CSC747 Assignment 2

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Textbook Questions

1.1) $\frac{1}{2}e^{j\pi}$
$\frac{1}{2} < \pi \quad \pi radians = 180$
$\frac{1}{2}(cos180 + jsin180) = \frac{1}{2}(-1 + j0) = -0.5$

$\frac{1}{2}e^{j\frac{\pi}{2}}$
$\frac{1}{2}(cos180 - jsin180) = \frac{1}{2}(-1 - j0) = -0.5$

$e^{j\frac{\pi}{2}}$
$\frac{\pi}{2} = \frac{180}{2} = 90$
$(cos90 + jsin90) = (0 + j1) = j$

$e^{-j\frac{\pi}{2}}$
$(cos90 - jsin90) = (0 - j1) = -j$

$e^{j5\frac{\pi}{2}}$
$5\frac{\pi}{2}) = 5\frac{180}{2}) = 5(90) = 450$
$(cos450 + jsin450) = (0 + j1) = j$

$\sqrt{2}e^{j\frac{\pi}{4}}$
$\frac{\pi}{4} = \frac{180}{4} = 45$
$\sqrt{2}(cos45 + jsin45) = \sqrt{2}(.707 + j.707) = .999 + .999j = 1 + j$

$\sqrt{2}e^{j9\frac{\pi}{4}}$
$9\frac{\pi}{4}) = 9\frac{180}{4}) = 9(45) = 405$
$\sqrt{2}(cos405 + jsin405) = \sqrt{2}(.707 + j.707) = .999 + .999j = 1 + j$

$\sqrt{2}e^{-j9\frac{\pi}{4}}$
$\sqrt{2}(cos405 - jsin405) = \sqrt{2}(.707 - j.707) = .999 - .999j = 1 - J$

$\sqrt{2}e^{-jpi/4}$

$\frac{\pi}{4} = \frac{180}{4} = 45$
$\sqrt{2}(cos45 - jsin45) = \sqrt{2}(.707 - j.707) = .999 - .999j = 1 - j$

1.2 $5 = 5e^{j0}$

$-2 = 2e^{j\pi}$

$-3j = 3e^{-j\frac{\pi}{2}}$

$\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{\pi}{3}}$

$1 + j = \sqrt{2}e^{j\frac{\pi}{4}}$

$(1-j)^2 = 2e^{-j\frac{\pi}{2}}$

$j(1-j) = \sqrt{2}e^{j\frac{\pi}{4}}$

$(1+j)(1-j) = e^{j\frac{\pi}{2}}$

$\sqrt{2} + j\sqrt{2}/(1 + j\sqrt{3}) = e^{-j\frac{\pi}{2}}$

1.4 a.$x[n-3]$
$x[n] = 0 \qquad \forall\{n < -2, n > 4\}$
$x[n-3] = 0 \qquad \forall\{n - 3 < -2, n - 3 > 4\}$
$x[n-3] = 0 \qquad \forall\{n < -2 + 3, n > 4 + 3\}$
$x[n-3] = 0 \qquad \forall\{n < 1, n > 7\}$

b.$x[n+4]$
$x[n+4] = 0 \qquad \forall\{n + 4 < -2, n + 4 > 4\}$
$x[n+4] = 0 \qquad \forall\{n < -2 - 4, n > 4 - 4\}$
$x[n+4] = 0 \qquad \forall\{n < -6, n > 0\}$

c.$x[-n]$
$x[-n] = 0 \qquad \forall\{-n < -2, -n > 4\}$
$x[-n] = 0 \qquad \forall\{n > +2, n < -4\}$
$x[-n] = 0 \qquad \forall\{n < -4, n > 2\}$

d.$x[-n+2]$
$x[-n+2] = 0 \qquad \forall\{-n + 2 < -2, -n + 2 > 4\}$

$x[-n+2] = 0 \qquad \forall\{-n < -2-2, -n > 4-2\}$
$x[-n+2] = 0 \qquad \forall\{-n < -4, -n > 2\}$
$x[-n+2] = 0 \qquad \forall\{n > 4, n < -2\}$
$x[-n+2] = 0 \qquad \forall\{n < -2, n > 4\}$

e.$x[-n-2]$
$x[-n-2] = 0 \qquad \forall\{-n-2 < -2, -n-2 > 4\}$
$x[-n-2] = 0 \qquad \forall\{-n < -2+2, -n > 4+2\}$
$x[-n-2] = 0 \qquad \forall\{-n < 0, -n > 6\}$
$x[-n-2] = 0 \qquad \forall\{n > 0, n < -6\}$
$x[-n-2] = 0 \qquad \forall\{n < -6, n > 0\}$

1.26 a. $x[n] = sin(\frac{6\pi}{7}N + 1)$

$x[n+N] = sin(\frac{6\pi}{7}(n+N) + 1)$
$x[n+N] = sin(\frac{6\pi}{7}n + \frac{6\pi}{7}N + 1)$

For the signal to be periodic x[n+N]=x[n]
$sin(\frac{6\pi}{7}n + \frac{6\pi}{7}N + 1) = sin(\frac{6\pi}{7}n + 1)$

Possible if
$\frac{6\pi}{7}N = 2\pi m$
$m = 3, N = 7$

Therefore
$x[n+N] = sin(2\pi + \frac{6\pi}{7}n + 1)$
$x[n+N] = sin(\frac{6\pi}{7}n + 1) = x[n]$

This signal is periodic if N=7.

b. $x[n] = cos(\frac{n}{8} - \pi)$

$x[n+N] = cos(\frac{n+N}{8} - \pi)$
$x[n+N] = cos(\frac{n}{8} + \frac{N}{8} - \pi)$

For the signal to be periodic x[n+N]=x[n]
$cos(\frac{n}{8} + \frac{N}{8} - \pi) = cos(\frac{n}{8} - \pi)$

Possible if
$\frac{n}{8} - \pi = 2\pi m$

$m = \frac{1}{2}\pi, N = 8$

The value of m is not an integer, so the signal is not periodic.

c. $x[n] = cos(\frac{n}{8}\pi^2)$

$x[n+N] = cos(\frac{\pi}{8}(n+N)^2$
$x[n+N] = cos(\frac{\pi}{8}n^2 + \frac{\pi}{8}N^2 + \frac{\pi}{8}(2nN))$

For the signal to be periodic x[n+N]=x[n]
$cos(\frac{\pi}{8}n^2 + \frac{\pi}{8}N^2 + \frac{\pi}{8}(2nN)) = cos(\frac{\pi}{8}n^2)$

Possible if
$\frac{\pi}{8}N = 2\pi m$ and $\frac{\pi}{8}N^2 = 2\pi k$
$m = 1, k = 4, N = 8$

$x[n+N] = cos(\frac{\pi}{8}n^2 + \frac{\pi}{8}8^2 + \frac{\pi}{8}(16n))$
$x[n+N] = cos(8\pi + \frac{\pi}{8}n^2 + 2\pi n) = cos(2\pi n + \frac{\pi}{8}n^2)$
$x[n+N] = cos(\frac{\pi}{8}n^2) = x[n]$

The signal is periodic if N=8.

d. $x[n] = cos(\frac{\pi}{2}n)cos(\frac{\pi}{4}n)$

$x[n+N] = cos(\frac{\pi}{2}(n+N))cos(\frac{\pi}{4}(n+N))$
$x[n+N] = cos(\frac{\pi}{2}n + \frac{\pi}{2}N)cos(\frac{\pi}{4}n + \frac{\pi}{4}N))$

Possible if
$\frac{pi}{2}N_1 = 2\pi m_1$, $\frac{pi}{4}N_2 = 2\pi m_2$,
$N_1 = 4, m_1 = 1, N_2 = 8, m_2 = 1$
N_1 and N_2 are period of $cos(\frac{\pi}{2}n)$ and $cos(\frac{\pi}{4}n)$ and m_1 and m_2 are positive integers.

Let $x_1[n] = cos(\frac{\pi}{2}n)$ and $x_2[n] = cos(\frac{\pi}{4}n$
$x_1[n+N_1] = cos(\frac{\pi}{2}n + \frac{\pi}{2}N_1) = cos(2\pi + \frac{\pi}{2}n) = cos(\frac{\pi}{2}n) = x_1[n]$
$x_1[n+N_1] = cos(\frac{\pi}{4}n + \frac{\pi}{2}N_2) = cos(2\pi + \frac{\pi}{4}n) = cos(\frac{\pi}{4}n) = x_2[n]$

The period of the signal x[x] will be L.C.M $N_!, N_2$.
LCM$N_!, N_2 =$ LCM(4, 8) = 8
The signal is period if N=8.

e. $x[n] = 2cos(\frac{\pi}{4}n) - 2cos(\frac{\pi}{2}n + \frac{\pi}{6})$

$x[n+N] = 2cos(\frac{\pi}{4}(n+N)) + sin(\frac{\pi}{8}(n+N)) - 2cos(\frac{\pi}{2}(n+N) + \frac{\pi}{6})$
$x[n+N] = 2cos(\frac{\pi}{4}n + \frac{\pi}{4}N) + sin(\frac{\pi}{8}n + \frac{\pi}{8}N) - 2cos(\frac{\pi}{2}n + \frac{\pi}{2}N + \frac{\pi}{6})$

$2cos(\frac{\pi}{4}n + \frac{\pi}{4}N) + sin(\frac{\pi}{8}n + \frac{\pi}{8}N) - 2cos(\frac{\pi}{2}n + \frac{\pi}{2}N + \frac{\pi}{6}) = 2cos(\frac{\pi}{4}n) + sin(\frac{\pi}{8}n) - 2cos(\frac{\pi}{2}n + \frac{\pi}{6})$

Possible if
$\frac{\pi}{4}N_1 = 2\pi m_1, \frac{\pi}{8}N_2 = 2\pi m_2, \frac{\pi}{2}N_3 = 2\pi m_3$

$N_1 = cos(\frac{\pi}{4}n), N_2 = sin(\frac{\pi}{8}n), N_3 = cos(\frac{\pi}{2}n)$ and $M-1, m_2, m_3$ are positive integers.

$N-1 = 8, m_1 = 1, N_2 = 16, m_2 = 1, N_3 = 4, m_3 = 1$

Let $x_1[n] = cos(\frac{\pi}{4}n), x_2[n] = sin(\frac{\pi}{8}n), x_3[n] = cos(\frac{\pi}{2}n),$

$x_1[n+N_1] = cos(\frac{\pi}{4}n + \frac{\pi}{4}N_1) = cos(2\pi + \frac{\pi}{4}n) = cos(\frac{\pi}{4}n) = x_1[n]$
$x_2[n+N_2] = sin(\frac{\pi}{8}n + \frac{\pi}{8}N_2) = sin(2\pi + \frac{\pi}{8}n) = sin(\frac{\pi}{8}n) = x_2[n]$
$x_3[n+N_3] = cos(\frac{\pi}{2}n + \frac{\pi}{2}N_3) = cos(2\pi + \frac{\pi}{2}n) = cos(\frac{\pi}{2}n) = x_3[n]$

The period of the signal x[x] will be L.C.M $N_!, N_2, N_3$.
LCM$N_!, N_2, N_3$ = LCM(8, 16, 4) = 16
The signal is period if N=16.

1.36 a. If $X[n]$ is periodic $e^{j\omega_0(n+N)T = e^{j\omega_0 nT}}$, where $\omega_0 = 2\pi/T_0$. This implies that $\frac{2\pi}{NT} = \frac{\omega_o}{k} => \frac{2\pi}{NT} = \frac{2\pi}{kT_0} => \frac{T}{T_0} = \frac{k}{N}$ = a rational number, (by applying Eq.(158) where use NT replaces N and k replaces m.

b. If $T/T_0 = p/q$ then $x[n] = e^{j2(p/p)}$. The fundamental period is $q/gcd(p/q)$ and the fundamental frequency is $\frac{2\pi}{q}gcd(p/q) = \frac{2\pi}{p}\frac{p}{q} = \frac{\omega_0}{p}gcd(p,q) = \frac{\omega_0 T}{p}gcd(p,q)$

c. $p/gcd(p,q)$ period of $x(t)$ are needed.

Signals and Systems Using MATLAB Questions
1.2

a. Finding the fundamental period:
$x_M[n] = sin(\frac{2\pi Mn}{N}$
$\Omega = \frac{2\pi M}{N}$
The Period is $N_0 = \frac{2\pi}{\Omega}k$
K is the smallest integer and N_0 is a positive integer. $N_0 = \frac{2\pi}{2}\pi M/Nk$
$N_0 = \frac{N}{M}k$
The fundamental period is $N_0 = \frac{N}{M}k$ and k is a positive integer so N_0 is a positive integer.

In general for a periodic signal $x_M[n] = sin(\frac{2}{N})$, the fundamental period should satisfy $2\pi Mn_0/N = 2 => n_0 = Nm/M$, where m is the smallest integer that makes n_0 an integer.

For $x4[n]$ where M=4 and N=12, the fundamental period is 3 when m=1, which is shown in the plot.
For $x5[n]$ where M=5 and N=12, the fundamental period is 12 when m=5, which is shown in the plot.
For $x7[n]$ where M=7 and N=12, the fundamental period is 12 when m=7, which is shown in the plot.
For $x10[n]$ where M=10 and N=12, the fundamental period is 6 when m=5, which is shown in the plot.
For $x15[n]$ where M=15 and N=12, the fundamental period is 4 when m=5, which is shown in the plot.

b. 1. There are 3 unique signals plotted.
2. X1[n] and x4[n] are identical, for $x1[n]\omega_k = 2\pi k/5 = 2\pi/5$ where k=6.
$x_6 = sin(\omega_k n) = sin(2\pi 6/5) = sin[2\pi(1 + 1/5)] = sin(2\pi/5 + 2\pi) = sin(2\pi/5) = x_1[n]$
since their ω_k is 2π away.

c. As shown in the plot, X2[n] is not periodic. Since for it to be periodic, it must satisfy the condition $2n/N = 2\pi n$, or $n = \pi Nm$ where N and m are integers and n is a rationall number. In this case, there are no such N and m to make n as a rational number.
On the other hand, $x_1[n] = cos(\frac{2\pi n}{N}) + 2cos(\frac{3\pi n}{N})$ and $x_3[n] = cos(\frac{2\pi}{N}) + 3sin(\frac{5\pi n}{N})$ are periodic. To find out their fundamental period, we need to consider the both terms of them.
For example, $x_1[n] = cos(\frac{2\pi n}{N}) + 2cos(\frac{3\pi n}{N} = cos(\omega_2 n)$, where $\omega_1 = 2\pi/N$ and $\omega_2 = 3\pi/N$. Reference tot he Equation (1.58) on page 28 of the main textbook, the fundamental period can be calculated by: $T_0 = m(2\pi/\omega_0)$.

1. For $cos(2\pi/N), \omega_1 = 2\pi/N, T_1 = m(2\pi/\omega_1) = m(2\pi N/2\pi) = 6m$. Let m=1 (the smallest m that makes T an integer), $T_1 = 6$.

2. For $cos(3\pi/N), \omega_1 = 3\pi/N, T_2 = m(2\pi/\omega_2) = m(2\pi N/3\pi) = 4m..$ Let m=1 (the smallest m that makes T an integer), $T_2 = 4.$

-The fundamental period for $x_1[n]$ is the Least Common Multiplier of 6 and 4 which is 12. LCM(4, 6) = 12. In the same way, you can calculate the fundamental period for $x_3[n]$ is 24 (where Let m=5, the smallest m that makes T an integer). LCM(6, 24) = 24

d. $x[n] = sin(\frac{n\pi}{4})cos(\frac{n\pi}{4}) = \frac{1}{2}sin2(\frac{2\pi}{4})\ \frac{1}{2}sin(\frac{n\pi}{4}$
The period of the signal is $\frac{2\pi}{\pi} = 4$

$x[n] = cos^2(\frac{n\pi}{4}) = \frac{1}{2}[1 + cos(\frac{n\pi}{2})]$
The period of the signal is $\frac{2\pi}{\pi}/2 = 4$

$x[n] = sin\frac{n\pi}{4})cos(\frac{n\pi}{8}) = \frac{1}{2}[sin(\frac{n\pi}{4} + sin\frac{n\pi}{8}) + sin(\frac{n\pi}{4} - \frac{n\pi}{8})] = \frac{1}{2}[sin\frac{3n\pi}{8} + sin\frac{n\pi}{8})]$
The period of the signal is the highest value of $\frac{2\pi}{\frac{3\pi}{8}}, \frac{2\pi}{\frac{\pi}{8}} = \frac{16}{3}, 16.$
The period is 16.