Assignment 4a (Main textbook, chapter2-part1)

2.1

Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ *and* $h[n] = 2\delta[n+1] + 2\delta[n-1]$,

Compute and plot each of the following convolutions:

(a). $y_1[n] = x[n] * h[n]$

Given that $y_1[n] = x[n] * h[n] = \Sigma_{k=-\infty}^{\infty} h[k]x[n-k]$

The signals $x[n]$ and $h[n]$ are as shown in Figure S2.1.

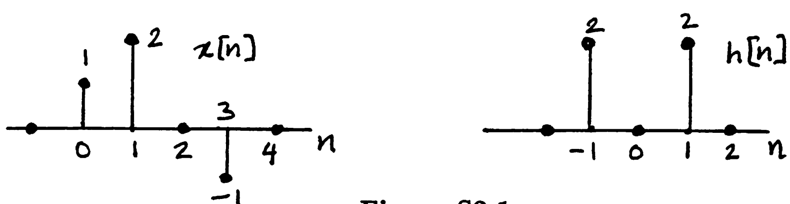


Figure S2.1

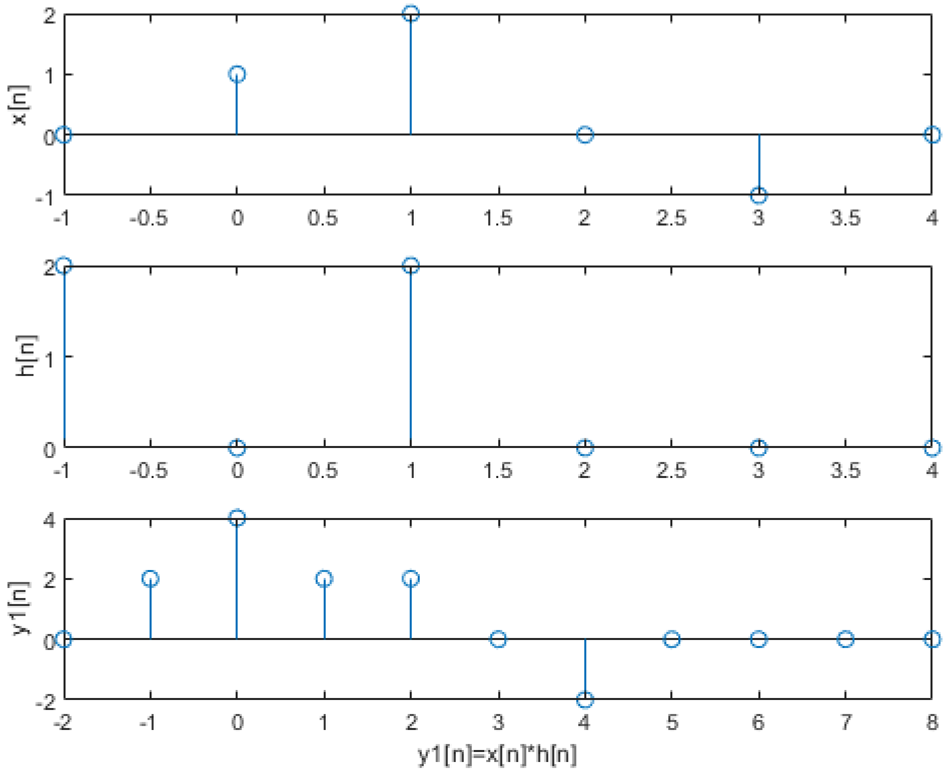
From the figure, since h[n] only has non-zero values at h[-1] and h[1], we can see that the above convolution sum can be reduced to

$$y_1[n] = h[-1]x[n+1] + h[1]x[n-1] = 2x[n+1] + 2x[n-1]$$

=>

$$y_1[n] = 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4]$$
$$= 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

```
% 2.1a
% Compute and plot y1[n]
clf;
n=[-1:4];
x=0*n;
x(1)=0; x(2)=1; x(3)=2; x(4)=0; x(5)=-1;    x(6)=0;
subplot(3,1,1);  stem(n,x); ylabel('x[n]');
h=n*0;
h(1)=2; h(2)=0; h(3)=2;
subplot(3,1,2); stem(n,h);  ylabel('h[n]');
y1 = conv(x,h);
ny=[-2:8];
subplot(3,1,3); stem(ny,y1);    xlabel('y1[n]=x[n]*h[n]');  ylabel('y1[n]');
```



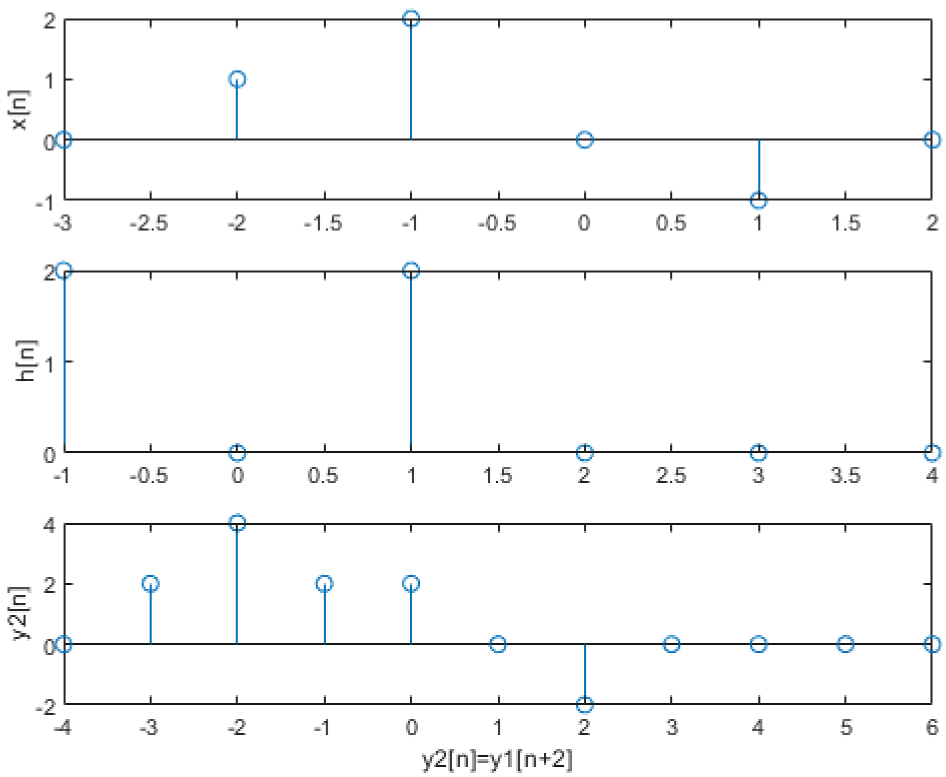
(b). $y_2[n] = x[n+2] * h[n]$

GIven $y_2[n] = x[n+2] * h[n]$ and assume the system is LTI,

Then comparing with $y_1[n] = x[n] * h[n]$, the input x[n+2] is x[n] advanced by 2.

So that $y_2[n] = y_1[n+2]$

```
% 2.1b
% Compute and plot y2[n]
clf;
nx=[-3:2];
x=0*n;
x(1)=0; x(2)=1; x(3)=2; x(4)=0; x(5)=-1; x(6)=0;
subplot(3,1,1);  stem(nx,x);  ylabel('x[n]');
nh=[-1:4];
h=n*0;
h(1)=2; h(2)=0; h(3)=2;
subplot(3,1,2); stem(nh,h); ylabel('h[n]');
y2 = conv(x,h);
ny2=[-4:6];
subplot(3,1,3); stem(ny2,y2);   xlabel('y2[n]=y1[n+2]');    ylabel('y2[n]');
```

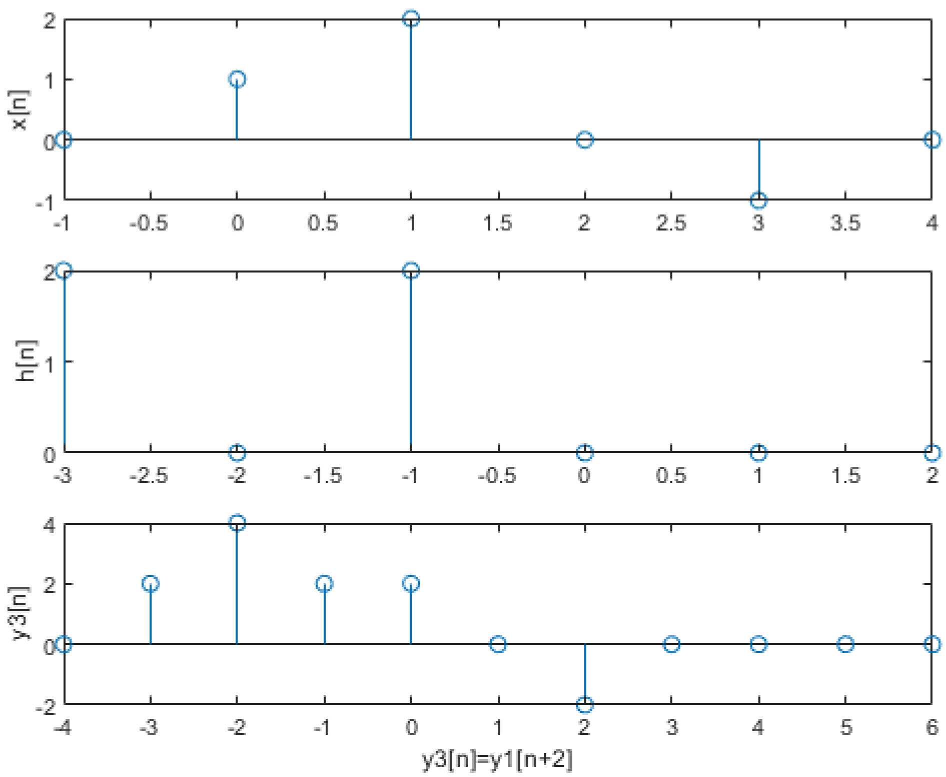


(c). $y_3[n] = x[n] * h[n+2]$

Assumet the system is LTI, comparing to $y_1[n] = x[n] * h[n]$, the input h[n+2] is h[n] advaced by 2..

So that $y_3[n] = y_1[n+2]$

```
%2.1c
% Compute and plot y3[n]
clf;
nx=[-1:4];
x=0*n;
x(1)=0; x(2)=1; x(3)=2; x(4)=0; x(5)=-1;    x(6)=0;
subplot(3,1,1);  stem(nx,x);  ylabel('x[n]');
nh=[-3:2];
h=n*0;
h(1)=2; h(2)=0; h(3)=2;
subplot(3,1,2); stem(nh,h); ylabel('h[n]');
y3 = conv(x,h);
ny3=[-4:6];
subplot(3,1,3); stem(ny3,y3);   xlabel('y3[n]=y1[n+2]');    ylabel('y3[n]');
```



2.4

Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1 & 3 \le n \le 8 \\ 0 & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1 & 4 \le n \le 15 \\ 0 & \text{otherwise} \end{cases}$$

Given that $y[n] = x[n] * h[n] = \Sigma_{k=-\infty}^{\infty} x[k]h[n-k]$

The signals x[n] and h[n] are as shown below:

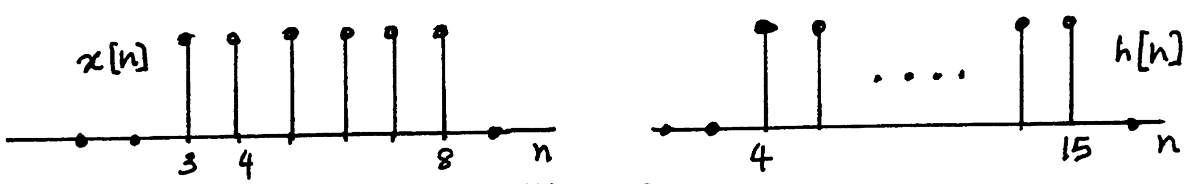


Figure S2.4

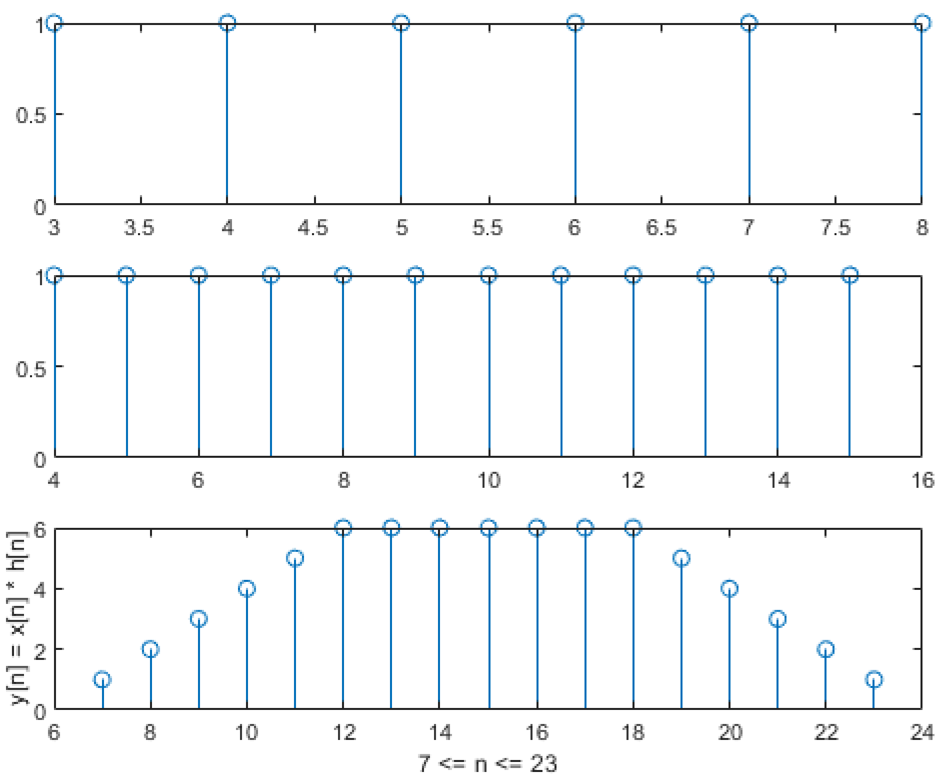
From this figure, we see that the above summartion can be reduced to:

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$

```
% 2.c
% The MATLAB code below will compute and plot y[n] as solution for problem 2.c
clf;
nx=[3:8];
x=nx*0;
x(1:end)=1;
subplot(3,1,1); stem(nx,x);
nh=[4:15];
h=nh*0;
h(1:end)=1;
subplot(3,1,2); stem(nh,h);
ny=[7:23];
y=ny*0;
y=conv(x,h);
subplot(3,1,3); stem(ny,y); xlabel('7 <= n <= 23');  ylabel('y[n] = x[n] * h[n]')
```



2.4 Results Analysis

The analytical results derived mathematically are in agreement with the results from MATLAB program computation and plots.

2.8

Determine and sketch the convolution of the following two signals

$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-1, & 1 < t \le 2 \\ 0, & elsewhere \end{cases}$$

$$h(t) = \delta(t+2) + 2\delta(t+1)$$

Analytical Solution:

Applying the convolution integral:

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral can be reduced to:

$$x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-2}^{-2} \delta(\tau+2)x(t-\tau)d\tau + 2\int_{-1}^{-1} \delta(\tau+1)x(t-\tau)d\tau$$

$$y(t) = x(t) * h(t) = x(t+2) + 2x(t+1)$$

The signals $x(t+2)$ and $2x(t+1)$ are plotted in Figure S2.8.

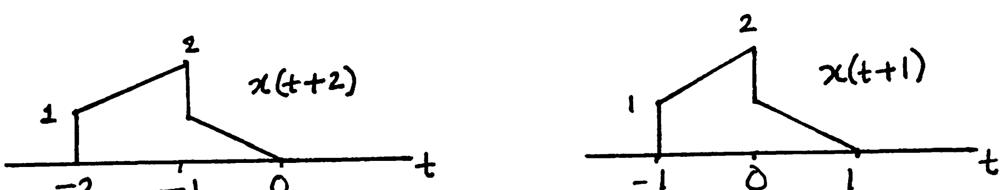
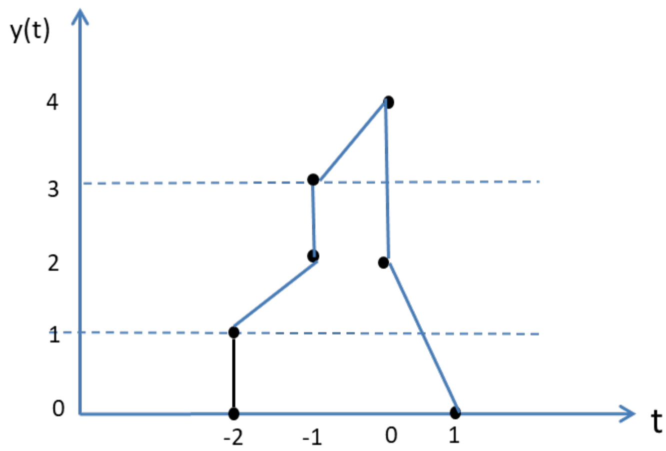


Figure S2.8

Using these plots, we can easily show that

$$y(t) = \begin{cases} t+3, & -2 < t \le -1 \\ t+4, & -1 < t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{otherwise} \end{cases}$$



2.11

Let $x(t) = u(t-3) - u(t-5)$ *and* $h(t) = e^{-3t}u(t)$

(a). Compute $y(t) = x(t) * h(t)$

From the given information, we see that $h(t)$ is non zero only for $0 \le t \le \infty$. Therefore,

$$\begin{aligned} y(t) &= x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \\ &= \int_{0}^{\infty} e^{-3\tau}(u(t-\tau-3) - u(t-\tau-5))d\tau \end{aligned}$$

We can easily show that $(u(t-\tau-3) - u(t-\tau-5))$ is non zero only in the range $(t-5) < \tau < (t-3)$. Therefore, for $t \le 3$, the above integral evaluates to zero. For $3 < t \le 5$, the above integral is

$$y(t) = \int_{0}^{t-3} e^{-3\tau}d\tau = \frac{1 - e^{-3(t-3)}}{3}$$

For $t > 5$, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau}d\tau = \frac{(1 - e^{-6})e^{-3(t-5)}}{3}$$

(b). Compute $g(t) = (dx(t)/dt) * h(t)$

By differentiating $x(t)$ with respect to time we get

$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$

Therefore,

$$g(t) = \frac{dx(t)}{dt} * h(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5).$$

(c). How is *g(t)* related to *y(t)*?

From the result of part (a), we may compute the derivative of $y(t)$ to be

$$\frac{dy(t)}{dt} = \begin{cases} 0, & -\infty < t \leq 3 \\ e^{-3(t-3)}, & 3 < t \leq 5 \\ (e^{-6}-1)e^{-3(t-5)}, & 5 < t \leq \infty \end{cases}$$

This is exactly equal to $g(t)$. Therefore, $g(t) = \frac{dy(t)}{dt}$.