

CSC 747 Assignment 3

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Textbook Problems

1.6 Determine whether or not each of the following signals is periodic:

a. $x_1(t) = 2e^{j(t+\pi/4)}u(t)$

A signal $x(t)$ is periodic if $x(t) = x(t+T)$, where T is the period of the signal, for every T units, the signal $x(t)$ will repeat over all the interval of time. The period T for a complex exponential signals is given as, $T = \frac{2\pi}{\omega}$, where the signal $x(t) = e^{j\omega_0 t}$. Given the signal $x_1(t) = e^{j(t+\pi/4)}u(t)$, we know that $u(t)=0 \forall \{t < 0\}$, which states that the signal $x_1(t)$ is present only for $t \geq 0$, which means the signal is not periodic

b. $x_2[n] = u[n] + u[-n]$

Given the signal, $x_2[n] = u[n] + u[-n]$,

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad (1)$$

and

$$u[-n] = \begin{cases} 1 & n \leq 0 \\ 0 & n > 0 \end{cases} \quad (2)$$

The signal $x_2[n] = 1$ for $n \neq 0$ and $x_2[n] = 2$ for $n = 0$, which means that the signal is not periodic.

c. $x_3[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-4k] - \delta[n-1-4k]\}$

From the figure, the signal $x_3[n]$ is periodic with a fundamental period of 4.

1.16 Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is $y[n] = x[n]x[n-2]$.

a. Is the system memoryless?

A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent only on the input at that

same time. For the given signal, the output $y[n]$ of the system depends upon the past value of the input $x[n]$. Hence, the system is not memoryless

b. Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

$y[n] = A[n](a\delta[n-2])$ $y[n] = a^2\delta[n]\delta[n-2]$. $\delta[n] = 1$ for $n = 0$ and $\delta[n-2] = 1$ for $n = 2$, which means that $\delta[n]\delta[n-2] = 0$. This means that the output of the system is $y[n] = 0$.

c. Is the system invertible?

A system is said to be invertible if distinct inputs lead to distinct outputs. The system output, $y[n]$, is always zero for any input of the form $\delta[n-k]$, $k \in \mathbb{Z}$. Therefore, the system is not invertible.

1.27 a. $y(t) = x(t-2) + x(2-t)$

(1) A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent only on the input at that same time, so if the output is $t=0$ and $y(0) = x(-2) + x(2)$, the output $y(0)$ at $t=0$ depends on the last value $x(-2)$ and the next value $x(2)$. So that means the system has memory.

(2) If there is a shift t_0 in the output $y(t)$, then $y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$ and $y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$. The input is shifted to t_0 , so then the output is $x(t-t_0-2) + y(t) = x(t-t_0-2) + x(2-t-t_0)$. This means that a shift of t_0 in the output does not have a corresponding shift in input, which means that the system is time-variant.

(3) If $y_1(t) = x_1(t-2) + x_1(2-t)$ and $y_2(t) = x_2(t-2) + x_2(2-t)$, and $x_3(t)$ is a linear combination of $x_1(t)$ and $x_2(t)$ which is $x_3(t) = ax_1(t) + bx_2(t)$, then the output $y_3(t)$ is

$$y_3(t) = x_3(t-2) + x_3(2-t)$$

$$y_3(t) = ax_1(t-2) + bx_2(t-2) + ax_1(2-t) + bx_2(2-t)$$

$$y_3(t) = ax_1(t-2) + ax_1(2-t) + bx_2(t-2) + bx_2(2-t)$$

$$y_3(t) = ax_1(t-2) + ax_1(2-t) + bx_2(t-2) + bx_2(2-t)$$

$$y_3(t) = a[x_1(t-2) + x_1(2-t)] + b[x_2(t-2) + x_2(2-t)]$$

$$y_3(t) = ay_1(t) + by_2(t)$$

Based on this, the system uses both additivity and homogeneity properties, which means the system is linear.

(4) A system is said to be casual if the output at any time depends only on the input at the present time and in the past, so if the output is $t=0$ and $y(0) = x(-2) + x(2)$, then the output $y(0)$ at $t=0$ depends on the last value $x(-2)$ and the next value $x(2)$. So that means the system is not

casual.

(5) If $|x(t)| < \infty$ for all t then
 $|y(t)| = |x(t-2) + x(2-t)|$
 $|y(t)| \leq |x(t-2)| + |x(2-t)|$
 $|y(t)| \leq \infty$
This means it is stable.

b. $y(t) = [\cos(3t)x(t)]$

(1) If the output is $t = t_1$ then $y(t_1) = [\cos(3(t_1))]x(t_1)$. So the output $y(t_1)$ at $t = t_1$ is dependent on the current value of $x(t_1)$, which means that the system does not have memory.

(2) If there is a shift t_0 in the output $y(t)$, then you get $y(t - t_0) = [\cos(3(t - t_0))]x(t - t_0)$ and $y(t - t_0) = [\cos(3t - 3t_0)]x(t - t_0)$. if the input is shifted to t_0 , then the output is $x(t - t_0) \rightarrow y(t) = [\cos(3t)]x(t - t_0)$. This shows that a shift of t_0 in the input doesn't have a corresponding shift in the output, which means that the system is time-variant.

(3) If $y_1(t) = [\cos(3t)]x_1(t)$ and $y_2(t) = [\cos(3t)]x_2(t)$ and x_3 is $x_3(t)$ and it is a linear combination of $x_1(t)$ and $x_2(t)$ then it's $x_3(t) = ax_1(t) + bx_2(t)$. So the output is

$$\begin{aligned} y_3(t) &= [\cos(3t)]x_3(t) \\ y_3(t) &= [\cos(3t)]ax_1(t) + [\cos(3t)]bx_2(t) \\ y_3(t) &= a[\cos(3t)]x_1(t) + b[\cos(3t)]x_2(t) \\ y_3(t) &= ay_1(t) + by_2(t) \end{aligned}$$

This shows that the system satisfies both additivity and homogeneity properties, which means the system is linear.

(4) If the output is $t = t_1$ then $y(t_1) = [\cos(3(t_1))]x(t_1)$. So the output $y(t_1)$ at $t = t_1$ is dependent on the current value of $x(t_1)$, which means that the system is casual.

(5) If $|x(t)| < \infty$ for all t , then
 $|y(t)| = |[\cos(3t)]x(t)|$
 $|y(t)| = |\cos(3t)||x(t)|$
 $|y(t)| = |x(t)|$
 $|y(t)| \leq \infty$

This shows that the system is stable.

c. $y(t) = \int_{-\infty}^{\infty} x(\tau) d\tau$

(1) If the output is at $t = t_1$, or $y(t_1) = \int_{-\infty}^{2t_1} x(\tau) d\tau$, then the output $y(t_1)$ at $t = t_1$ is dependent on the past value $-\infty$ and the future value

$t_1 + 1t < 2t_1$, which means the system has memory.

(2) If there a shift of t_0 in the output $y(t)$, then $y(t - t_0) = \int_{-\infty}^{2(t-t_0)}(\tau)d\tau$ and $y(t - t_0) = \int_{-\infty}^{2t-2t_0}(\tau)d\tau$. If the input is shift to t_0 , then the output is $x(t - t_0) \rightarrow y(t) = \int_{-\infty}^{2t}(\tau - t_0)d\tau$. This shows that a shift of t_0 does not have a corresponding shift in the output, which means the system is time-variant.

(3) If $y_1(t) = \int_{-\infty}^{2t} x_1(\tau)d\tau$ and $y_2(t) = \int_{-\infty}^{2t} x_2(\tau)d\tau$ and x_3 is $x_3(t)$ and is a linear combination of $x_1(t)$ and $x_2(t)$, then it's $x_3(t) = ax_1(t) + bx_2(t)$. Then the output is

$$y_3(t) = \int_{-\infty}^{2t} x_3(\tau)d\tau$$

$$y_3(t) = \int_{-\infty}^{2t} ax_1(t) + bx_2(t)d\tau$$

$$y_3(t) = a \int_{-\infty}^{2t} x_1(\tau)d\tau + b \int_{-\infty}^{2t} x_2(\tau)d\tau$$

$$y_3(t) = ay_1(t) + by_2(t)$$

This shows that the system satisfies both additivity and homogeneity properties, which means the system is linear.

(4) If the output is at $t = t_1$, or $y(t_1) = \int_{-\infty}^{2t_1}(\tau)d\tau$, then the output $y(t_1)$ at $t = t_1$ is dependent on the past value $-\infty$ and the future value $t_1 + 1t < 2t_1$, which means the system is not casual.

(5) If $|x(t)| < \infty$ for all t , then $|y(t)| = |\int_{-\infty}^{2t}(\tau)d\tau|$ and from triangle inequality for integrals, for every function $g(t)$, $|y(t)| = |\int_{-\infty}^{2t}(\tau)d\tau| \Rightarrow |y(t)| \leq |\int_{-\infty}^{2t} |x(\tau)|d\tau|$. If $|x(t)| < \infty$, the integral values depends on the value of t and it varies with t . The output is not bounded even if a bounded input is applied, which means the system is not stable.

- 1.31 a.
b.

MATLAB Programming Book Questions

1. 1.4.
e. $y[n] = x^3[n]$
 $y(0) = 0x(0) \quad y(1) = 1x(1)$
f. $y[n] = nx[n]$

g. $y[n] = x[2n]$

Linear:

$$x_1[n] \rightarrow y_1[n] = x_1[2n]$$

$$x_2[n] \rightarrow y_2[n] = x_2[2n]$$

$$ax_1[n] + bx_2[n] = x_3[n]$$

$$y_3[n] = x_3[2n]$$

$$y_3[n] = ax_1[2n] + bx_2[2n]$$

$$y_3[n] = ay_1[n] + by_2[n]$$

The system is linear.

Time-Invariant: $x_1[n] \rightarrow y_1[n] = x_1[2n]$

$$x_1[n - n_0] \rightarrow y_2[n] = x_2[2n] \quad y_2 = x_1[2n - n_0]$$

$$y_1[n - n_0] = x_1[2(n - n_0)] \quad y_1[n - n_0] = x_1[2n - 2n_0]$$

$$y_2[n] \neq y_1[n - n_0] \text{ for } x_2[n] = x_1[n - n_0].$$

The system is time-variant.

Casual:

The system is casual.

Stability:

$$x[n] \leq 0$$

$$x[2n] \leq 0$$

$$y[n] \leq 0$$

The system is stable.