Assignment-3a Solution

Textbook Problems

1.6 Determine whether or not each of the following signals is periodic:

(a) $x_1(t) = 2e^{j(t+\pi/4)}u(t)$

(b) $x_2[n] = u[n] + u[-n]$

(c) $x_3[n] = \Sigma_{k=-\infty}^{\infty}(\delta[n-4k] - \delta[n-1-4k])$

a) $x_1(t) = 2e^{j(t+\frac{\pi}{4})}u(t)$

By definition, if $x_1(t)$ is perioidc, it much be $x_1(t) = x_1(t+T)$, where *T* is the peroid which can be negtive and positime. Since $x_1(t) = 0 \ \ for \ \ t < 0$,

so it is not periodic.

b) $x_2[n] = u[n] + u[-n]$ for composite signals, each one has a period N in discrete time singal

Since $x_2[n] = 1 \ \ for \ \ all \ \ n$. Therefore, it is periodic with a fundament period of infinity, or with a fundamental frequency of zero.

c) $x_3[n] = \sum_{k=-\infty}^{\infty}\{\delta[n-4k] - \delta[n-1-4k]\}$

$x_3[n]$ can be ploted as the image below:

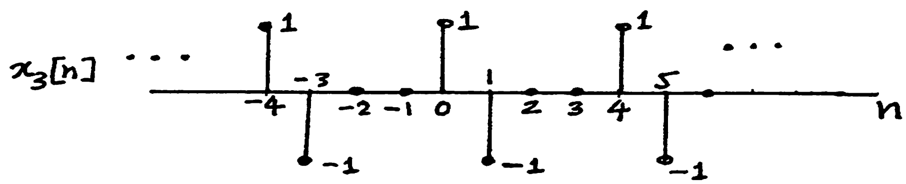


Figure S1.6

Therefore, it is peridic with a fundamental period of 4.

1.16 Consider a discrete-time system with inout x[n] and output y[n]. The input-output relationshipp for this system is: $y[n] = x[n]x[n-2]$. Answer the follwing questions:

(a) Is the sysem memoryless?

Answer: *A system is said to be* ***memoryless*** *if its output for each value of the independent variable at a given time is dependent on the*

input at only that same time. Because y[n] depends on past values of x[n], i.e. x[n-2] is the value x[n] delayed by 2.

(b) Determine the output of the system when the input is $A\delta[n]$, where A is an real or complex number.

Answer: *The output of the system will be* $y[n] = \delta[n]\delta[n-2] = 0$

(c) Is the system invertible?

Answer: *From the result of (b), we conclude that the system out is always zero for inputs of the form* $\delta[n-k], k \in I$ *(Integer). Therefore the system is not invertible.*

1.22 A discrete-time signal is shown in Fighure P1.22. Sketch and label carefully each of the following signals:

(a) x[n-4]

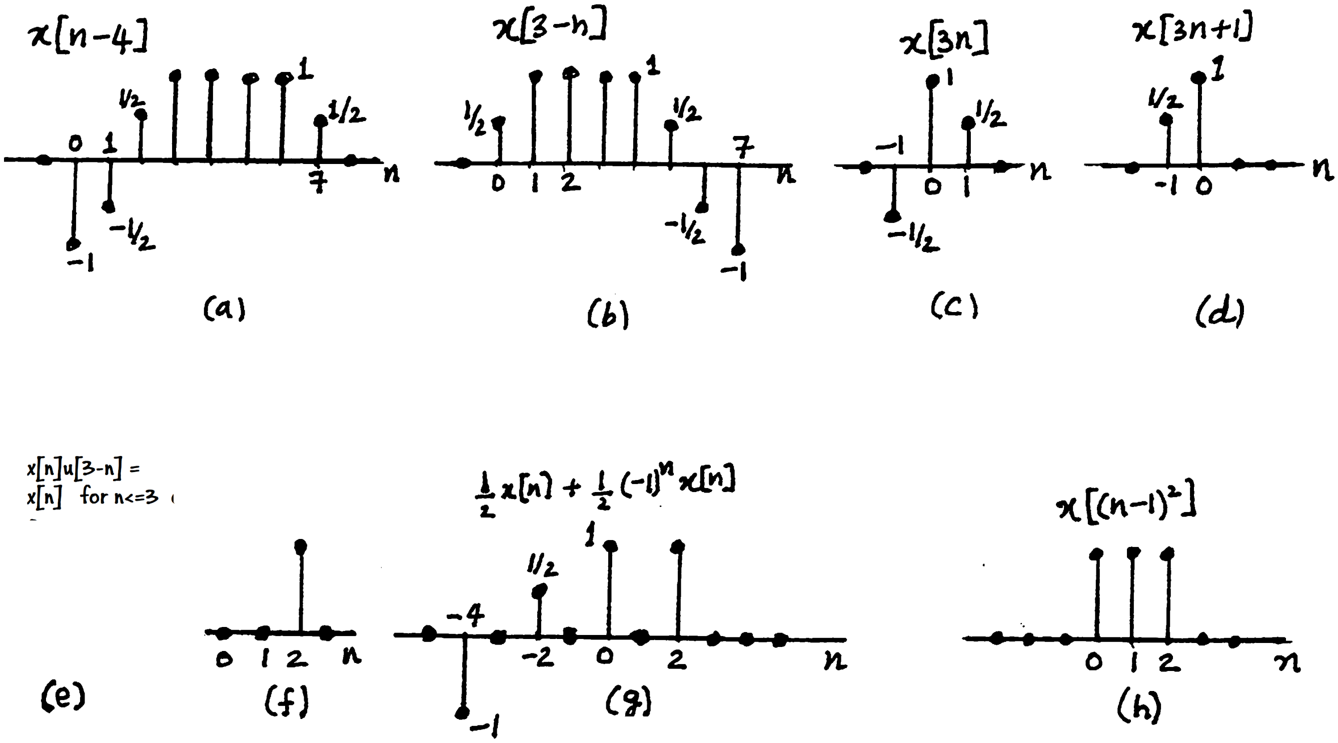
(b) x[3-n]

(c) x[3n]

(d) x[3n+1]

(e) x[n]u[3-n]

(f) $x[n-2]\delta[n-2]$



Sketchs for Problem 1.22

Ans **wer:**

1.27 In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

1. **Memoryless**
2. **Time invariant**
3. **Linear**
4. **Causal**
5. **Stable**

Determine which of these properties hold and which do not hold. Justify at least one of which properties **Does Not** hold by a counter example. In each example, y(t) denote the system output and x(t) is the system input.

(a) $y(t) = x(t-2) + x(2-t)$

(b) $y(t) = cos(3t)x(t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

Answer:

(a) The system is linear and stable. **It is not memoryless**, because y(t) depends on past values of x[t],

(b) The system is Memoryless, Linear, Causal and Stable. **It is not Invertibility.**

A system is said to be invertible if distinct inputs **lead to** distinct outputs. For example, let x(t) =u(t), then $y(t) = cos(3t),\ for\ \ t \geq 0$. Therefore $y(t) = cos(3t) = cos(3(t + 2\pi/3))$, Therefore this is a counter-example.

(c) The system is Linear. **It is not Stable.**

A system is said to be stable if a system is one in which small inputs lead to responses that do not diverge. For example, let x(t) =u(t), then $y(t) = \int_{-\infty}^{2t} d\tau = \tau \mid_{-\infty}^{2t} = \infty$, Therefore it is not stable.

1.31 In this problem, w illustrate one of the most important consequences of the properties of linearity and time invariance. Speciffically, once we know the response of a linear system or a linear time-invariant (LTI) system to a singal input or the response to several inputs, we directly compute the responses to many other input signals. Much of the remindr of this book deals with a thorough expoitation of this fact in order to develop results and techniques for analyzig and synthesizing LTI systems.

(a) Consider an LTI system whose response to the signal $x_1(t)$ in the Figure P1.31(a) is the signal $y_1(t)$ illustrated in Figure P1.31(b). Determine and sketch arefully the response of the system to the input $x_2(t)$ depicted in Figure P1.3(c).

Answer: Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in the sketch below.

(b) Determine and sketch the response of the system consider in part (a) to the input $x_3(t)$ shown in Figure P1.31(d).

Answer: Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in the sketch blow.