Krótkie wprowadzenie do metody FDTD (cz. 2(2D))

Równania Maxwella

- $\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon}(\nabla \times \boldsymbol{H} - \boldsymbol{J})$
- $\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu}\nabla \times \boldsymbol{E}$

$$\boldsymbol{J} = \boldsymbol{J}_{source} + \sigma\boldsymbol{E}$$

Mikroskopowe prawo Ohma

Źródło prądu

- $\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_x - \sigma E_x\right)$
- $\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} - J_y - \sigma E_y\right)$
- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z - \sigma E_z\right)$
- $\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right)$
- $\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)$
- $\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$

Równania Maxwella – ograniczenie do 2D

Zakładamy niezmienniczość ze względu na translację wzdłuż osi z (pole EM nie zmienia się w kierunku z)

- $\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \cancel{\frac{\partial H_y}{\partial z}} - J_x - \sigma E_x\right)$
- $\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - \cancel{\frac{\partial H_x}{\partial z}} - J_y - \sigma E_y\right)$
- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z - \sigma E_z\right)$

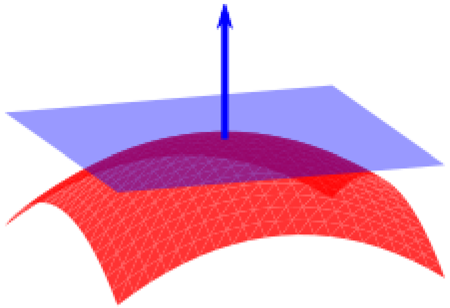
- $\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\cancel{\frac{\partial E_y}{\partial z}} - \frac{\partial E_z}{\partial y}\right)$
- $\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\cancel{\frac{\partial E_x}{\partial z}} - \frac{\partial E_z}{\partial x}\right)$
- $\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$

Równania Maxwella – ograniczenie do 2D

Zakładamy niezmienniczość ze względu na translację wzdłuż osi z

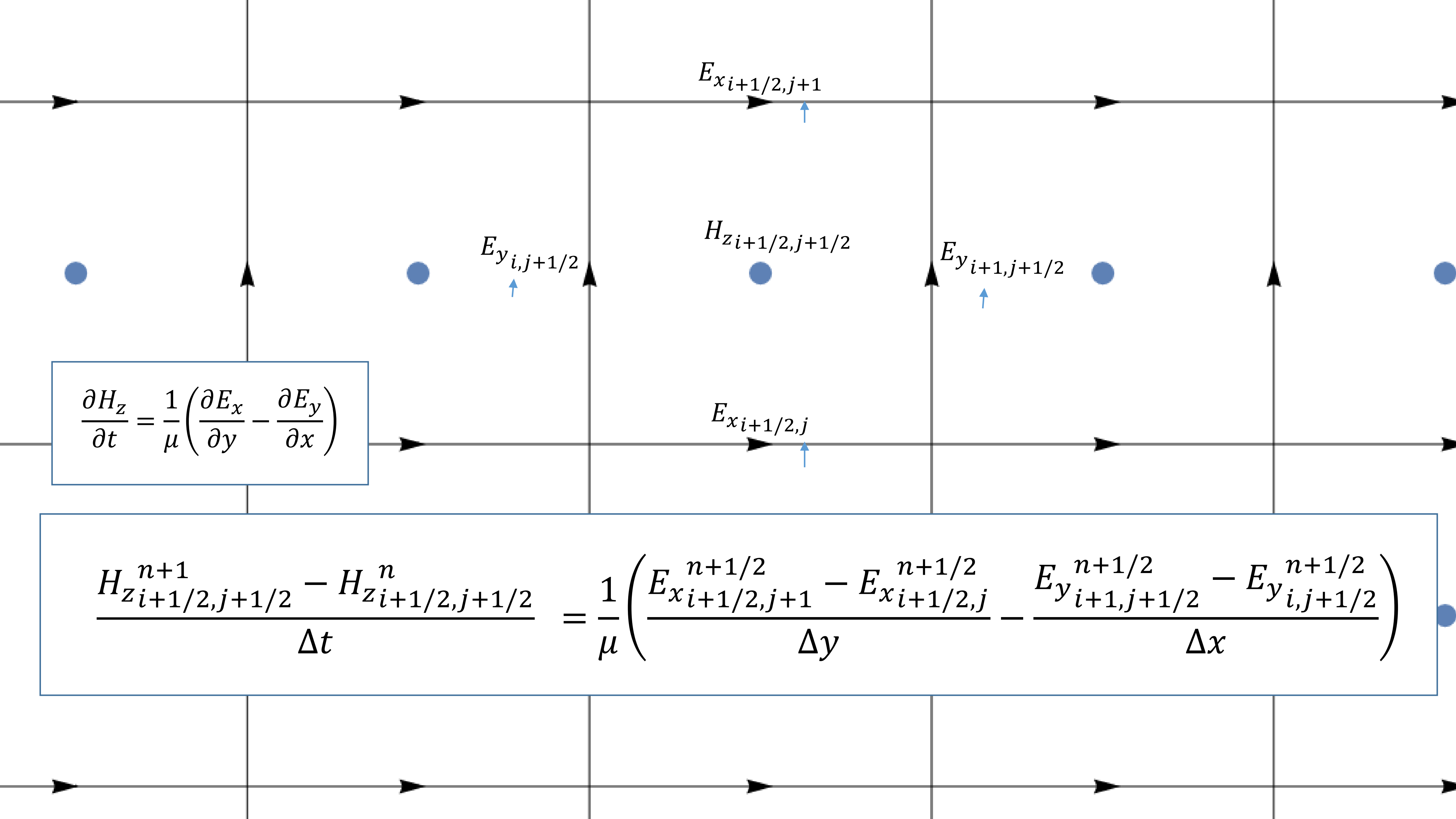
Odpowiednie grupowania daje dwa układy równań całkowicie **niezależne** i opisujące różne zjawiska.
Na przykład weźmy mały fragment powierzchni Ziemi:

- założymy niezmienne pole EM wraz z wysokością nad powierzchnią Ziemi
- używamy jednego z układów równań – niebieskiego lub zielonego
- jeśli założymy prąd płynący wzdłuż osi z to używamy zielonego



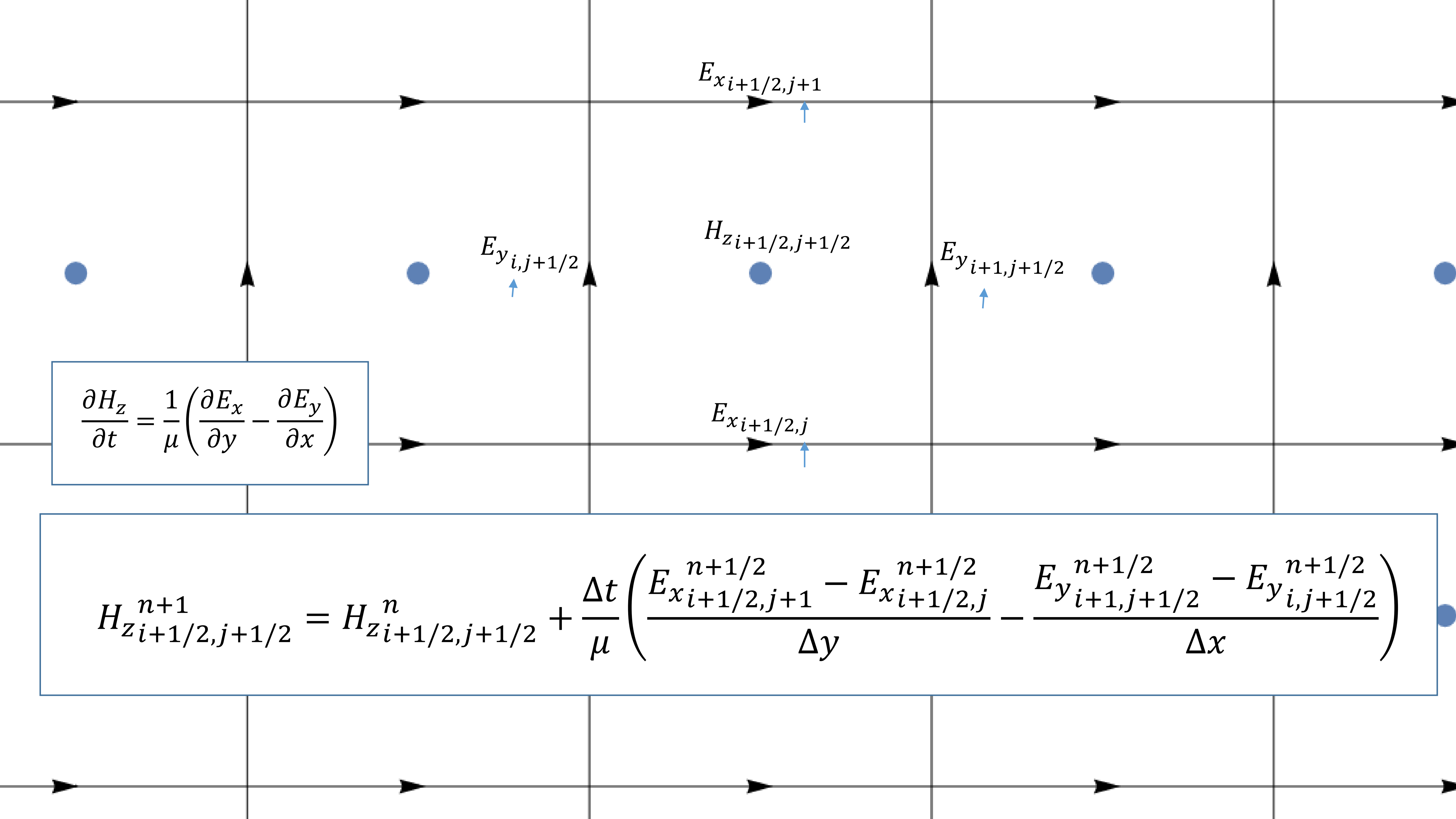
- $\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \cancel{\frac{\partial H_y}{\partial z}} - J_x - \sigma E_x\right)$
- $\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - \cancel{\frac{\partial H_x}{\partial z}} - J_y - \sigma E_y\right)$
- $\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z - \sigma E_z\right)$

- $\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\cancel{\frac{\partial E_y}{\partial z}} - \frac{\partial E_z}{\partial y}\right)$
- $\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\cancel{\frac{\partial E_x}{\partial z}} - \frac{\partial E_z}{\partial x}\right)$
- $\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$



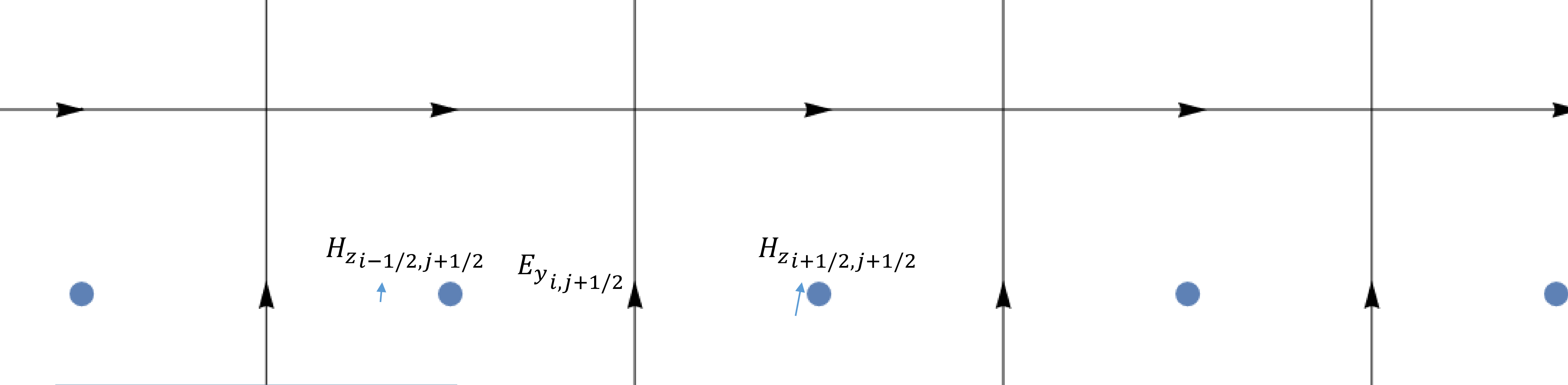
$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

$$\frac{{H_z}^{n+1}_{i+1/2,j+1/2} - {H_z}^{n}_{i+1/2,j+1/2}}{\Delta t} = \frac{1}{\mu}\left(\frac{{E_x}^{n+1/2}_{i+1/2,j+1} - {E_x}^{n+1/2}_{i+1/2,j}}{\Delta y} - \frac{{E_y}^{n+1/2}_{i+1,j+1/2} - {E_y}^{n+1/2}_{i,j+1/2}}{\Delta x}\right)$$



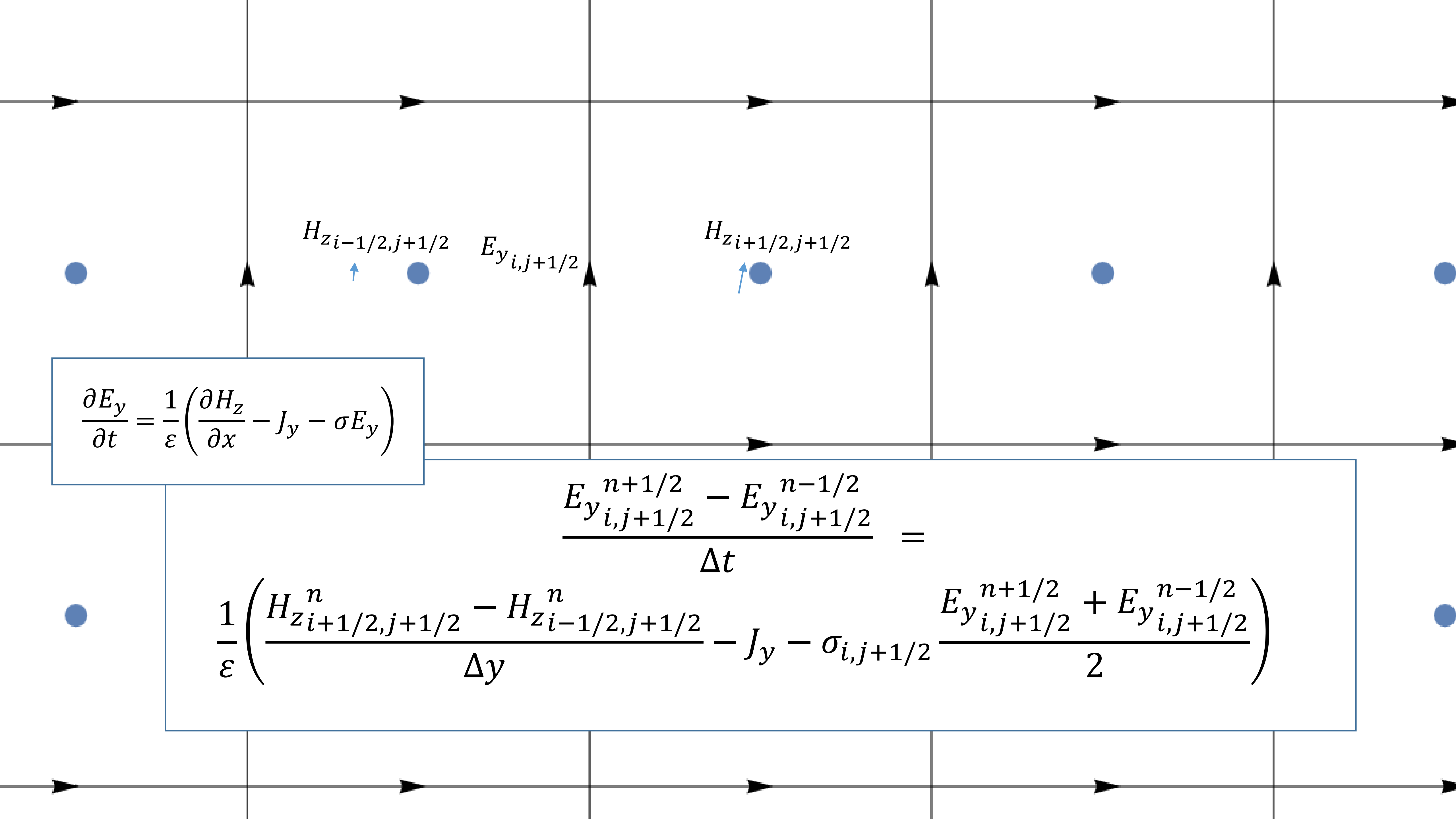
$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right)$$

$$H_{z_{i+1/2,j+1/2}}^{\;n+1} = H_{z_{i+1/2,j+1/2}}^{\;n} + \frac{\Delta t}{\mu}\left(\frac{E_{x_{i+1/2,j+1}}^{\;n+1/2} - E_{x_{i+1/2,j}}^{\;n+1/2}}{\Delta y} - \frac{E_{y_{i+1,j+1/2}}^{\;n+1/2} - E_{y_{i,j+1/2}}^{\;n+1/2}}{\Delta x}\right)$$



$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - J_y - \sigma E_y\right)$$

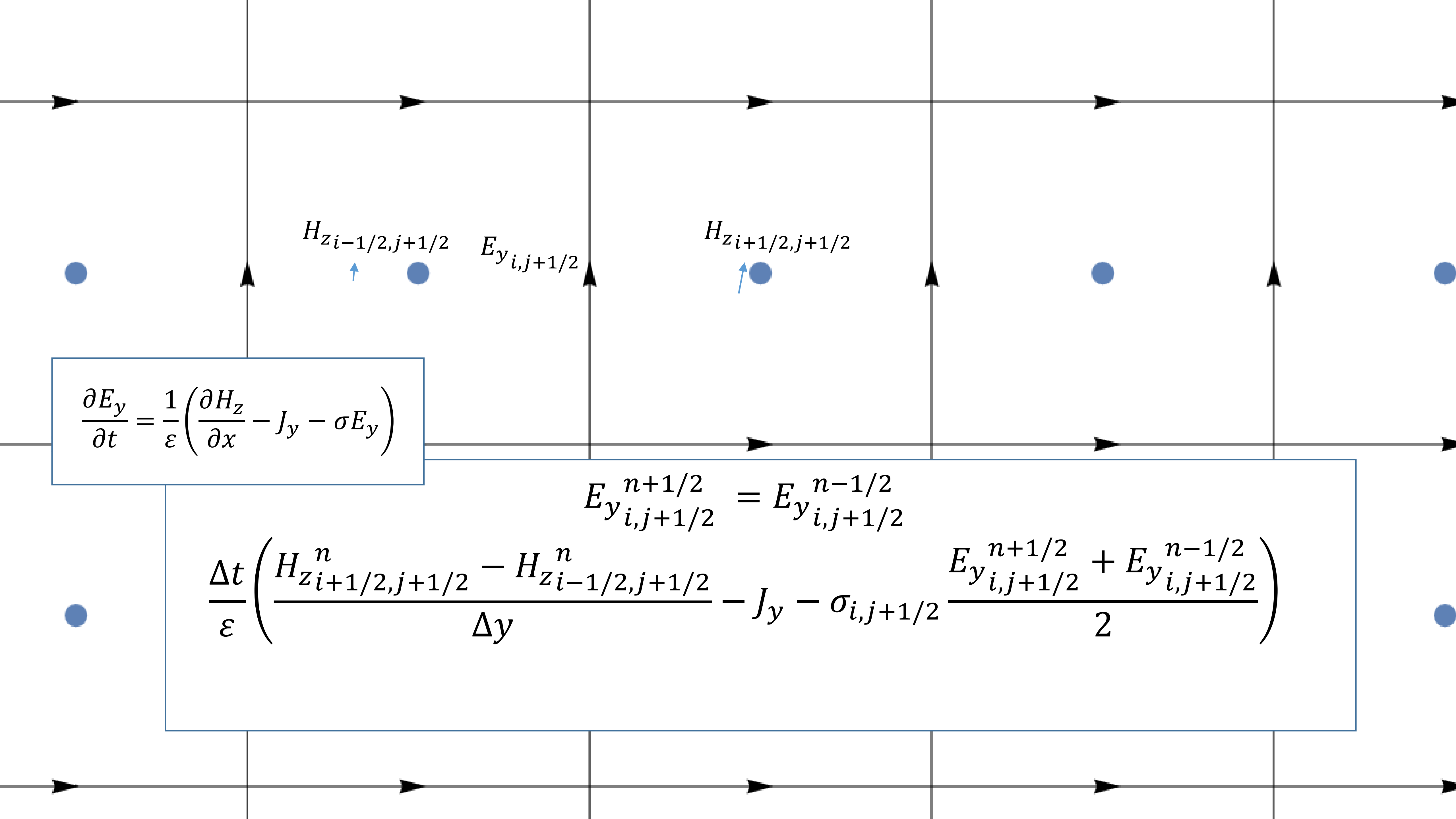
$$\frac{E_y{}^{n+1/2}_{i,j+1/2} - E_y{}^{n-1/2}_{i,j+1/2}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{H_z{}^{n}_{i+1/2,j+1/2} - H_z{}^{n}_{i-1/2,j+1/2}}{\Delta y} - J_y - \sigma_{i,j+1/2} E_y{}^{n}_{i,j+1/2}\right)$$



$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - J_y - \sigma E_y\right)$$

$$\frac{E_{y\,i,j+1/2}^{\,n+1/2} - E_{y\,i,j+1/2}^{\,n-1/2}}{\Delta t} =$$

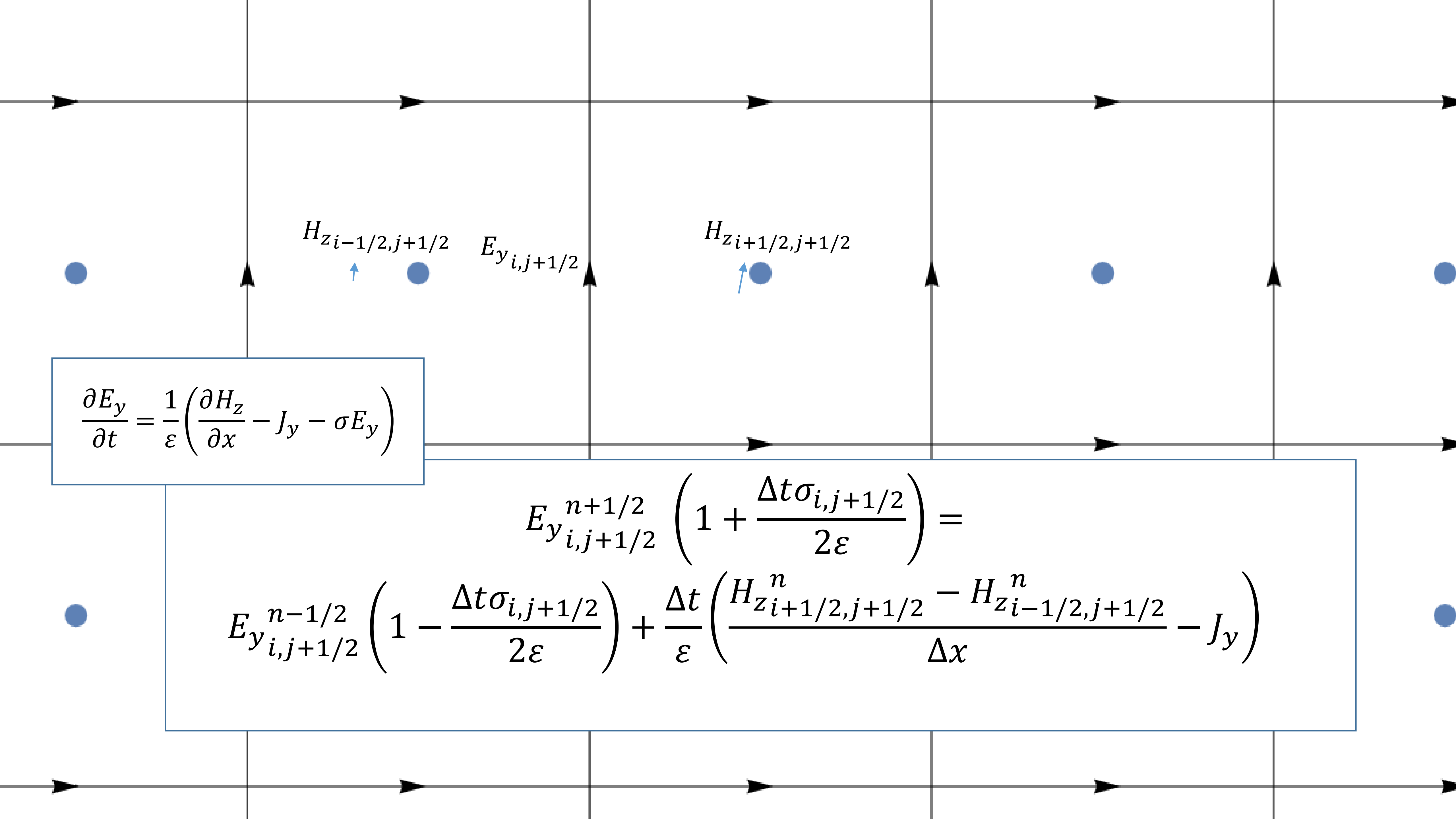
$$\frac{1}{\varepsilon}\left(\frac{H_{z\,i+1/2,j+1/2}^{\,n} - H_{z\,i-1/2,j+1/2}^{\,n}}{\Delta y} - J_y - \sigma_{i,j+1/2}\frac{E_{y\,i,j+1/2}^{\,n+1/2} + E_{y\,i,j+1/2}^{\,n-1/2}}{2}\right)$$



$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - J_y - \sigma E_y\right)$$

$$E_{y\,i,j+1/2}^{\,n+1/2} = E_{y\,i,j+1/2}^{\,n-1/2}$$

$$\frac{\Delta t}{\varepsilon}\left(\frac{H_{z\,i+1/2,j+1/2}^{\,n} - H_{z\,i-1/2,j+1/2}^{\,n}}{\Delta y} - J_y - \sigma_{i,j+1/2}\frac{E_{y\,i,j+1/2}^{\,n+1/2} + E_{y\,i,j+1/2}^{\,n-1/2}}{2}\right)$$



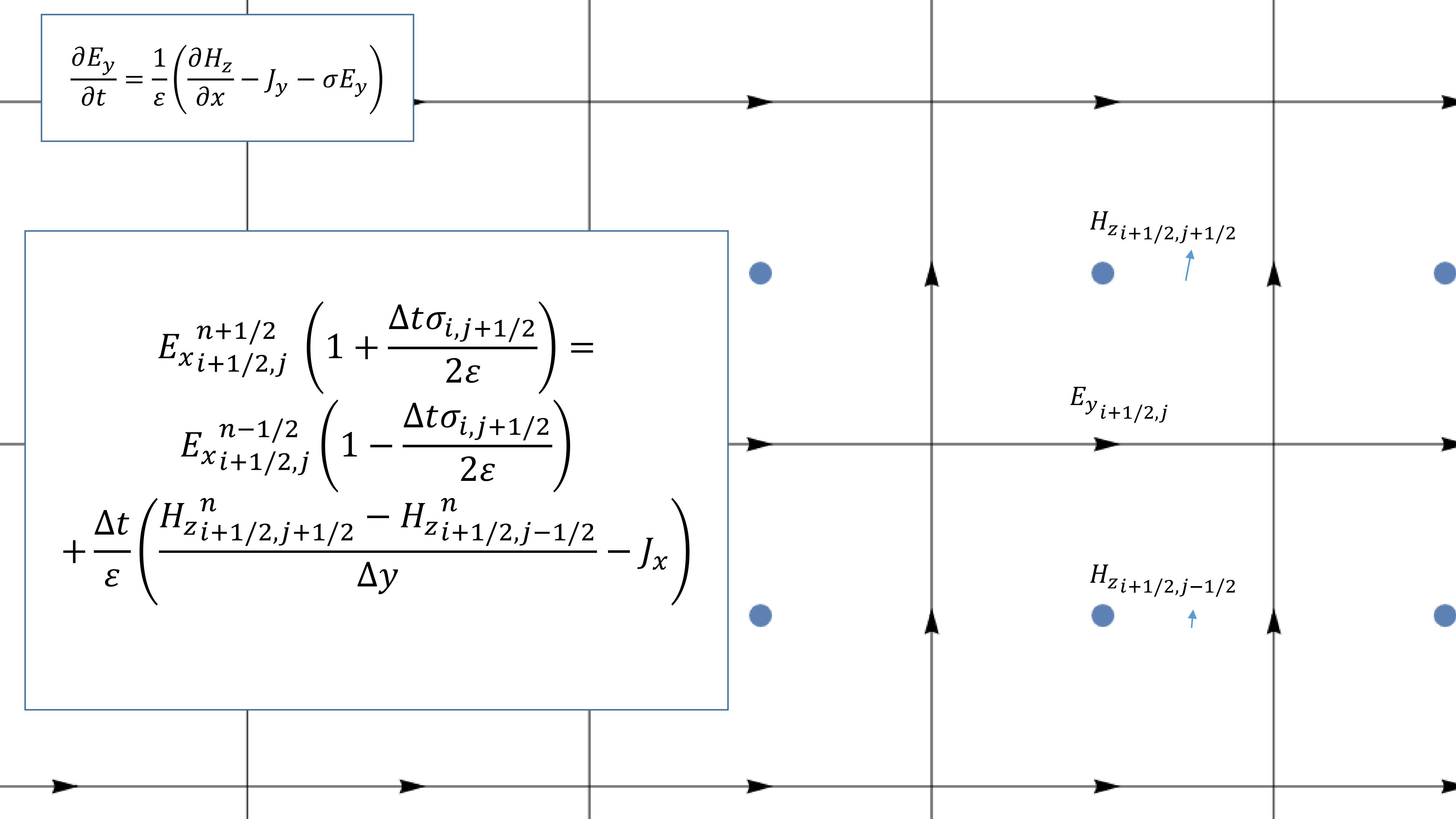
$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - J_y - \sigma E_y\right)$$

$$E_{y\,i,j+1/2}^{\,n+1/2}\left(1 + \frac{\Delta t \sigma_{i,j+1/2}}{2\varepsilon}\right) =$$

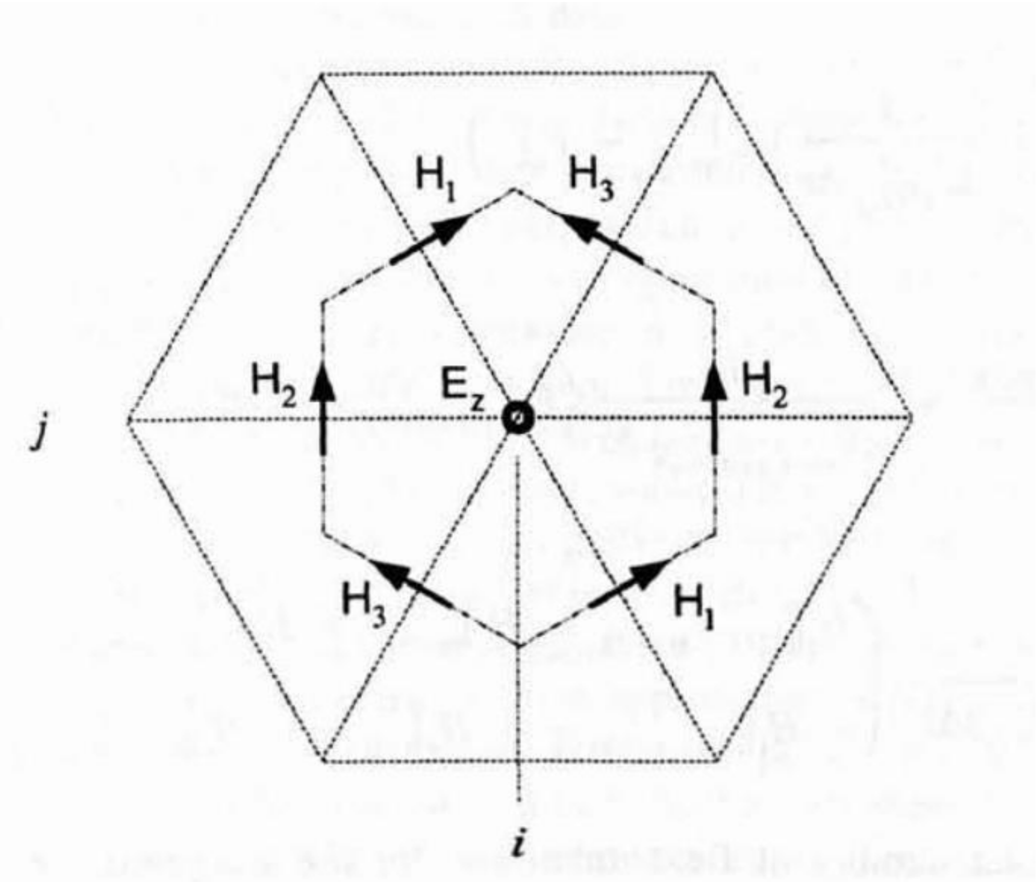
$$E_{y\,i,j+1/2}^{\,n-1/2}\left(1 - \frac{\Delta t \sigma_{i,j+1/2}}{2\varepsilon}\right) + \frac{\Delta t}{\varepsilon}\left(\frac{H_{z\,i+1/2,j+1/2}^{\,n} - H_{z\,i-1/2,j+1/2}^{\,n}}{\Delta x} - J_y\right)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} - J_y - \sigma E_y\right)$$

$$E_{x\,i+1/2,j}^{\,n+1/2}\left(1 + \frac{\Delta t \sigma_{i,j+1/2}}{2\varepsilon}\right) =$$
$$E_{x\,i+1/2,j}^{\,n-1/2}\left(1 - \frac{\Delta t \sigma_{i,j+1/2}}{2\varepsilon}\right)$$
$$+\frac{\Delta t}{\varepsilon}\left(\frac{H_{z\,i+1/2,j+1/2}^{\,n} - H_{z\,i+1/2,j-1/2}^{\,n}}{\Delta y} - J_x\right)$$



A co dla siatek skomplikowanych?



źr: Allen Taflove, Susan C. Hagness, *Computational electrodynamics – FDTD method*

Równania Maxwella

- $\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$
- $\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} = -\nabla \times \boldsymbol{E}$

$$\boldsymbol{J} = \boldsymbol{J}_{source} + \sigma \boldsymbol{E}$$

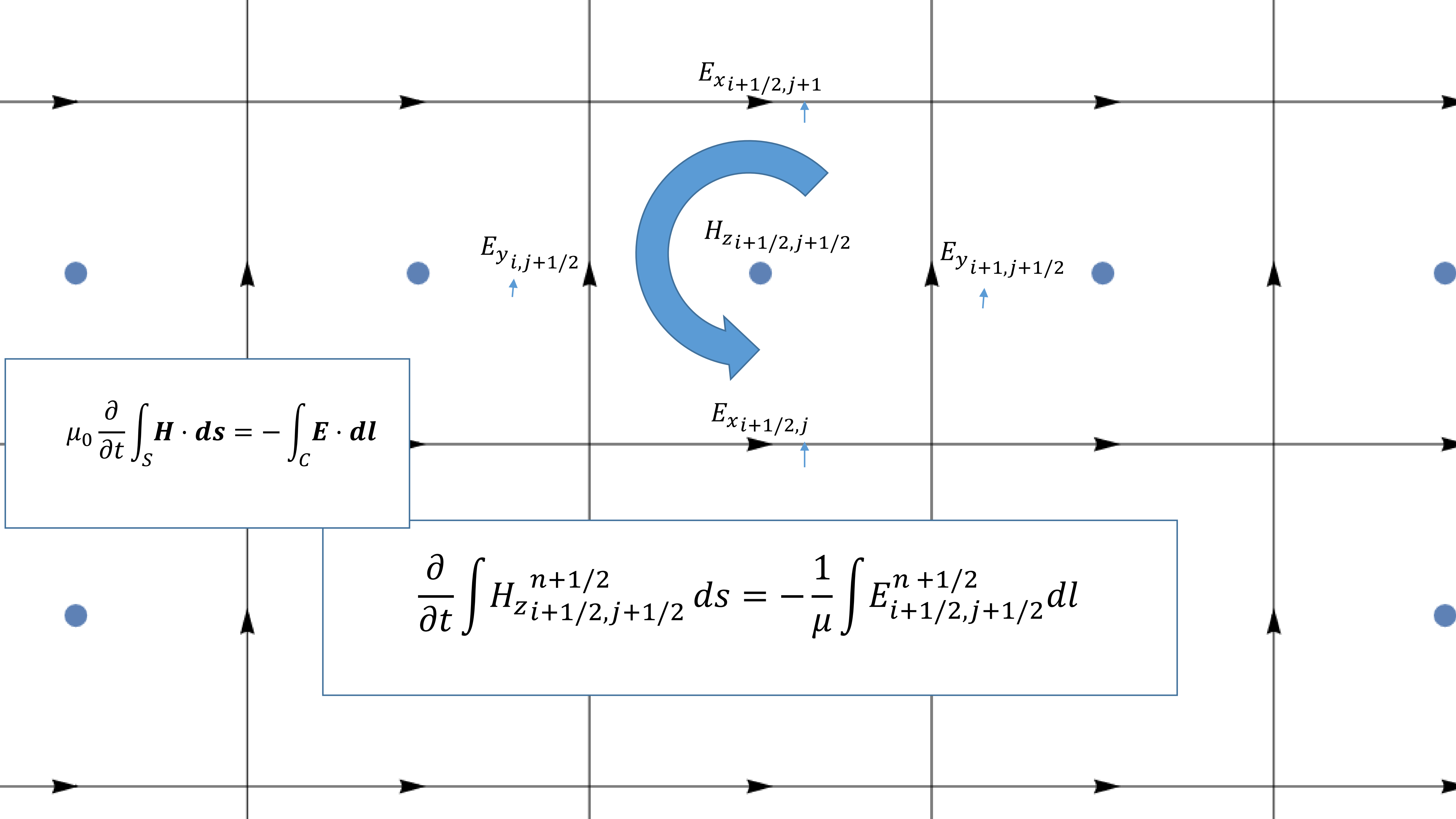
Źródło prądu

Mikroskopowe prawo Ohma

$\Updownarrow$

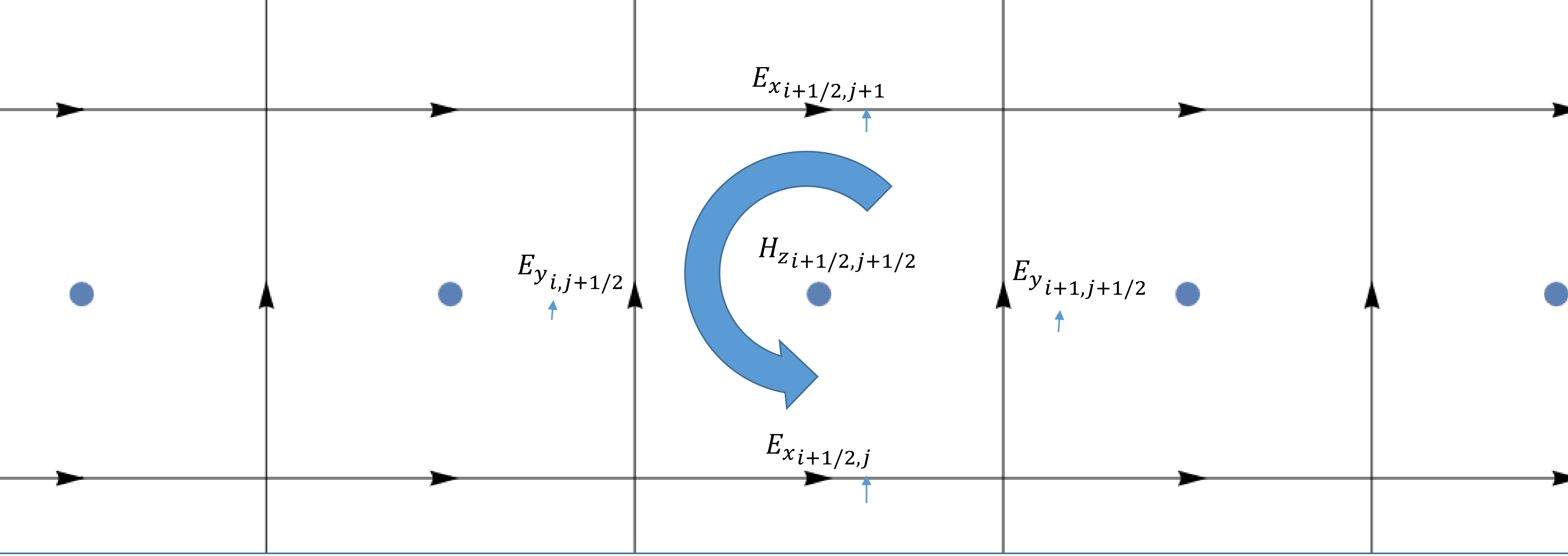
- $\varepsilon_0 \frac{\partial}{\partial t} \int_S \boldsymbol{E} \cdot \boldsymbol{ds} = \int_C \boldsymbol{H} \cdot \boldsymbol{dl} - \boldsymbol{I}$
- $\mu_0 \frac{\partial}{\partial t} \int_S \boldsymbol{H} \cdot \boldsymbol{ds} = -\int_C \boldsymbol{E} \cdot \boldsymbol{dl}$

$$\boldsymbol{I} = \int_S \boldsymbol{J} \cdot \boldsymbol{ds}$$

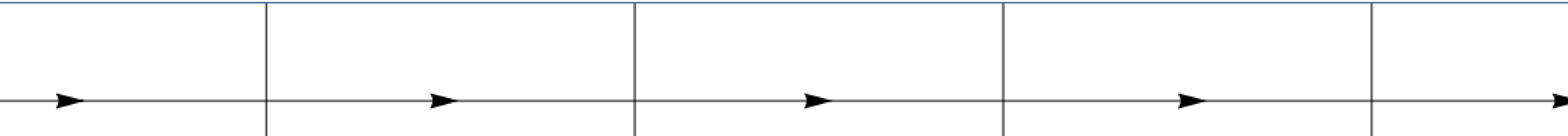


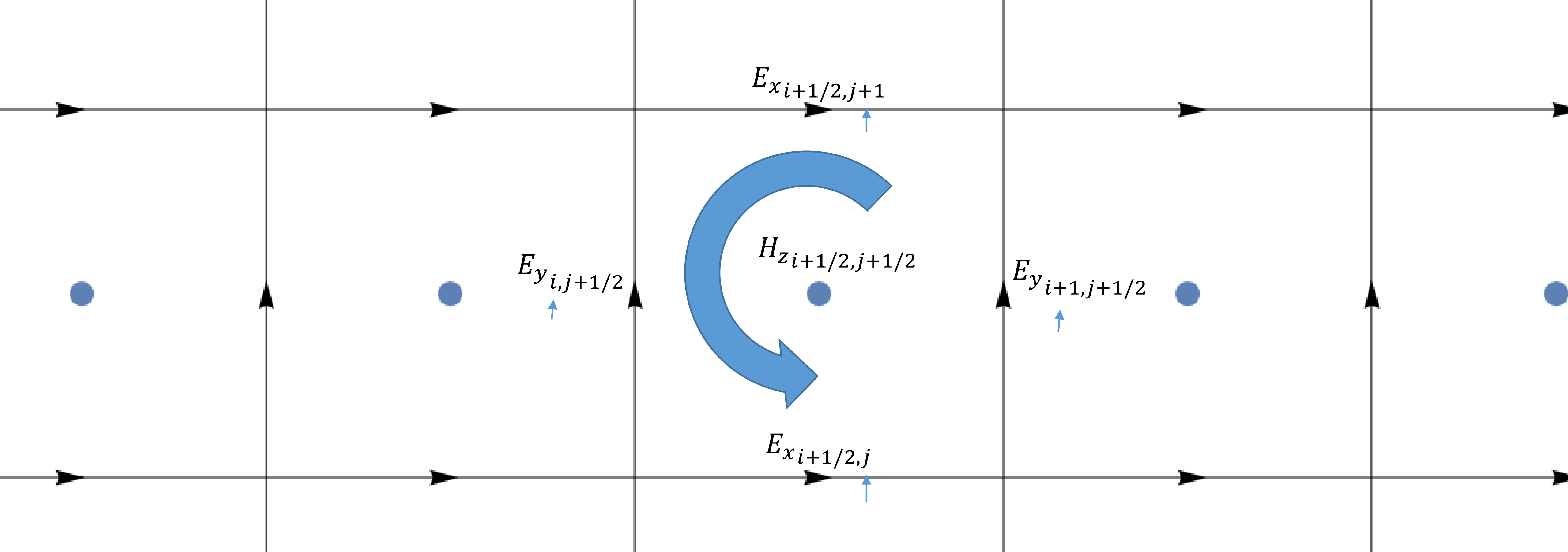
$$\mu_0 \frac{\partial}{\partial t}\int_S \boldsymbol{H}\cdot \boldsymbol{ds} = -\int_C \boldsymbol{E}\cdot \boldsymbol{dl}$$

$$\frac{\partial}{\partial t}\int H_{z_{i+1/2,j+1/2}}^{n+1/2}\, ds = -\frac{1}{\mu}\int E_{i+1/2,j+1/2}^{n+1/2} dl$$

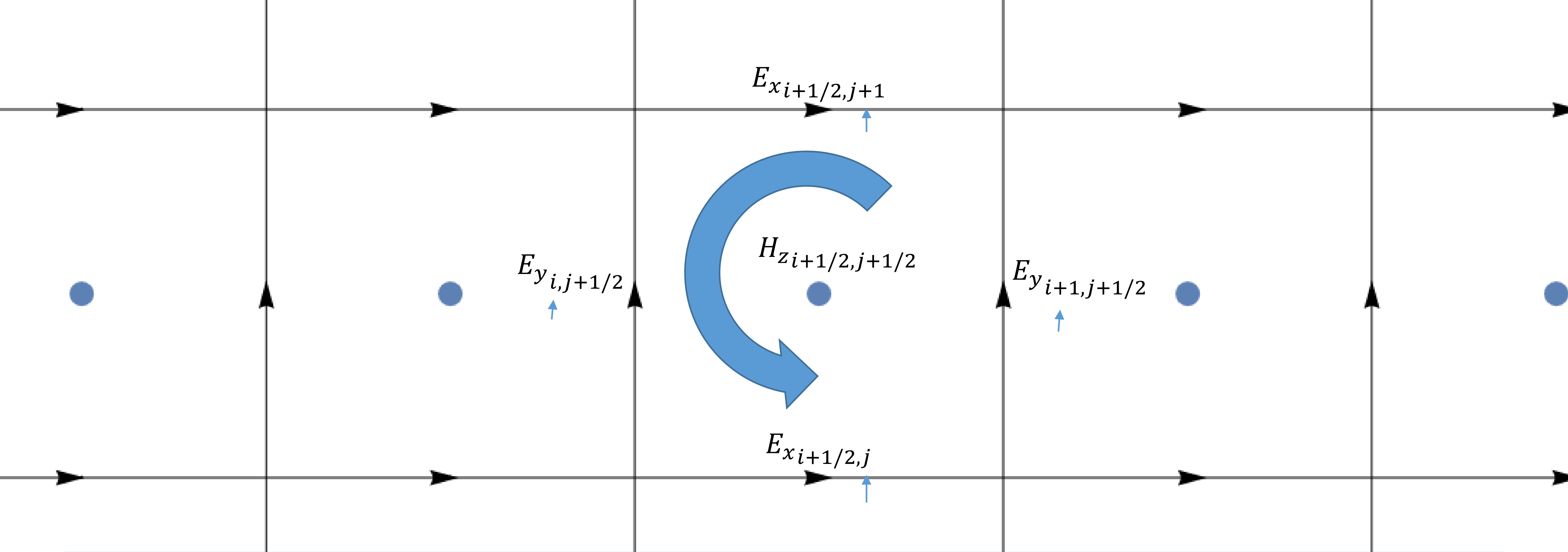


$$\frac{\partial}{\partial t}\Delta x\Delta y H_{z_{i+1/2,j+1/2}}^{n+1/2} = \frac{1}{\mu}\left(\Delta x\left(E_{x_{i+1/2,j+1}}^{n+1/2} - E_{x_{i+1/2,j}}^{n+1/2}\right) + \Delta y\left(E_{y_{i+1,j+1/2}}^{n+1/2} - E_{y_{i,j+1/2}}^{n+1/2}\right)\right)$$

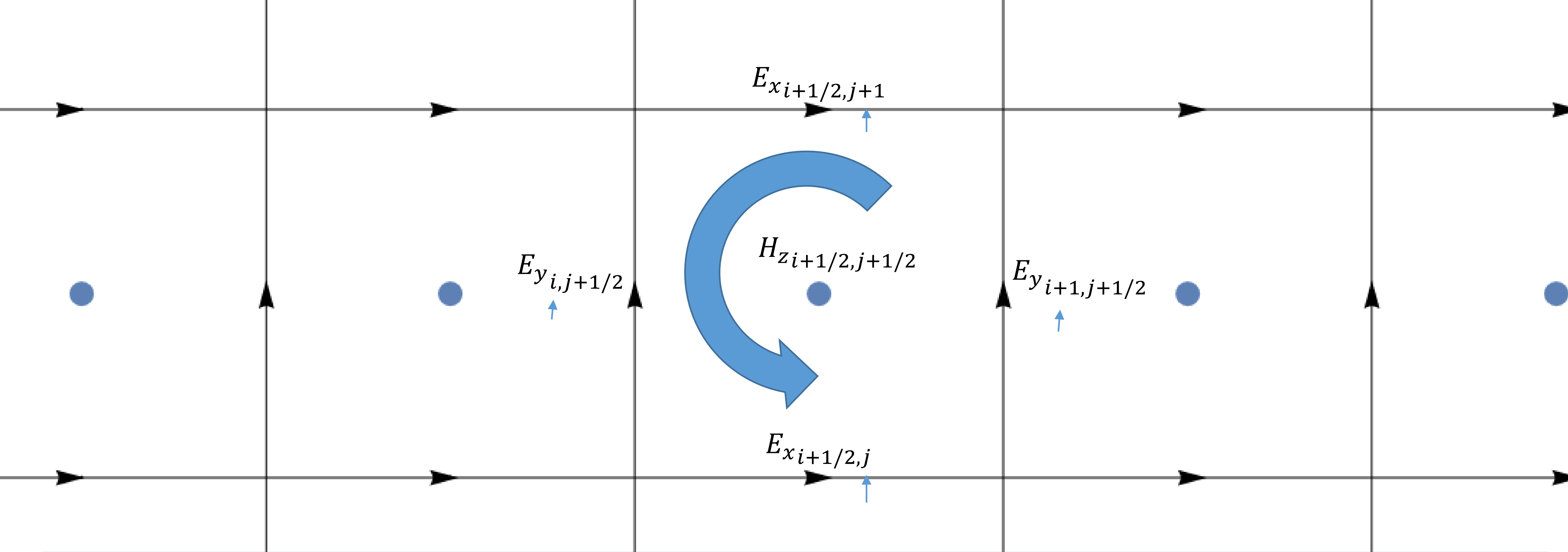




$$\Delta x \Delta y \frac{\partial}{\partial t} H_{z\,i+1/2,j+1/2}^{\,n+1/2} = \frac{1}{\mu}\left(\Delta x\left(E_{x\,i+1/2,j+1}^{\,n+1/2} - E_{x\,i+1/2,j}^{\,n+1/2}\right) + \Delta y\left(E_{y\,i+1,j+1/2}^{\,n+1/2} - E_{y\,i,j+1/2}^{\,n+1/2}\right)\right)$$



$$\Delta x \Delta y \frac{H_{z\,i+1/2,j+1/2}^{\,n+1} - H_{z\,i+1/2,j+1/2}^{\,n}}{\Delta t} = \frac{1}{\mu}\left(\Delta x\left(E_{x\,i+1/2,j+1}^{\,n+1/2} - E_{x\,i+1/2,j}^{\,n+1/2}\right) + \Delta y\left(E_{y\,i+1,j+1/2}^{\,n+1/2} - E_{y\,i,j+1/2}^{\,n+1/2}\right)\right)$$



$$\frac{H_{z_{i+1/2,j+1/2}}^{n+1} - H_{z_{i+1/2,j+1/2}}^{n}}{\Delta t} = \frac{1}{\mu}\left(\frac{E_{x_{i+1/2,j+1}}^{n+1/2} - E_{x_{i+1/2,j}}^{n+1/2}}{\Delta y} + \frac{E_{y_{i+1,j+1/2}}^{n+1/2} - E_{y_{i,j+1/2}}^{n+1/2}}{\Delta x}\right)$$