Assignment-7a (Chapter 3 Problems)

3.12

Each of the two sequences $x_1[n]$ *and* $x_2[n]$ has a period ***N = 4***, and the corresponding Fourier series coefficients are specified as

$x_1[n] <==> a_k$ $x_2[n] <==> b_k$.

where

$a_0 = a_3 = \frac{1}{2}a_1 = \frac{1}{2}a_2 = 1$ *and* $b_0 = b_1 = b_2 = b_3 = 1$.

Using the multiplication property in Table 3.1, determine the Fourier series coeffcients c_k for the signal $g[n] = x_1[n]x_2[n]$.

$x_1[n] --> a_k \qquad x_2[n] --> b_k$

$$g[n] = x_1[n]x_2[n] --> c_k = \sum_{t=0}^{N-1} a_1 b_{k-t}$$

$a_0 = a_3 = 1$

$a_1 = a_2 = 2$

$b_0 = b_1 = b_2 = b_3 = 1$

$b_{-4} = b_{-3} = b_{-2} = b_{-1} = 1$

$b_k = 1$ for all values of k

$$c_k = \sum_{t=0}^{3} a_t b_{k-t}$$

$c_k = a_o b_k + a_1 b_{k-1} + a_2 b_{k-2} + a_3 b_{k-3}$

$c_k = 1 * b_k + 2 * b_{k-1} + 1 * b_{k-2} + 2 * b_{k-3}$

$k = 1$

$c_k = 1 * b_1 + 2 * b_0 + 1 * b_{-1} + 2 * b_{-2}$

$c_k = 1 * 1 + 2 * 1 + 1 * 1 + 2 * 1$

$c_k = 6$

b_k is 1 and $c_k = 1$ for all values of k.

$g[n] = x_1[n]x_2[n] --> 6$

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Revision:

$x_1[n]x_2[n] <-> \sum_{l=<N>} a_l b_{k-l} = \sum_{k=0}^{3} a_l b_{k-l}$

$x_1[n]x_2[n] <-> a_0 b_k + a_1 b_{k-1} + a_2 b_{k-2} + a_3 b_{k-3}$

g[n] has period N=4 and coefficients g[n] <==> c_K are:

$c_k = b_k + 2b_{k-1} + 2b_{k-2} + b_{k-3}$ for k = 0, 1, 2, 3

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3.14 When the impulse train

$x[n] = \Sigma_{k=-\infty}^{\infty} \delta[n - 4k]$

Is the imput to a particular LTI system with frequency response $H(e^{j\omega})$, the output of the system is found to be

$y[n] = cos(\frac{5\pi}{2} n + \frac{\pi}{4})$.

Determine the values of $H(e^{jk\pi/2})$ for $k = 0, 1, 2$ and 3.

$F x[n] = X(j\omega) = 2\pi a_k$

$T = 4$

$a_k = \frac{1}{T} \int_{\underline{-T/2}}^{\frac{T}{2}} \delta(t) e^{-j\omega t} dt$

$$a_k = \frac{1}{4}\int_{\frac{4}{2}}^{\frac{-4}{2}} \delta\delta(t)e^{-j\omega t}dt$$

$$a_k = \frac{1}{4} * 1$$

$$a_k = \frac{1}{4}$$

$$X(j\omega) = 2\pi a_k$$

$$X(j\omega) = 2\pi * \frac{1}{4}$$

$$X(j\omega) = \frac{\pi}{2}$$

$$F y[n] = Y(j\omega)$$

$$Y(j\omega) = F\{cos(\frac{5\pi}{2}n + \frac{\pi}{4})\}$$

$$Y(j\omega) = \pi[\delta(\omega - frac5\pi 2) + \delta(\omega + \frac{5\pi}{2})]$$

$$H(j\omega)e^{j\omega} = \frac{Y(j\omega)}{X(j\omega)}$$

$$H(j\omega)e^{j\omega} = \frac{\pi[\delta(\omega - \frac{5\pi}{2}) + \delta(\omega + \frac{5\pi}{2})]}{\frac{\pi}{2}}$$

$$H(j\omega)e^{j\omega} = 2[\delta(\omega - \frac{5\pi}{2}) + \delta(\omega + \frac{5\pi}{2})]$$

k=0

$$He^{jk\pi/2} = H(j\omega)_{\omega=0} = \frac{2[\delta(\omega - \frac{5\pi}{2}) + \delta(\omega + \frac{5\pi}{2})]}{e^j\omega}$$

$$He^{jk\pi/2} = 0$$

k=1

$$He^{jk\pi/2} = H(j\omega)_{\omega=\frac{5\pi}{2}} = \frac{2[\delta(\omega - \frac{5\pi}{2}) + \delta(\omega + \frac{5\pi}{2})]}{e^j\omega}$$

$$He^{jk\pi/2} = 2$$

k=2

$$He^{jk\pi/2} = H(j\omega)_{\omega=2\frac{5\pi}{2}} = \frac{2[\delta(\omega - \frac{5\pi}{2}) + \delta(\omega + \frac{5\pi}{2})]}{e^j\omega}$$

$$He^{jk\pi/2} = \infty$$

k=3

$$He^{jk\pi/2} = H(j\omega)_{\omega=3\frac{5\pi}{2}} = \frac{2[\delta(\omega - \frac{5\pi}{2}) + \delta(\omega + \frac{5\pi}{2})]}{e^j\omega}$$

$$He^{jk\pi/2} = 0$$

3.27

A discrete-time periodic signal $x[n]$ is real valued and has a fundamental ***N = 5***. The nonzero Fourier series coefficients for $x[n]$ are

$a_0 = 2,\ a_2 = a_2^* = 2e^{j\pi/6},\ a_4 = a_4^* = e^{j\frac{\pi}{3}}$.

Express $x[n]$ in the form

$x[n] = A_0 + \Sigma_{k=1}^{\infty} A_k sin(\omega_k n + \phi_k)$.

$\omega_k = \dfrac{\pi}{6}$ for all k

$$x[n] = a_0 + a_2 sin(\frac{\pi}{6}n + \phi_2) + a_2 sin(\frac{\pi}{6}n + \phi_{-2}) + a_4 sin(\frac{\pi}{6}n + \phi_4) + a_{-4} sin(\frac{\pi}{6}n + \phi_{-4})$$

$a_0 = 2, a_2 = 2, a_{-2} = 2, a_4 = 1, a_{-4} = 1, \phi_2 = \frac{\pi/6}{,} \phi_{-2} = \frac{-\pi}{6}, \phi_4 = \frac{\pi}{3}, \phi{-4} = \underline{-\pi/3}$

$x[n] = 2 + 2sin(\frac{\pi}{6}n + \frac{\pi}{6}) + 2sin(\frac{\pi}{6}n - \frac{\pi}{6}) + sin(\frac{\pi}{6}n + \frac{\pi}{3}) + sin(\frac{\pi}{6}n - \frac{\pi}{3})$

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Revision:

$x[n] = a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2}e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4}e^{-j4(2/pi/N)n}$

$x[n] = 2 + 2e^{j\pi/6}e^{-j(4\pi/5)n} + 2e^{-j\pi/6}e^{-j(4\pi/5)n} + e^{j\pi/3}e^{j(8\pi/5)n} + e^{-j\pi/3}e^{-j(8\pi/5)n}$

$x[n] = 2 + 4cos[(4\pi n/5) + \pi/6] + 2cos[(8\pi n/5) + \pi/3]$

$x[n] = 2 + 4sin[(4\pi n/5) + 2\pi/3] + 2sin[(8\pi n/5) + 5\pi/6]$

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3.54

Consider the function

$a[k] = \Sigma_{n=0}^{N-1} e^{j(2\pi/N)kn}$

(a) Show that $a_k = N$ for $k = 0, \pm N, \pm 2N, \pm 3N, \ldots$

k=N

$$a[N] = \sum_{n=0}^{N-1} e^{j(2\pi n)} = e^{j(2\pi)0} + e^{(j2\pi)1} + ... + e^{j2\pi)N-1} = 1 + 1 + ... + 1 = N$$

$a[N] = N$

k=0

$$a[0] = \sum_{n=0}^{N-1} R^0 = \sum_{n=0}^{N-1} (1) = N$$

(b) Show that $a[k] = 0$ whenever k is not a integer multiple of N. (Hint: Use the finite sum formula.)

Let $k = \frac{N}{2}$ where N is even

$$a[\frac{N}{2}] = \sum_{n=0}^{N-1} e^{j(\pi n)} = e^{j\pi 0} + e^{j\pi} + e^{j2\pi} + ... + e^{j\pi(N-1)} = 1 - 1 + 1 - 1 + ... - 1 = 0$$

$$a[\frac{N}{2}] = 0$$

(c) Repeat parts (a) and (b) if

$a[k] = \Sigma_{n=<N>} e^{j(2\pi/N)kn}$.

$$a[k] = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} e^{j(\frac{2\pi}{N})kn}$$

Let k = 0

$$a[0] = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} 1 = N + 1$$

$a[k] = N + 1$ if $k = 0, + - N, + - 2N, + - 3N$

Let $k = \frac{N}{2}$, N is even

$$a[\frac{N}{2}] = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} e^{j\pi n} = e^{-j\pi\frac{N}{2}} + ... + 1 + ... + e^{j\pi\frac{N}{2}}$$

$k = \frac{N}{2}$, N = even, if N=2

$$a[\frac{N}{2}] = \sum_{n=-1}^{1} e^{j\pi n} = e^{-j\pi} + e^{0} + e^{j\pi} = -1 + 1 - 1 = -1$$

N=4

$$a[\frac{N}{2}] = \sum_{-2}^{2} e^{j\pi n} = e^{-2j\pi} + e^{-j\pi} + e^{-j0} + e^{j\pi} + e^{j2\pi} = 1 - 1 + 1 - 1 + 1 = 1$$

$a[k] = + - 1$ is k is not an integer of N

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Revision:

$$a[k] = \sum_{n=<N>} e^{j(2\pi/N)kn}$$

Let

$$a[k] = \sum_{n=q}^{q+N-1} e^{j(2\pi/N)kn}$$

where q is some arbitrary integerr. By putting k=pN, we may again easily show that

$$a[pN] = \sum_{n=q}^{q+N-1} e^{j(2\pi/N)pNn} = \sum_{n=q}^{q+N-1} e^{j2\pi pn} = \sum_{n=q}^{q+N-1} 1 = N$$

Now

$$a[k] = e^{j(2\pi/N)kq} \sum_{n=1}^{N-1} e^{j(2\pi/N)kn}$$

Using part (b), we may argue that a[k] = 0 for $k \neq pN, p\epsilon I$

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