Assignment 4a (Main textbook, chapter2-part1)

2.1

Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ *and* $h[n] = 2\delta[n+1] + 2\delta[n-1]$,

Compute and plot each of the following convolutions:

(a). $y_1[n] = x[n] * h[n]$

(b). $y_2[n] = x[n+2] * h[n]$

(c). $y_3[n] = x[n] * h[n+2]$

a)

$$y_1[n] = x[n] * h[n] = \sum_{n=-\infty}^{\infty} x[k]h[n-k]$$

$$y_1[n] = \sum_{n=-\infty}^{\infty} x[k]2\delta[n-k+1] + 2\delta[n-k-1]$$

$$y_1[n] = 2\sum_{n=-\infty}^{\infty} x[k]\delta[n+1-k] + 2\sum_{n=-\infty}^{\infty} x[k]\delta[n-1-k])$$

$$y_1[n] = 2x[n+1] + 2x[n-1]$$

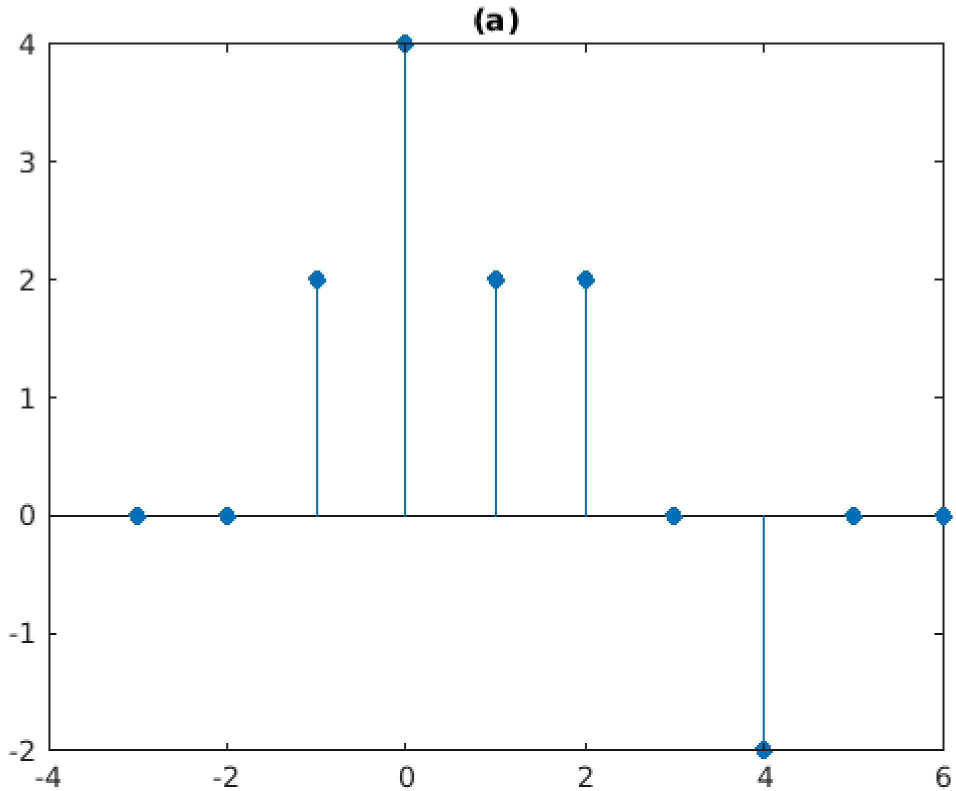
$$y_1[n] = 2\delta[n+1] + 2\delta[n+1-1] - \delta[n+1-3] + 2\delta[n-1] + 2\delta[n-1-1] - \delta[n-1-3]$$

$$y_1[n] = 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4]$$

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

```
clf;
a = [0.0 0.0 2.0 4.0 2.0 2.0 0.0 -2.0 0.0 0.0];
n1 = -3:6;

%subplot(3,1,1);
stem(n1,a,'filled');
title('(a)');
```



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REVISION

From the figure, since h[n] only has non zero values h[-1] and h[1], we can see that the above convolution sum can be reduced to $y[n] = k[-1]x[n+1] + h[1]x[n-1] = 2x[n+1] + 2x[n-1]$

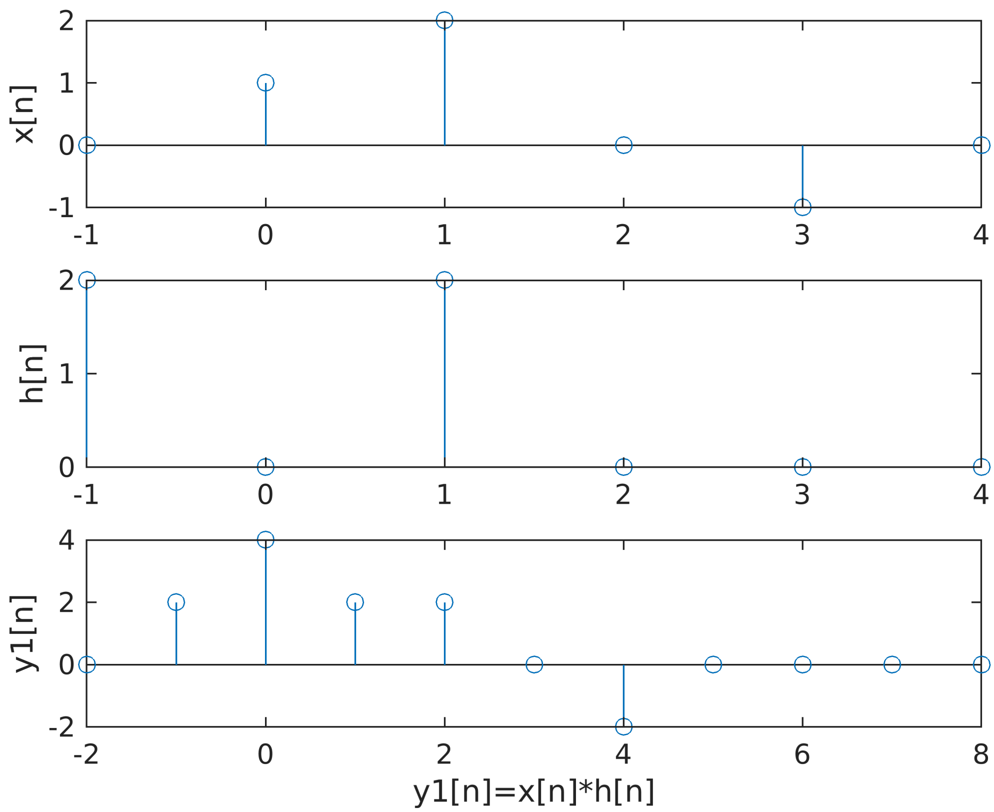
=> $y_1[n] = 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4]$

$= 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$

```
clf;

n=[-1:4];
x=0*n;
x(1)=0;
x(2)=1;
x(3)=2;
x(4)=0;
x(5)=-1;
x(6)=0;
subplot(3, 1, 1);
stem(n, x);
ylabel('x[n]');
h=n*0;
h(1)=2;
```

```
h(2)=0;
h(3)=2;
subplot(3, 1, 2);
stem(n, h);
ylabel('h[n]');
y1=conv(x, h);
ny=[-2:8];
subplot(3, 1, 3);
stem(ny, y1);
xlabel('y1[n]=x[n]*h[n]');
ylabel('y1[n]');
```



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b)

$$y_2[n] = x[n+2] * h[n] = \sum_{n=-\infty}^{\infty} x[k+2]h[n-k]$$

$$y_2[n] = \sum_{n=-\infty}^{\infty} x[k+2]2\delta[n-k+1] + 2\delta[n-k-1]$$

$$y_2[n] = 2\sum_{n=-\infty}^{\infty} x[k+2]\delta[n+1-k] + 2\sum_{n=-\infty}^{\infty} x[k+2]\delta[n-1-k]$$

$y_2[n] = 2x[n+3] + 2x[n+1]$

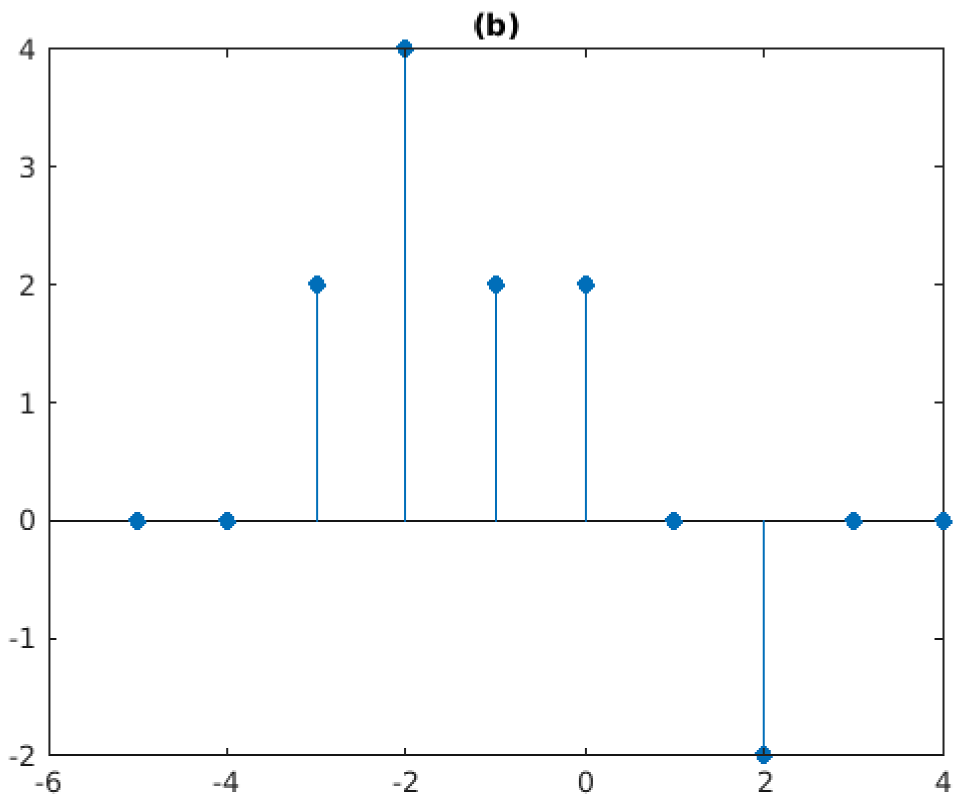
$y_2[n] = 2\delta[n+3] + 2\delta[n+3-1] - \delta[n+3-3] + 2\delta[n+1] + 2\delta[n+1-1] - \delta[n+1-3]$

$y_2[n] = 2\delta[n+3] + 4\delta[n+2] - 2\delta[n] + 2\delta[n+1] + 4\delta[n] - 2\delta[n-2]$

$y_2[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]$

```
clf;
b = [0.0 0.0 2.0 4.0 2.0 2.0 0.0 -2.0 0.0 0.0];
n2 = -5:4;

%subplot(3,1,2);
stem(n2,b,'filled');
title('(b)');
```



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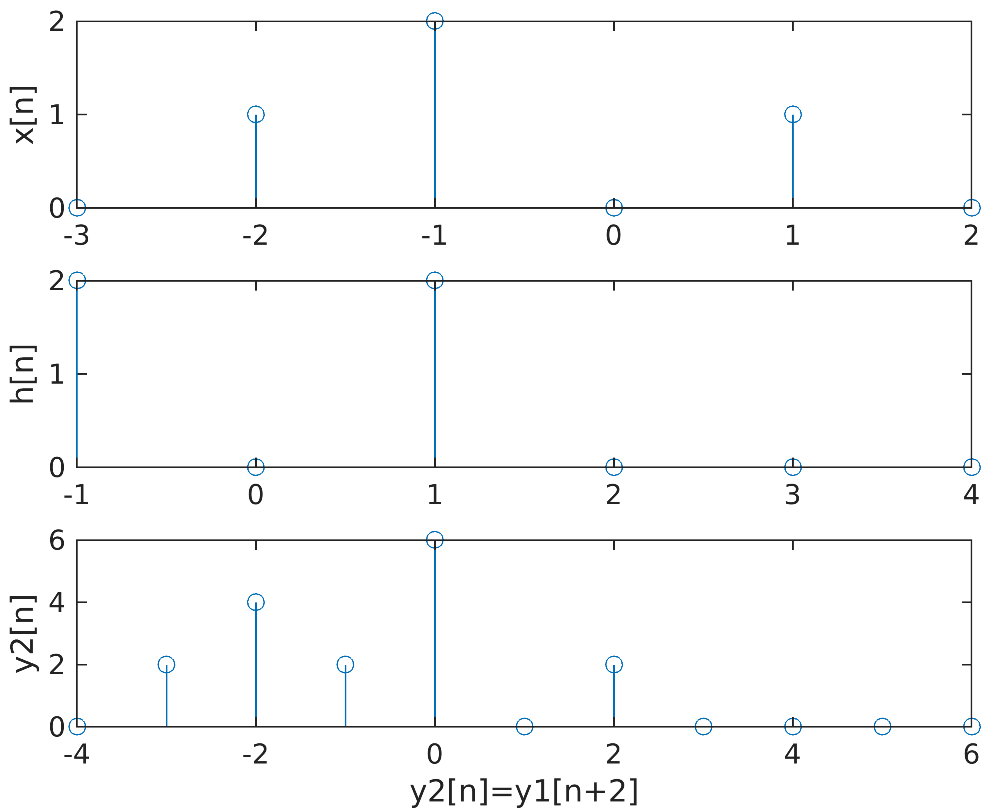
REVISION

Given $y_2[n] = x[n+2] * h[n]$ and assume the system is LTI, then comparing with $y_1[n] = x[n] * h[n]$, the input x[n +2] is x[n] advanced by 2. So that $y_2[n] = y_1[n+2]$

```
clf;
nx=[-3:2];
```

```
x=0*n;
x(1)=0;
x(2)=1;
x(3)=2;
x(4)=0;
x(5)=1;
x(6)=0;
subplot(3, 1, 1);
stem(nx, x);
ylabel('x[n]');
nh=[-1:4];
h=n*0;
h(1)=2;
h(2)=0;
h(3)=2;
subplot(3, 1, 2);
stem(nh, h);
ylabel('h[n]');
y2=conv(x, h);
ny2=[-4:6];
subplot(3, 1, 3);
stem(ny2, y2);
xlabel('y2[n]=y1[n+2]');
ylabel('y2[n]');
```



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c)

$$y_3[n] = x[n] * h[n+2] = \sum_{n=\infty}^{\infty} x[k]h[n-k+2]$$

$$y_3[n] = \sum_{n=\infty}^{\infty} x[k]2\delta[n-k+2+1] + 2\delta[n-k+2-1]$$

$$y_3[n] = 2\sum_{n=\infty}^{\infty} x[k]\delta[n+3-k] + 2\sum_{n=\infty}^{\infty} x[k]\delta[n+1-k]$$

$$y_3[n] = 2x[n+3] + 2x[n+1]$$

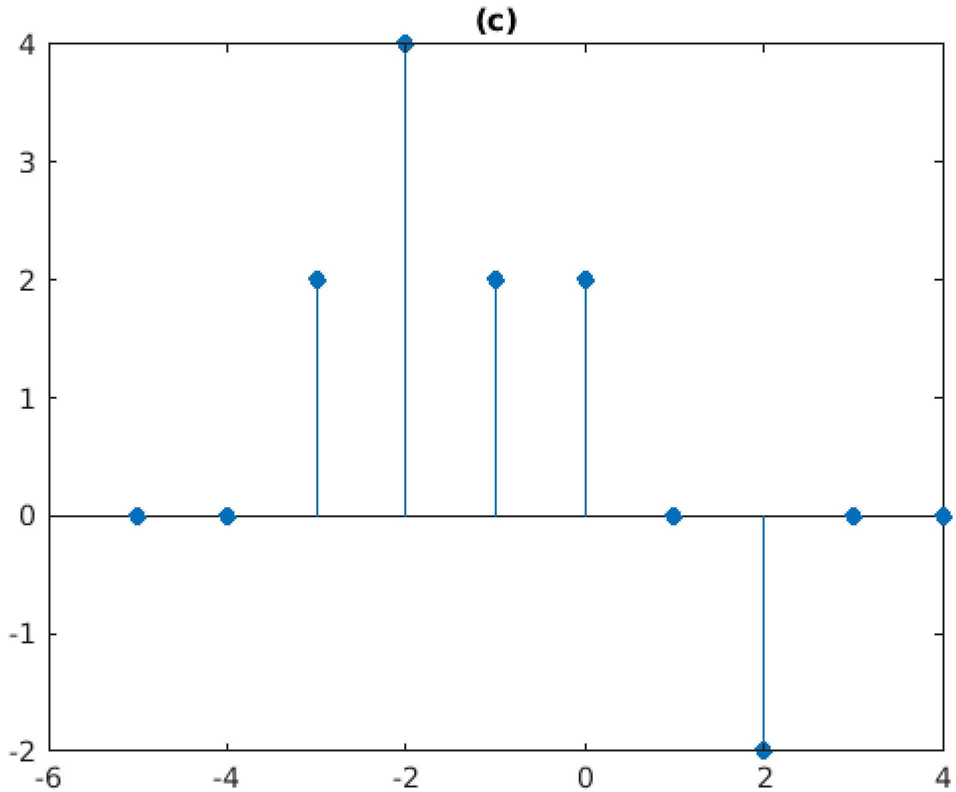
$$y_3[n] = 2\delta[n+3] + 2\delta[n+3-1] - \delta[n+3-3] + 2\delta[n+1] + 2\delta[n+1-1] - \delta[n+1-3]$$

$$y_3[n] = 2\delta[n+3] + 4\delta[n+2] - 2\delta[n] + 2\delta[n+1] + 4\delta[n] - 2\delta[n-2]$$

$$y_3[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]$$

```
clf;
c = [0.0 0.0 2.0 4.0 2.0 2.0 0.0 -2.0 0.0 0.0];
n3 = -5:4;

%subplot(3,1,3);
stem(n3,c,'filled');
title('(c)');
```



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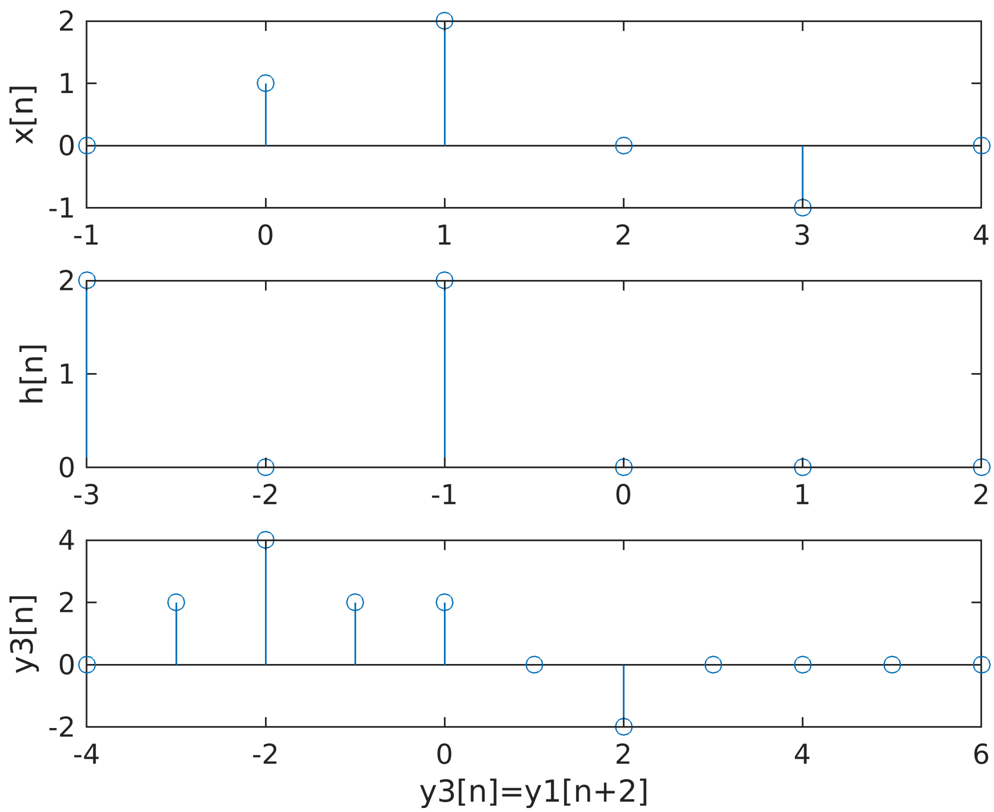
REVISION

Assume the system is LTI, comparing to $y_1[n] = x[n] * h[n]$, the input h[n+2] is h[n] advanced by 2 so that $y-3[n] = y_1[n+2]$

```
clf;
nx=[-1:4];
x=0*n;
x(1)=0;
x(2)=1;
x(3)=2;
x(4)=0;
x(5)=-1;
x(6)=0;
subplot(3, 1, 1);
stem(nx, x);
ylabel('x[n]');
nh=[-3:2];
h=n*0;
h(1)=2;
h(2)=0;
h(3)=2;
subplot(3, 1, 2);
stem(nh, h);
```

```
ylabel('h[n]');
y3=conv(x, h);
ny3=[-4:6];
subplot(3, 1, 3);
stem(ny3, y3);
xlabel('y3[n]=y1[n+2]');
ylabel('y3[n]');
```



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2.4

Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1 & 3 \le n \le 8 \\ 0 & \text{otherwise} \end{cases}$$

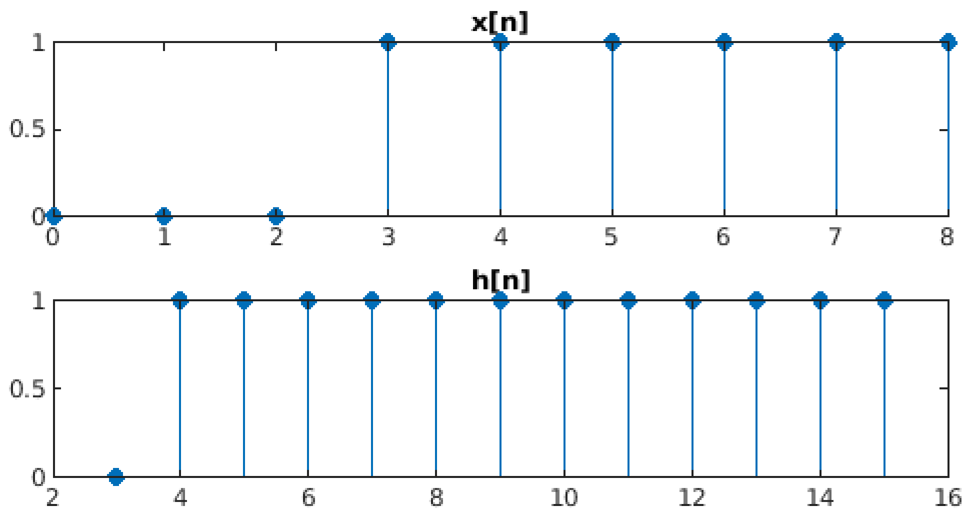
$$h[n] = \begin{cases} 1 & 4 \le n \le 15 \\ 0 & \text{otherwise} \end{cases}$$

```
clf;
x = [0.0 0.0 0.0 1.0 1.0 1.0 1.0 1.0 1.0];
n1 = 0:8;

subplot(3,1,1);
stem(n1,x,'filled');
title('x[n]');

h = [0.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0];
n2 = 3:15;

subplot(3,1,2);
stem(n2,h,'filled');
title('h[n]');
```



$y[n] = x[n] * h[n]$. $y[n]$ starts from $n_1 + n_2 = 3 + 4 = 7$ to $n_1 + n_2 = 8 + 15 = 23$

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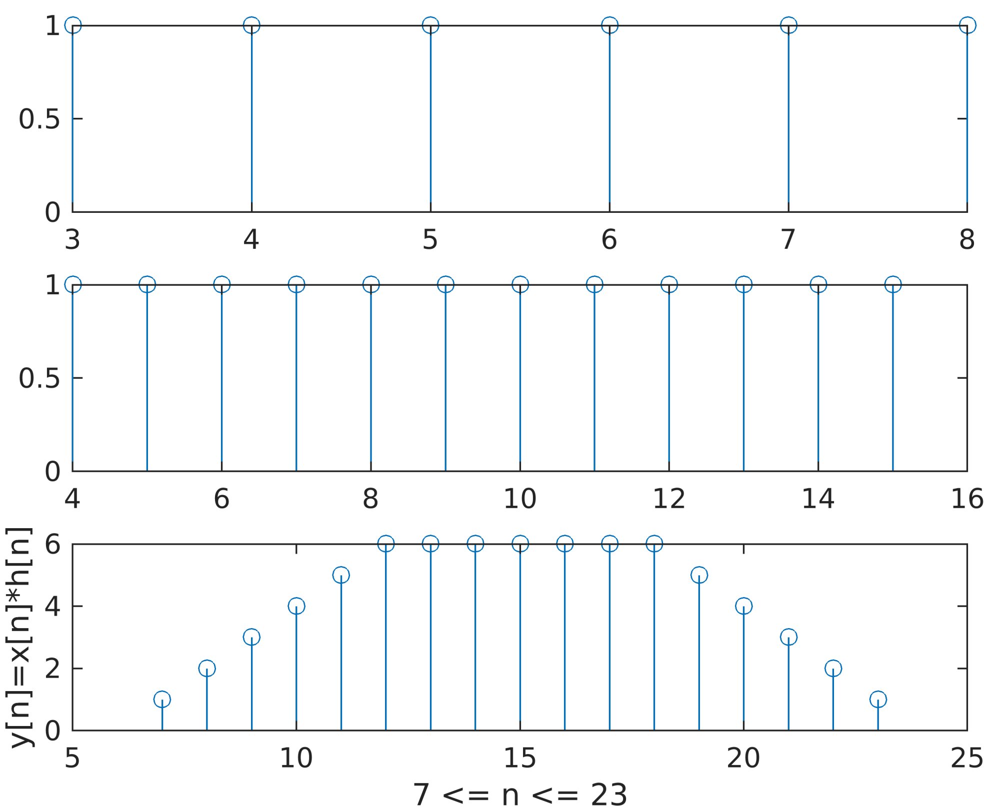
REVISION

From the figure, we see that the above summaration can be reduced to:

$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$ this gives y[n] { n-6, 7 <=n<=11; 6, 12 <=n<=18; 24-n, 19<=n<=23; 0, otherwise

```
clf;
nx=[3:8];
x=nx*0;
x(1:end)=1;
subplot(3, 1, 1);
stem(nx, x);
nh=[4:15];
h=nh*0;
h(1:end)=1;
subplot(3, 1, 2);
stem(nh, h);
ny=[7:23];
y=ny*0;
y=conv(x, h);
subplot(3, 1, 3);
stem(ny, y);
xlabel('7 <= n <= 23');
ylabel('y[n]=x[n]*h[n]');
```



Results Analysis: The analytical results derived mathematically are in agreement with the results from MATLAB program computation and plots

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2.8

Determine and sketch the convolution of the following two signals

$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-1, & 1 < t \le 2 \\ 0, & elsewhere \end{cases}$$

$$h(t) = \delta(t+2) + 2\delta(t+1)$$

$$y(t) = x(t) * h(t)$$

$$y(t) = x(t) * \{\delta(t+2) + 2\delta(t+1)\}$$

$$y(t) = x(t) * \delta(t+2) + 2x(t) * \delta(t+1)$$

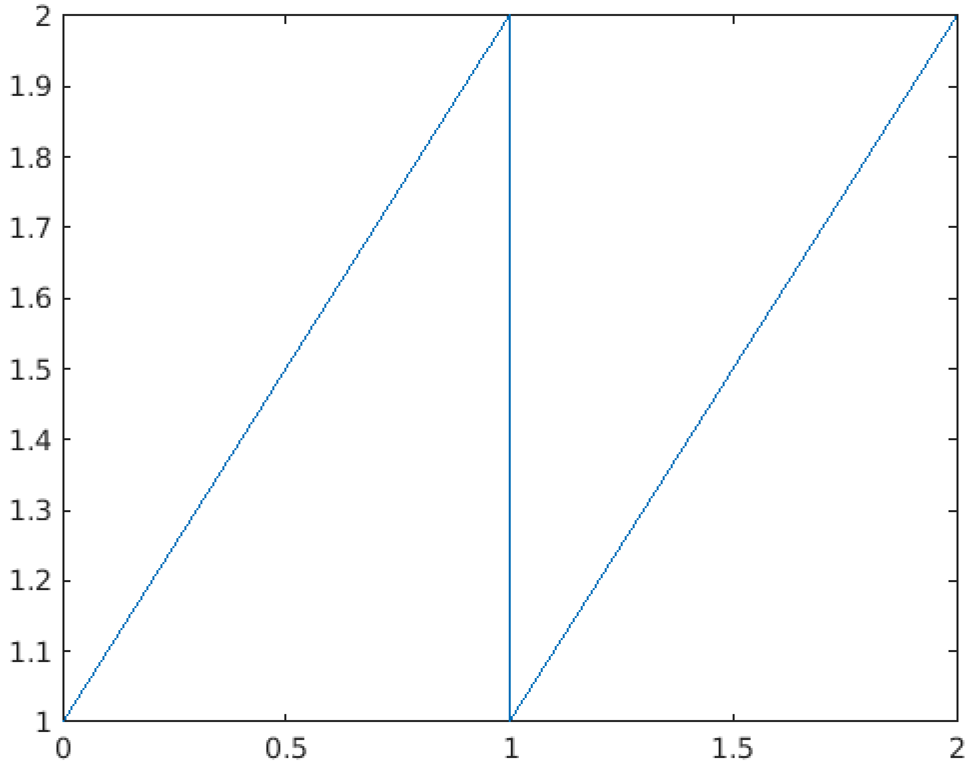
$$y(t) = x(t+2) + 2x(t+1)$$

x(t) at t=0, x(t) = 1. At t=1, x(t)=2. This is a linearly increasing function.

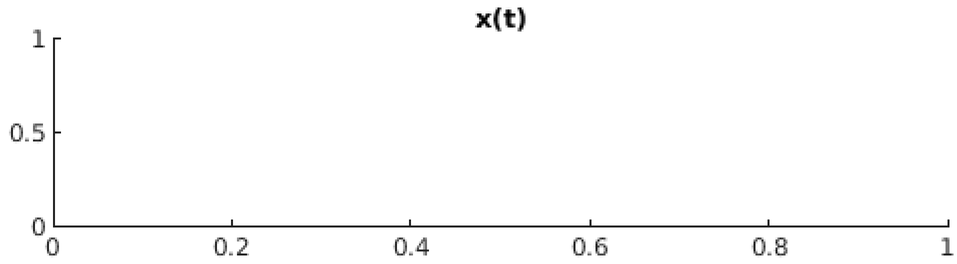
x(t) at 1<t<=2, t=1, x(t) =1, which means that x(t) falls rom 2 to 1 at t=1. At t=2, x(t) =0. This is a linearly descreasing function.

```
clf;

% x(t)
x = [1 2 1 2];
t = [0 1 1 2];
plot(t, x);
```




```
subplot(3,1,1);
title('x(t)');
```




```
%x(t+2)
```

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REVISION

Analytical Solution: Applying the convolution integral:

$x(t) * h(t) = int_{-\infty}^{\infty} x(\tau)h(t-\tau) = int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral can be reduced to:

$x(t) * h(t) = int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = int_{-2}^{-2}\delta(\tau+2)x(t-\tau)d\tau + 2int_{-1}^{-1}\delta(\tau+1)x(t-\tau)d\tau$

$y(t) = x(t) * h(t) = x(t+2) + 2x(t+1)$

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2.11

Let $x(t) = u(t-3) - u(t-5) \;\; and \;\; h(t) = e^{-3t}u(t)$

(a). Compute $y(t) = x(t) * h(t)$

(b). Compute $g(t) = (dx(t)/dt) * h(t)$

(c). How is *g(t)* related to *y(t)*?

$$x(t) = u(t-3) - u(t-5)$$

$$x(s) = \frac{1}{s}e^{-3s} - \frac{1}{s}e^{-5s}$$

$$h(t) = e^{-3t}u(t)$$

$$h(s) = \frac{1}{s+3}$$

a)

$$y(t) = x(t) * h(t)$$

$$y(s) = x(s) * h(s)$$

$$y(s) = \frac{1}{s}(e^{-3s} - e^{-5s}) * \frac{1}{s+3}$$

$$y(s) = \frac{1}{s(s+3)}(e^{-3s} - e^{-5s})$$

$$y(s) = (\frac{1/2}{s} - \frac{1/2}{s+3})(e^{-3s} - e^{-5s})$$

$$y(s) = \frac{1}{2}[\frac{1}{s}e^{-3s} - \frac{1}{s}e^{-5s} - \frac{1}{s+3}e^{-3s} + \frac{1}{s+3}e^{-5s}]$$

$$y(t) = \frac{1}{2}[u(t-3) - u(t-5) - e^{-3(t-3)}u(t-3) + e^{-3(t-5)}u(t-5)]$$

$$y(t) = \frac{1}{2}[[1 - e^{-3(t-3)}]u(t-3) - [1 - e^{-3(t-5}]u(t-5)]$$

==

REVISION

From the given information, we see that h(t) is non zero only for 0<=t<= infinity. Therefore,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\text{\textbackslash Tau})x(t - \text{\textbackslash Tau})d\text{\textbackslash Tau}$$

$$= \int_{0}^{\infty} e^{-3\tau}(u(t-\tau-3) - u(t-\tau-5))d\tau$$

We can easily show that \$u(t-\tau-3)-u(t-\tau-5)) is non zero only in the range (t-5) < tau < (t-3). Therefore, for t<=3, the above integral evaluates to zero. For 3 < t <= 5, the above integral is

$$y(t) = \int_0^{t-3} e^{-3\tau} d\tau = \frac{1 - e^{-3(t-3}}{3}$$

For t>5, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} d\tau = \frac{(1 - e^{-6}) e^{-3(t-5)}}{3}$$

==

b)

$$g(t) = \frac{d}{dt} x(t) * h(t)$$

$$\frac{d}{dt} x(t) => sx(s)$$

$$L\{g(t)\} = L[\frac{d}{dt} x(t)] * L[h(t)]$$

$$g(s) = sx(s) * h(s)$$

$$g(s) = s[\frac{1}{s}(e^{-3s} - e^{-5s}) * \frac{1}{s+3}]$$

$$g(s) = \frac{1}{s+3}[e^{-3s} - e^{-5s}]$$

$$g(s) = \frac{1}{s+3} e^{-3s} - \frac{1}{s+3} e^{-5s}$$

$$g(t) = e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)$$

c)

$$y(t) = x(t) * h(t)$$

$$y(s) = x(s) * h(s)$$

$$g(t) = \frac{d}{dt} x(t) * h(t)$$

$$g(s) = sx(s) * h(s)$$

$$g(s) = s(x(s) * h(s))$$

$$g(s) = s(y(s))$$

$$g(t) = \frac{d}{dt} y(t)$$