Inferring stellar ages by combining isochrone fitting with gyrochronology

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ABSTRACT

The ages of main sequence stars are difficult to infer because their outward appearances change subtly and slowly during their hydrogen burning lifetimes. In the era of *Gaia*, where precise parallaxes are available for millions of stars, isochrone fitting can be used to provide a constraint on stellar ages. In addition, for those stars with observed rotation periods, gyrochronal ages may also be available. By combining two sets of observable stellar properties and dating methods that are sensitive to different evolving processes in stars, it may be possible to infer more precise and accurate ages than using either method in isolation. In this investigation, the spectroscopic properties of main sequence stars ($T_{\rm eff}$, [Fe/H] and log g) are combined with their *Kepler* rotation periods, using a hierarchical Bayesian model, to infer their ages as predicted from *both* stellar evolution models and gyrochronology. This is a pilot study, not aiming to produce a state-of-the-art dating model, rather to explore the process of combining two heterogeneous dating methods. Combining two heterogeneous dating methods can illuminate flaws in one or both, although without ground truth it can be difficult to identify the cause of inconsistencies. Although calibration is not the main purpose of this exploratory investigation and the parameters of our gyrochronology model are fixed, only a slight modification to our algorithm would be required to perform a calibration. We provide open source code that calculates stellar ages from spectroscopic parameters and/or apparent magnitudes, and rotation periods.

1. Introduction

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The formation and evolution of the Milky Way (MW) and the planetary systems within it are two topics of significant interest to the astronomical community today. Both of these fields require precise and accurate ages of thousands of stars. Recent advances in galactic archaeology have made use of the ages of red giant stars, some derived from asteroseismology and some from spectroscopy. The age distribution of stellar populations in the MW have been explored using these stellar ages. Red giants are highly luminous and can be observed to great distances, thus providing age information on the scale of tens of kilo-parsecs. Main sequence (MS) stars on the other hand, although fainter, are more numerous and their ages may provide new insights into the formation and evolution of the Solar neighborhood. MS star ages are also of great interest for studying the formation and evolution of planetary systems. Almost all exoplanets discovered to date orbit MS stars and it is therefore the ages of MS stars that are needed to capture snapshots of planet evolution. Unfortunately, the very property that makes MS stars good hosts for habitable planets also makes them difficult to date: they do not change substantially over time.

A star like the Sun will increase in luminosity by only around a factor of two before turning off the MS. In addition, the Sun's temperature will only increase by around 100 K during its $\sim$8 billion year MS lifetime. Luminosity and temperature are not sensitive proxies for age and can also be difficult to measure with their precision highly sensitive to their distance and the amount of extincting dust along the line of sight. On the other hand, Sun's rotation period will vary by almost an order of magnitude over its MS lifetime. Stellar rotation periods are much more sensitive to age than luminosity or temperature and can be measured precisely with little dependence on distance and none on extinction. Incorporating rotation period measurements into isochrone fitting methods provides additional information that allows for much more precise age inference.

In addition to the difficulties imposed by the slow timescale for change within MS stars that results in poor age precision, different dating methods often produce inconsistent predictions for the age of a star as a result of model inaccuracies. For example, an asteroseismic age will not necessarily agree with a isochronal or rotational age. Even isochronal ages derived from different stellar evolution models can be inconsistent. This arises from...

Due to the abundance of rotation periods for MS stars already provided by *Kepler/K2* and the many more expected from future photometric surveys, rotation-dating, or 'gyrochronology' is one of the most readily available methods for inferring stellar ages. Magnetic braking in MS stars was observed by Skumanich (1972) who, using observations of young clusters and the Sun, found that the rotation periods of Solar-type stars decay with the square-root of time. Now, the relation between rotation period and age is well-established and has been studied in detail CITATIONS. The convenient characteristic of stars that allows their ages to be inferred from their *current* rotation periods, not their primordial ones, comes from the steep dependence of spin-down rate on rotation period. Observations of young clusters indicate that stellar angular momentum loss rate is proportional to the cube of the angular velocity. This means that a star spinning with high angular velocity will experience a much greater angular momentum loss rate than a slowly spinning star. For this reason, no matter the initial rotation period of a Sun-like star, after around the age of the Hyades, ($\sim$ 600 Myr) stellar rotation periods appear to converge onto a tight sequence. After this time, the age of a star can be inferred, to first order, from its mass and rotation period alone.

Despite significant advances in theoretical models of stellar spin-down as well as new calibrations of empirical models, the gyrochronology relations have not yet been finalized. In particular, they suffer from a lack of suitable calibration stars at old ages and low masses. These regions of parameter space are particularly important because some evidence suggests that rotational evolution changes at old ages and low masses. For example, recent results show that old *Kepler* asteroseismic stars rotate more rapidly than expected given their age (*e.g.* Angus et al. 2015; van Saders et al. 2016). These data can be reproduced with a model that relaxes magnetic breaking at a critical Rossby number, *Ro* (the ratio of rotation period to the convective overturn timescale) of around the Solar value. As stellar rotation periods lengthen and stars cross this *Ro* threshold, they maintain a constant rotation period after that time. The gyrochronology model described in van Saders et al. (2016) and **?** includes weakened magnetic braking after stars reach the Solar *Ro* threshold.

The models developed and calibrated in Epstein and Pinsonneault (2014); **?**); van Saders et al. (2016); **?** are expensive to compute and, just as with most isochrones and stellar evolution tracks, are usually pre-computed over a grid of stellar parameters in order to perform tractable inference. In order to infer an age from these models, one would effectively perform isochrone fitting but in this case, rotation period would be added as an additional parameter and the ages inferred would therefore be more precise and accurate. In liu of these models becoming readily available to the community, we develop here a complementary method that combines isochrones with an empirical gyrochronology model using a hierarchical Bayesian model. The methodology presented here is related to the family of models described above in that both use a combination of rotation periods and other observable properties that track evolution on the Hertzprung-Russell diagram in concert. A major difference is that the gyrochronology model used here is an entirely empirically calibrated one, as opposed to a physically derived one. One major advantage of using a physically motivated gyrochronology model over an empirically calibrated one is the ability to rely on physics to interpolate or extrapolate over parts of parameter space with sparse data coverage. However, rotational spin-down is a complex process that is not yet fully understood and currently no physical model can accurately reproduce all the data available. For this reason, even physically motivated gyrochronology models cannot always be used to reliably extrapolate into unexplored parameter space. Although we use a simple version of an empirical gyrochronology model in this work, which, like the physical gyrochronology models, cannot yet reproduce all the observed data, several simple modifications could be made to this model that *would* produce significant improvements. For example, including and allowing for outliers; stars with anomalously fast or slow rotation periods, could be incorporated into our model. Ultimately, the model we present here will provide a baseline against which more physically motivated models, e.g. the van Saders et al. (2016) models, can be compared.

2. Method

In Bayesian statistics, combining information from different models can be relatively simple, as long as the processes being modeled, that generated the data, are independent. In this case, we are combining information that relates to the burning of hydrogen in the core (this is the process that drives the slow increase in $T_{\rm eff}$ and luminosity over time) with information about the magnetic braking history of a star (rotation period). We can assume that, to first order, these two processes are independent: the hydrogen fraction in the core does not affect a star's rotation period and vice versa. In practise we can never be entirely sure that two such processes are independent but, at least within the uncertainties, any dependence here is unlikely to affect our results. If this assumption is valid, then the likelihoods calculated using each model can be multiplied together.

The desired end product of this method is an estimate of the non-normalized posterior probability density function (PDF) over the age of a star,

$$p(A|\mathbf{m_x}, T_{\rm eff}, \log g, \hat{F}, P_{rot}, \bar{\omega}), \tag{1}$$

where A is age, $\mathbf{m_x}$ is a vector of apparent magnitudes in various bandpasses, $\hat{F}$ is the *observed* bulk metallicity, and $P_{\rm rot}$ is the rotation period and $\bar{\omega}$ is parallax.

In order to calculate a posterior over age, we must marginalize over parameters that relate to age, but are not of interest in this study: mass (M), distance (D), V-band extinction (A_V) and an *inferred* bulk metallicity, F. The marginalization involves integrating over these extra parameters,

$$\begin{gathered} p(A|\mathbf{m_x}, T_{\rm eff}, \log g, \hat{F}, P_{rot}, \bar{\omega}) \propto \\ \int p(\mathbf{m_x}, T_{\rm eff}, \log g, \hat{F}, P_{rot}, \bar{\omega}|A, M, D, A_V, F)\ p(A)p(M)p(D)p(A_V)p(F)dMdDdA_VdF. \end{gathered} \tag{2}$$

This equation is a form of Bayes' rule,

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}, \tag{3}$$

where the likelihood of the data given the model is,

$$p(\mathbf{m_x}, T_{\rm eff}, \log g, \hat{F}, P_{rot}, \bar{\omega}|A, M, D, A_V, F), \tag{4}$$

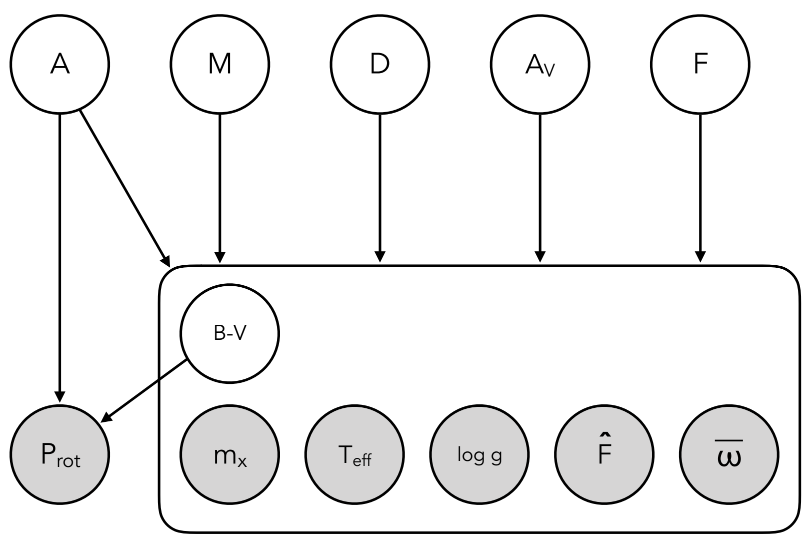
and the prior PDF over parameters is,

$$p(A)p(M)p(D)p(A_V)p(F). \tag{5}$$

Not all of the observables on the left of the | in the likelihood depend on all of the parameters to the right of the |. For example, rotation period, $P_{\rm rot}$ doesn't depend on V-band extinction, A_V. It is useful to factorize the likelihood wherever there are conditional independencies. For example, instead of the likelihood we wrote in equation 2, where every observable depends on every parameter, we could have written,

$$p(\mathbf{m_x}, T_{\rm eff}, \log g, \hat{F}, P_{rot}, \bar{\omega}|A, M, D, \tag{6}$$

Fig. 1.— A probabilistic graphical model (PGM) showing the conditional dependencies between the parameters (white nodes) and observables (gray nodes) in our model. Apparent magnitude, m_x, effective temperature, $T_{\rm eff}$, surface gravity, log g, observed bulk metallicity, $\hat{F}$, and parallax, $\bar{\omega}$ are determined by the mass, M, age, A, distance, D, extinction, A_V and bulk metallicity, F, of a star. These dependencies are indicated by arrows that start at a 'parent' node and end at the dependent observable, or 'child' node. The box drawn around some of the nodes indicates that everything inside it depends on every parameter that points toward it. For example, log g depends on A, M, D, A_V, and F. In our model, rotation period, $P_{\rm rot}$, only depends on age and a B-V color that is a latent parameter, predicted from the isochronal model. In our model, rotation period does not directly depend on distance, extinction, metallicity or mass, only age and B-V color. This PGM is a representation of the factorized joint PDF over parameters and observables which is written in equation **??**.



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