Assignment 5a (chapter2-part2)

2.16

For each of the following statements, determine whether ti is true or false (justify your answers).

(a). If $x[n] = 0 \ \ forn < T_1 \ \ and \ \ h[n] = 0 \ \ for \ \ n < N_2$, then $x[n] * h[n] = 0 \ \ for \ \ n < N_1 + N_2$.

TRUE.

We can see than h[n] has to move toward the right by N1+N2 units before x[n] and h[n] overlap with the non-zero areas. So that $x[n] * h[n] = 0$ for $n < N_1 + N_2$

(b). If $y[n] = x[n] * h[n]$, then $y[n-1] = x[n-1] * h[n-1]$.

FALSE.

Consider:

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

From this,

$$y[n-1] = \sum_{k=-\infty}^{\infty} x[k]h[n-1-k]$$

$$y[n] = x[n] * h[n-1]$$

This shows that the given statement is false.

(c). If $y[n] = x[n] * h[n]$, then $y(-t) = x(-t) * h(-t)$.

TRUE.

Consider:

$$y(t) = x(t) * h(t) = \int_{\infty}^{-\infty} x(\tau)h(t-\tau)d\tau$$

From this,

$$y(-t) = \int_{-\infty}^{\infty} x(\tau)h(-t-\tau)d\tau$$

$$y(-t) = \int_{-\infty}^{\infty} x(-\tau)h(-t+\tau)d\tau$$

$$y(-t) = x(-t) * h(-t)$$

This shows that the given statement is true.

(d). If $x(t) = 0 \ \ for \ \ t > T_1 \ \ and \ \ h(t) = 0 \ \ for \ \ t > T_2$, then $x(t) * h(t) = 0 \ \ for \ \ t > T_1 + T_2$.

TRUE.

This can be explained by considering:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

In Figure S2.16, we plot $x(\tau)$ and $h(t-\tau)$ under the assumptions that (1) x(t) = 0 for t > T_1 and (2) h(t) = 0 for t > T_2. Clearly, the product $x(\tau)h(t-\tau)$ is zero if $t - T_2 > T_1$. Therefore, $y(t) = 0$ for $t > T_1 + T_2$.

2.18

Consider a causal LTI system whose input x[n] and output y[n] are leated by the difference equation

$$y[n] = \frac{1}{4}y(n-1) + x[n]$$

Determine $y[n] \ \ if \ \ x[n] = \delta[n-1]$

Since the system is casual, $y[n] = 0$ fpr $n < 1$. Now,

$$y[1] = \frac{1}{4}y[0] + x[1] = 0 + 1 = 1$$

$$y[2] = \frac{1}{4}y[1] + x[2] = \frac{1}{4} + 0 = \frac{1}{4}$$

$$y[3] = \frac{1}{4}y[2] + x[3] = \frac{1}{16} + 0 = \frac{1}{16}$$

... $y[m] = (\frac{1}{4})m - 1$ ...

Therefore,

$$y[n] = (\frac{1}{4})^{n-1}u[n-1]$$

2.19

Consider the cascade of the following two system S_1 *and* S_2, as depicted in Figure P2.19:

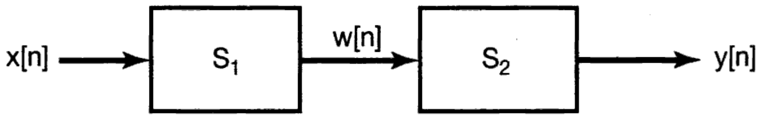


Figure P2.19

S_1 : causal LTI,

$w[n] = \frac{1}{2}w[n-1] + x[n]$;

S_2 : causal LTI,

$y[n] = \alpha y[n-1] + \beta w[n]$.

The difference quation relating x[n] and y[n] is:

$y[n] = \frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$.

(a). Determine α *and* β.

$y[n] = \alpha y[n-1] + \beta w[n]$

$w[n] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1]$

$w[n-1] = \frac{1}{\beta}y[n-1] = \frac{\alpha}{\beta}y[n-1]$

$w[n] = \frac{1}{2}w[n-1] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2]$

$\frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2] = x[n]$

$y[n] = (\alpha + \frac{1}{2})y[n-1] - \frac{\alpha}{2}y[n-2] + \beta x[n]$

$\alpha = \frac{1}{4},\ \beta = 1$

(b). Show the impulse response of the cascade connection of S_1 *and* S_2.

$w[n] = \frac{1}{2}w[n-1] + x[n]$ and $y[n] = \frac{1}{4}y[n-1] + w[n]$

$h_1[n] = (\frac{1}{2})^n u[n]$

$h_2[n] = (\frac{1}{4})^n u[n]$

$$h[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k]$$

$$h[n] = \sum_{k=0}^{\infty} (\frac{1}{2})^k (\frac{1}{4})^{n-k} u[n-k]$$

$$h[n] = \sum_{k=0}^{\infty} (\frac{1}{2})^k (\frac{1}{4})^{n-k} = \sum_{k=0}^{n} (\frac{1}{2})^{2(n-k)}$$

$$h[n] = [2(\frac{1}{2})^n - (\frac{1}{4})^n] u[n]$$

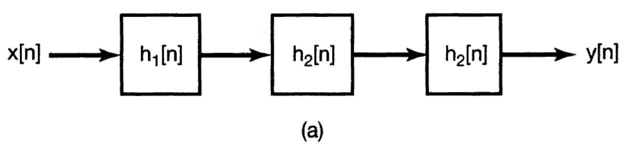
2.24

Consider the cascade interconnection of three causal LTI systems, illustatred in Figure P2.24(a).

The impulse resonse $h_2[n]$ is

$h_2[n] = u[n] - u[n-2]$,

and the overall impulse response is as shown in Figure P2.24(b).



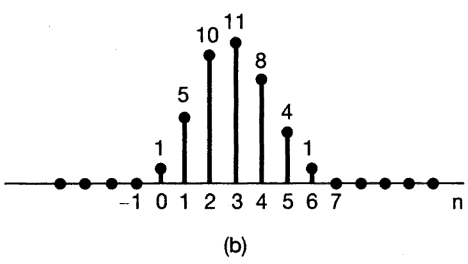


Figure P2.24

(a). Find the impulse response $h_1[n)$

$h_2[n] = \delta[n] + \delta[n-1]$

$h_2[n] * h_2[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$h[n] = h_1[n] * [h_2[n] * h_2[n]]$

$h[n] = h_1[n] + 2h_1[n-1] + h_1[n-2]$

$h[0] = h_1[0] => h_1[0] = 1$

$h[1] = h_1[1] + 2h_1[0] => h_1[1] = 3$

$h[2] = h_1[2] + 2h_1[1] + h_1[0] => h_1[2] = 3$

$h[3] = h_1[3] + 2h_1[2] + h_1[1] => h_1[3] = 2$

$h[4] = h_1[4] + 2h_1[3] + h_1[2] => h_1[4] = 1$

$h[5] = h_1[5] + 2h_1[4] + h_1[3] => h_1[5] = 0$

$h_1[n] = 0$ for $n < 0$ and $n \geq 5$

(b). Find the response of the overall system to the imput $x[n] = \delta[n] - \delta[n-1]$.

$y[n] = x[n] * h[n] = h[n] - h[n-1]$

2.31

Consider the LTI system initially at rest and described by the difference equation

$y[n] + 2y[n-1] = x[n] + 2x[n-2]$

Find the response of this system to the input despicted in Figure P2.31 by solving the difference equation recursively.

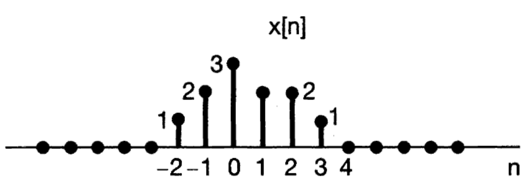


Figure P2.31

$y[n] = 0$ for $n < -2$

$y[n] = x[n] + 2x[n-2] - 2y[n-1]$

$y[-2]=1, y[-1]=0, y[0]=5, y[1]=-4, y[4]=56, y[5]=-110(-2)^{n-5}$ for $n \geq 5$