Assignment 5a (chapter2-part2)

2.16

For each of the following statements, determine whether ti is true or false (justify your answers).

(a). If $x[n]=0 \;\; for \;\; n<N_1 \;\; and \;\; h[n]=0 \;\; for \;\; n<N_2$, then $x[n]*h[n]=0 \;\; for \;\; n<N_1+N_2$.

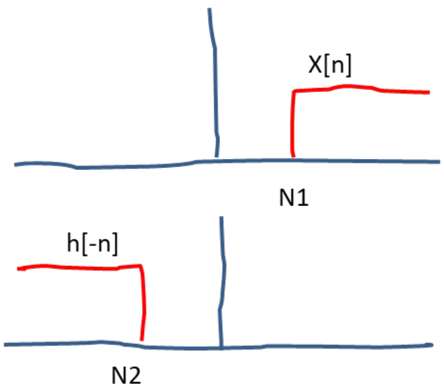
(b). If $y[n]=x[n]*h[n]$, then $y[n-1]=x[n-1]*h[n-1]$.

(c). If $y[n]=x[n]*h[n]$, then $y(-t)=x(-t)*h(-t)$.

(d). If $x(t)=0 \;\; for \;\; t>T_1 \;\; and \;\; h(t)=0 \;\; for \;\; t>T_2$, then $x(t)*h(t)=0 \;\; for \;\; t>T_1+T_2$.

(a) Answer: True

The figure below show the relation between x[n] and h[n] regarding the convolution of them.



We can see that h[n] has to move toward right by N1 + N2 units before x[n] and h[n] overlap with the non-zero areas. So that $x[n]*h[n]=0 \;\; for \;\; n<N_1+N_2$

(b) Answer: False

Consider:

$$\begin{aligned} y[n] &= x[n]*h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \end{aligned}$$

From this,

$$\begin{aligned} y[n-1] &= \sum_{k=-\infty}^{\infty} x[k]h[n-1-k] \\ &= x[n]*h[n-1] \end{aligned}$$

This shows that the given statement is false.

(c) Answer: True

Consider:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau.$$

From this,

$$\begin{aligned} y(-t) &= \int_{-\infty}^{\infty} x(\tau)h(-t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} x(-\tau)h(-t+\tau)d\tau \\ &= x(-t) * h(-t) \end{aligned}$$

This shows that the given statement is true.

(d) Answer: True

This can be explain by considering

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau.$$

In Figure S2.16, we plot $x(\tau)$ and $h(t-\tau)$ under the assumptions that (1) $x(t) = 0$ for $t > T_1$ and (2) $h(t) = 0$ for $t > T_2$. Clearly, the product $x(\tau)h(t-\tau)$ is zero if

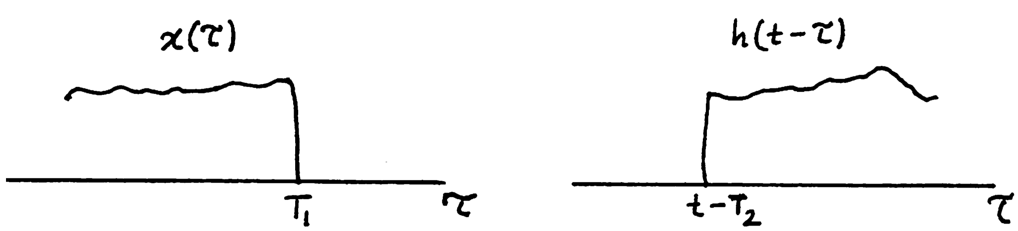


Figure S2.16

$t - T_2 > T_1$. Therefore, $y(t) = 0$ for $t > T_1 + T_2$.

2.18

Consider a causal LTI system whose input x[n] and output y[n] are leated by the difference equation

$$y[n] = \frac{1}{4}y(n-1) + x[n]$$

Determine $y[n]$ *if* $x[n] = \delta[n-1]$

Answer:

Since the system is causal, $y[n] = 0$ for $n < 1$. Now,

$$\begin{aligned} y[1] &= \frac{1}{4}y[0] + x[1] = 0 + 1 = 1 \\ y[2] &= \frac{1}{4}y[1] + x[2] = \frac{1}{4} + 0 = \frac{1}{4} \\ y[3] &= \frac{1}{4}y[2] + x[3] = \frac{1}{16} + 0 = \frac{1}{16} \\ &\vdots \\ y[m] &= (\frac{1}{4})^{m-1} \\ &\vdots \end{aligned}$$

Therefore,

$$y[n] = (\frac{1}{4})^{n-1}u[n-1]$$

2.19

Consider the cascade of the following two system S_1 *and* S_2, as depicted in Figure P2.19:

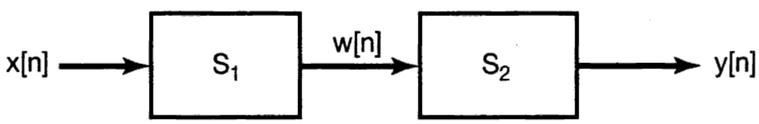


Figure P2.19

S_1 : causal LTI,

$w[n] = \frac{1}{2}w[n-1] + x[n]$;

S_2 : causal LTI,

$y[n] = \alpha y[n-1] + \beta w[n]$.

The difference quation relating x[n] and y[n] is:

$y[n] = \frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$.

(a). Determine α *and* β.

(b). Show the impulse response of the cascade connection of S_1 *and* S_2.

(a) Answer:

Consider the difference equation relating $y[n]$ and $w[n]$ for S_2:

$$y[n] = \alpha y[n-1] + \beta w[n]$$

From this we may write

$$w[n] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1]$$

and

$$w[n-1] = \frac{1}{\beta} y[n-1] - \frac{\alpha}{\beta} y[n-2]$$

Weighting the previous equation by $1/2$ and subtracting from the one before, we obtain

$$w[n] - \frac{1}{2} w[n-1] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2]$$

Substituting this in the difference equation relating $w[n]$ and $x[n]$ for S_1,

$$\frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2] = x[n]$$

That is,

$$y[n] = (\alpha + \frac{1}{2}) y[n-1] - \frac{\alpha}{2} y[n-2] + \beta x[n]$$

Comparing with the given equation relating $y[n]$ and $x[n]$, we obtain

$$\alpha = \frac{1}{4}, \qquad \beta = 1$$

(b) Answer:

The difference equations relating the input and output of the systems S_1 and S_2 are

$$w[n] = \frac{1}{2}w[n-1] + x[n] \qquad \text{and} \qquad y[n] = \frac{1}{4}y[n-1] + w[n]$$

From these, we can use the method specifed in Example 2.15 to show that the impulse responses of S_1 and S_2 are

$$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_2[n] = \left(\frac{1}{4}\right)^n u[n],$$

respectively. The overall impulse response of the system made up of a cascade of S_1 and S_2 will be

$$\begin{aligned} h[n] &= h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k]h_2[n-k] \\ &= \sum_{k=0}^{\infty} (\frac{1}{2})^k (\frac{1}{4})^{n-k} u[n-k] \\ &= \sum_{k=0}^{n} (\frac{1}{2})^k (\frac{1}{4})^{n-k} = \sum_{k=0}^{n} (\frac{1}{2})^{2(n-k)} \\ &= [2(\frac{1}{2})^n - (\frac{1}{4})^n]u[n] \end{aligned}$$

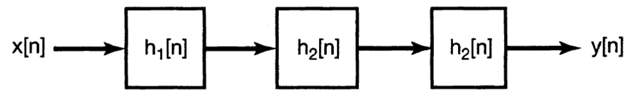
2.24

Consider the cascade interconnection of three causal LTI systems, illustatred in Figure P2.24(a).

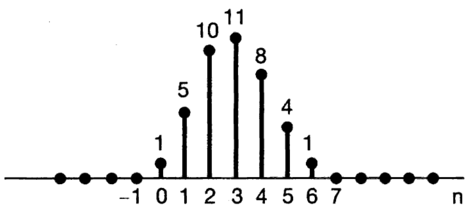
The impulse resonse $h_2[n]$ is

$h_2[n] = u[n] - u[n-2]$,

and the overall impulse response is as shown in Figure P2.24(b).



(a)



(b)

Figure P2.24

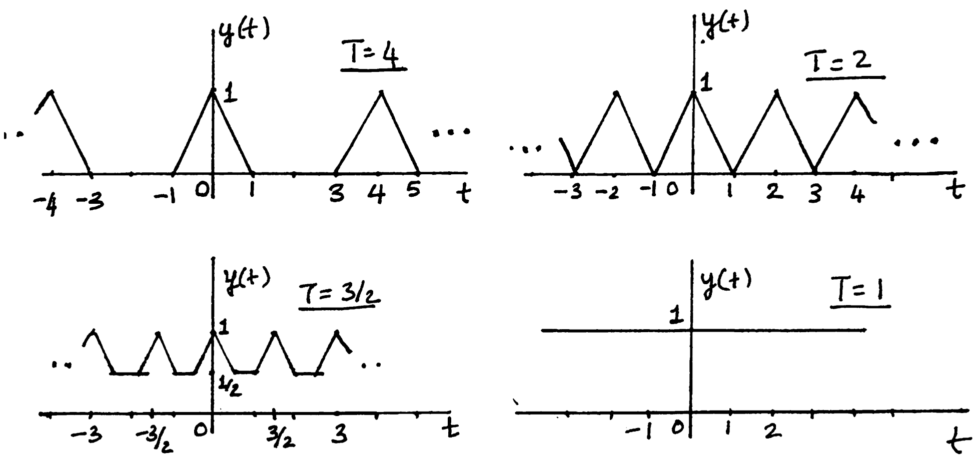
(a). Find the impulse response $h_1[n)$

(b). Find the response of the overall system to the imput $x[n] = \delta[n] - \delta[n-1]$.

(a) Answer:

We are given that $h_2[n] = \delta[n] + \delta[n-1]$. Therefore,

$$h_2[n] * h_2[n] = \delta[n] + 2\delta[n-1] + \delta[n-2].$$



Since

$$h[n] = h_1[n] * [h_2[n] * h_2[n]],$$

we get

$$h[n] = h_1[n] + 2h_1[n-1] + h_1[n-2].$$

Therefore,

$$\begin{aligned}
h[0] &= h_1[0] & \Rightarrow \quad & h_1[0] = 1, \\
h[1] &= h_1[1] + 2h_1[0] & \Rightarrow \quad & h_1[1] = 3, \\
h[2] &= h_1[2] + 2h_1[1] + h_1[0] & \Rightarrow \quad & h_1[2] = 3, \\
h[3] &= h_1[3] + 2h_1[2] + h_1[1] & \Rightarrow \quad & h_1[3] = 2, \\
h[4] &= h_1[4] + 2h_1[3] + h_1[2] & \Rightarrow \quad & h_1[4] = 1 \\
h[5] &= h_1[5] + 2h_1[4] + h_1[3] & \Rightarrow \quad & h_1[5] = 0.
\end{aligned}$$

$h_1[n] = 0$ for $n < 0$ and $n \geq 5$.

(b) Answer:

In this case,

$$y[n] = x[n] * h[n] = h[n] - h[n-1].$$

2.31

Consider the LTI system initially at rest and described by the difference equation

$$y[n] + 2y[n-1] = x[n] + 2x[n-2]$$

Find the response of this system to the input despicted in Figure P2.31 by solving the difference equation recursively.

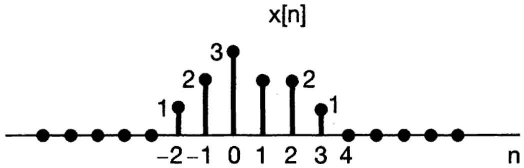


Figure P2.31

Answer:

Initial rest implies that $y[n] = 0$ for $n < -2$. Now

$$y[n] = x[n] + 2x[n-2] - 2y[n-1].$$

Therefore,

$$y[-2] \;=\; 1, \quad y[-1] = 0, \quad y[0] = 5, \quad y[1] = -4$$

$y[4] = 56, y[5] = -110, \quad y[n] = -110(-2)^{n-5} \quad$ for $n \geq 5$.