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CTMC/DTMC Conversion and aPRAM

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1. Introduction

This document explores the aPRAM format proposed by the University of Pittsburgh, it's relation to the space of dynamic models, translations to and from Petri-nets and aPRAM, and characterizes the error implied by the aPRAM formalism.

2. Petri-net to aPRAM

In order to translate Petri-nets to aPRAMs we need to examine a Petri-net as a series of `Act` or `Mod` choices at any given point of time. To do so, we consider a state variable in the Petri-net, and the set of events that can transition "tokens" in a Petri-net state variable to other state variables.

Figre 1 shows an open Petri-net, focusing on the tokens in `init`. Each token in this case are equivalent to an aPRAM "agent". In this case, the agent represented by these tokens transitions from `init` to `S` with rate a, and from `init` to `I` with rate b.

To translate this to an aPRAM, we take the time step for aPRAM δt, and use the following properties.

Axiom 2.1. *Markovian property Events whose rates are exponential are Markovian. Markovian*

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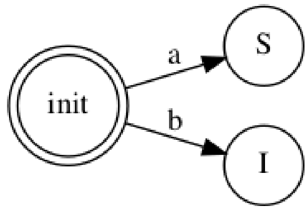

Figure 1: An open Petri-net, considering the state variable `init` and the enabled events that can act on "tokens" in that state variable.

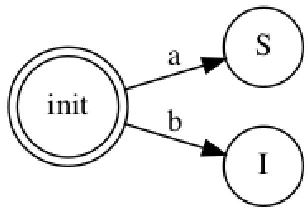

Figure 2: An open Petri-net, considering the state variable `init` and the enabled events that can act on "tokens" in that state variable.

processes are memoryless *the next-event time for an event is independent of the time which has elapsed, and depends only on the current state of the system.*

Axiom 2.2. *Exponential probability distribution function The probability distribution function of an exponentially distributed random variable with rate λ is given as*

$$f(x) = \lambda e^{-\lambda x}.$$

The probability distribution function gives the probability that the next event time is exactly x.

Axiom 2.3. *Exponential cumulative distribution function The cumulative distribution function (CDF) of an exponentially distributed random variable with rate λ is given as*

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} \lambda e^{-\lambda x} dt && (1) \\ &= 1 - e^{-lambdax} && (2) \end{aligned}$$

The CDF gives the probability that the next event time is less than or equal to x.

In order to convert a Petri-net to an aPRAM our goal is to go from a representation like figure 1, with rates a and b to a representation like figure 2, with probabilities x, y, and z. More generally, assume an agent is represented by a token in a Petri-net. This token can transition to other state variables with rates given by the vector $R = [r_1, r_2, \ldots]$ which has size $|R| = n$. The aPRAM representation of the agent this token models is given by a vector $P = [p_0, p_1, \ldots]$ which has size $|P| = n + 1$. We will assume p_i is the probability the transition with rate r_i is chosen during an interval of length δt, and p_0 is the probability the agent does not transition during an interval of length δt. Axiom 2.1 allows us to consider all intervals of length δt without concern for the elapsed simulation time.