Inferring stellar ages by combining isochrone fitting with gyrochronology

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ABSTRACT

The ages of main sequence stars are difficult to infer because their outward appearances change subtly and slowly during their hydrogen burning lifetimes. In the era of *Gaia*, where precise parallaxes are available for millions of stars, isochrone fitting can be used to provide a constraint on stellar ages. In addition, for those stars with observed rotation periods, gyrochronal ages may also be available. By combining two sets of observable stellar properties and dating methods that are sensitive to different evolving processes in stars, it may be possible to infer more precise and accurate ages than using either method in isolation. In this investigation, the spectroscopic properties of main sequence stars ($T_{\rm eff}$, [Fe/H] and log g) are combined with their *Kepler* rotation periods, using a hierarchical Bayesian model, to infer their ages as predicted from *both* stellar evolution models and gyrochronology. This is a pilot study, not aiming to produce a state-of-the-art dating model, rather to explore the process of combining two heterogeneous dating methods. Combining two heterogeneous dating methods can illuminate flaws in one or both, although without ground truth it can be difficult to identify the cause of inconsistencies. Although calibration is not the main purpose of this exploratory investigation and the parameters of our gyrochronology model are fixed, only a slight modification to our algorithm would be required to perform a calibration. We provide open source code that calculates stellar ages from spectroscopic parameters and/or apparent magnitudes, and rotation periods.

1. Introduction

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The formation and evolution of the Milky Way (MW) and the planetary systems within it are two topics of significant interest to the astronomical community today. Both of these fields require precise and accurate ages of thousands of stars. Recent advances in galactic archaeology have made use of the ages of red giants, some calculated from asteroseismology and some from spectroscopy. The age distribution of stellar populations in the MW have been explored using the ages of these stars. Red giants are highly luminous and can be observed to great distances, thus providing age information on the scale of tens of kilo-parsecs. Main sequence (MS) stars on the other hand, although fainter, are more numerous and their ages may provide new insights into the formation and evolution of the Solar neighborhood. MS star ages are also of great interest for studying the formation and evolution of planetary systems. Almost all exoplanets discovered to date orbit MS stars and it is therefore the ages of MS stars that are needed to capture snapshots of planet evolution. Unfortunately, the very property that makes MS stars good hosts for habitable planets also makes them difficult to date: they do not change substantially over time.

Unlike the spectra or photometric colors of red giants, MS star spectra do not contain a significant amount of age information. This can be understood as due to the spacing of isochrones on a Hertsprung-Russell or color-magnitude diagram. On the MS, isochrones are tightly spaced and, even with very precise measurements of effective temperature and luminosity, the position of a MS on the HR diagram might be consistent with range of isochrones spanning several billion years. On the giant branch however, isochrones are spread further apart, so that sufficiently precisely measured temperatures and luminosities may yield ages that are precise to within 20% or better. Look at typical age uncertainties from APOGEE. Asteroseismology can provide precise ages of both red giant and MS stars however, due to the greater abundance of observation suitable for *red giant* asteroseismology, precise red giant asteroseismic ages once again outnumber MS ages. The typical periods of red giant asteroseismic pulsations are long and can be detected using *Kepler*'s long cadence mode of one observation per thirty minutes, which is its most common observing mode. MS stars, on the other hand, oscillate with periods of just a few minutes, and the long cadence *Kepler* observations, taken once every half-hour are not capable of resolving these pulsations. Instead, they must be observed in *Kepler*'s short cadence mode (one observation every minute), as has been done for around two thousand stars. However, since the amplitude of pulsation scales with stellar radius, the majority of successfully measured ages using short-cadence observations (currently around 500) are for subgiants; a relatively small number of these stars are truly on the MS. This may change now that the short cadence *Kepler* light curves have been reprocessed and new precise ages for the full sample of two thousand stars may be measured shortly.

An alternative dating method that uses stellar rotation periods, gyrochronology, has the potential to provide MS star ages that are precise to around 20%. Due to the abundance of rotation periods for MS stars already provided by *Kepler*/*K2* and the many more expected from future photometric surveys, gyrochronology is one of the most readily available methods for inferring stellar ages and, as such, has gained interest over the last few years. Magnetic braking in MS stars was observed by Skumanich (1972) who, using observations of young clusters and the Sun, found that the rotation periods of Solar-type stars decay with the square-root of time. It has since been established that the rotation period of a star depends, to first order, only on its age and mass. This means that by measuring a star's rotation period and a mass proxy (B-V color is the most commonly used mass-proxy in the literature), one can determine its age. The convenient characteristic of stars that allows their ages to be inferred from their *current* rotation periods, not their primordial ones, comes from the steep dependence of spin-down rate on rotation period. Observations of young clusters indicate that stellar angular momentum loss rate is proportional to the cube of the angular velocity. This means that a star spinning with high angular velocity will experience a much greater angular momentum loss rate than a slowly spinning star. For this reason, no matter the initial rotation period of a Sun-like star, after around the age of the Hyades, ($\sim$ 600 Myr) stellar rotation periods appear to converge onto a tight sequence. After this time, the age of a star can be inferred, to first order, from its mass and rotation period alone.

The relation between age, rotation period and mass has been studied in detail CITATIONS, and several different models have been developed to capture the rotation evolution of stars. Some of these models are theoretical and model angular momentum loss as a function of the properties of magnetic field and stellar wind. Others are empirical and attempt to capture the behavior of stars from a purely observational standpoint, using simple functional forms that can reproduce the data. Both types of model, theoretical and empirical, must be calibrated using observations since even the theoretical models are highly sensitive to stellar properties that are not measurable, mass-loss rate and magnetic field geometry, for example. Despite significant advances in theoretical models of stellar spin-down as well as new calibrations of empirical models, the gyrochronology relations have not yet been finalized. In particular, they suffer from a lack of suitable calibration stars at old ages and low masses. These regions of parameter space are particularly important because some evidence suggests that rotational evolution changes at old ages and low masses. For example, recent results show that old *Kepler* asteroseismic stars rotate more rapidly than expected given their age (*e.g.* Angus et al. 2015; van Saders et al. 2016). These data can be reproduced with a model that relaxes magnetic breaking at a critical Rossby number, *Ro* (the ratio of rotation period to the convective overturn timescale) of around the Solar value. As stellar rotation periods lengthen and stars cross this *Ro* threshold, they maintain a constant rotation period after that time. The gyrochronology model described in van Saders et al. (2016) and **?** includes weakened magnetic braking after stars reach the Solar *Ro* threshold.

The models developed and calibrated in Epstein and Pinsonneault (2014); **?**); van Saders et al. (2016); **?** are expensive to compute and, just as with most isochrones and stellar evolution tracks, are usually pre-computed over a grid of stellar parameters in order to perform tractable inference. In order to infer an age from these models, one would effectively perform isochrone fitting but in this case, rotation period would be added as an additional parameter and the ages inferred would therefore be more precise and accurate. We present here a complementary method that combines isochrones with an empirical gyrochronology model using a Bayesian framework. The methodology is related to the family of models described above in that both use a combination of rotation periods and other observable properties that track stellar evolution on the HR diagram in concert. A major difference is that the gyrochronology model used here is an entirely empirically calibrated one, as opposed to a physically derived one. One major advantage of using a physically motivated gyrochronology model over an empirically calibrated one is the ability to rely on physics to interpolate or extrapolate over parts of parameter space with sparse data coverage. However, rotational spin-down is a complex process that is not yet fully understood and currently no physical model can accurately reproduce all the data available. For this reason, even physically motivated gyrochronology models cannot always be used to reliably extrapolate into unexplored parameter space. Although we use a simple version of an empirical gyrochronology model in this work, which, like the physical gyrochronology models, cannot yet reproduce all the observed data, several simple modifications could be made to this model that *would* produce significant improvements. For example, including and allowing for outliers; stars with anomalously fast or slow rotation periods, could be incorporated into our model. Ultimately, the model we present here will provide a baseline against which more physically motivated models, e.g. the van Saders et al. (2016) models, can be compared.

A star like the Sun will increase in luminosity by only around a factor of two before turning off the MS. In addition, the Sun's temperature will only increase by around 100 K during its $\sim$8 billion year MS lifetime. Luminosity and temperature are not sensitive proxies for age and can also be difficult to measure with their precision highly sensitive to their distance and the amount of extincting dust along the line of sight. On the other hand, Sun's rotation period will vary by almost an order of magnitude over its MS lifetime. Stellar rotation periods are much more sensitive to age than luminosity or temperature and can be measured precisely with little dependence on distance and none on extinction. Incorporating rotation period measurements into isochrone fitting methods provides additional information that allows for much more precise age inference.

2. Method

In Bayesian statistics, combining information from different models can be relatively simple, as long as the processes being modeled, that generated the data, are independent. In this case, we are combining information that relates to the burning of hydrogen in the core (this is the process that drives the slow increase in T_{eff} and luminosity over time) with information about the magnetic braking history of a star (rotation period). We can assume that, to first order, these two processes are independent: the hydrogen fraction in the core does not affect a star's rotation period and vice versa. In practise we can never be entirely sure that two such processes are independent but, at least within the uncertainties, any dependence here is unlikely to affect our results. If this assumption is valid, then the likelihoods calculated using each model can be multiplied together.

The desired end product of this method is an estimate of the non-normalized posterior probability density function (PDF) over the age of a star,

$$p(A|\mathbf{m_x}, T_{\text{eff}}, \log g, \hat{F}, P_{rot}, \bar{\omega}), \tag{1}$$

where A is age, $\mathbf{m_x}$ is a vector of apparent magnitudes in various bandpasses (in our model $\mathbf{m_x} = [m_J, m_H, m_K]$), $\hat{F}$ is the *observed* bulk metallicity, P_{rot} is the rotation period and $\bar{\omega}$ is parallax. In order to calculate a posterior PDF over age, we must marginalize over parameters that relate to age, but are not of interest in this study: mass (M), distance (D), V-band extinction (A_V) and an *inferred* bulk metallicity, F. The marginalization involves integrating over these extra parameters,

$$p(A|\mathbf{m_x}, T_{\text{eff}}, \log g, \hat{F}, P_{rot}, \bar{\omega}) \propto \tag{2}$$
$$\int p(\mathbf{m_x}, T_{\text{eff}}, \log g, \hat{F}, P_{rot}, \bar{\omega}|A, M, D, A_V, F)\ p(A)p(M)p(D)p(A_V)p(F)dMdDdA_VdF.$$

This equation is a form of Bayes' rule,

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}, \tag{3}$$

where the likelihood of the data given the model is,

$$p(\mathbf{m_x}, T_{\text{eff}}, \log g, \hat{F}, P_{rot}, \bar{\omega}|A, M, D, A_V, F), \tag{4}$$

and the prior PDF over parameters is,

$$p(A)p(M)p(D)p(A_V)p(F). \tag{5}$$

Not all of the observables on the left of the | in the likelihood depend on all of the parameters to the right of the |. For example, rotation period, P_{rot} doesn't depend on V-band extinction, A_V. In our model, we make use of conditional independence in the problem in hand and use this to factorize the likelihood. Instead of the likelihood we wrote in equation 2, where every observable depends on every parameter, our model can be factorized as,

$$p(\mathbf{m_x}, T_{\text{eff}}, \log g, \hat{F}, \bar{\omega}, B-V | A, M, D, A_V, F)\ p(P_{\text{rot}} | A, B-V), \tag{6}$$

where we have introduced a new parameter, $B-V$, which is the $B-V$ color often used as a mass proxy in the literature. In our model $B-V$ is not measured but inferred; it is a latent parameter. We infer $B-V$ because *Kepler* stars have 2MASS photometry in J, H and K bands but do not all have directly observed B-V colors. However, the gyrochronology model we use is calibrated to B-V color, not J-K or otherwise. The probabilistic graphical model depicting the joint probability over parameters and observables is shown in figure 2. It describes the conditional dependencies between parameters (in white circles) and observables (in grey circles) with arrows leading from the causal processes to the dependent processes. For example, it is the mass, age, metallicity, extinction and distance that determines the observed spectroscopic properties (T_{eff}, $\log g$ and [Fe/H]) and apparent magnitudes (m_j, m_h and m_K). These parameters also determine the B-V color of a star. In turn, it is a star's age and B-V color that determines its rotation period. Note that, written this way, stellar rotation periods do not directly depend on stellar mass. Mass determines B-V and B-V, along with age determines rotation period. The purpose of this PGM is not to designed to depict the physical realities of stellar evolution, it is only a visual description of the structure of the model we use. It may well be that rotation period depends directly on mass and metallicity in reality, but it is more practical for us to assume that these dependencies are weak enough not to significantly affect the ages that we ultimately infer.

The factorization of the likelihood described in equation 2 and depicted in figure 2 allows us to multiply two separate likelihood functions together: one computed using an isochronal model and one computed using a gyrochronal model. The isochronal likelihood function is,

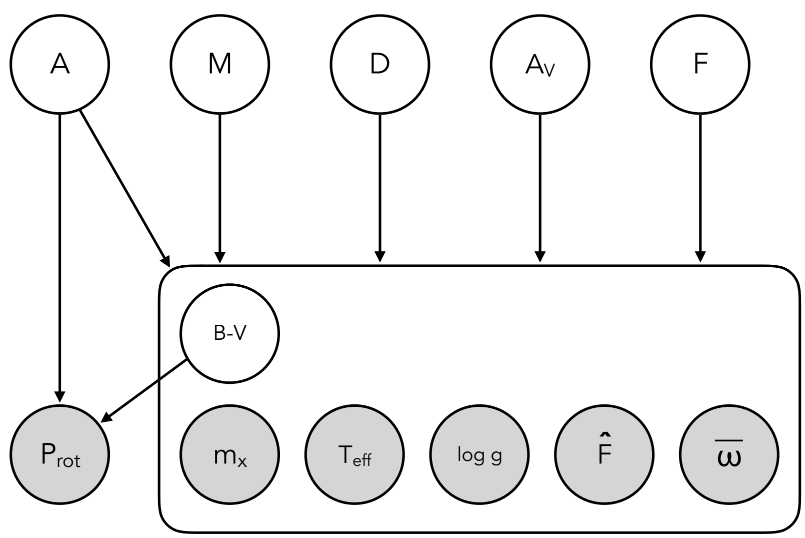
$$\mathcal{L} = p(\mathbf{m_x}, T_{\text{eff}}, \log g, \hat{F}, \bar{\omega}, B-V | A, M, D, A_V, F) \tag{7}$$

$$= \frac{1}{\sqrt{(2\pi)^n \det(\sum)}} \exp\left(-\frac{1}{2}(\mathbf{O_I} - \mathbf{I})^T \sum^{-1} (\mathbf{O_I} - \mathbf{I})\right), \tag{8}$$

where $\mathbf{O_I}$ is the vector of n observables: T_{eff}, $\log g$, $\hat{F}$, $\bar{\omega}$, m_j, m_h and m_k, $\mathbf{I}$ is the vector of corresponding predictions, calculated using an isochrone model, and $\sum$ is the covariance matrix of the set of observables. We assume there is no covariance between these observables and so this covariance matrix consists of individual parameter variances along the diagonal with zeros everywhere else. To calculate $\mathbf{I}$, the vector of predicted observables, we use the `isochrones.py` package. A detailed description of the isochronal model is provided later in this manuscript. The gyrochronal likelihood function is,

$$\mathcal{L} = p(P_{\text{rot}} | A, B-V) \tag{9}$$

Fig. 1.— A probabilistic graphical model (PGM) showing the conditional dependencies between the parameters (white nodes) and observables (gray nodes) in our model. Apparent magnitude, m_x, effective temperature, $T_{\rm eff}$, surface gravity, log g, observed bulk metallicity, $\hat{F}$, and parallax, $\bar{\omega}$ are determined by the mass, M, age, A, distance, D, extinction, A_V and bulk metallicity, F, of a star. These dependencies are indicated by arrows that start at a 'parent' node and end at the dependent observable, or 'child' node. The box drawn around some of the nodes indicates that everything inside it depends on every parameter that points toward it. For example, log g depends on A, M, D, A_V, and F. In our model, rotation period, $P_{\rm rot}$, only depends on age and a B-V color that is a latent parameter, predicted from the isochronal model. In our model, rotation period does not directly depend on distance, extinction, metallicity or mass, only age and B-V color. This PGM is a representation of the factorized joint PDF over parameters and observables which is written in equation **??**.



$$= \frac{1}{\sqrt{(2\pi)\det(\sum_P)}} \exp\left(-\frac{1}{2}(\mathbf{P_O} - \mathbf{P_P})^T \sum^{-1}(\mathbf{P_O} - \mathbf{P_P})\right), \quad (10)$$

where P_O is a 1-D vector of observed rotation periods, P_P is the vector of corresponding predicted rotation periods, calculated using the ith age and B-V values predicted by the isochronal model. The full likelihood function used in our model is the product of these two likelihood functions,

$$\mathcal{L} = \frac{1}{\sqrt{(2\pi)^n \det(\sum)}} \exp\left(-\frac{1}{2}(\mathbf{O_I} - \mathbf{I})^T \sum^{-1}(\mathbf{O_I} - \mathbf{I})\right) \quad (11)$$

$$\times \frac{1}{\sqrt{(2\pi)\det(\sum_P)}} \exp\left(-\frac{1}{2}(\mathbf{P_O} - \mathbf{P_P})^T \sum^{-1}(\mathbf{P_O} - \mathbf{P_P})\right), \quad (12)$$

A common approach to age-dating a star is to make separate age predictions using separate sets of observables. For example, if the rotation period, parallax and apparent magnitudes in a range of bandpasses were available, one could compute both the gyrochronal age of a star and its isochronal age. How these two age predictions are later combined is then a difficult choice. Is it best to average these predictions or just use the more precise of the two or the one believed to be more accurate? The methodology described here provides an objective method for combining age estimates. There is, after all only one age for each star. If the two or more dating methods do not agree on the same age then one or models must be inaccurate.

The gyrochronology model we use to predict P_P is,

$$P = A^\eta \alpha (B - V - \delta)^\beta, \quad (13)$$

where η, α, β and δ are taken from **?**. Although this gyrochronology model does not provide a good fit to all the available data, we reiterate that no single model *is* able to reproduce all the data, and that there is utility in using an extremely simple functional form. Again, this is not meant as a calibration exercise: in this paper we are more concerned with introducing a new framework which allows an improved gyrochronology model to easily be swapped in for this one.

3. Results

4. Discussion

5. Conclusion

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