Eq lineari 1° ordine

$$y' = -a(x)y + g(x)$$

1) considero EOA $\quad y' = -a(x)y$

soluzioni: $\quad Ke^{-A(x)} \qquad A(x) = \int a(x)\,dx$

2) Considero la somma

$$\bar{y} + Ke^{-A(x)}$$

dove $\bar{y}$ è soluzione di E.C.

3) Considero $\quad \bar{y} = \gamma(x)\, e^{-A(x)}$

4) trovo $\gamma(x)$ $\qquad \gamma(x) = \int g(x)\, e^{A(x)}\,dx$

5) Trovato $\bar{y}$ e EOA $\left[+Ke^{-A(x)}\right]$

$$y(x) = \bar{y} + Ke^{-A(x)}$$

Esempio

$$y' = -\frac{2x}{x^2+1} + \frac{1}{x(x^2+1)}$$

$$a(x) = \frac{2x}{1+x^2}$$

$$g(x) = \frac{1}{x(x^2+1)}$$

Passo 1) $y' = -\frac{2x}{x^2+1}$

soluzioni: $K e^{-A(x)}$ $\quad A(x) = \int \frac{2x}{1+x^2} = \ln(x^2+1)$

$$K e^{-\ln(x^2+1)} = K \frac{1}{x^2+1}$$

Passo 2) $\bar{y} + K \frac{1}{x^2+1}$

3) $\bar{y} = \gamma(x)\, e^{-A(x)} = \gamma(x) \frac{1}{x^2+1}$

4) $\gamma(x) = \int g(x)\, e^{A(x)}\, dx = \int \frac{1}{x(x^2+1)} (x^2+1)\, dx = \ln|x|$

5) $\bar{y}(x) = \ln|x| \cdot \frac{1}{x^2+1}$ $\quad g(x)$

$$\text{Sol(EDA)} = K \frac{1}{x^2+1}$$

Somma

$$y(x) = \ln|x| \cdot \frac{1}{x^2+1} + K \frac{1}{x^2+1}$$

✓