9.1) $A = \begin{pmatrix} 2 & 4 \\ -2 & -4 \end{pmatrix}$ $\quad A - \lambda E = \begin{pmatrix} 2-\lambda & 4 \\ -2 & -4-\lambda \end{pmatrix}$

$\mathrm{def}(A - \lambda E) = (2-\lambda)\cdot(-4-\lambda) - 4\cdot(-2) =$

$= -8 - 2\lambda + 4\lambda + \lambda^2 + 8 = \lambda^2 + 2\lambda = \lambda(\lambda+2)$

$\mathrm{def}(A - \lambda E) = 0 \Longrightarrow \lambda_1 = 0$

$\lambda_2 = 2$

$\lambda = \lambda_1 = 0$

$\begin{cases} 2x + 4y = 0 \\ -2 - 4y = 0 \end{cases}$ $\begin{cases} 2x = -4y \\ \underline{x = -2y} \end{cases}$ для $\lambda = 0$: $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\lambda = \lambda_2 = -2$

$\begin{cases} (2+2)x - 4y = 0 \\ -2x + (-4-2) = 0 \end{cases}$ $\begin{cases} 4x - 4y = 0 \\ -2x + 2y = 0 \end{cases}$

$x = y$; $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ для $\lambda = 2$

$\vec{g}_2$ invertible matrix $P_{[x,y]} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$
diagonalizes A

$$P^{-1}AP = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \longrightarrow A = P \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} P^{-1}$$

$$A^n = \left(P \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} P^{-1}\right)^n = P \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}^n P^{-1} =$$

$$= P \begin{pmatrix} 0 & 0 \\ 0 & (-2)^n \end{pmatrix} P^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & (-2)^n \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & (-2)^n \\ 0 & (-2)^n \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} (-2)^n & (-2)^n \\ (-2)^n & (-2)^n \end{pmatrix}$$

(10.)

$$\left\{ V_1 = \begin{vmatrix} 1 \\ 1 \\ 1 \\ -1 \end{vmatrix} ; V_2 = \begin{vmatrix} 2 \\ -1 \\ -1 \\ 1 \end{vmatrix} ; V_3 = \begin{vmatrix} -1 \\ 2 \\ 2 \\ 1 \end{vmatrix} \right.$$

$b_1 = V_1$

$$b_2 = V_2 - \frac{\langle a_2\ b_1 \rangle}{\langle b_1\ b_1 \rangle} b_1 =$$

$$= \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10/4 \\ -5/4 \\ -5/4 \\ 5/4 \end{pmatrix} \sim \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} \ b_2 \perp b_1$$

$$b_3 = V_3 - Pr_{b_1}(V_3) - Pr_{b_2}(V_3) =$$

$$= V_3 - \frac{\langle V_3 ; b_1 \rangle}{\langle b_1 ; b_1 \rangle} \cdot V_3 - \frac{\langle V_3 ; b_2 \rangle}{\langle b_2 ; b_2 \rangle} \cdot V_3 =$$

$$= \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\rangle}{4} \cdot V_3 - \frac{\left\langle \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} ; \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle} \cdot V_3 =$$

(10_2)

$$= \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} - (-1) \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 + \frac{1}{2} - 1 \\ 2 - 1 + 2 \\ 2 - 1 + 2 \\ 1 - 1/2 + 1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ 3 \\ 3 \\ 3/2 \end{pmatrix} \sim \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$

Normalize b_1, b_2, b_3

$$b_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$ length of $b_1 = \sqrt{1+1+1+1} = \sqrt{4} = 2$

$$\hat{b} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ -1/2 \end{pmatrix}$$

$$b_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$ length of $b_2 = \sqrt{2^2+1+1+1} = \sqrt{7}$

$$\hat{b}_2 = \begin{pmatrix} 2/\sqrt{7} \\ -1/\sqrt{7} \\ -1/\sqrt{7} \\ 1/\sqrt{7} \end{pmatrix}$$

(10_3) $b_3 = \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix}$ length of $b_3 = \sqrt{1+2^2+2^2+1} =$
$= \sqrt{10}$

$$\hat{b}_3 = \begin{pmatrix} -1/\sqrt{10} \\ 2/\sqrt{10} \\ 2/\sqrt{10} \\ 1/\sqrt{10} \end{pmatrix}$$

I'm sorry for your eays ;(