CSC 747 Assignment 3

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Textbook Problems

1.6 Determine whether or not each each of the following signals is periodic:
a. $x_1(t) = 2e^{j(t+\pi/4)}u(t)$
A signal x(t) is periodic if x(t) = x(t+T), where T is the period of the signal, for every T units, the signal x(t) will repeat over all the interval of time. The period T for a complex exponential signals is given as, $T = \frac{2\pi}{\omega}_0$, where the signal $x(t) = e^{j\omega_0 t}$. Given the signal $x_1(t) = e^{j(t+\pi/4)}u(t)$, we know that u(t)=0 $\forall\{t < 0\}$, which states that the signal $x_1(t)$ is present only for t ¿ 0, which means the signal is not periodic

b. $x_2[n] = u[n] + u[-n]$
Given the signal, $x_2[n] = u[n] + u[-n]$,

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1}$$

and

$$u[-n] = \begin{cases} 1 & n \leq 0 \\ 0 & n > 0 \end{cases} \tag{2}$$

The signal $x_2[n] = 1$ for $n \neq 0$ and $x_2[n] = 2$ for $n =$, which means that the signal is not periodic.

$x_2 = u[n] + u[-n]$ for composite signals, each one has a period N in discrete time signal. Since $x_2[n] = 1$ for all n. Therefore, it is periodic with a fundamental period of infinity, or with a fundamental frequency of zero.

c. $x_3[n] = \sum_{k=-\infty}^{\infty}\{\delta[n-4k] - \delta[n-1-4k]\}$
From the figure, the signal $x_3[n]$ is periodic with a fundamental period of 4.

1.16 Consider a discrete-time system with input x[n] and output y[n]. The input-output relationship for this system is $y[n] = x[n]x[n-2]$.

a. Is the system memoryless?
A system is said to be memoryless if its output for each value of the independent vaariable at a given time is dependent only on the input at that same time. For the given signal, the output y[n] of the system depends upon the past value of the input x[n]. Hence, the system is not memoryless

b. Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.
$y = [n] = A[n](a\delta[n-2])$ $y = [n] = a^2\delta[n]\delta[n-2]$. $\delta[n] = 1$ for $n = 0$ and $\delta[n-2] = 1$ for $n = 2$, which means that $\delta[n]\delta[n-2] = 0$. This means that the output of the system is $y[n] = 0$.

c. Is the system invertible?
A system is said to be invertible if distinct inputs lead to distinct outputs. The system output, y[n], is always zero for any input of the form $\delta[n-k], k \in Z$. Therefore, the system is not invertible.

1.27 a. $y(t) = x(t-2) + x(2-t)$
(1) A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent only on the input at that same time, so if the output is t=0 and $y(0) = x(-2) + x(2)$, the output y(0) at t=0 depends on the last value x(-2) and the next value x(2). So that means the system has memory.

(2) If there is a shift t_0 in the output y(t), then $y(t-t_0) = x(t-t)0) + x(2-(t-t_0))$ and $y(t-t_0) = x(t-t)0)+x(2-t+t_0))$. The input is shifted to t_0, so then the output is $x(t-t_0)->y(t) = x(t-t_0)+x(2-t-t_0)$. This means that a shift of t_0 in the output does not have a corresponding shift in input, which means that the system is time-variant.

(3) If $y_1(t) = x_1(t-2)+x_1(2-t)$ and $y_2(t) = x_2(t-2)+x_2(2-t)$, and $x_3(t)$ is a linear combination of $x_1(t)$ and $x_2(t)$ which is $x_3(t) = ax_1(t)+bx_2(t)$, then the output $y_3(t)$ is
$y_3(t) = x_3(t-2) + x_3(2-t)$
$y_3(t) = ax_1(t-2) + bx_2(t-2) + ax_1(2-t) + bx_2(2-t)$
$y_3(t) = ax_1(t-2) + ax_1(2-t) + bx_2(t-2) + bx_2(2-t)$
$y_3(t) = ax_1(t-2) + ax_1(2-t) + bx_2(t-2) + bx_2(2-t)$
$y_3(t) = a[x_1(t-2) + x_1(2-t)] + b[x_2(t-2) + x_2(2-t)$
$y_3(t) = ay_1(t) + by_2(t)$

Based on this, the system uses both additivity and homogeneity properties, which means the system is linear.

(4) A system is said to be casual if the output at any time depends only on the input at the present time and in the past, so if the output is t=0 and $y(0) = x(-2) + x(2)$, then the output y(0) at t=0 depends on the last value x(-2) and the next value x(2). So that means the system is not casual.

(5) If $|x(t)| < \infty$ for all t then
$|y(t)| = |x(t-2) + x(2-t)|$
$|y(t)| \leq |x(t-2)| + |x(2-t)|$
$|y(t)| \geq \infty$
This means it is stable.

b. $y(t) = [cos(3t)x(t)]$
(1) If the output is $t = t_1$ then $y(t_1) = [cos(3(t_1))]x(t_1)$. So the output $y(t_1)$ at $t = t_1$ is dependent on the current value of $x(t_1)$, which means that the system does not have memory.

(2) If there is a shift t_0 in the output y(t), then you get $y(t - t_0) = [cos(3(t - t_0))]x(t - t_0)$ and $y(t - t_0) = [cos(3t - 3t_0))]x(t - t_0)$. if the input is shifted to t_0, then the output is $x(t-t_0)- > y(t) = [cos(3t)]x(t-t_0)$. This shows that a shift of t_0 in the input doesn't have a corresponding shift in the output, which means that the system is time-variant.

(3) If $y_1(t) = [cos(3t)]x_1(t)$ and $y_2(t) = [cos(3t)]x_2(t)$ and x_3 is $x_3(t)$ and it is a linear combination of $x_1(t)$ and $x_2(t)$ then it's $x_3(t) = ax_1(t) + bx_2(t)$. So the output is
$y_3(t) = [cos(3t)]x_3(t)$
$y_3(t) = [cos(3t)]ax_1(t) + bx_2(t)$
$y_3(t) = a[cos(3t)ax_1(t) + bx_2(t)$
$y_3(t) = ay_1(t) + by_2(t)$
This shows that the system satisfies both additivity and homogeneity properties, which means the system is linear.

(4) If the output is $t = t_1$ then $y(t_1) = [cos(3(t_1))]x(t_1)$. So the output $y(t_1)$ at $t = t_1$ is dependent on the current value of $x(t_1)$, which means that the system is casual.

(5) If $|x(t)| < \infty$ for all t, then
$|y(t)| = |[cos(3t)]x(t)|$
$|y(t)| = |cos(3t)||x(t)|$
$|y(t)| = |x(t)|$

$|y(t)| = \infty$
This shows that the system is stable.

c. $y(t) = \int_{-\infty}^{\infty}(\tau)d\tau$
(1) If the output is at $t = t_1$, or $y(t_1) = \int_{-\infty}^{2t_1}(\tau)d\tau$, then the output $y(t_1)$ at $t = t_1$ is dependent on the past value $-\infty$ and the future value $t_1 + 1t < 2t_1$, which means the system has memory.

(2) If there a shift of t_0 in the output y(t), then $y(t-t_0) = \int_{-\infty}^{2(t-t_0}(\tau)d\tau$ and $y(t-t_0) = \int_{-\infty}^{2t-2t_0}(\tau)d\tau$. If the input is shift to t_0, then the output is $x(t-t_0) -> y(t) = \int_{-\infty}^{2t}(\tau - t_0)d\tau$. This shows that a shift of t_0 does not have a corresponding shift in the output, which means the system is time-variant.

(3) If $y_1(t) = \int_{-\infty}^{2t} x_1(\tau)d\tau$ and $y_2(t) = \int_{-\infty}^{2t} x_2(\tau)d\tau$ and x_3 is $x_3(t)$ and is a linear combination of $x_1(t)$ and $x_2(t)$, then it's $x_3(t) = ax_1(t)+bx_2(t)$. Then the output is
$y_3(t) = \int_{-\infty}^{2t} x_3(\tau)d\tau$
$y_3(t) = \int_{-\infty}^{2t} ax_1(t) + bx_2(t)d\tau$
$y_3(t) = a\int_{-\infty}^{2t} x_1(\tau)d\tau + b\int_{-\infty}^{2t} x_2(\tau)d\tau$
$y_3(t) = ay_1(t) + by_2(t)$
This shows that the system satisfies both additivity and homogeneity properties, which means the system is linear.

(4) If the output is at $t = t_1$, or $y(t_1) = \int_{-\infty}^{2t_1}(\tau)d\tau$, then the output $y(t_1)$ at $t = t_1$ is dependent on the past value $-\infty$ and the future value $t_1 + 1t < 2t_1$, which means the system is not casual.

(5) If $|x(t)| < \infty$ for all t, then $|y(t)| = |\int_{-\infty}^{2t}(\tau)d\tau|$ and from triangle inequality for integrals, for every function g(t), $|y(t)| = |\int_{-\infty}^{2t}(\tau)d\tau| =>$ $|y(t)| \leq |\int_{-\infty}^{2t} |x(\tau)|d\tau|$. If $|x(t)| < \infty$, the integral values depends on the value of t and it varies with t. The output i snot bounded even if a bounded input is applied, which means the system is not stable.

1.31 a. Note that $x_2(t) = x_1(t) - x_1)(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in the sketch.
67 b. Note that $x_3(t) = x_1(t) + x_1(t-1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in the sketch.

MATLAB Programming Book Questions

1. 1.4.
   e. $y[n] = x^3[n]$
   $y(0) = 0x(0) \;\; y(1) = 1x(1)$

   Given system $y[n] = x_3[n]$, we set $x_1[n] = u[n]$ and $x_2[n] = \delta[n]$. Given $y_1[n]$ and $y_2[n]$ are the system response to $x_1[n]$ and $x_2[n]$, we let $x_3[n] = x1[n] + x2[n]$, then for $y[n] = x^3[n]$ to be linear. It must satisfy that $y[n] = y_1[n] + y_2[n]$ are the same. As shown in the plot, this is not the case. Therefore, $y[n] = x_3[n]$ is not linear.

   f. $y[n] = nx[n]$
   Given the system $y[n] = nx[n]$. Let $x[n] = u[n]$ be bounded input. If the y[n] is a stable system, then it would be bounded as well. The plot shows y[n] is not bounded. Therefore the system is not stable.

   g. $y[n] = x[2n]$
   Linear:
   $x_1[n] -> y_1[n] = x_1[2n]$
   $x_2[n] -> y_2[n] = x_2[2n]$

   $ax_1[n] + bx_2[n] = x_3[n]$
   $y_3[n] = x_3[2n]$
   $y_3[n] = ax_1[2n] + bx_2[2n]$
   $y_3[n] = ay_1[n] + by_2[n]$

   The system is linear.

   Time-Invariant: $x_1[n] -> y_1[n] = x_1[2n]$
   $x_1[n - n_0] = x_2[n] -> y_2[n] = x_2[2n] \;\; y_2 = x1[2n - n_0]$
   $y_1[n - n_0] = x_1[2(n - n_0)] \;\; y_1[n - n_0 = x_1[2n - 2n_0]$

   $y_2[n] \neq y_1[n - n_0$ for $x_2[n] = x_1[n - n_0]$.

   The system is time-variant.

   Casual:
   The system is casual.

   Stability:
   x[n] ¡ 0
   x[2n] ¡ 0
   y[n] ¡ 0

The system is stable.