1a

$$x_1 - 2x_2 + 2x_3 = 4 \quad (1)$$
$$x_1 - x_2 = -2 \quad (2)$$
$$-x_1 + x_2 + 3x_3 = 4 \quad (3)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 1 & -1 & 0 & -2 \\ -1 & 1 & 3 & 4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 1 & -2 & -6 \\ 0 & -1 & 5 & 8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 1 & -2 & -6 \\ 0 & 0 & 3 & 2 \end{array}\right)$$

$$3x_3 = 2 \rightarrow \underline{x_3 = \frac{2}{3}}$$

$$x_2 - \frac{4}{3} = -6 \rightarrow \underline{x_2} = \frac{4}{3} - \frac{18}{3} = \underline{-\frac{14}{3}}$$

$$x_1 + \frac{28}{3} + \frac{4}{3} = 4 \rightarrow \underline{x_1} = \frac{12}{3} - \frac{4}{3} - \frac{28}{3} = \underline{-\frac{20}{3}}$$

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$$x_1 + 2x_2 + 4x_3 = 5$$
$$x_1 + x_2 + 3x_3 = 3$$
$$2x_1 + x_2 + 5x_3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 1 & 1 & 3 & 3 \\ 2 & 1 & 5 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 1 & 2 & 4 & 5 \\ 2 & 1 & 5 & 2 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

inconsistent equation!!!

(2) $2x_1 - x_2 + x_3 + 2x_4 = 0$

$x_1 + 2x_2 + x_3 + 3x_4 = 0$

$5x_1 - 5x_2 + 2x_3 + 3x_4 = 0$

$$\begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 5 & -5 & 2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 1 & -1 & 1 & 2 \\ 5 & -5 & 2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -5 & -1 & -4 \\ 0 & -15 & -3 & -12 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -5 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\underline{x_3 = -4x_4 - 5x_2}$

$\underline{x_1} = 2x_2 - 5x_2 - 4x_4 + 3x_4 = \underline{-3x_2 - x_4}$

let $x_2 = a$, $x_4 = b$ $\quad x_1 = -5a - 4b$, $\quad x_3 = -3a - b$

basis: $a\begin{pmatrix} -5 \\ 1 \\ -3 \\ 0 \end{pmatrix} + b\begin{pmatrix} -4 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

3 $\left[\begin{pmatrix}1\\2\\1\\-1\end{pmatrix}, \begin{pmatrix}0\\5\\4\\-5\end{pmatrix}, \begin{pmatrix}3\\1\\-1\\2\end{pmatrix}, \begin{pmatrix}2\\-1\\-2\\3\end{pmatrix} \right] \sim$

$$\sim \begin{pmatrix}1 & 0 & 3 & 2\\ 2 & 5 & 1 & -1\\ 1 & 4 & -1 & -2\\ -1 & -5 & 2 & 3\end{pmatrix} \sim \begin{pmatrix}1 & 4 & -1 & -2\\ 2 & 5 & 1 & 1\\ 1 & 0 & 3 & 2\\ -1 & -5 & 2 & 3\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}1 & 4 & -1 & -2\\ 0 & -3 & 3 & 5\\ 0 & -4 & 4 & 4\\ 0 & -1 & 1 & 1\end{pmatrix} \sim \begin{pmatrix}1 & 4 & -1 & -2\\ 0 & -1 & 1 & 1\\ 0 & -3 & 3 & 5\\ 0 & -4 & 4 & 4\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}1 & 4 & -1 & -2\\ 0 & 1 & -1 & -1\\ 0 & 0 & 0 & 2\\ 0 & 0 & 0 & 0\end{pmatrix} \begin{pmatrix}1 & 4 & -1 & -2\\ 0 & 1 & -1 & -1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{pmatrix}$$

Bazis: $\begin{pmatrix}1\\2\\1\\-1\end{pmatrix}$ i $\begin{pmatrix}0\\5\\4\\-5\end{pmatrix}$ i $\begin{pmatrix}2\\-1\\-2\\3\end{pmatrix}$

4) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = -2x_2 - x_3 = 2u - v$

$x_2 = 4$

$x_3 = v$

$u \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + v \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ - Answer

5a) $A_1 = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

1) $|A_1| = 5 \cdot 1 - 2 \cdot 3 = -1$

$|A_1| \neq 0$, so A is invertible

2) Minor matrix $M = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

3) Cofactors matrix $A = \begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix}$

4) $A^T = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$

5) $A^{-1} = \frac{1}{|A|} \cdot A^T = -\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$

5b $A_2 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 3 \end{bmatrix}$

$|A_2| = 1\cdot1\cdot3 + 2\cdot2\cdot3 + 1\cdot2\cdot1 -$

$- 1\cdot1\cdot3 - 2\cdot2\cdot3 - 1\cdot2\cdot1 = 0$

$|A_2| = 0 \Rightarrow A_2$ isn't invertable

5c $A_3 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ -1 & -2 & 1 \end{bmatrix}$

1) $|A_3| = -1\cdot2\cdot1 + 1\cdot1\cdot(-1) + 0\cdot1\cdot(-2) -$

$- 0\cdot2\cdot(-1) - 1\cdot1\cdot1 - (-1)\cdot2\cdot(-1) =$

$= -2 - 1 - 1 - 2 = -6$

$|A_3| \neq 0$ A_3 is invertable

2) Minor matrix $M = \begin{bmatrix} 4 & 2 & 0 \\ 1 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix}$

3) $\overline{A_3} = \begin{bmatrix} 4 & -2 & 0 \\ -1 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix}$

4) $\overline{A}_3^{1} = \begin{bmatrix} 4 & -1 & 1 \\ -2 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

5) $A_3^{-1} = \frac{1}{|B|} \cdot A_3^T = \frac{1}{-6} \begin{bmatrix} 4 & -1 & 1 \\ -2 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

(6) $A = \begin{vmatrix} 1 & 2 & 2 \\ 1 & 1 & x \\ 2 & 1 & x^2 \end{vmatrix}$

$|A| = 1\cdot1\cdot x^2 + 2\cdot x\cdot 2 + 2\cdot1\cdot1 -$

$-2\cdot1\cdot2 - 2\cdot1\cdot x^2 - 1\cdot x\cdot 1 =$

$= -x^2 + 3x - 2$

if $A \neq 0$, then it invertible

$-x^2 + 3x - 2 = 0$

$D = 9 - 8 = 1$

$x_1 = 2$

$x_2 = 1$

$|A| \neq 0$; if $x \in (-\infty, 1)$ or $x \in (2, +\infty)$, or $x \in (1; 2)$

(7) $B = \left\{ \begin{vmatrix} 3 \\ 2 \end{vmatrix}, \begin{vmatrix} 1 \\ 1 \end{vmatrix} \right\}$

$a\begin{vmatrix} 3 \\ 2 \end{vmatrix} + b\begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 5 \\ 2 \end{vmatrix}$

$\begin{cases} 3a + b = 5 \\ 2a + b = 2 \end{cases}$

$b = 2 - 2a$

$3a + 2 - 2a = 5$;

Answer:

$a = 3, b = 4$

$$A = \begin{pmatrix} 2 & 4 & 2 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

8/1

$$A - \lambda E = \begin{pmatrix} 2-\lambda & 4 & 2 \\ 1 & -1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{pmatrix}$$

$$\det(A-\lambda E) = (2-\lambda)(-1-\lambda)(1-\lambda) + 4\cdot 1\cdot 0 + 2\cdot 1\cdot 0 - 2\cdot(-1-\lambda)\cdot 0 - 4\cdot 1\cdot(1-\lambda) - (2-\lambda)\cdot 1\cdot 0 =$$

$$= (1-\lambda)\big((2-\lambda)(-1-\lambda)-4\big)$$

$\det(A-\lambda E) = 0$!

1) $1-\lambda = 0 \Rightarrow \lambda = 1$

2) $(2-\lambda)(-1-\lambda)-4 = 0$

$-2-2\lambda+\lambda+\lambda^2-4=0$

$\lambda^2-\lambda-6=0$

$D = 1+24 = 25$

$\lambda_2 = 3$, $\lambda_3 = -2$, $\lambda_1 = 1$

82) for $\lambda = \lambda_1 = 1$

$$\begin{pmatrix} 2-1 & 4 & 2 \\ 1 & -1-1 & 1 \\ 0 & 0 & 1-1 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 2 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 2 \\ 0 & 1 & \frac{1}{6} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2-\frac{4}{6} \\ 0 & 1 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \begin{cases} y = \frac{1}{6}z \\ x = -\frac{4}{3}z \end{cases} \quad \begin{pmatrix} -\frac{1}{6}z \\ -\frac{4}{3}z \\ z \end{pmatrix}$$

$\lambda = \lambda_2 = -2$

$$A - \lambda E = \begin{pmatrix} 2+2 & 4 & 2 \\ 1 & -1+2 & 1 \\ 0 & 0 & 1+2 \end{pmatrix} \sim \begin{pmatrix} 4 & 4 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{cases} 4x+4y+2z=0 \\ z=0 \\ 3z=0 \end{cases} \quad \begin{cases} z=0 \\ x=-y; \end{cases} \quad \begin{pmatrix} -y \\ y \\ 0 \end{pmatrix}$$

$\lambda = \lambda_3 = 3$

$$A - \lambda E = \begin{pmatrix} 2-3 & 4 & 2 \\ 1 & -1-3 & 1 \\ 0 & 0 & 1-3 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & 2 \\ 1 & -4 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\begin{cases} -x+4y+2z=0 \\ x-4y+z=0 \end{cases} \quad \frac{z=0}{x=4y} \quad \begin{pmatrix} 4y \\ y \\ 0 \end{pmatrix}$$

$$\begin{cases} -x+4y=0 \\ x-4y=0 \end{cases}$$