$y'' - 2y' - 3y = e^x(\cos x - 3\sin x)$ $1\, e^x \cos x \quad -3 \sin x$

- $y'' - 2y' - 3y = e^x \cos x$ $\quad y$
- $y'' - 2y' - 3y = e^x \sin x$ $\quad z$

$1 \cdot y + (-3) z$

$y'' - 2y' - 3y = e^x \cos x$

$y_1(x) = e^{-3x} \quad y_2(x) = e^x$

$f(x) = e^x \cos(x)$ $\quad e^x \cos x = Re\left[e^{(1+i)x}\right]$

$e^{(a+ib)x} = e^{ax}\left[\cos(bx) + i \sin(bx)\right]$

$\boxed{Re = e^{ax}\cos(bx)}$ $\quad Im = e^{ax}\sin(bx)$

$y'' - 2y' - 3y = Re\left[e^{(1+i)x}\right]$

$y(x) = Re\, w(x)$

$\Downarrow$

$w'' - 2w' - 3w = e^{(1+i)x}$

$\lambda = 1+i$ non rad.

$\bar{w}(x) = Q(x) e^{\lambda x} = a e^{\lambda x} = a e^{(1+i)x}$

$\bar{w}'(x) = a(1+i) e^{(1+i)x}$

$\bar{w}''(x) = a(1+i)^2 e^{(1+i)x}$

$a(1+i)^2 \cancel{e^{(1+i)x}} - 2a(1+i)\cancel{e^{(1+i)x}} - 3a\cancel{e^{(1+i)x}} = \cancel{e^{(i+1)x}}$

$a(\cancel{1} + 2i \cancel{-1}) - 2a - 2ai - 3a = 1$

$2ai - 2a - 2ai - 3a = 1$

$-5a = 1 \quad a = -\frac{1}{5}$

$\bar{w}(x) = -\frac{1}{5} e^{(1+i)x}$ $\quad Re\left[e^{(1+i)x}\right]$

$y(x) = Re\, \bar{w}(x)$ $\quad \boxed{-\frac{1}{5} e^x \cos x}$

$\boxed{y(x) = -\frac{1}{5} e^x \cos x}$ $\quad e^x \cos x$

$y'' - 2y' - 3y = e^x \sin x$ $\quad e^{(1+i)x}$

$\Downarrow$

$w'' - 2w' - 3w = e^{(1+i)x}$

$\bar{w}(x) = -\frac{1}{5} e^{(1+i)x}$ $\quad -\frac{1}{5} e^x \sin x$

$y(x) = Im\, \bar{w}(x)$ $\quad y(x) = -\frac{1}{5} e^x \sin x$

$y(x) = -\frac{1}{5} e^x \cos x$

$z(x) = -\frac{1}{5} e^x \sin x$

$1 \cdot y + (-3) z$

$\bar{y}(x) - \frac{1}{5} e^x \cos x + \frac{3}{5} e^x \sin x$

$y_1(x) = e^{-3x} \quad y_2(x) = e^x$

$y(x) = -\frac{1}{5} e^x \cos x + \frac{3}{5} e^x \sin x + k_1 e^{-3x} + k_2 e^x$