Limiti Notevoli e Forme Indeterminate

Limiti per $x \to 0$ di $f : \mathbb{R} \to \mathbb{R}$:

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$$

$$\lim_{x \to 0} \frac{(1+x)^{\alpha} - 1}{x} = \alpha \quad \alpha \neq 0$$

$$\lim_{x \to 0^+} x^x = 1$$

$$\lim_{x \to 0^+} x^{\frac{1}{x}} = 0$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log a$$

Funzioni logaritmiche:

$$\lim_{x \to 0^+} x^b \log x = 0 \quad \forall b > 0$$

$$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}$$

Funzioni trigonometriche:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\arctan x}{x} = 1$$

Limiti per $x \to \infty$ di $f : \mathbb{R} \to \mathbb{R}$:

$$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to +\infty} \frac{a^x}{x^b} = +\infty \quad \forall a > 1, b > 0$$

$$\lim_{x \to -\infty} a^x |x|^b = 0 \quad \forall a > 1, b > 0$$

$$\lim_{x \to +\infty} \sqrt[x]{x} = \lim_{x \to +\infty} x^{\frac{1}{x}} = 1$$

$$\lim_{x \to \infty} \frac{e^x}{x^b} = \infty$$

$$\lim_{x \to +\infty} \frac{\log x}{x^b} = 0 \quad \forall b > 0$$

$$\lim_{x \to +\infty} \frac{\log x}{e^x} = 0$$

Forme indeterminate:

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^{∞}, 0^0, $(\pm\infty)^0$, $+\infty - \infty$.

Confronto di infiniti e infinitesimi:

Se

$$\lim_{n \to \infty} |a_n| = \infty$$

Allora:

$$\log_a n \leq n^b \leq c^n \leq n! \leq n^n \quad \text{con } a, b, c > 1$$

Se

$$\lim_{n \to \infty} |a_n| = 0$$

Allora:

$$\frac{1}{\log_a n} \geq \frac{1}{n^b} \geq \frac{1}{c^n} \geq \frac{1}{n!} \geq \frac{1}{n^n} \quad \text{con } a, b, c > 1$$