Métodos Numéricos

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1 Planteo del problema

2 Formulación del sistema

Partiendo de las ecuaciones (1)-(6) tenemos:

$$0 = \frac{t_{j-1,k}-2t_{j,k}+t_{j+1,k}}{(\Delta r)^2} + \frac{t_{j,k}-t_{j-1,k}}{r\Delta r} + \frac{t_{j,k-1}-2t_{j,k}+t_{j,k+1}}{r^2(\Delta\Theta)^2}$$
$$= t_{j-1,k}(\frac{1}{(\Delta r)^2} + \frac{-1}{r\Delta r}) + t_{j,k}(\frac{-2}{(\Delta r)^2} + \frac{1}{r\Delta r} + \frac{-2}{(r\Delta\Theta)^2}) + t_{j+1,k}\frac{1}{(\Delta r)^2} + t_{j,k-1}\frac{1}{(r\Delta\Theta)^2} + t_{j,k+1}\frac{1}{(r\Delta\Theta)^2}$$

Para simplificar la notación tomo:

$$\alpha = \frac{1}{(\Delta r)^2} + \frac{-1}{r\Delta r}$$
$$\beta = \frac{-2}{(\Delta r)^2} + \frac{1}{r\Delta r} + \frac{-2}{(r\Delta\Theta)^2}$$
$$\gamma = \frac{1}{(\Delta r)^2}$$
$$\chi = \frac{1}{(r\Delta\Theta)^2}$$

Por lo que tengo:

$$\alpha t_{j-1,k} + \beta t_{j,k} + \gamma t_{j+1,k} + \chi t_{j,k-1} + \chi t_{j,k+1} = 0 \iff$$
$$\beta t_{j,k} + \gamma t_{j+1,k} + \chi t_{j,k-1} + \chi t_{j,k+1} = -\alpha t_{j-1,k} \iff$$
$$\alpha t_{j-1,k} + \beta t_{j,k} + \chi t_{j,k-1} + \chi t_{j,k+1} = -\gamma t_{j+1,k} \iff$$
$$\beta t_{j,k} + \chi t_{j,k-1} + \chi t_{j,k+1} = -\alpha t_{j-1,k} - \gamma t_{j+1,k}$$

2.1 Generalización

3 Modelado

3.1 "Matriz en banda"