$E[e^{tX}]$ ⟨ $\sum_i p_i e^{tx_i}$; $\int_{-\infty}^{+\infty} e^{tx_i} f(x_i)\,dx$

FME generatrice

Bernoulli $p^k q^{1-k}$

$m_X(t) = q + pe^t$ $E[X]$, $E[X^2]$

$\mu = E[X] = \frac{d}{dt} m_X(t)\big|_{t=0} =$

$= 0 + pe^t = pe^t\big|_{t=0} = p$

$V = E[X^2] - E[X]^2$

$E[X^2] = \frac{d}{dt} pe^t\big|_{t=0} = pe^t\big|_{t=0} = p$

$p - p^2 = p(1-p) = pq$

Binomiale $\binom{n}{k} p^k q^{n-k}$

$m_X(t) = \sum_{i=1}^{n} p_i e^{tx_i}$ Come Bernoulli ma fatto n volte

$m_X(t) = (q + pe^t)^n$

$E[X] \leftarrow \frac{d}{dt} m_X(t)\big|_{t=0}$

$E[X^2] \leftarrow \frac{d^2}{dt^2}$ //

$\frac{d}{dt} m_X(t)\big|_{t=0} = n(q+pe^t)^{n-1} pe^t\big|_{t=0}$

$= n(p+q)^{n-1} p = np$ $p+q=1$

$\frac{d}{dt}\, n(q+pe^t)^{n-1} pe^t =$

$= n(n-1)p\cancel{e^t}p\cancel{e^t} + n\cancel{(q+pe^t)^{n-1}}p\cancel{e^t} =$

$= n(n-1)p^2 + np = n^2p^2 - np^2 + np =$

$= n(np^2 - p^2 + p)$

$E[X^2] = n(np^2 - p^2 + p)$ $E[X]^2 = n^2p^2$

$n(np^2 - p^2 + p) - n^2p^2 =$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2} = np - np^2 =$

$= n(p - p^2) = n((1-p)p) = nqp$

Geometrica $p\,q^{k-1}$

$m_X(t) = \sum_{k=1}^{\infty} p q^{k-1} e^{t\cdot k} =$

$p \sum_{k=1}^{\infty} q^{k-1} e^{tk} = p \sum_{k=1}^{\infty} q^{k-1} e^t e^{-t} e^{tk} =$

$= pe^t \sum_{k=1}^{\infty} q^{k-1} e^{tk} e^{-t} =$

$= pe^t \sum_{k=1} q^{k-1} e^{t(k-1)} = pe^t \sum_{k=1}^{\infty} (qe^t)^{k-1}$

$pe^t \sum_{k=0}^{\infty} (qe^t)^k = pe^t \frac{1}{1-qe^t}$

- $E[X] = \frac{d}{dt} m_X(t)\big|_{t=0}$

$E[X^2] = \frac{d^2}{dt^2} m_X(t)\big|_{t=0}$

$\frac{pe^t}{1-qe^t} = \frac{pe^t(1-qe^t) - pe^t(-qe^t)}{(1-qe^t)^2} =$

$= \frac{p\cancel{e^t}(1-q\cancel{e^t}) + p\cancel{e^{2t}}q}{(1-q\cancel{e^t})^2} =$

$= \frac{p(1-q) + pq}{(1-q)^2} = \frac{\cancel{p} - \cancel{pq} + \cancel{pq}}{p^{\cancel{2}}}$

$= 1/p$

$\frac{d}{dt} \frac{pe^t(1-qe^t) - pe^t(-qe^t)}{(1-qe^t)^2} =$

$= \frac{\qquad}{(1-qe^t)^4}$

Poisson $\binom{n}{k} p^k q^{n-k} =$

$= \frac{n!}{k!(n-k)!} \frac{1}{n^k} (np)^k (1-p)^n \frac{1}{(1-p)^k}$ $k!$ $\to 1$ μ^k $e^{-\mu}$ $\to 1$

$n \to \infty$
$p \to 0$
$np \to \mu$

$\frac{n!}{(n-k)!} \frac{1}{n^k} \to 1$ $(np)^k \to \mu^k$

$(1-p)^n = \left(1 - \frac{np}{n}\right)^n$ $np \to \mu$ $\left(1 - \frac{\mu}{n}\right)^n = e^{-\mu}$

$$\frac{\mu^k e^{-\mu}}{k!}$$

$m_X(t) = E[e^{tX}] = \sum_{k=1}^{\infty} e^{tk} \frac{\mu^k e^{-\mu}}{k!} =$

$= e^{-\mu} \sum_{k=1}^{\infty} \frac{(e^t \mu)^k}{k!} = e^{\mu(e^t - 1)}$

$E[X] = \frac{d}{dt} m_X(t)\big|_{t=0} = e^{\mu(e^t-1)} \cdot \mu e^t\big|_{t=0} = \mu$

$E[X^2] = \frac{d}{dt} e^{\mu(e^t-1)} \cdot \mu e^t\big|_{t=0} =$

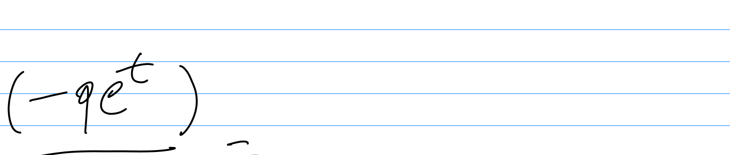
$= \cancel{e^{\mu(e^t-1)}} \mu e^t \mu e^t + \cancel{e^{\mu(e^t-1)}} \mu e^t =$

$\mu\mu + \mu = \mu^2 + \mu = \mu(1+\mu)$

$E[X^2] - E[X]^2 = \mu(1+\mu) - \mu^2 =$

$= \cancel{\mu^2} + \mu - \cancel{\mu^2} = \mu$

Esponenziale



Prob 1° successo entro t

$F_X(t) = P(X \le t)$

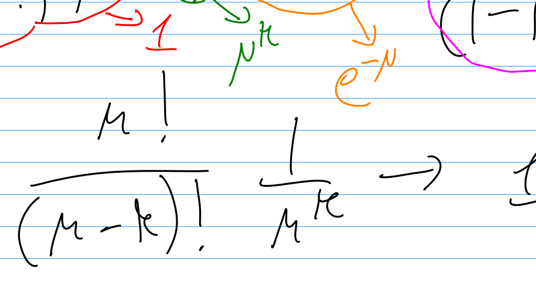
$P(X > t) = 1 - P(X \le t) = 1 - F_X(t)$

$P(X \le t) = 1 - P(X > t) = 1 - P(N_t = 0)$

$= 1 - e^{-\lambda t}$

Se si deriva si ottiene $f_X(x)$

Weibull



$Y \sim Exp(\lambda)$
$X = Y^{1/\alpha}$ $\Rightarrow X \sim Wei(\alpha, \lambda)$

$F_X(x) = P(X \le x) = P(Y^{1/\alpha} \le x) =$

$= P(Y \le x^\alpha) = F_Y(x^\alpha) = 1 - e^{-\lambda x^\alpha}$

Derivando si ottiene

Standardizzazione la Normale $\mu = 0$, $\sigma^2 = 1$

$Z = \frac{X - \mu}{\sigma}$ $X = Z\sigma + \mu$
$dx = dz \cdot \sigma$

Integriamo la densità e otteniamo la standard