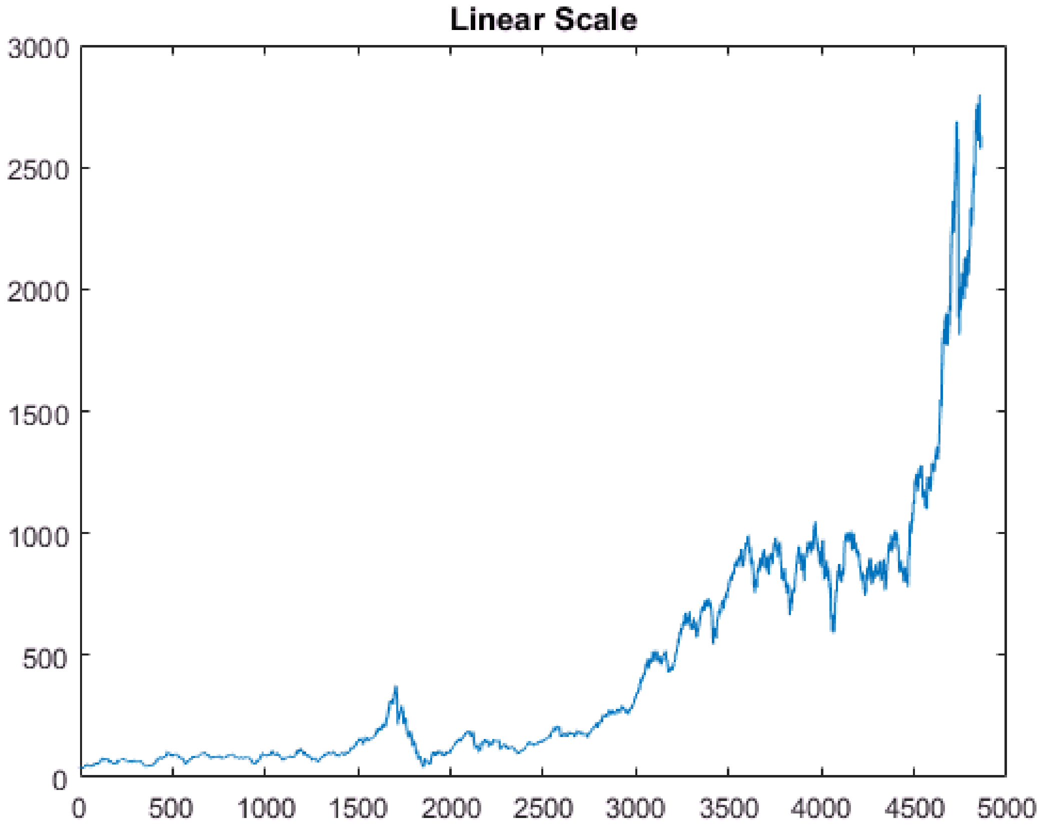
Case 1: Using a RNG between 1 and 6 to change the rates

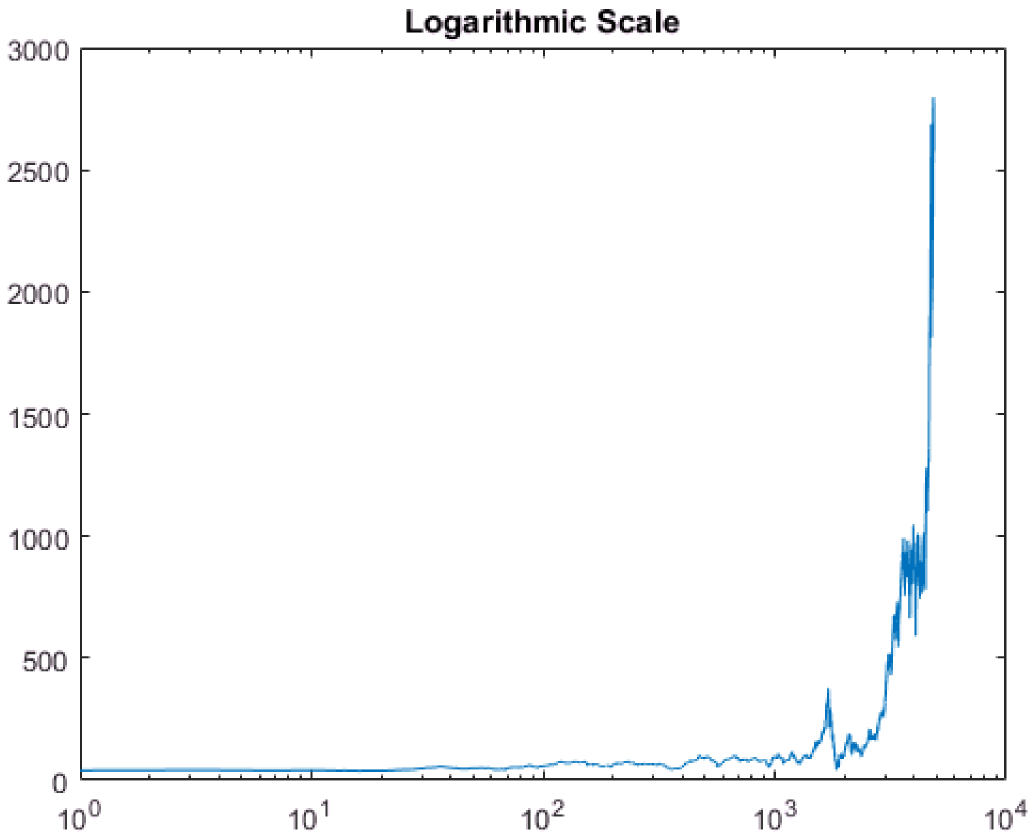
(a) Plot the DJIA data on both a linear and a semi-logarithmic scale. Assuming that you started with $1000 and invested all of your money in the DJIA, how much money would you have at the end of the investment interval (4861 weeks)? If you had put all of your money in the bank at 3% annual percentage rate (APR), compounded weekly,

what rate would you need to achieve the same level of performance? If r = 0.03 is the APR, then the bank balance after N weeks from a weekly compounded interest bearing account is equal to g = (1 + r/52)^N the initial balance.

```
load djia.mat
%RNG to pick different rates for the bank
rng_num = randi(6);
rates = [0.01, 0.02, 0.03, 0.04, 0.05, 0.06];

%1a
figure;
plot(djia)
title('Linear Scale')
```



```
semilogx(djia)
title('Logarithmic Scale')
```



```
%Investing in DJIA
djia_investment_rate = djia(end)/djia(1);
djia_investment  = 1000*djia_investment_rate;
txt1 = ['The investment of $1000 into DJIA will yeild ', num2str(djia_investment)];
disp(txt1);
```

```
The investment of $1000 into DJIA will yeild 64394.9045
```

```
%Investing in the bank
N = 4861;
bank_interest = (1 +rates(rng_num)/52)^N;
bank_investment = 1000 + (1000 * bank_interest);
txt1_2 = ['The investment of $1000 into DJIA will yeild ', num2str(bank_investment), '. The ir
disp(txt1_2);
```

```
The investment of $1000 into DJIA will yeild 107889.1982. The interest rate is 106.8892
```

```

% To find r, the profit rate
N = length(djia);
r = ((djia_investment_rate)^(1/N)-1)*52;
txt2 = ['To have the same yeild as investing in DJIA, we will need a rate of ',num2str(r*100),
disp(txt2)
```

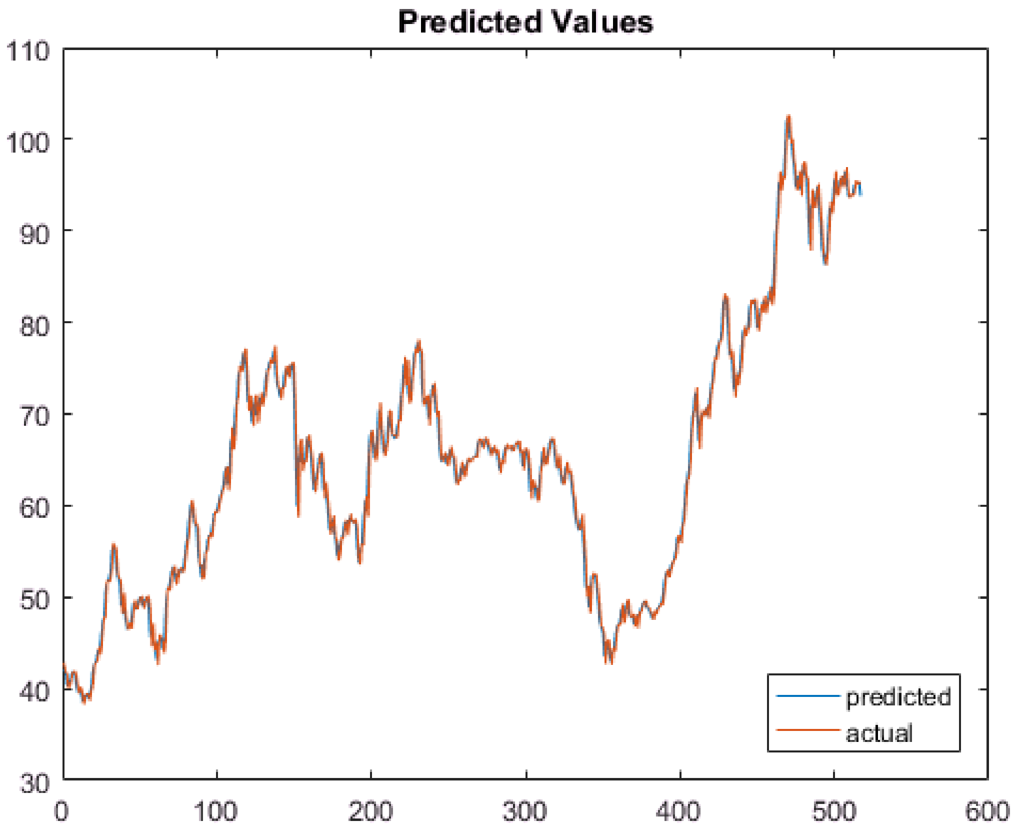
```
To have the same yeild as investing in DJIA, we will need a rate of 4.4574%
```

(b) Assume that p = 3 and create the vector x and matrix X in Eq. (6.29) from the first decade of data, i.e., use N = 520 weeks. The MATLAB \ operator can be used to solve for the vector a that minimizes the inner product e J *e in Eq. (6.29). Solve for the linear predictor coefficients using the MATLAB \ operator by a=-X\x.

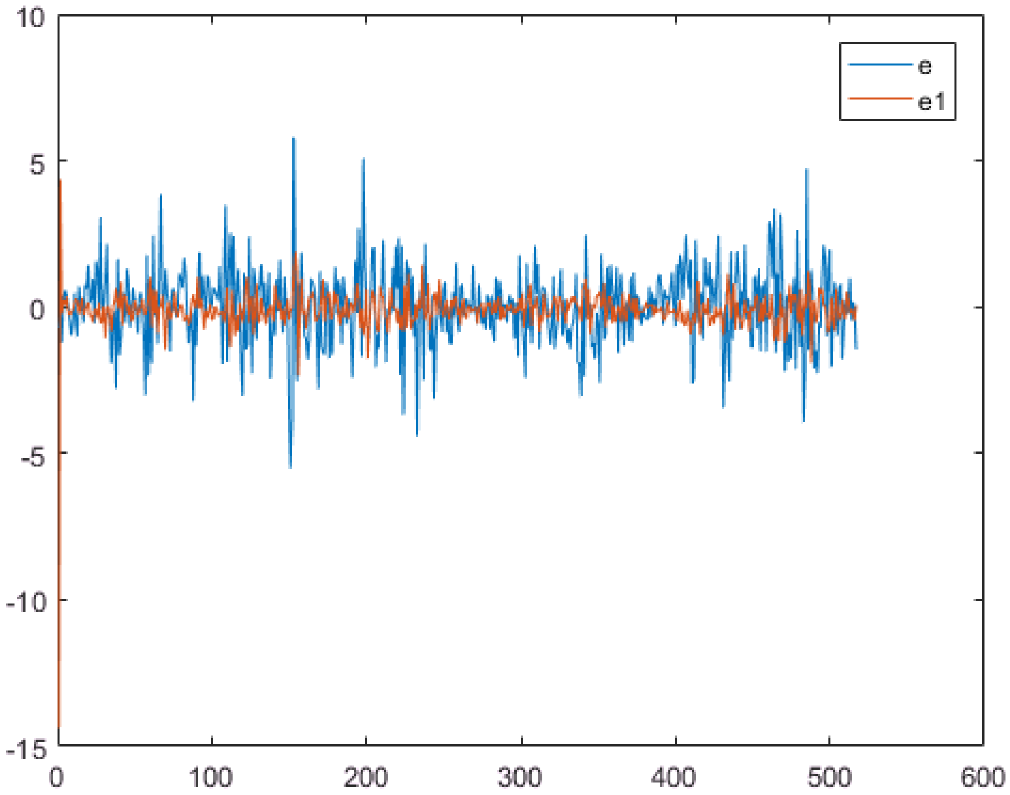
```
%1b
p = 3;
NN = 520;

% Defining X
for i = 1:NN-p
    for j = 1:p
        X(i,j) = djia(i+j-1);
    end
end

% Defining x
for i = p+1:NN
    x(i-p,1) = djia(i);
end
% Solve for coefficients using X and x
a = -X\x;
```

(c) Create the vector of predicted values for the first decade of data using xhat1=-X*a. Also create the vector xhat2 by appropriately using filter on the sequence djia. Note that the coefficients in the vector a are in the reverse of the order required by filter. Plot the predicted values on the same set of axes as the actual weekly average. Also determine the total squared error between the predicted and actual values. As a check, do this two ways. First use e=x+X*a to compute the prediction error, and L1 then calculate the error by subtracting your predicted sequence xhat2 from the actual values and make sure that these are the same.

```
%1c
xhat1 = -X*a;
xhat2 = filter(-flip(a),1,djia(1:NN-p));
figure
plot(djia(4:520))
hold on
plot(xhat1)
legend('predicted','actual','location','southeast')
title('Predicted Values')
hold off
```



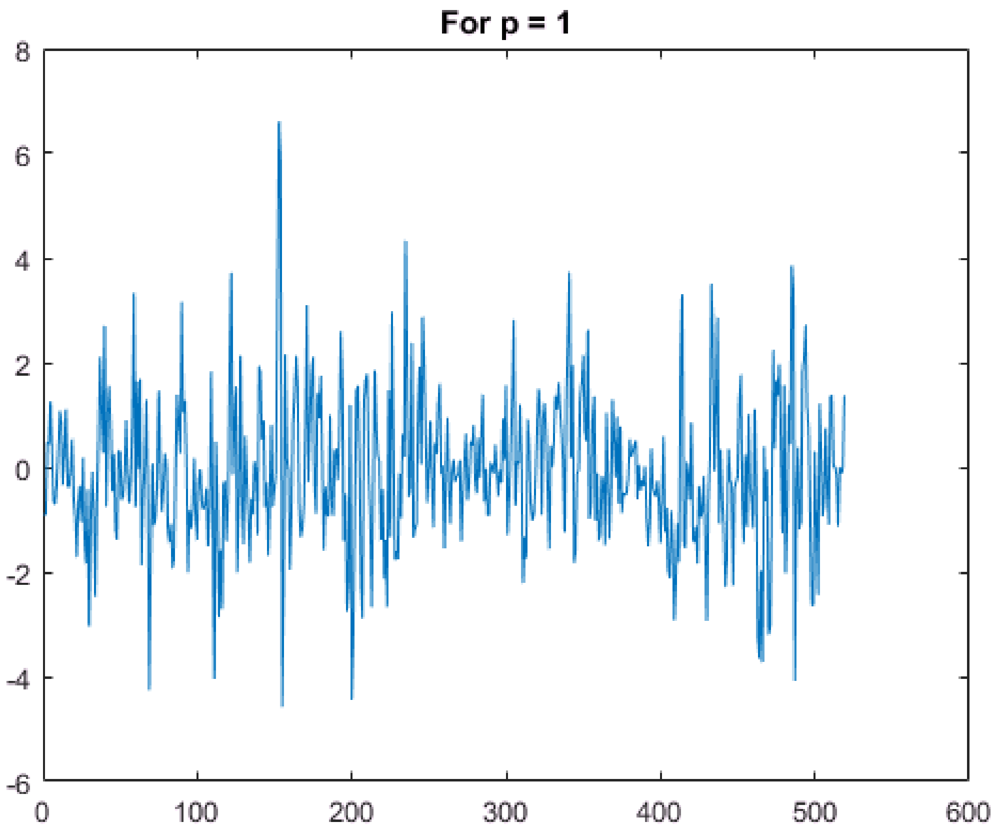
```
% Total squared error
e = x+X*a; % prediction error
e1 = djia(1:517)-xhat2;
figure
plot(e)
hold on
plot(e1)
legend('e','e1')
```

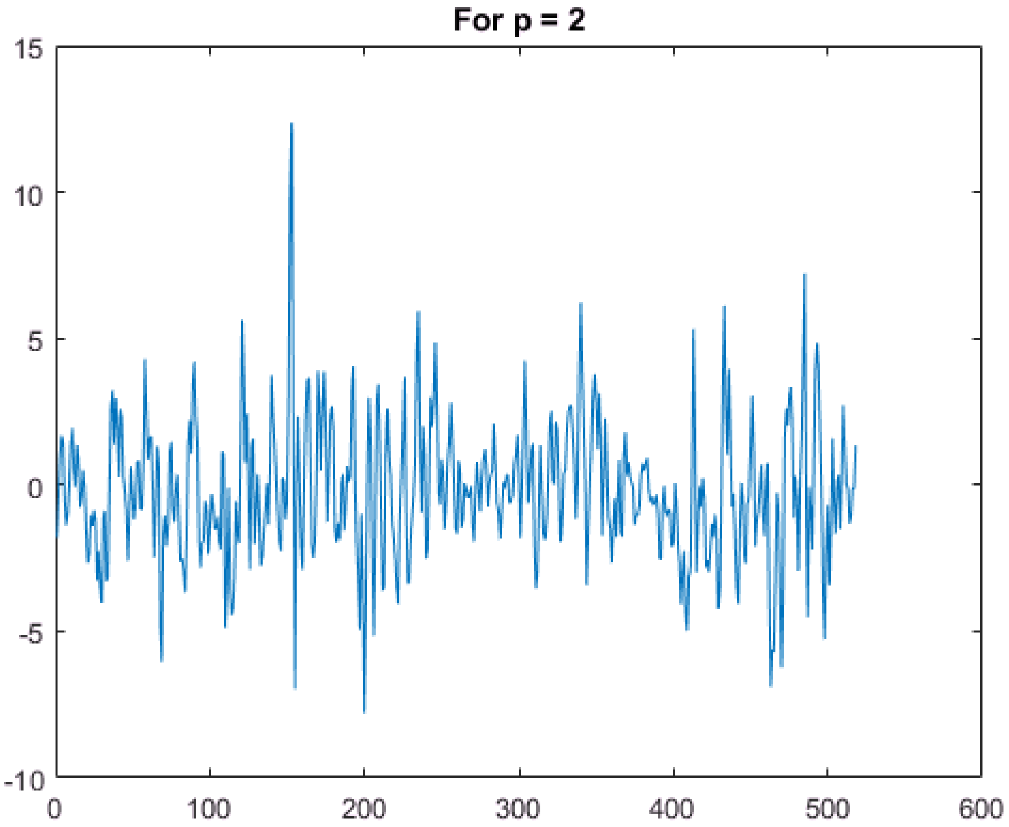


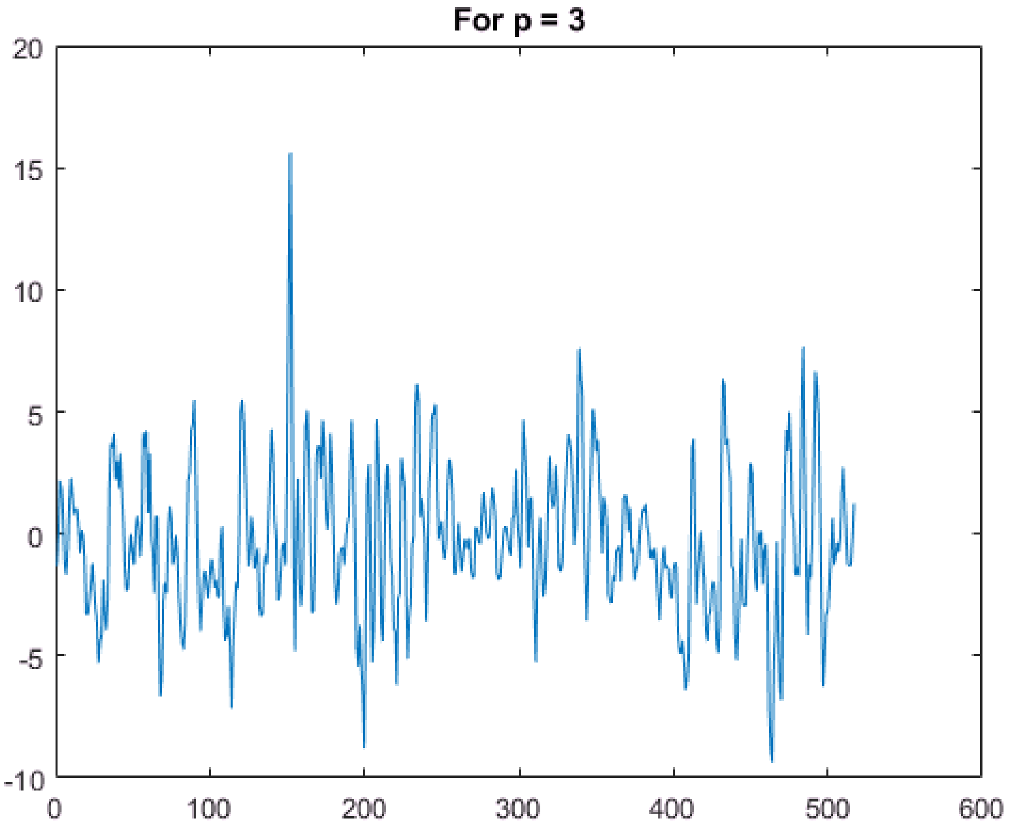
(d) Calculate and plot the total squared prediction error as a function of p for p = 1, . . . ,10. You will have to find the predictor coefficients a1, . . . , a10 for each model order p, and then calculate each of the prediction errors. What is an appropriate value for p, i.e., is there a value of p after which the decrease in prediction error is negligible?

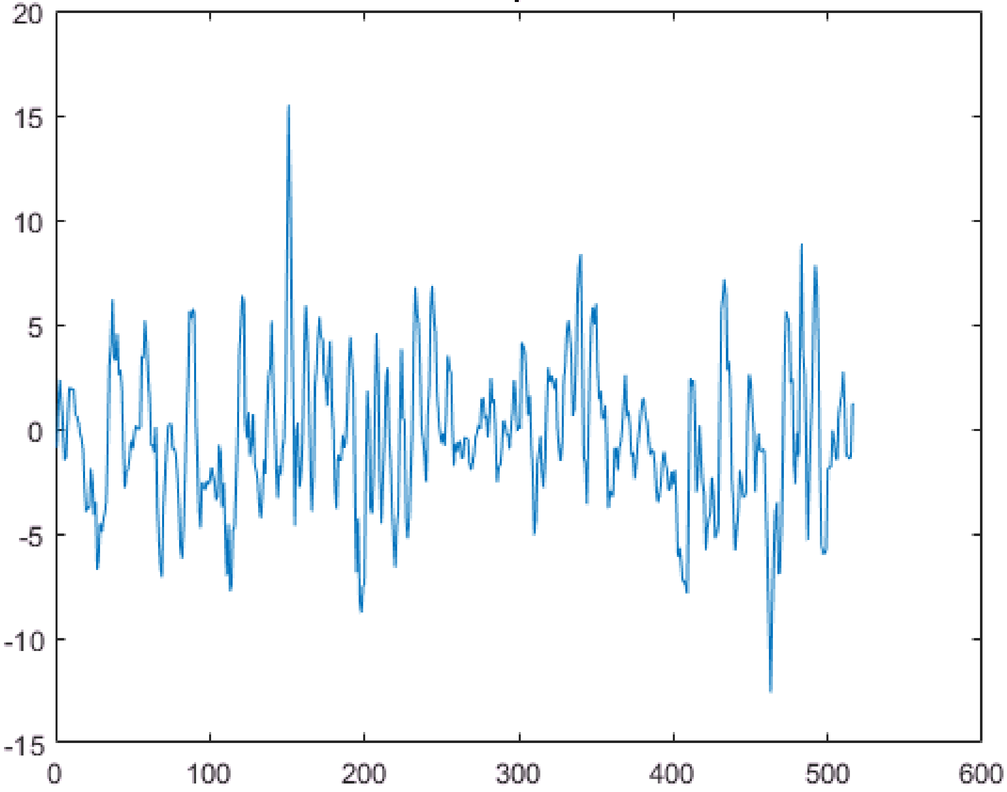
```
cells = cell(1,10);
for p = 1:10
for i = 1:NN-p
    for j = 1:p
        X(i,j) = djia(i+j-1);
    end
end

cells{1,p} = X;
clear X
end
cells = cell(1,10);
for p = 1:10
for i = p+1:NN
    x(i-p,1) = djia(i);

end
cells{1,p} = x;
clear x
end
% Value of coefficients
Cellsa = cell(1,10);
for p = 1:10
    Cellsa{1,p} = -cells{1,p}\cells{1,p};
end
```

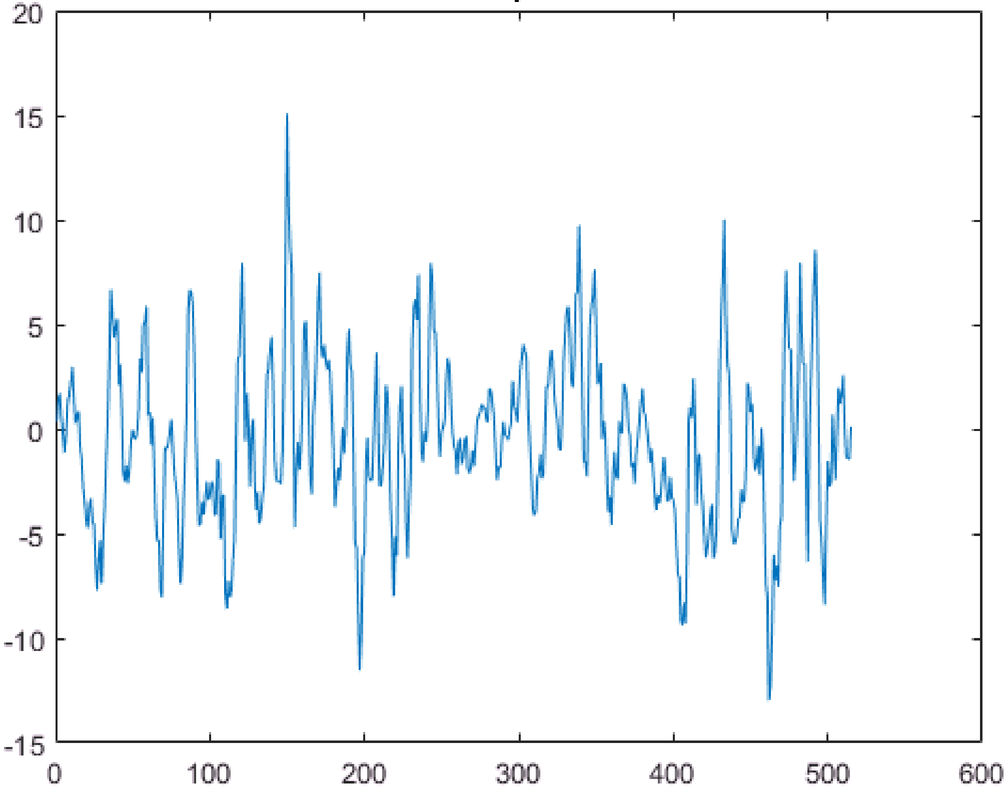
```
xhat = cell(1,10);
for p = 1:10
xhat{1,p} = -cells{1,p}*Cellsa{1,p};
end
Cdjia = cell(1,10);
for p = 1:10
    Cdjia{1,p} = djia(1:NN-p);
end
Cerror = cell(1,10);
for p = 1:10
    Cerror{1,p} = Cdjia{1,p} - xhat{1,p};
    figure
    plot(Cerror{1,p})
   x = [num2str(p)];
   title(['For p = ',num2str(p)])
end
```

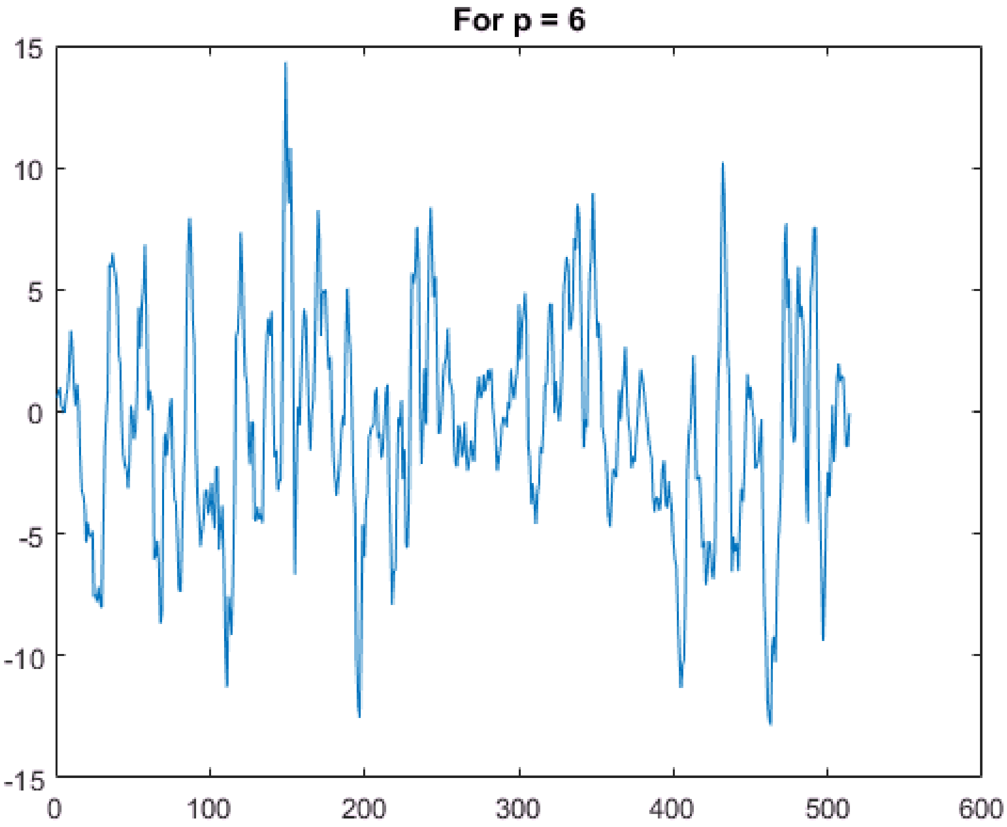



For p = 2
15
10
5
0
-5
-10
0
100
200
300
400
500
600


For p = 3
20
15
10
5
0
-5
-10
0
100
200
300
400
500
600

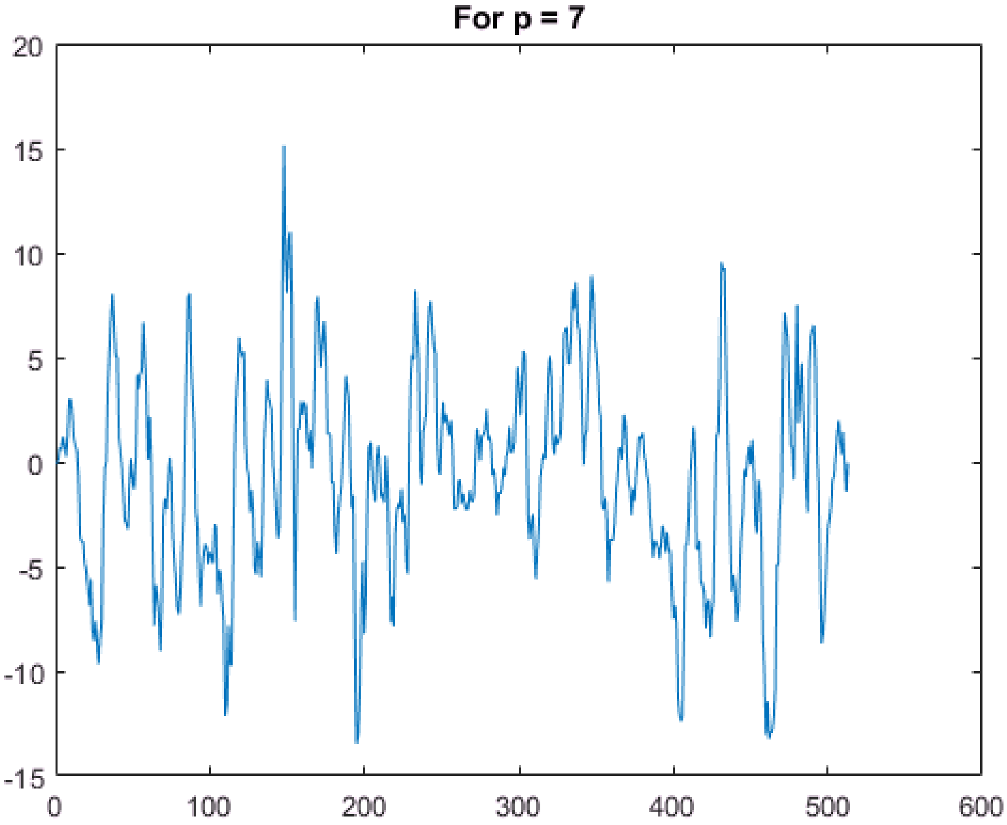

For p = 4
20
15
10
5
0
-5
-10
-15
0
100
200
300
400
500
600


For p = 5
20
15
10
5
0
-5
-10
-15
0
100
200
300
400
500
600



For p = 6

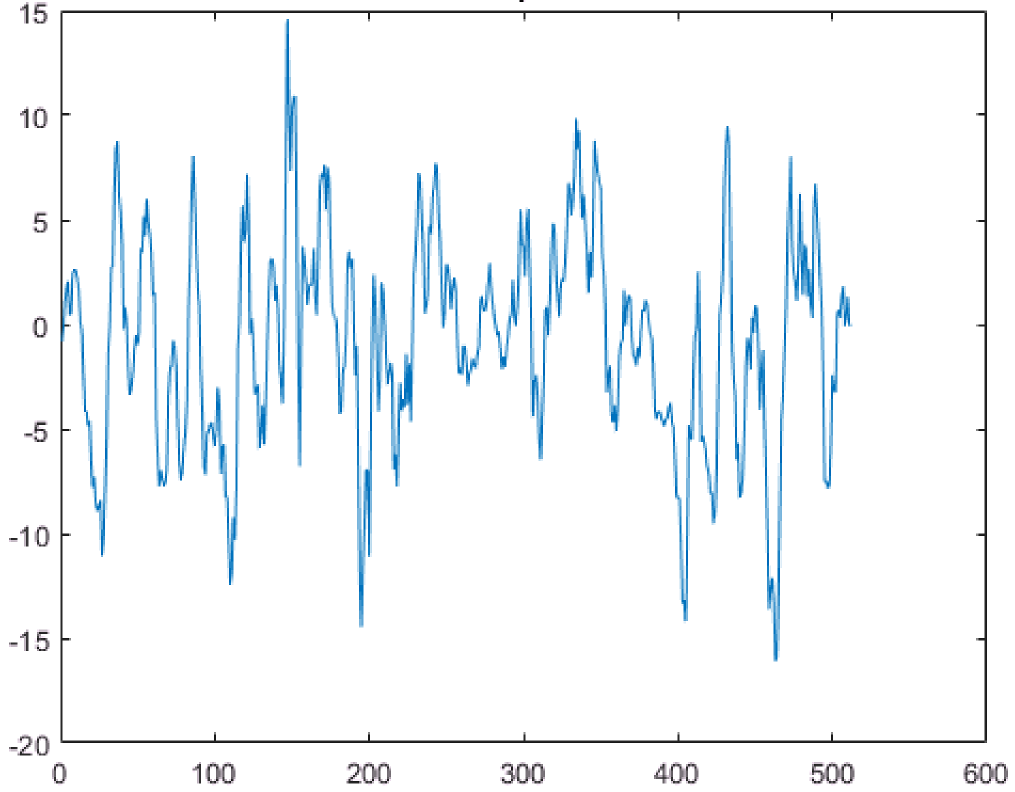
15
10
5
0
-5
-10
-15
0
100
200
300
400
500
600



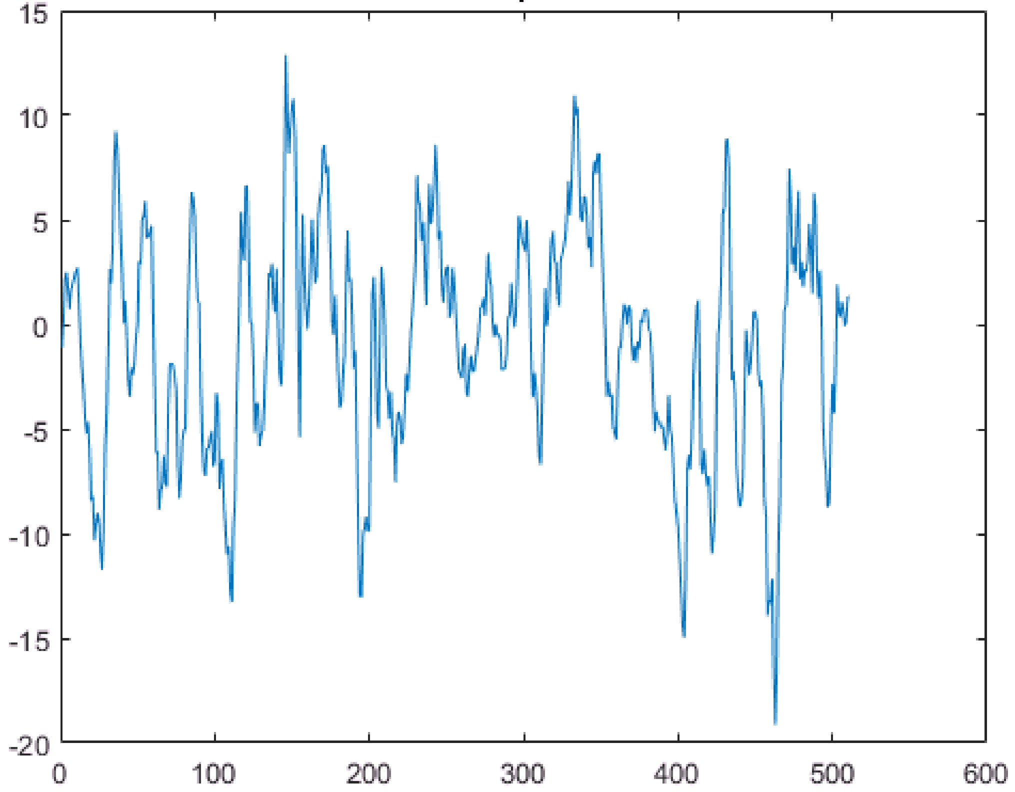
For p = 7

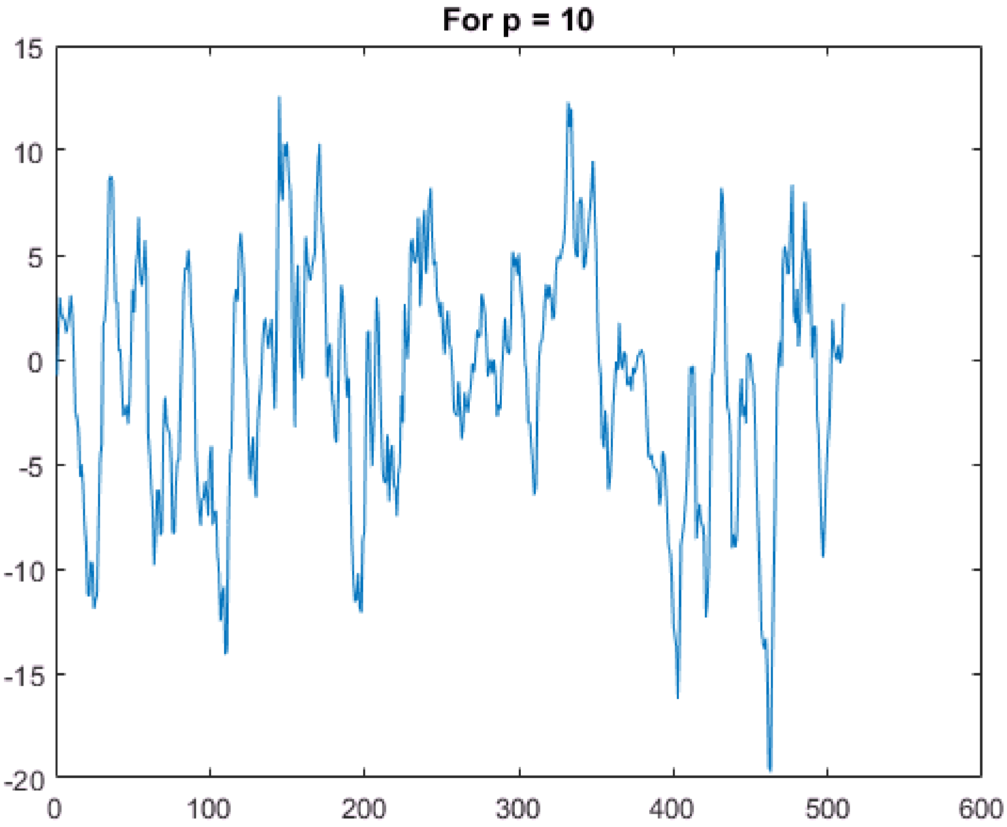
20
15
10
5
0
-5
-10
-15
0
100
200
300
400
500
600

For p = 8


15
10
5
0
-5
-10
-15
-20
0
100
200
300
400
500
600

For p = 9


15
10
5
0
-5
-10
-15
-20
0
100
200
300
400
500
600



```
disp(['We have a decrease in prediction error for p = 1. We can say that for p = 1,'...
    'this prediction error is negligible'])
```

```
We have a decrease in prediction error for p = 1. We can say that for p = 1, this prediction error is ne
```

(e) Given the predictor you designed based on the first decade of data and the model order you have selected from Part (d), you will now test the investment strategy outlined in the introduction. Give yourself $1000 at the end of the pth week, and make 520 trading decisions based on the output of your predictor. First, determine an upper bound on the amount of money you could make. This would be how much you could make if you were omniscient, i.e., if you knew which direction the stock market was going each week and were always invested in the better of either the bank or the DJIA. Now, as a lower bound, calculate how much money you would make if you left all of your money in the bank and earned a gain of (1 + 0.03152) each week. As another lower bound, determine how much you would make with the "buy-and-hold" strategy, where you put all your money in the DJIA every week. Finally, calculate how much money you would make with your predictor. What is the equivalent APR that the bank would have had to pay you to achieve the same gain as your predictor?

```
%1e
xhat_u = max(xhat1)/xhat1(1);
upper_bound = 1000*xhat_u;
txt3 = ['Based on our prediction, in 520 weeks we will have ',...
    num2str(upper_bound)];
disp(txt3)
```

```
Based on our prediction, in 520 weeks we will have 2395.5802
```

```
lower_bound = 1000*(1+rates(rng_num)/52)^520;
txt4 = [' Within 520 weeks with ', num2str(rates(rng_num)), ' the minimal amount we can make i
    num2str(lower_bound)];
```

```
disp(txt4)
```

```
  Within 520 weeks with 0.05 the minimal amount we can make is  1648.3252
```

```
lower_bound2 = 1000*xhat1(end)/xhat1(1);
txt5 = [' If we were to buy and hold, using $1000 we would yeild  ',...
    num2str(lower_bound2)];
disp(txt5)
```

```
  If we were to buy and hold, using $1000 we would yeild  2223.1287
```

```
rr = ((xhat1(end)/xhat1(1))^(1/NN)-1)*52;
txt6 = ['To have the same, the APR rate should be ',num2str(rr*100),'%'];
disp(txt6)
```

```
 To have the same, the APR rate should be 7.9953%
```

(f) Now use your prediction strategy on the most recent decade in the data, i.e., the last 520 weeks of the DJIA. Calculate how you did and how each of the bounds perform: best-possible, all in the bank account, and buy-and-hold. Also calculate the equivalent APR for your predictor.

```
xhat_u = max(xhat1)/xhat1(1);
upper_bound = 1000*xhat_u;
txt7 = ['The maximum amount of money that can be made is $',...
    num2str(upper_bound)];
disp(txt7)
```

```
 The maximum amount of money that can be made is $2395.5802
```

```

lower_bound = 1000*(1+0.03/52)^520;
txt8 = [' The minimum amount we can make with ',num2str(rates(rng_num)), ' and in 520 weeks is
    num2str(lower_bound)];
disp(txt8)
```

```
  The minimum amount we can make with 0.05 and in 520 weeks is $1349.742
```

```

lower_bound2 = 1000*xhat1(end)/xhat1(1);
txt9 = [' If we were to buy and hold, using $1000 we would yeild  ',...
    num2str(lower_bound2)];
disp(txt9)
```

```
  If we were to buy and hold, using $1000 we would yeild  2223.1287
```

```

rr = ((xhat1(end)/xhat1(1))^(1/NN)-1)*52;
txt10 = ['To have the same, the APR rate should be ',num2str(rr*100),'%'];
disp(txt10)
```

```
 To have the same, the APR rate should be 7.9953%
```