Assignment-3b Solution

Companion Book MATLAB Problems

1.4 Properties of Discrete-Time Systems

In this exericise you will practice using MATLAB to construct such counter-examples demonstrate that the system fails to satisfy the property in question.

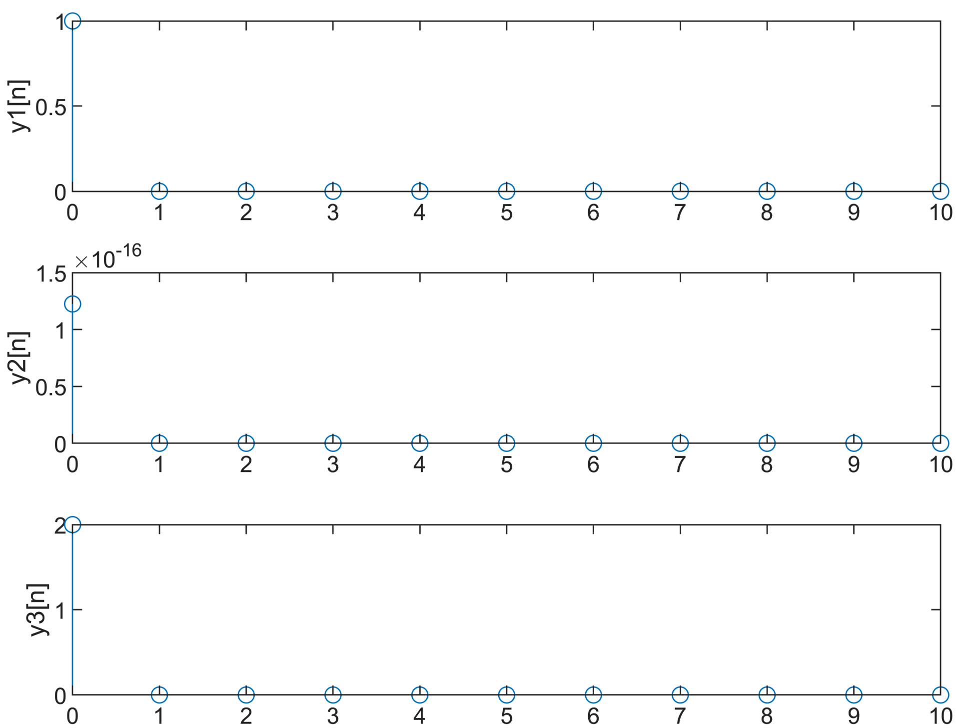
(a) The system $y[n] = sin(\pi/2x[n])$ is not linear. Use the signals $x_1[n] = \delta[n]\ and\ x_2[n] = 2\delta[n]$ to demonstrate how the system violates linearity.

(b) The system $y[n] = x[n] + x[n+1]$ is not causal. Use the singal $x[n] = u[n]$ to demonstrate this . Define the MATLAB vectors x and y to represent the input on the interval $-5 \le n \le 9$, and the output on the interval $-6 \le n \le 9$.

(c) The system $y[n] = log(x[n])$ is not stable

(d) The system given in Part (a) is not invertible

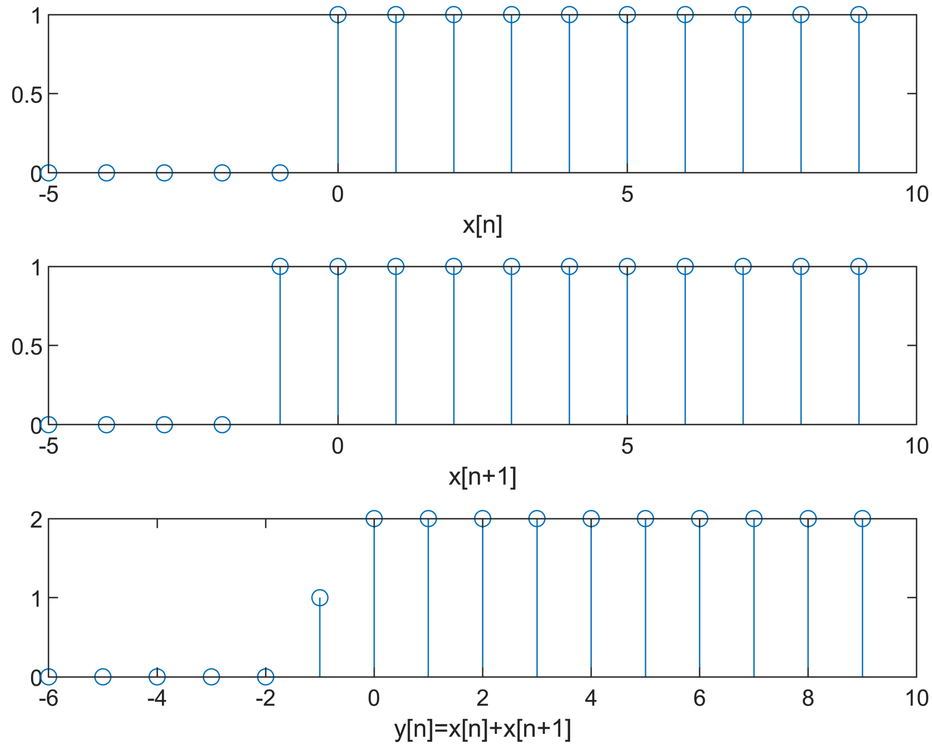
```
% L1_4a.m
clf;
n=[0:10];
x1=n*0;
x1(1)=1;
x2=2*x1;
y1=sin(pi/2 .*x1);
y2=sin(pi/2 .*x2);
% if the system is linear, then y3=y2.
y3=2*y1;
%plot
subplot(3,1,1); stem(n,y1); ylabel('y1[n]');
subplot(3,1,2); stem(n,y2); ylabel('y2[n]');
subplot(3,1,3); stem(n,y3); ylabel('y3[n]');
```



1.4a Results analysis:

As shown in the above plot, the system is not linear, since the plot shows a couner example how the system violates linearity, given $y_3[n] \neq y_2[n]$

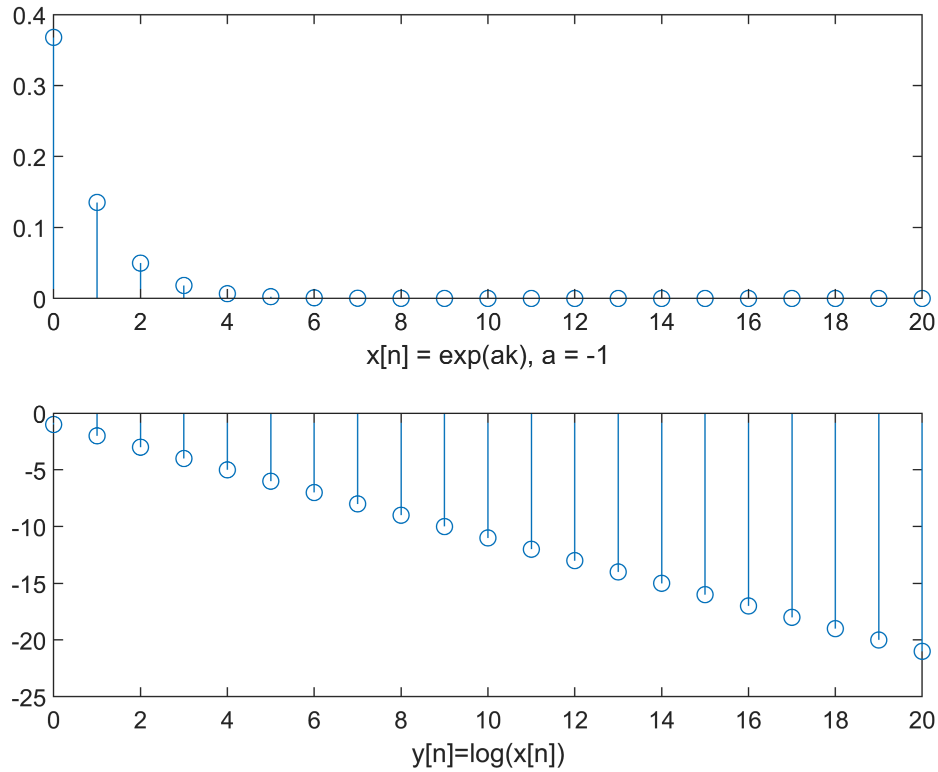
```
% L1_4b.m
% y[n]=x[n]+x[n+1] is not causal
clf
nx=[-5:9];
x=nx*0;
x(6:end)=1;
subplot(3,1,1); stem(nx,x)
xlabel('x[n]');

x1=nx*0;
x1(5:end)=1;
subplot(3,1,2); stem(nx,x1);
xlabel('x[n+1]');

ny=[-6:9];
y=ny*0;
y(6)=1;
y(7:end)=2;
subplot(3,1,3); stem(ny, y);
xlabel('y[n]=x[n]+x[n+1]');
```



1.4(b) Results Analysis:

As the plot shows, y[n] is depend of the future value of x[n], so it is not causal.

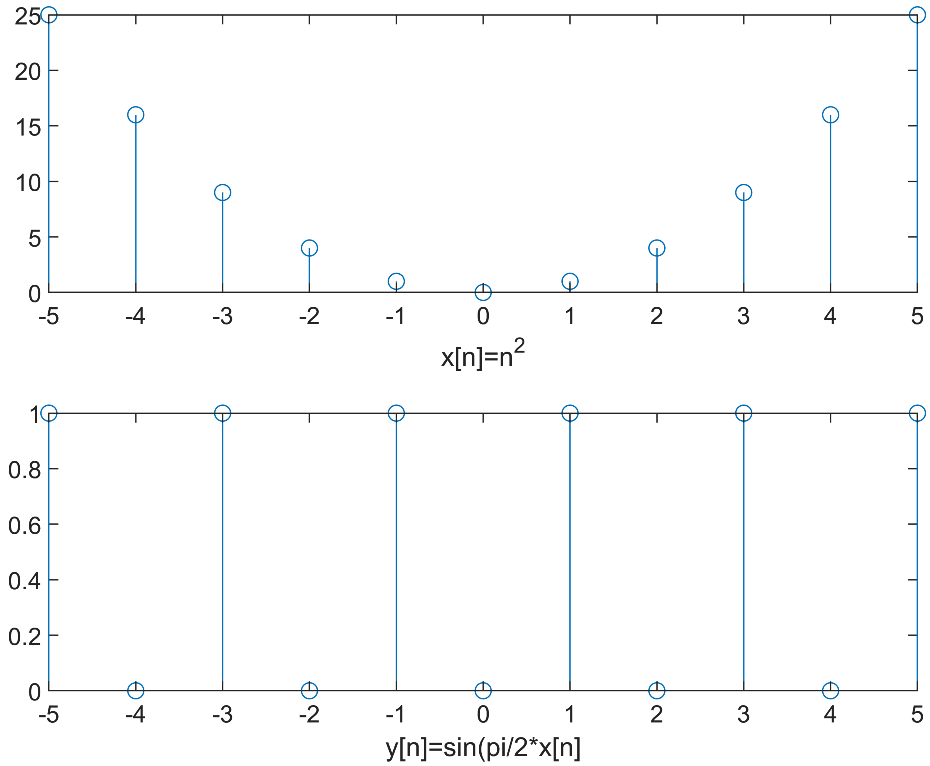
```
% L1_4c
% when a<0, x[n]=exp(ak) is bounded in (0, 1].
% while y[n]=ak is not bunded. so it is not stable.

clf
n=[0:20];
x=n*1.0;
a=-1;
for k=1:21
    x(k)=exp(a*k);
end
subplot(2,1,1); stem(n,x);
xlabel('x[n] = exp(ak), a = -1')
y=log(x);
subplot(2,1,2); stem(n,y);
xlabel('y[n]=log(x[n])')
```



1.4c Results Analysis:

As the above plot shows, given a bounded input $x[n] = e^{ak}$ with a = -1, the y becomes unbounded. So this is a counter example showing y[n] is not stable.

```
% L1_4d.m
% Given y[n]=sin(pi/2*x[n]), the system is not invertible.
%
clf;
n=[-5:5];
x=n.^2;
subplot(2,1,1); stem(n, x);
xlabel('x[n]=n^2')
y=sin(pi/2 .*x);
subplot(2,1,2); stem(n,y);
xlabel('y[n]=sin(pi/2*x[n]')
```



1.4d Results Analysis:

A system is invertible if distinct imputs lead to disinct outputs. The above plot is a counter example shows for two distinct inputs at $x[-2]$ *and* $x[2]$, the system outputs have the same value (not distinctable). Therefore the system is not invertible.

Advanced Problems

For each of the flollowing systems, state whether of not the system is linear, time-invariant, causal, stable, and invertible. You only need to show at least one preporty that system does ot possecess by a conuter-example using MATLAB.

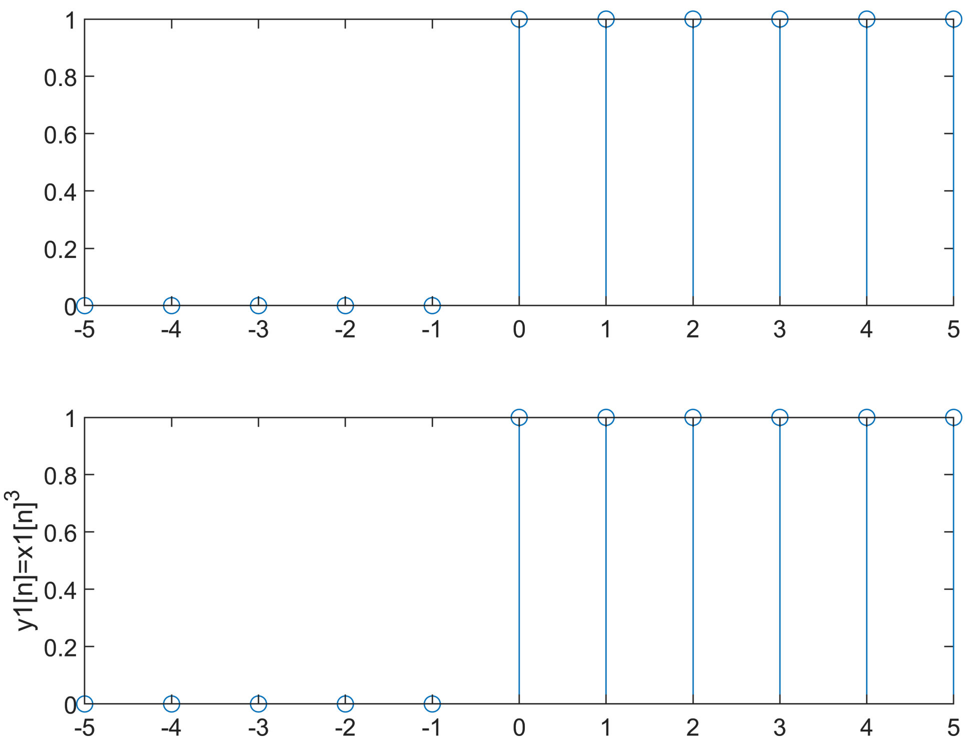
(e) $y[n] = x^3[n]$. Hint: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(f) $y[n] = nx[n]$

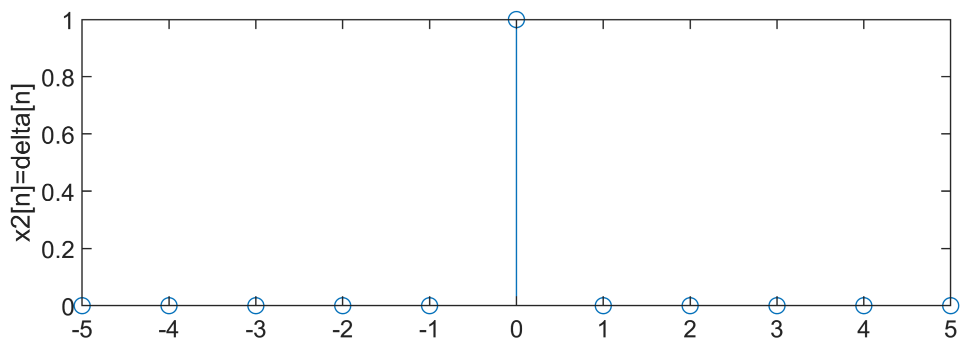
(g) $y[n] = x[2n]$

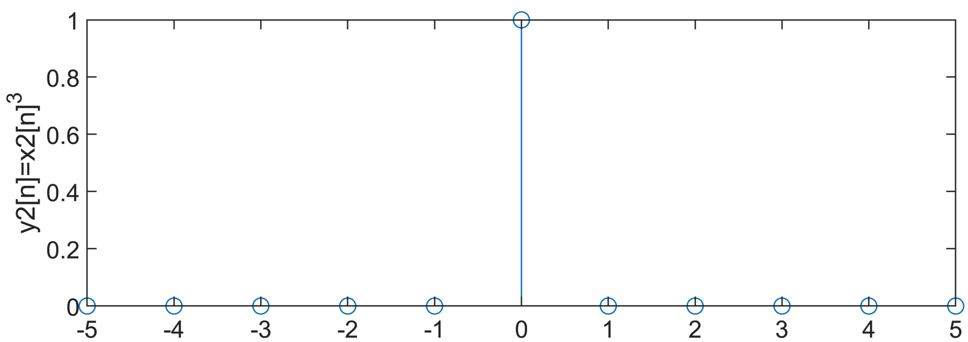
```
% L1_4e.m
% y[n]=x3[n]^3 is not linear

clf;

n=[-5:5];
% set x1[n]=u[n]
```

```
x1=n*0;
x1(6:end)=1;
y1=x1.^3;
figure;
subplot(2,1,1); stem(n,x1);
subplot(2,1,2); stem(n,y1);
ylabel('y1[n]=x1[n]^3');
```

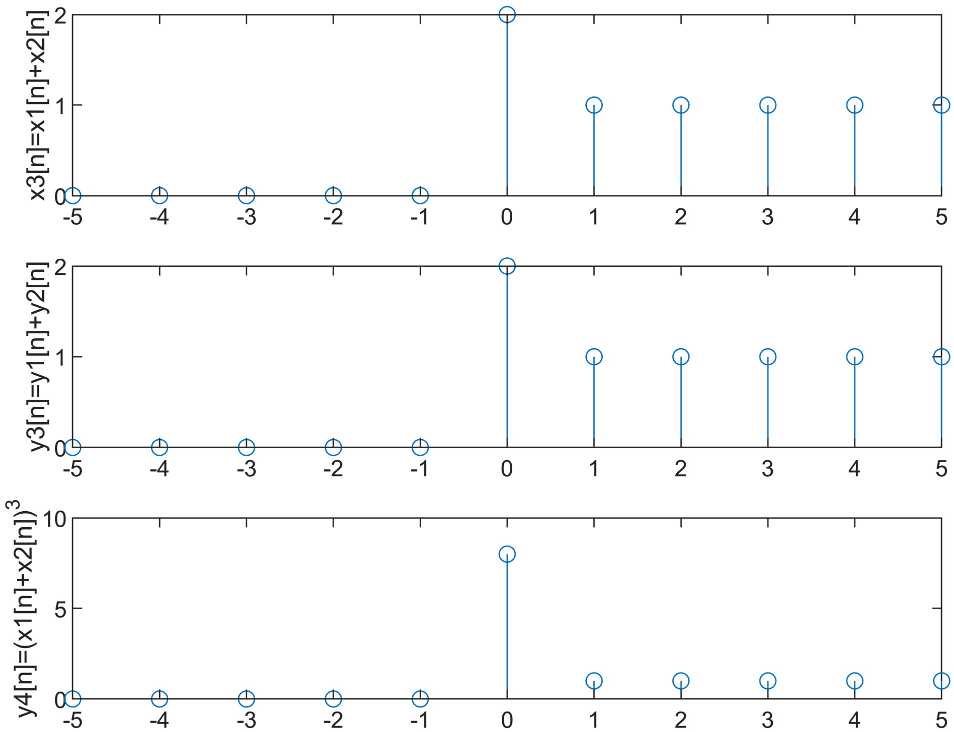



```
% set x2[n]=delta[n]
x2=n*0;
x2(6)=1;
y2=x2.^3;
figure;
subplot(2,1,1); stem(n,x2);
ylabel('x2[n]=delta[n]');
subplot(2,1,2); stem(n,y2);
ylabel('y2[n]=x2[n]^3');
```






```
% set x3[n]=x1[n]+x2[n]
x3=x1+x2;
% y[n]=y1[n]+y2[n]
y3=y1+y2;

figure;
subplot(3,1,1); stem(n,x3);
ylabel('x3[n]=x1[n]+x2[n]');
subplot(3,1,2); stem(n,y3);
ylabel('y3[n]=y1[n]+y2[n]');

% y[n]=x3[n]^3
%yy=x1.^3 + 3*x1.^2.*x2 + 3*x1.*x2.^3 + x2.^3
y4=(x1+x2).^3;
subplot(3,1,3); stem(n,y4);
ylabel('y4[n]=(x1[n]+x2[n])^3')
```



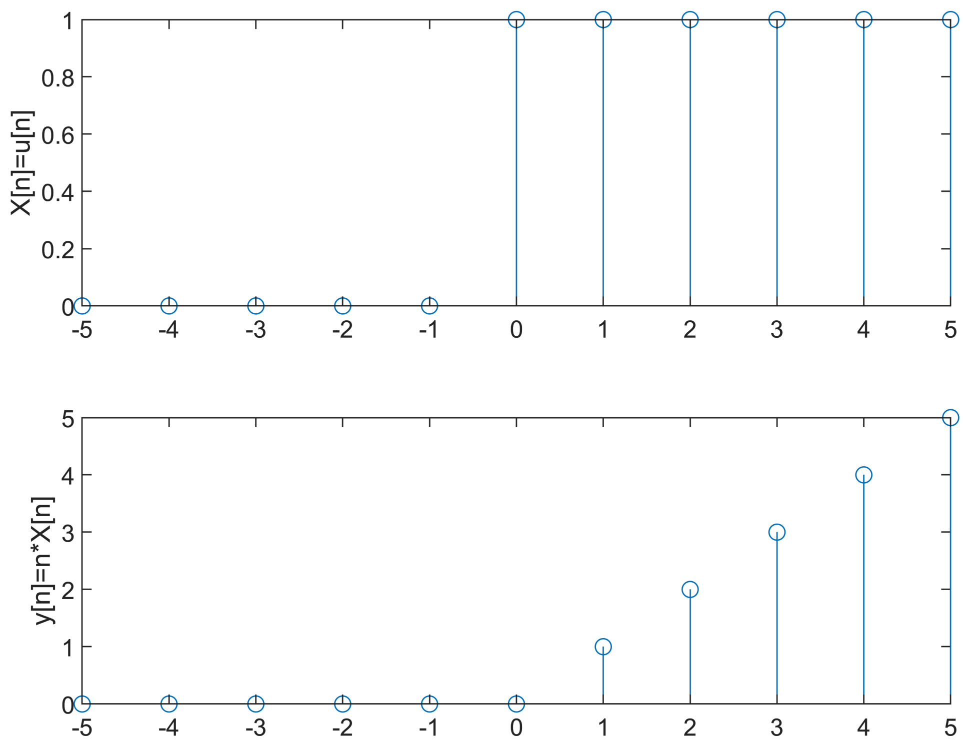
1.4e Result Analysis:

Given system $y[n]=x^3[n]$, we set $x1[n]=u[n]$ and $x2[n]=\delta[n]$.

Given $y_1[n]$ *and* $y_2[n]$ are the system response to $x_1[n]$ *and* $x_2[n]$, if we let $x3[n]=x1[n]+x2[n]$, then for $y[n]=x^3[n]$ to be linear. It is must satisfy that $y[n]=x_3[n]^3$ *and* $y[n]=y_1[n]+y_2[n]$ are the same.

As shown in the above plot, this is not the case. Therefore $y[n]=x^3[n]$ is not linear.

```
% L1_4f.m
% y[n]=n*x[n] is not stable

clf;

n=[-5:5];
% set x[n]=u[n]
x=n*0;
x(6:end)=1;
y=n.*x;
figure;
subplot(2,1,1); stem(n,x);
ylabel('X[n]=u[n]');
subplot(2,1,2); stem(n,y);
ylabel('y[n]=n*X[n]');
```

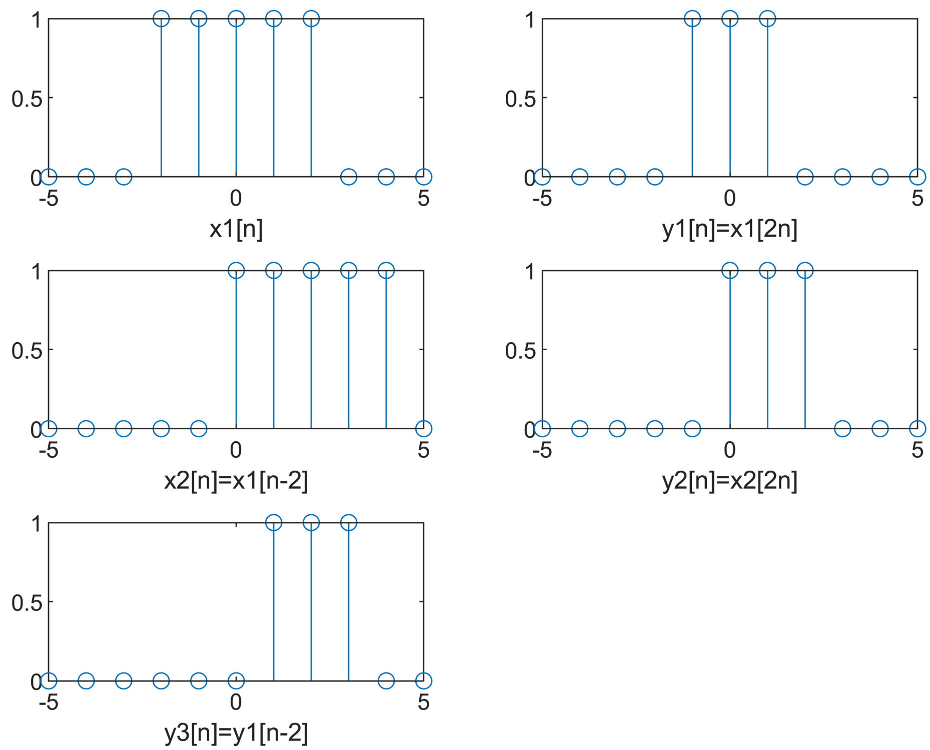


1.4f Result Analsis:

Given the system $y[n] = nx[n]$. Let $x[n] = u[n]$ be bounded input. If the y[n] is a stable system, then it would be bounded as well. The plot shows y[n] is not bounded. Therefore the system is not stable.

```
% L1_4g.m
% the system is not time invariant
clf;
n=[-5:5];

% x1[n]
x1=n*0;
x1(4:8)=1;
subplot(3,2,1);  stem(n,x1);
xlabel('x1[n]');

% y1[n]=x1[2n]
y1=n*0;
y1(5:7)=1;
subplot(3,2,2);  stem(n,y1);
xlabel('y1[n]=x1[2n]');

% x2[n]=x1[n-2]
x2=n*0;
```

```
x2(6:10)=1;
subplot(3,2,3);  stem(n,x2);
xlabel('x2[n]=x1[n-2]');

% y2[n]=x2[2n]
y2=n*0;
y2(6:8)=1;
subplot(3,2,4);  stem(n,y2);
xlabel('y2[n]=x2[2n]');

%y3[n]=y1[n-2]
y3=n*0;
y3(7:9)=1;
subplot(3,2,5);  stem(n,y3);
xlabel('y3[n]=y1[n-2]');
```



1.4g Result Analysis:

For a given $y[n] = x[2n]$, if y[n] is a time-invariant system then it must satisfy the follow:

For a input $x_1[n] = u[-2] - u[3]$, let $x_2[n] = x_1[n-2]$, a delayed $x_1[n]$ by . Given $y_1[n]$ *and* $y_2[n]$ are the system responses for $x_1[n]$ *and* $x_2[n]$ respectively, then $y_2[n]$ must be the same as the shift of the system output $y_3[n] = y_1[n-2]$. The above plots show this is not the case. Therefore, the system is not time-invariant.

1.6 Continous-Time Complex Exponential Signals

Complex exponentials are particularly useful for analyzing signals and systems, since they form the building blocks for a large class of signals. Two familar signals which can be expressed as a sum of complex exponentials are consine and sine. Let consider the continous-time complex expoential signals which have the form e^{st}.   For example, by setting $s = \pm i\omega t$, we obtain:

$$cos(\omega t) = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t}) \tag{1.7}$$

$$sin(\omega t) = \frac{1}{2i}(e^{i\omega t} - e^{-i\omega t}) \tag{1.8}$$

In this exercise, you will be asked to use the Symbolic Math Toolbox.

Basic Problems

(a). Consider the continous-time sinusoid, $x(t) = sin(2\pi t/T)$.   A symbolic expression can be created to represent $x(t)$ within MATLAB by executing

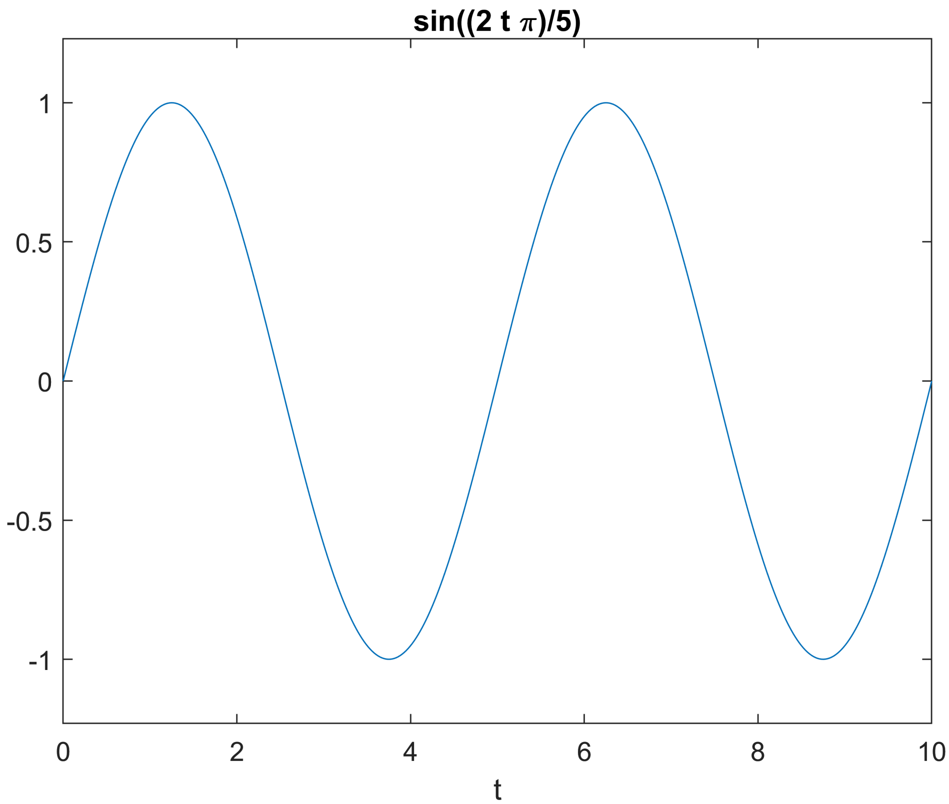
>> x = sym(sin(2*pi*t/T));

If you desire T=5, you can use bubs as follows:

>> x5 = subs(x, T, 5);

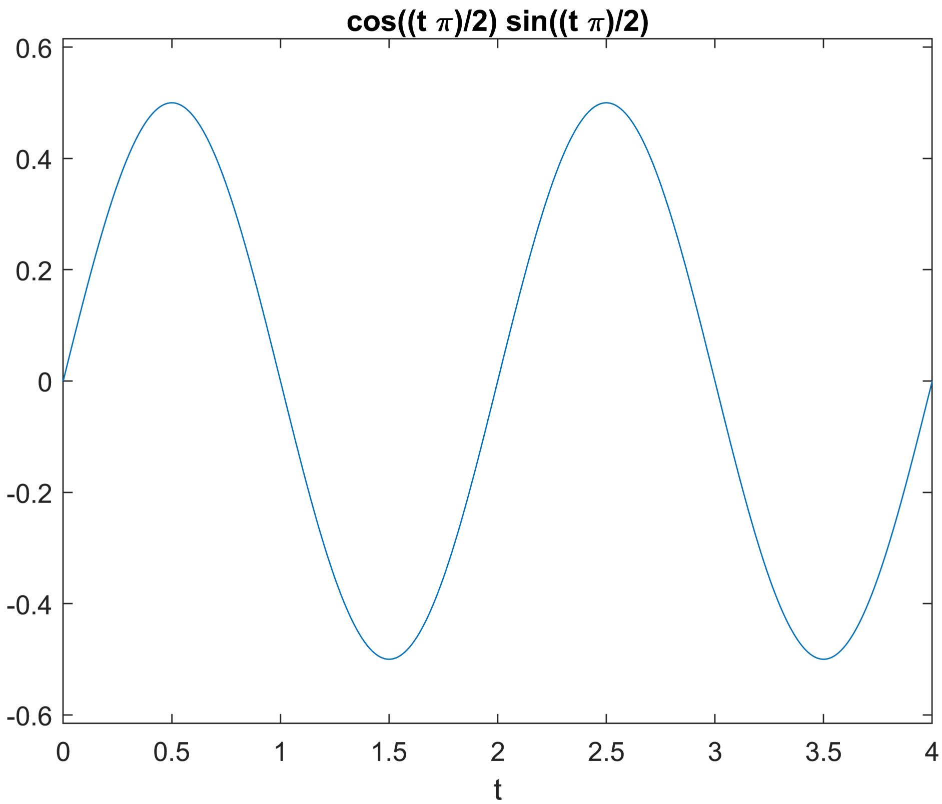
Note: both lines of code are slitly different from the companion book to avoid a compiling error.

```
% 1.6a
clf;
syms t;
syms T;
x = sym(sin(2*pi*t/T));
x5=subs(x,T,5);
ezplot(x5,[0 10]);
```

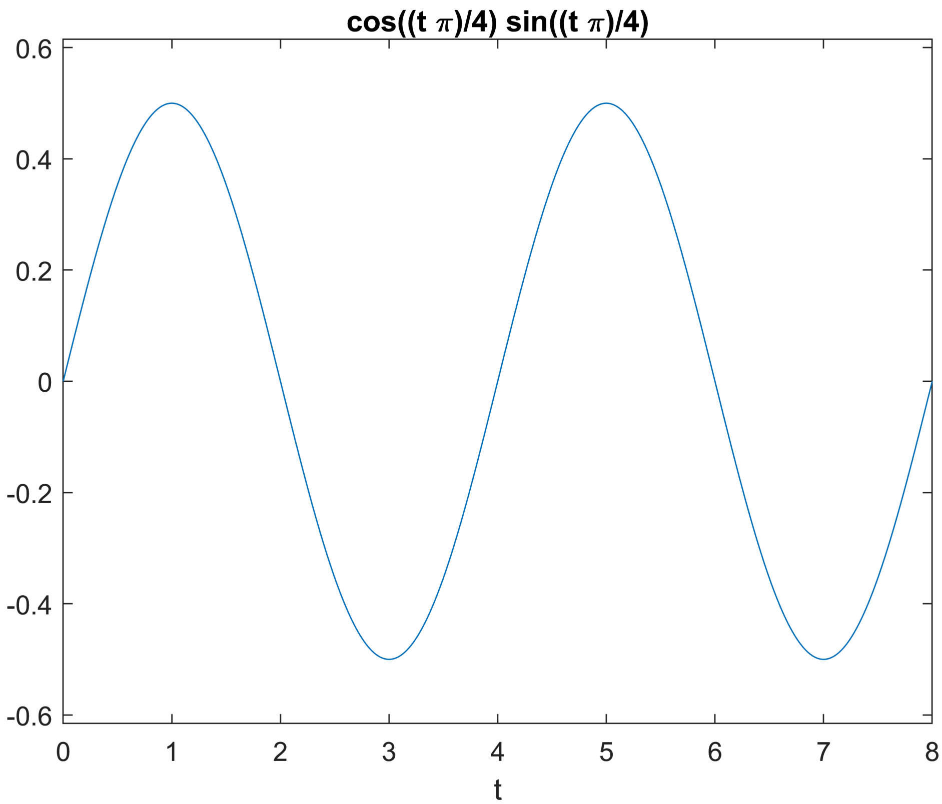


(b). Create a symbolic expression for the signal $x(t) = cos(2\pi t/T)sin(2\pi t/T)$. For T=4, 8, and 16, use ezplot to plot the signal on the intrval $0 \le t \le 32$. What is the fundamental period of x(t) in terms of T?

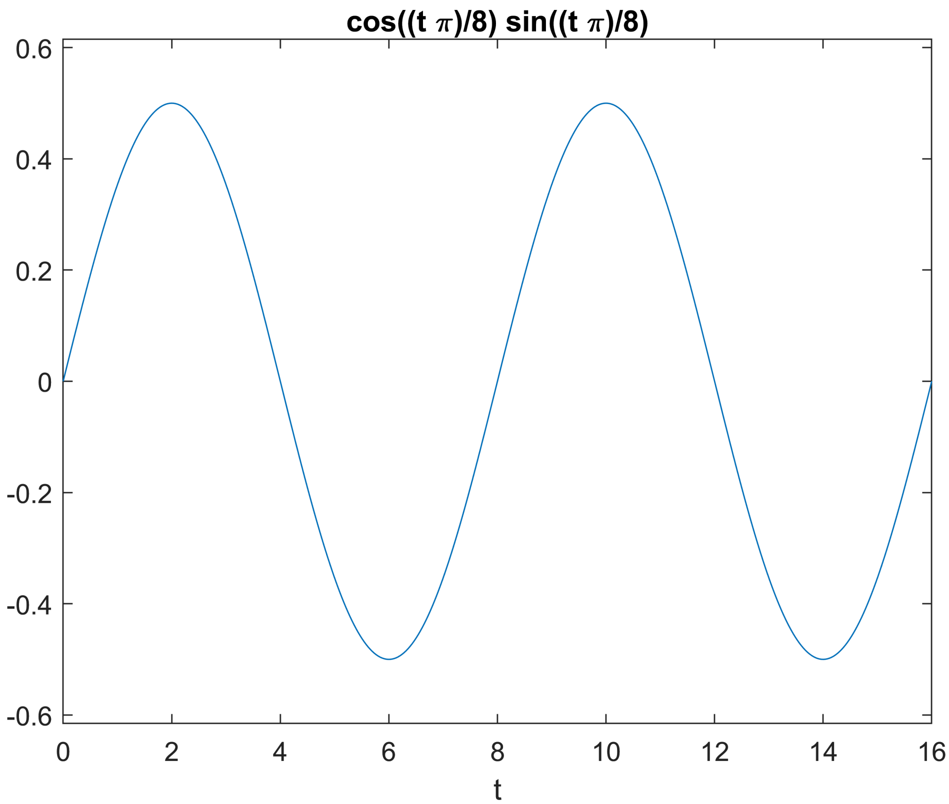
```
% 1.6b
clf;
syms t;
syms T;
x = sym(cos(2*pi*t/T)*sin(2*pi*t/T));
x4=subs(x,T,4);
ezplot(x4,[0 4]);
```




```
x8=subs(x,T,8);
ezplot(x8,[0 8]);
```



```
x16=subs(x,T,16);
ezplot(x16,[0 16]);
```



1.6b Resulta Analysis

As shown in the above plots, the fundamental period $T_0 = 2, 4 \ \ and \ \ 8$, respectively, for T=4, 8, and 16.

Mathmetically, $x(t) = cos(2\pi t/T)sin(2\pi t/T) = \frac{1}{2}sin[2(2\pi t/T)] = \frac{1}{2}sin(4\pi t/T)$, where $\omega_0 = \frac{4\pi}{T}$. So to compute fundamental period using $T_0 = \frac{2\pi}{\omega_0} \ m = T/2$, We got $T_0 = 2, 4 \ \ and \ \ 8$, respectively, for T=4, 8, and 16. We alo noticed that the amplitude is 1/2.

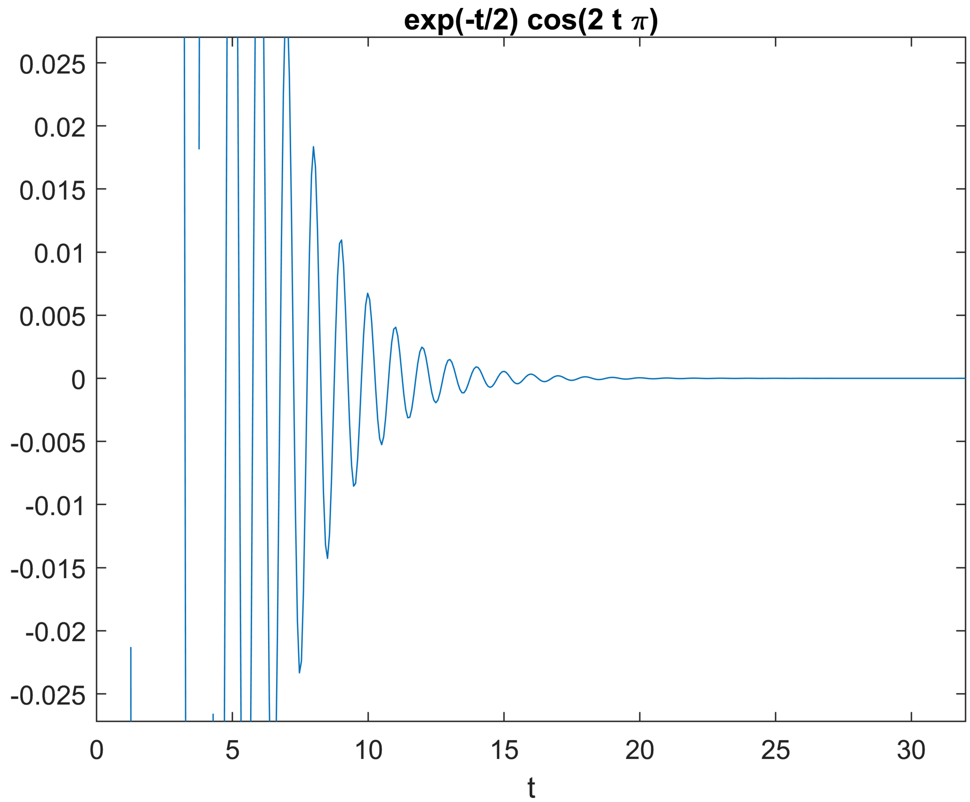
Therefore the above plots agree with the mathmetical analysis.

Intermediate Problem

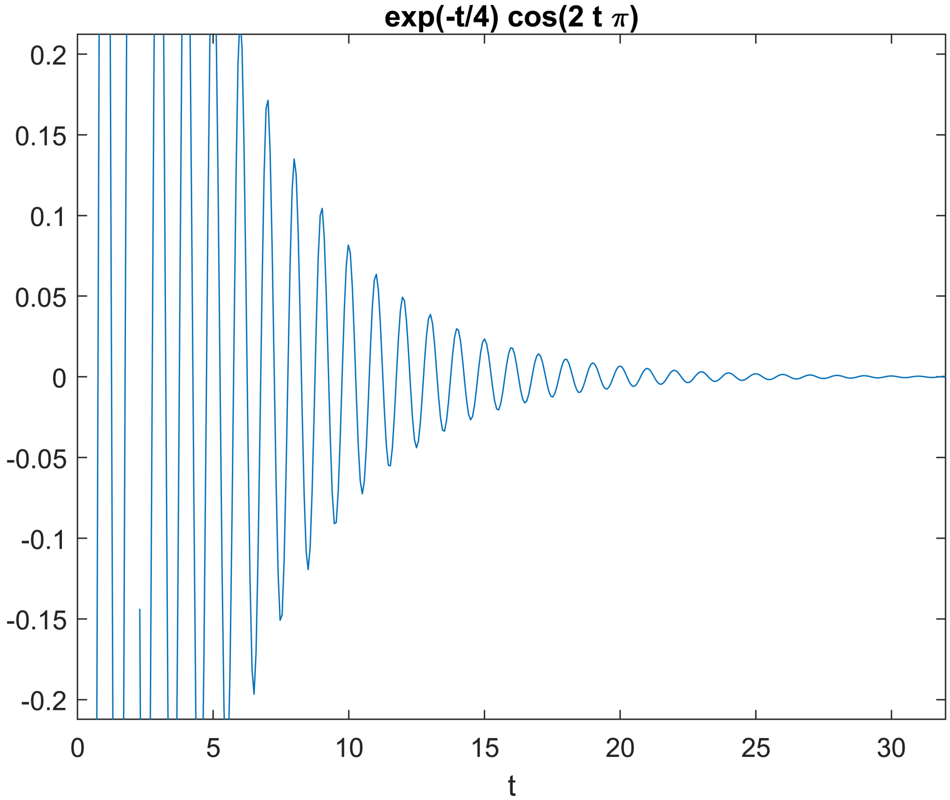
(c). Create a symbolic expression for th signal $x(t) = e^{-at}cos(2\pi t)$. For $a = 1/2, 1/4 \ \ and \ \ 1/8$, use ezplot to determine t_d, time time at which $|x(t)|$ last crossed 0.1. Define t_d as the time at which the signal dies out. Use ezplot to determine for each value of a how many complete periods of the cosine occur before the signal diew out. Does the number of periods appear to be proportional to Q?

```
% 1.6c
```

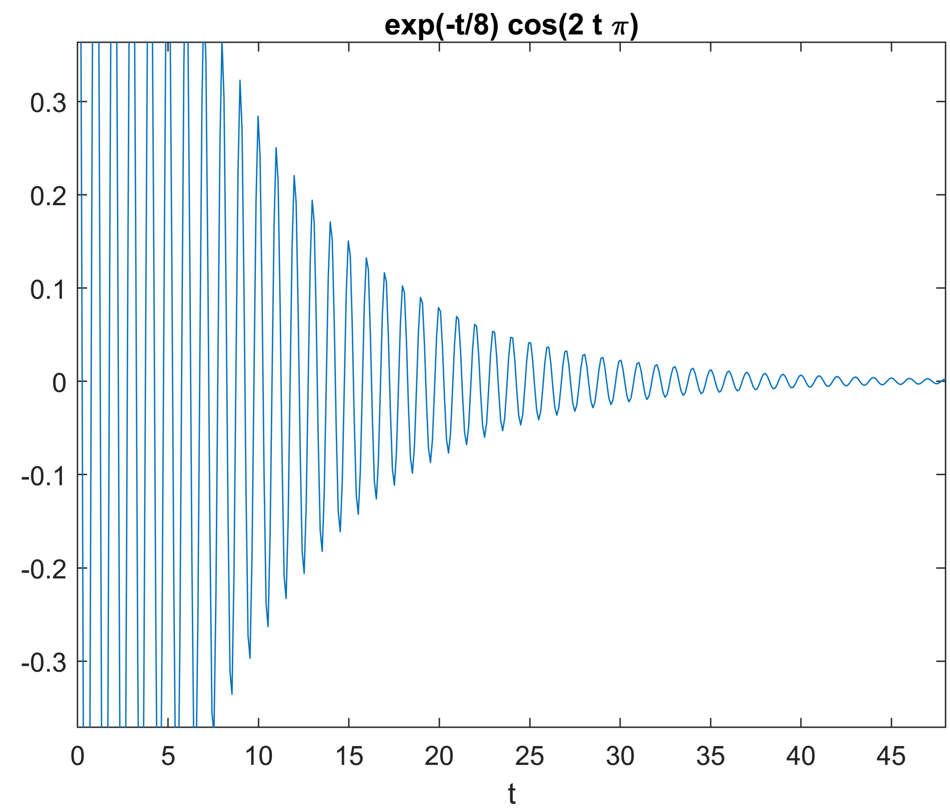
```
clf;
syms t;
syms a;
x = sym(exp(-a*t)*cos(2*pi*t));
xh=subs(x,a,1/2);
ezplot(xh,[0 32]);
```




```
xq=subs(x,a,1/4);
ezplot(xq,[0 32]);
```




```
xe=subs(x,a,1/8);
ezplot(xe,[0 48]);
```



1.6c Results and Analysis

As shown in above plots, by observation, for $a = 1/2, 1/4 \ \ and \ \ 1/8$, we have $t_d = 18, 22, 44$ respectively. The number of complete periods before the signal dies out are 12, 24, 48.

Yes, the number of periods appear to be propertional to Q (the greater Q is, the greater the number of periods are).