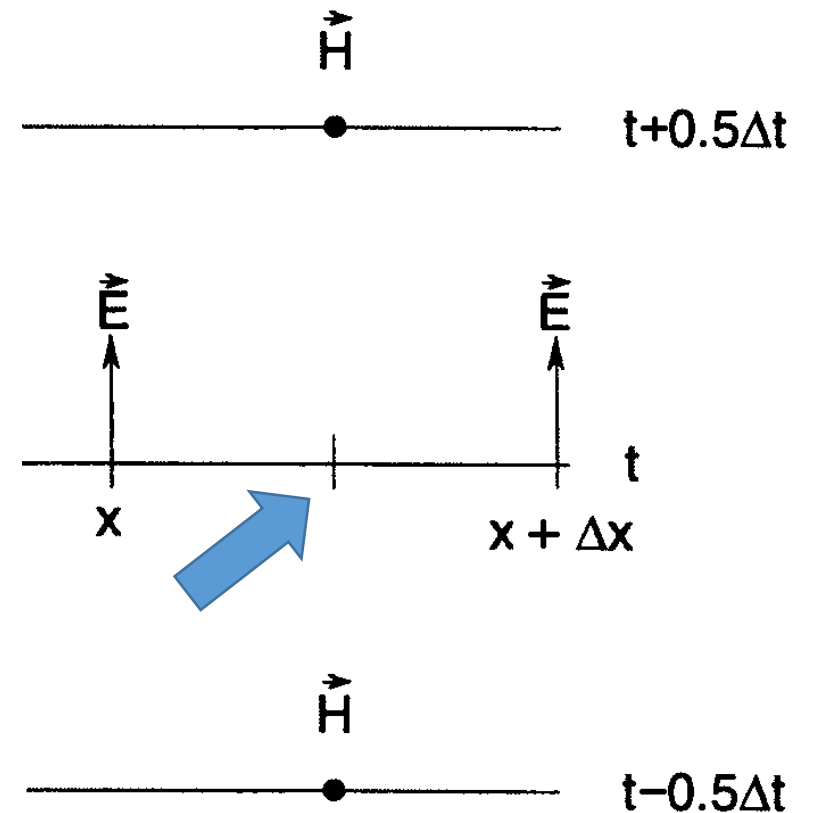


źr: Allen Taflove, Susan C. Hagness,  
*Computational electrodynamics –  
 FDTD method*

- $\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x}$

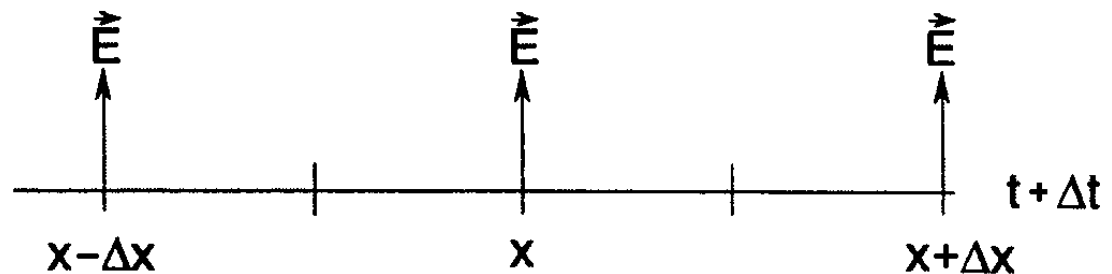
- $\frac{H_{y_{i+1/2}}^{n+1/2} - H_{y_{i+1/2}}^{n-1/2}}{\Delta t} = \frac{1}{\mu} \left( \frac{E_{z_{i+1}}^n - E_{z_i}^n}{\Delta x} \right)$

- $H_{y_{i+1/2}}^{n+1/2} = H_{y_{i+1/2}}^{n-1/2} + \frac{\Delta t}{\mu \Delta x} (E_{z_{i+1}}^n - E_{z_i}^n)$



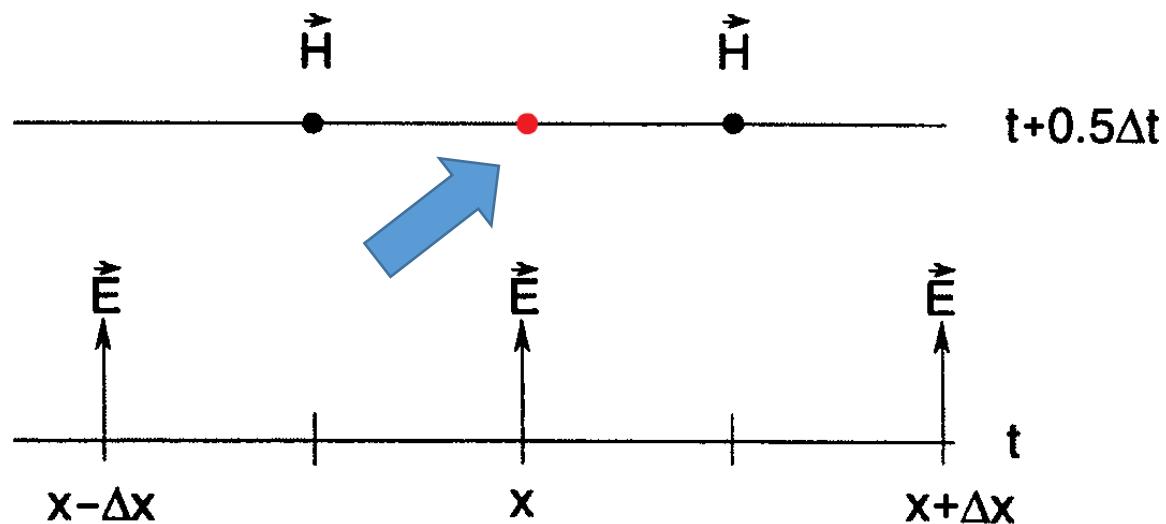
- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} - J_z - \sigma E_z \right)$

- $\frac{E_{zi}^{n+1} - E_{zi}^n}{\Delta t} = \frac{1}{\varepsilon} \left( \frac{H_{y i+1/2}^{n+1/2} - H_{y i-1/2}^{n+1/2}}{\Delta x} - J_i - \sigma E_{zi}^{n+1/2} \right)$



A blue arrow points from the term  $E_{zi}^{n+1/2}$  in the equation above to the following expression:

$$E_{zi}^{n+1/2} = \frac{E_{zi}^{n+1} + E_{zi}^n}{2}$$





- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} - J_x - \sigma E_z \right)$
- $\frac{E_{zi}^{n+1} - E_{zi}^n}{\Delta t} = \frac{1}{\varepsilon} \left( \frac{H_{y_{i+1/2}}^{n+1/2} - H_{y_{i-1/2}}^{n+1/2}}{\Delta x} - J_i^{n+1/2} - \sigma_i E_{zi}^{n+1/2} \right)$

- $E_{zi}^{n+1} = \frac{\varepsilon - \sigma_i \Delta t / 2}{\varepsilon + \sigma_i \Delta t / 2} E_{zi}^n + \frac{\Delta t}{\varepsilon + \sigma_i \Delta t / 2} \left( \frac{H_{y_{i+1/2}}^{n+1/2} - H_{y_{i-1/2}}^{n+1/2}}{\Delta x} - J_i^{n+1/2} \right)$
- $H_{y_{i+1/2}}^{n+1/2} = H_{y_{i+1/2}}^{n-1/2} + \frac{\Delta t}{\mu \Delta x} (E_{z_{i+1}}^n - E_{zi}^n)$

# Przykład programu