CSC747 Digital Signal Processing

Exam 1

Echo Cancellation in Communication Networks

Jessica Gallo

October 3, 2020

1 Report Summary

My understanding of the problem is that echo cancellation helps with communication over telephones and the math used is continuous time unit impulses which are used with convolutions and programming to examine the sounds of the signals and phrases. Echo cancellation allows microphones and telephones to output the sound intended to be inputed instead of including the sound that is bounced off of walls. Convergence helps with echo cancellation to find the impulse response. The concept of what an echo cancellation is not hard at all. The math is okay once I really review it and so is the MATLAB programming; however, part c was the hardest for me. I was unable to finish it. Also, the last part of the MATLAB program was a bit difficult as well. In my view, solving the analytical part and then doing the MATLAB part did help for the math part of the MATLAB section, but in my opinion, it did not help with the programming.

2 Reading and Self-Test

1. Echo-suppression devices, developed in the late 1950s, were first employed to manage echo generated primarily in satellite circuits.
   TRUE

2. The two distinct areas of echo cancellation are acoustic and hybrid echo cancellation.
   TRUE

3. Hybrid echo results from converting four-wire trunk circuits to two-wire local cabling using a device called a "hybrid".
   TRUE

4. Today's digital cellular network technologies, namely TDMA, CDMA, and GSM/PCS, require significantly less processing power to transmit signal paths through channels.
   FALSE

5. When a mobile user calls a fixed network customer, unacceptable call quality can result at the fixed end of the link. In addition, certain digital handsets add an offset to the transmit signal, causing degradation of digital cellular's speech compression performance.
   FALSE

6. The explosive growth in the wireless telecommunications industry in this decade has been a result of which of the following?

d. All of the above

7. Which of the following is the term for the amount of time that the canceller has to hold the model of the echo in order to recognize and cancel it?
   d. convergence time

8. Although not a factor in itself on digital cellular networks, which of the following becomes a problem in PSTN-originated called?
   c. hybrid echo

9. Digital processing delays are encountered as signals are processed through various network routes, such as which of the following?
   c. All of the above

10. Which factor is expected to bring digital wireless telephony much closer to matching wireline quality?
    d. All of the above

3 Analytical Component

2.64 One important use of inverse systems is in situations in which one wishes to remove distortions of some type. A good example of this is the problem of removing echoes from acoustic signals. For example, if an auditorium has a perceptible echo, then an initial acoustic impulse will be followed by attenuated versions of the sound at regularly spaced intervals. Consequently, an often-used model for this phenomenon is an LTI system with an impulse response consisting of a train of impulses, i.e.,

$h(t) = \sum_{k=0}^{\infty} h_k \delta(t - kT)$.

Here the echoes occur T seconds apart, and h_k represents the gain factor on the kth echo resulting form an initial acoustic impulse.

$h(t) = \sum_{k=0}^{\infty} h_k \delta(t - kT)$ = LTI system
h_k = fain factor
T = time separation.

(a) Suppose that x(t) represents the original acoustic signal (the music produced by an orchestra, for example) and that $y(t) = x(t) * h(t)$ is the actual signal that is heard if no processing is done to remove the echoes.

In order to remove the distortion introduced by the echoes, assume that a microphone is used to sense y(t) and that the resulting signal is transduced into an electrical signal. We will also use y(t) to denote this signal, as it represents the electrical equivalent of the acoustic signal, and we can go from one to the other via acoustic-electrical conversion systems. The important point to note is that the system with impulse response given by eq. 2.64 is invertible. Therefore, we can find an LTI system with impulse response g(t) such that

$y(t) * g(t) = x(t)$

and thus, by processing the electrical signal y(t) in this fashion and then converting back to an acoustic signal, we can remove the troublesome echoes. The required impulse response g(t) is also an impulse train:

$g(t) = \sum_{k=0}^{\infty} \delta_k \delta(t - kT)$.

Determine the algebraic equations that the successive g_k must satisfy, and solve these equations for g_0, g_1, g_2 in terms of h_k.

$x(t) =$ original acoustic signal
$y(t) = x(t) * h(t) =$ actual signal
$y(t) =$ signal
$g(t) = \sum_{k=0}^{\infty} g_k \delta(t - kT)$

$x(t) * h(t) * g(t) = x(t)$
$h(t) * g(t) = \delta(t)$

$h(t) * g(t) = [\sum_{k=0}^{\infty} h_k \delta(t - kT)] * [\sum_{k=0}^{\infty} g_k \delta(t - kT)]$
$\delta(t) = \sum_{k=0}^{n} h_k g_{n-k} \delta(t - kT)$

$n = 0$
$\delta(0) = h_0 g_0$
$1 = h_0 g_0$
$\frac{1}{h_0} = g_0$

$n = 1$
$\delta(1) = \sum_{k=0}^{1} h_k g_{n-k} \delta(t - k)$
$0 = h_0 g_{1-0} + h_1 g_{1-1}$
$0 = h_0 g_1 + h_1 g_0 1$
$g_1 = \frac{-h_1}{h_0} g_0$

$g_1 = \frac{-h_1}{h_0^2}$

$n = 1$
$\delta(2) = \sum k = 0^2 h_k g_{n-k}$
$0 = h_0 g_{2-0} + h_1 g_(2-1) + h_2 g_{2-2}$
$0 = h_0 g_2 + h_1 g_1 + h_2 g_0$
$-h_1 g_1 - h_2 g_0 = h_0 g_2$
$\frac{-h_1 g_1}{h_0} - \frac{h_2}{h_0} g_0 = g_2$
$\frac{-h_1}{h_0} (\frac{-h_1}{h_0^2}) - \frac{h_2}{h_0} (\frac{1}{h_0}) = g_2$
$\frac{h_1^2}{h_0^3} - \frac{h_0}{h_0^2} = g_2$

(b) Suppose that $h_0 = 1, h_1 = \frac{1}{2} and h_i = 0$ for all $i \geq 2$. What is g(t) in this case?

$h_0 = 1 \quad h_1 = \frac{1}{2}$
$h_i = 0 \quad i \geq 2$

$g_0 = \frac{1}{h_0} = g_0 = 1$
$g_1 = \frac{-h_1}{h_0^2} = g_1 = \frac{-\frac{1}{2}}{1^2} = g_1 = -\frac{1}{2}$

$g_2 = \frac{h_1^2}{h_0^3} - \frac{h_2}{h_0^2} = g_2 = \frac{\frac{1}{2}^2}{1^3} - \frac{0}{1^2} = g_2 = \frac{\frac{1}{4}}{1} - \frac{0}{1} = g_2 = \frac{1}{4}$

$g(t) = \sum_{k=0}^{\infty} g_k \delta(t - kT)$
$g(t) = g_0 \delta(t) + g_1 \delta(t-T) + g_2 \delta(t-T)$
$g(t) = \delta(t) + (-\frac{1}{2}) \delta(t-T) + \frac{1}{4} \delta(t-T)$

(c) A good model for the generation of echoes is illustrated in Figure 2.64. Hence, each successive echo represents a fed-back version of y(t), delayed by T seconds and scaled by α. Typically, $0 < \alpha < 1$, as successive echoes are attenuated.

(i) What is the impulse response of this system? (Assume initial rest, i.e., y(t)=0 for $t < 0$ if x(t)=0 for $t < 0$.

(ii)Show that the system is stable if $0 < \alpha < 1$ and unstable if $\alpha > 1$.

(iii)What is g(t) in this case? Construct a realization of the inverse system using adders, coefficient multipliers, and T-second delay elements.

(d) Although we have phrased the preceding discussion in terms of continuous-time systems because of the application we have been considering, the same general ideas hold in discrete time. That is, the LTI system with impulse response

$h[n] = \sum_{k=0}^{\infty} h_k \delta[n - kN]$

is invertible and has as its inverse an LTI system with impulse response

$g[n] = \sum g_k \delta[n - kN]$.

It is not difficult to check that the g_k staisfy the same algebraic equations as in part (a). Consider now the discrete-time LTI system with impulse response

$h[n] = \sum_{k=-\infty}^{\infty} \delta[n - kN]$ $h[n] = \sum_{k=-\infty}^{\infty} \delta[h - kN]$

This system is not invertible. Find two inputs that produce the same output.

$x_1 = \delta[n]$
$y[n] = h[n] * x_1[n]$
$y[n] = h[n] * \delta[n]$
$y[n] = h[n]$

$x_2[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n - N]$
$y[n] = h_2[n] * x_2[n]$
$y[n] = h_2[n] * (\frac{1}{2}\delta[n] + \frac{1}{2}\delta[n - N])$
$y[n] = h[n] * \frac{1}{2}\delta[n] + \frac{1}{2}h[n - N]$
$y[n] = \frac{1}{2}h[n] + \frac{1}{2}h[n - N]$
$\frac{1}{2}(h[n] + h[n - N])$

$h[n] = \sum_{k=-\infty}^{\infty} \delta[n - Nk]$

$y[n] = \frac{1}{2}(\sum_{k=-\infty}^{\infty} + \sum_{k=-\infty}^{\infty} \delta[n - N - Nk])$
$y[n] = \frac{1}{2}2\sum_{k=-\infty}^{infty} \delta[n - Nk]$
$y[n] = \sum_{k=-\infty}^{\infty} \delta[n - Nk]$

$y[n] = k[n]$

4 Works Cited

https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.414.2937rep=rep1type=pdf