**3.2** A discrete-time periodic signal x[n] is real valued and has a fundamental period N=5. The nonzero Fourier series coefficients for x[n] are

$$a_0 = 1,\ a_2 = a_{-2}^* = e^{j\pi/4},\ a_4 = a_{-4}^* = 2e^{j\pi/3}$$

$$x[n] = A_0 + \sum_{k=-\infty}^{\infty} A_k sin(w_k n + \phi_k)$$

$$e^{\frac{j\pi}{4}} = cos\frac{\pi}{4} + jsin\frac{\pi}{4} = \frac{1+j}{\sqrt{2}}$$

$$2e^{\frac{j\pi}{3}} = 2cos\frac{\pi}{3} + 2jsin\frac{\pi}{3} = 1 + j\sqrt{3}$$

$$w_0 = \frac{2\pi}{N} = \frac{2\pi}{5}$$

$$x[n] = \sum_{k=-4} a_k e^{jkw_0 n} = a_{-4}e^{-j4w_0 n} + a_{-2}e^{-j2w_0 n} + a_0 + a_2 e^{j2w_0 n} + a_4 e^{j4w_0 n}$$

$$a_2 = e^{\frac{j\pi}{4}} => a_{-2} => e^{-\frac{j\pi}{4}}$$

$$a_4 => 2e^{\frac{j\pi}{3}} => a_{-4} => 2e^{-\frac{j\pi}{3}}$$

$$x[n] = a_{-4}e^{-j4w_0 n} + a_{-2}e^{-j2w_0 n} + a_0 + a_2 e^{j2w_0 n} + a_4 e^{j4w_0 n}$$

$$x[n] = 2e^{-\frac{j\pi}{3}} * e^{-j4w_0 n} + e^{-\frac{j\pi}{4}} * e^{-j2w_0 n} + 1 + e^{\frac{j\pi}{4}} * e^{j2w_0 n} + 2e^{\frac{j\pi}{3}} * e^{j4w_0 n} \text{cos()}$$

$$x[n] = (1 - j\sqrt{3})e^{-\frac{j8\pi n}{5}} + \frac{(1-j)}{\sqrt{2}}e^{-\frac{j4\pi n}{5}} + 1 + \frac{(1+j)}{\sqrt{2}}e^{\frac{j4\pi n}{5}} + (1+\sqrt{3}j)e^{\frac{j8\pi n}{5}}$$

$$x[n] = 1 + (e^{\frac{j8\pi n}{5}} + e^{-\frac{j8\pi n}{5}}) + \sqrt{3}j(e^{\frac{j8\pi n}{5}} - e^{-\frac{j8\pi n}{5}}) + \frac{1}{\sqrt{2}}(e^{\frac{j4\pi n}{5}} + e^{-\frac{j4\pi n}{5}}) + \frac{j}{\sqrt{2}}(e^{\frac{j4\pi n}{5}} - e^{-\frac{j4\pi n}{5}})$$

$$x[n] = 1 + 2cos(\frac{8\pi}{5}n) - 2\sqrt{3}sin(\frac{8\pi}{5}n) + \sqrt{2}cos(\frac{4\pi}{5}n) - \sqrt{2}(\frac{4\pi}{5}n)$$

$$x[n] = 1 + cos(\frac{\pi}{3})cos(\frac{8\pi}{5}n) - 4sin(\frac{\pi}{3})sin(\frac{8\pi}{5}n) + 2cos(\frac{\pi}{4})cos(\frac{4\pi}{5}n) - 2sin(\frac{\pi}{4})sin(\frac{4\pi}{5}n)$$

$$x[n] = 1 + 2cos(\frac{4\pi}{5}n + \frac{\pi}{4}) + 4cos(\frac{8\pi}{5}n + \frac{\pi}{3})$$

$$x[n] = 1 + 2cos(\frac{4\pi}{5}n + \frac{\pi}{4}) + 4cos(\frac{8\pi}{5}n + \frac{\pi}{3})$$

$$x[n] = 1 + 2sin(\frac{\pi}{2} + \frac{4\pi}{5} + \frac{\pi}{4}) + 4sin(\frac{\pi}{2} + \frac{8\pi}{5}n + \frac{\pi}{3})$$

$$x[n] = 1 + 2sin(\frac{4\pi}{5}n + 3\frac{3\pi}{4}) + 4sin(\frac{8\pi}{5}n + \frac{5\pi}{6})$$

**3.4** Use the Fourier series analysis equation (3.39) to calculate the coefficients $a_k$ for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5 & 0 \leq t \leq 1 \\ -1.5 & 1 \leq t \leq 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

$$T = \frac{2\pi}{w_0} = \frac{2\pi}{\pi} = 2$$

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jnw_0t}dt = a_n$$

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jnw_0t}dt$$

$$a_k = \int_0^1 1.5e^{-jn\pi t}dt + \int_1^2 (-1.5)e^{-jn\pi t}dt$$

$$a_n = \frac{1}{2}[1.5x\frac{e^{-jn\pi t}}{-jn\pi}\int_0^1 - 1.5x\frac{e^{-jn\pi t}}{-jn\pi}\int_1^2]$$

$$a_n = \frac{1}{2}[1.5x\frac{1 - e^{-jn\pi}}{jn\pi} + 1.5\frac{e^{-j2n\pi} - e^{-jn\pi}}{jn\pi}]$$

$$a_n = \frac{1}{2}[\frac{1.5}{jn\pi} - 3\frac{e^{-jn\pi}}{jn\pi} + 1.5\frac{e^{-j2n\pi}}{jn\pi}]$$

$$a_n = \frac{1}{2jn\pi}[1.5 - 3e^{-jn\pi} + 1.5e^{-j2n\pi}]$$

**3.25** Consider the following three continuous-time signals with a fundamental period of T = 1/2:

$x(t) = cos(4\pi t),$

$y(t) = sin(4\pi t),$

$z(t) = x(t)y(t)$

(a) Determine the Fouries serier coefficients of x(t).

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{\frac{1}{2}}\int_0^{\frac{1}{2}} cos(4\pi t)dt = 2[\frac{sin(4\pi t)}{4\pi}] = \frac{1}{2\pi}[sin(2\pi) - 0] = 0$$

$$a_k = \frac{2}{T}\int_0^1 x(t)cos(nw_0 t)dt$$

$$a_k = \frac{2}{\frac{1}{2}}\int_0^{V_2} cos(4\pi t) * cos(nw_0 t)dt$$

$$a_k = 4\int_0^{\frac{1}{2}} \frac{cos(4\pi t + nw_0 t) + cos(4\pi t - nw_0 t}{2} dt$$

$$a_k = 2[\frac{sin((4\pi + nw_0)t) + sin((4\pi - nw_0)t)}{(4\pi + nw_0)(4\pi - nw_0)}]$$

$$a_k = 2[\frac{sin(\frac{4\pi + nw_0}{2})}{4\pi + nw_0} + \frac{sin(\frac{4\pi - nw_0}{2})}{4\pi - nw_0}]$$

$$b_k = \frac{2}{T}\int_0^T x(t) * sin(nw_0 t)dt$$

$$b_k = 4\int_0^{\frac{1}{2}} cos(4\pi t) * sin(nw_0 t)dt$$

$$b_k = 4\int_0^{\frac{1}{2}} \frac{-sin(4\pi t - nw_0 t) + sin(4\pi t + nw_0 t)}{2}$$

$$b_k = 2[\frac{cos(\frac{4\pi - nw_0}{2})}{4\pi - nw_0} - \frac{cos(\frac{4\pi + nw_0}{2})}{4\pi + nw_0}]$$

(b) Determine the Fouries serier coefficients of y(t).

$a_0 = 0$

$$a_k = \frac{2}{T}\int_0^T sin(4\pi t) * cos(nw_0 t)dt$$

$$a_k = 4\int_0^{\frac{1}{2}} cos(nw_0 t) * sin(4\pi t)dt$$

$$a_k = 2[\frac{cos\frac{nw_0 - 4\pi}{2}}{nw_0 - 4\pi} - \frac{cos\frac{w_0 n + 4\pi}{2}}{nw_0 + 4\pi}]$$

$$b_k = \frac{2}{T}\int_0^T sin(4\pi t) * sin(nw_0 t)dt$$

$$b_k = 4\int_0^{\frac{1}{2}} \frac{-cos(4\pi t + nw_0 t) + cos(4\pi t - nw_0 t)}{2}$$

$$b_k = 2[\frac{sin(\frac{4\pi - nw_0}{2})}{4\pi - nw_0} - \frac{\frac{4\pi + nw_0}{2}}{4\pi + nw_0}]$$

(c) Use the resuls of parts (a) and (b), along with the multiplicaton property of the continuous-time Fourier serier, to determine the Fourier series coefficients of z(t)=x(t)y(t).

Let $x(t) -> c_n$ & $y(t) -> D_n$

$C_n = D_n = \frac{a_n - jb_n}{2}$ & $C_0 = D_0 = a_0$

$$x(t)y(t) -> \sum_{n=-\infty}^{\infty} C_n D_{k-a}$$

(d) Determine the Fourier series of z(t) through direct expansion of z(t) in trigonometric form, and compare your result with that of part (c ).

$$z(t) = x(t) * y(t) = sin(4\pi t) * cos(4\pi t) = \frac{sin(8\pi t}{2} = \frac{1}{2}sin(8\pi t)$$

$a_0 = 0$

$$a_k = [\frac{cos\frac{nw_0 - 8\pi}{2}}{nw_0 - 8\pi} - \frac{cos\frac{nw_0 + 8\pi}{2}}{nw_0 + 8\pi}]$$

**3.32** Consider the signal x[n] depicted in Figure P3.32. This signal is periodic with period N=4. The signal can be expressed in terms of a discrete-time Fourier seriers as

$$x[n] = \Sigma_{k=0}^{3} a_k e^{jk(2\pi/4)n}$$

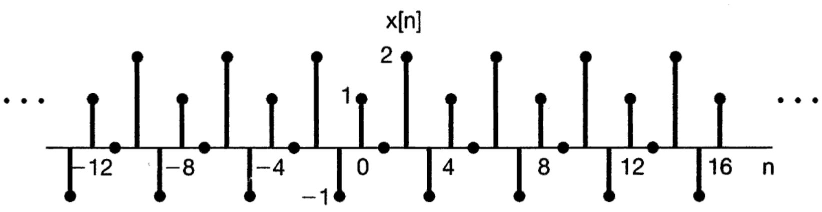


**Figure P3.32**

As mentioned in the text, one way to determine the Fourier series coefficients is to treat eq. (P3.32-1) as a set of four linear equations (for n=0, 1, 2, 3) in four unknowns $(a_0, a_1, a_2, a_3)$.

(a) Write out these four equations explicityly, and solve them directly using any standard technique for solving foour equations in four unknowns. (Be sure first to reduce the foregoing complex exponentials to the simplest form.)

$$a_0 + a_1 + a_2 + a_3 = 1$$

$$a_0 + ja_1 - a_2 - ja_3 = 0$$

$$a_0 - a_1 + a_2 - a_3 = 2$$

$$a_0 - ja_1 - a_2 + ja_3 = -1$$

$$a_0 = 1/2$$

$$a_1 = \frac{1 + j}{4}$$

$$a_2 = -1$$

$$a_3 = -\frac{1 - 2}{4}$$

(b) Check your anser by calculationg the $a_k$ directly, using the discrete-time Fourier series analysis equaiton

$$a_k = \frac{1}{4}\Sigma_{n=0}^{3} x[n] e^{-jk}(2\pi/4)n$$

$$a_k = \frac{1}{4}[1 + 2e^{-jk\pi} - e^{-jk3\pi/2}]$$