

Compiling Proof Obligations

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1 Introduction

In this chapter, we will look at the relationship between the verification conditions generated for a Java like source programming language and the verification conditions generated for the bytecode language as defined in Chapter ?? . More particularly, we argue that they are syntactically equivalent if the compiler is nonoptimizing and satisfies certain conditions.

First, we would like to give the context and motivations for studying the relationship between source and bytecode verification.

Security becomes an important issue for the overall software quality. This is especially the case for mobile code or what so ever untrusted code. A solution proposed by G.Necula (see his thesis ??) is the PCC framework which brings the possibility to a code receiver to verify if an unknown code or untrusted code respects certain safety conditions before being executed by the code receiver. In this framework the code client annotates the code automatically, generates verification conditions automatically and proves them automatically. The program accompanied by the proof is sent to the code receiver who will typecheck the proof against the verification conditions that he will generate. Because of its full automation this architecture fails to deal with complex functional or security properties.

The relation of the verification conditions over bytecode and source code can be used for building an alternative PCC architecture which can deal with complex security policy. More particularly, such an equivalence can be exploited by the code producer to generate the certificate interactively over the source code. Because of the equivalence between the proof obligations over source and bytecode programs (modulo names and types), their proof certificates are also the same and thus the certificate generated interactively over the source code can be sent along with the code.

In the following, Section 1.1 presents an overview of existing work on this subject. In Section 2, we introduce the source programming language. As we shall see, this programming language has the basic features of Java as it supports object creation and manipulation, like instance field access and update, exception throwing and handling, method calls as well as subroutines.

Section 3 presents a simple non optimizing compiler from the source language to the bytecode language presented already in Chapter ?? . In this section, we will discuss certain properties of the compiler which are necessary conditions for establishing this equivalence.

Next, Section 4 presents the weakest precondition predicate transformer which we define over the source language.

Section 5 introduces a new formulation of the weakest precondition for bytecode, which is defined over the compilation of source expressions and statements. This formulation of the weakest precondition is helpful for establishing the desired relation between bytecode and source proof obligations. In this section, we also discuss upon what conditions the weakest precondition given before in Chapter ?? and this new version are equivalent.

In Section 6, we proceed with the proof of equivalence between the proof obligations generated by the weakest precondition defined in Chapter ?? . To do this, we first establish the equivalence between the source weakest precondition and the bytecode weakest precondition defined over the compilation of statements and expressions. We exploit also the equivalence between the two

bytecode weakest preconditions to conclude that the source verification condition generator and the bytecode verification condition generator presented in Chapter ??

1.1 Related Work

Several works dealing with the relation between source and its compilation verification condition generated.

Barthe, Rezk and Saabas in ?? also argue that proof obligations produced over source code and bytecode produced by a nonoptimizing compiler are equivalent. The source language supports method invocation, exception throwing and handling. They do not consider instructions that may throw runtime exceptions. Note that because of this

However, in the article they do not discuss at all what are the assumptions or properties of the compiler which will guarantee this equivalence. We claim that their proof for the verification condition preserving compilation holds only if their non optimizing compiler has the properties discussed here in Section 3.

2 Source language

We present a source Java-like programming language which supports the following features: object manipulation and creation, method invocation, throwing and handling exceptions, subroutines etc. The first definition that we give hereafter presents all the constructs of our language which evaluate to a value.

Definition 2.1 (Expression) *The grammar for source expressions is defined as follows*

$$\begin{aligned}
 \mathcal{E}^{src} ::= & \text{constInt} \\
 & | \text{true} \\
 & | \text{false} \\
 & | \mathcal{E}^{src} \text{ op } \mathcal{E}^{src} \\
 & | \mathcal{E}^{src}.f \\
 & | \text{var} \\
 & | (Class) \mathcal{E}^{src} \\
 & | \text{null} \\
 & | \text{this} \\
 & | \mathcal{E}^{\mathcal{R}} \\
 & | \mathcal{E}^{src}.m() \\
 & | \text{new } Class(\mathcal{E}^{src}) \\
 \\
 \mathcal{E}^{\mathcal{R}} ::= & \mathcal{E}^{src} \mathcal{R} \mathcal{E}^{src} \\
 & | \mathcal{E}^{src} \text{ instanceof } Class \\
 \\
 \mathcal{R} \in & \{\leq, <, \geq, >, =, \neq\}
 \end{aligned}$$

We now give an informal description of the meaning of the expressions of the above grammar:

- **constInt** is any integer literal

because they do not expose those conditions, they cannot use the induction hypothesis because they have not proven that the condition to apply the induction hold

- **true** and **false** are the unique boolean constants
- **constRef** is a reference to an object in the memory heap
- $\mathcal{E}^{src} \text{ op } \mathcal{E}^{src}$ which stands for an arithmetic expression with any of the arithmetic operators $+, -, div, rem, *$
- $\mathcal{E}^{src}.f$ is a field access expression where the field with name f is accessed
- the cast expression $(Class)\mathcal{E}^{src}$ which is applied only to expressions from a reference type
- the expression **null** stands for the null reference which does not point to any location in the heap
- **this** refers to the current object
- $\mathcal{E}^{src}.m()$ stands for a method invocation expression. Note that here we consider only methods without arguments which return a value
- **new** $Class(\mathcal{E}^{src})$ stands for an object creation expression of class $Class$. We consider only constructors which take only one argument for the sake of readability

The language is also provided with relational expressions, which evaluate to the boolean values:

- $\mathcal{E}^{src} \mathcal{R} \mathcal{E}^{src}$ where $\mathcal{R} \in \{\leq, <, \geq, >, =, \neq\}$ stands for the relation between two expressions
- \mathcal{E}^{src} **instanceof** $Class$ states that \mathcal{E}^{src} has as type the class $Class$ or one of its subclasses

The expressions can be of object types or basic types. Formally the types are

$\text{JavaType} ::= Class, Class \in \text{ClassTypes} \mid \text{int} \mid \text{boolean}$

The next definition gives the control flow constructs of our language as well as the expressions that have a side effect

Definition 2.2 (Statement) *The grammar for expressions is defined as follows :*

$$\begin{aligned}
 \text{STMT} ::= & \text{STMT}; \text{STMT} \\
 & \mid \text{if } (\mathcal{E}^{\mathcal{R}}) \text{ then } \{\text{STMT}\} \text{ else } \{\text{STMT}\} \\
 & \mid \text{try } \{\text{STMT}\} \text{ catch } (\text{Exc}) \{\text{STMT}\} \\
 & \mid \text{try } \{\text{STMT}\} \text{ finally } \{\text{STMT}\} \\
 & \mid \text{throw } \mathcal{E}^{src} \\
 & \mid \text{while } (\mathcal{E}^{\mathcal{R}})[\text{INV}, \text{modif}] \{\text{STMT}\} \\
 & \mid \text{return } \mathcal{E}^{src} \\
 & \mid \mathcal{E}^{src} = \mathcal{E}^{src}
 \end{aligned}$$

From the definition we can see that the language supports also the following constructs :

est-ce que
je dois dire
qu'on con-
sidere un
sousemble
de Class qui
represente les
exceptions ?

- $STMT; STMT$, i.e. statements that execute sequentially
- $\text{if } (\mathcal{E}^R) \text{ then } \{STMT\} \text{ else } \{STMT\}$ which stands for an if statement. The semantics of the construct is the standard one, i.e. if the relation expression \mathcal{E}^R evaluates to true then the statement in the **then** branch is executed, otherwise the statement in the **else** branch is executed
- $\text{try } \{STMT\} \text{ catch } (Class) \{STMT\}$ which states that if the statement following the **try** keyword throws an exception of type **Exc** then the exception will be caught by the statement following the **catch** keyword
- $\text{try } \{STMT\} \text{ finally } \{STMT\}$
- $\text{while } (\mathcal{E}^R)[INV, \text{modif}] \{STMT\}$ states for a loop statement where the body statement $STMT$ will be executed until the relational expression \mathcal{E}^R evaluates to true.
- $\text{return } \mathcal{E}^{src}$ is the statement by which the execution will be finished
- $\mathcal{E}^{src} = \mathcal{E}^{src}$ stands for an assignment expression, where the value of the left expression is updated with the value of the right expression
- finally, every expression \mathcal{E}^{src} is a statement

3 Compiler

We now turn to specify a simple compiler from the source language presented in Section 2 into the bytecode language. The compiler does not perform any optimizations.

The compiler is the triple

$$\langle \lceil *, *, * \rceil, \text{addExHandler}, \text{addLoopSpec} \rangle$$

The first element in the triple is a function $\lceil \cdot \rceil$ which transforms statements and expressions into a sequence of bytecode instructions, the second element is the procedure **addExHandler** which keeps track of the exception handlers. The third element of the compiler is the procedure **addLoopSpec** which manages the compilation of loop specification, namely loop invariants and the modified expressions.

In the following, in the next Section 3.1 we define the procedure for exception handlers, in Section 3.2, we will present the procedure for compiling loops and in Sections ?? and 3.4 we will proceed with the definition of the function $\lceil \cdot \rceil$ for expressions and statements.

3.1 Exception handler table

Our source language contains exception handler constructions, and thus, when compiling a method body the compiler should keep track of the exception handlers by adding information in the exception handler table array **excHndls** (presented in Section ??) every time it sees one. To do this, the compiler is provided with a procedure with the following name and signature:

the exception handler function

$\text{addExcHandler} : \text{nat} \rightarrow \text{nat} \rightarrow \text{nat} \rightarrow \mathbf{Class}_{exc} \rightarrow \mathbf{Method}$

The definition of the procedure is the following:

$$\begin{aligned} \text{addExcHandler}(start, end, h, \mathbf{Exc}, \mathbf{m}) = \\ \mathbf{m}.excHndIS := \{(start, end, h, \mathbf{Exc}), \mathbf{m}.excHndIS\} \end{aligned}$$

The function adds a new element in the exception handler table of a method \mathbf{m} . The meaning of the new element is that every exception of type \mathbf{Exc} thrown in between the instructions $start \dots end$ can be handled by the code starting at index h .

We can remark that when the function addExcHandler adds a new element in the exception handler table array of a method \mathbf{m} , the new element is added at the beginning of the exception handler table $\mathbf{m}.excHndIS$.

3.2 Compiling loop invariants

When compiling a method \mathbf{m} , the compiler will also take care of the loop specification in the source loops by adding it in the loop specification table $\mathbf{m}.loopSpecS$ (defined in Section ??) of the method \mathbf{m} .

This is done by the procedure addLoopSpec which has the following signature

$\text{addLoopSpec} : \text{nat} \rightarrow \mathcal{F}^{bc} \rightarrow \text{list } \mathcal{E} \rightarrow \mathbf{Method}$

The definition of the procedure is hereafter:

$$\begin{aligned} \text{addLoopSpec}(i, \text{INV}, \text{modif}, \mathbf{m}) = \\ \mathbf{m}.loopSpecS := \{(i, \text{INV}, \text{modif}), \mathbf{m}.loopSpecS\} \end{aligned}$$

3.3 Compiling expressions in bytecode instructions

As we stated in the beginning the function for compiling statements in bytecode instruction is named $\lceil \cdot \rceil$

$$\lceil \cdot \rceil : \text{nat} * (\mathcal{STMT} \cup \mathcal{E}^{src}) * \text{nat} \longrightarrow \mathbf{I}[\]$$

The compiler function takes three arguments: a natural number s from which the labeling of the compilation of \mathcal{STMT} starts, the statement \mathcal{STMT} to be compiled and a natural number which is the greatest label in the compilation of \mathcal{STMT} and returns an array of bytecode instructions. In the next we look at the definition of the compiler over expressions.

- integer or boolean constant access

- integer constant access

$$\lceil s, \mathbf{constInt}, s \rceil = s : \text{push } \mathbf{constInt}$$

- boolean constant access

$$\lceil s, \mathbf{true}, s \rceil = s : \text{push } 1$$

$$\lceil s, \mathbf{false}, s \rceil = s : \text{push } 0$$

Note: the source boolean expressions are compiled down to integers

- method invocation

$$\begin{aligned} \lceil s, \mathcal{E}_1^{src}.m(\mathcal{E}_2^{src}), e \rceil &= \begin{array}{l} \lceil s, \mathcal{E}_1^{src}, e' \rceil; \\ \lceil e' + 1, \mathcal{E}_2^{src}, e - 1 \rceil; \\ e : \text{invoke } m \end{array} \end{aligned}$$

- field access

$$\lceil s, \mathcal{E}^{src}.f, e \rceil = \begin{array}{l} \lceil s, \mathcal{E}^{src}, e - 1 \rceil; \\ e : \text{getfield } f \end{array}$$

- local variable access

$$\lceil s, \mathbf{var}, s \rceil = s : \text{load } \mathbf{reg}_i$$

where \mathbf{reg}_i is the local variable at index i

- arithmetic expressions

$$\lceil s, \mathcal{E}_1^{src} \text{ op } \mathcal{E}_2^{src}, e \rceil = \begin{array}{l} \lceil s, \mathcal{E}_1^{src}, e' \rceil; \\ \lceil e' + 1, \mathcal{E}_2^{src}, e - 1 \rceil; \\ e : \text{arith_op} \end{array}$$

- cast expression

$$\lceil s, (\mathbf{Class}) \mathcal{E}^{src}, e \rceil = \begin{array}{l} \lceil s, \mathcal{E}^{src}, e - 1 \rceil; \\ e : \text{checkcast } \mathbf{Class} ; \end{array}$$

- instanceof expression

$$\lceil s, \mathcal{E}^{src} \mathbf{instanceof} \mathbf{Class}, e \rceil = \begin{array}{l} \lceil s, \mathcal{E}^{src}, e - 1 \rceil; \\ e : \text{instanceof } \mathbf{Class}; \end{array}$$

- null expression

$$\lceil s, \mathbf{null}, s \rceil = s : \text{push } \mathbf{null}$$

- object creation

$$\begin{aligned} \lceil s, \mathbf{new} \mathbf{Class}(\mathcal{E}^{src}), e \rceil &= \begin{array}{l} s : \text{new } \mathbf{Class}; \\ s + 1 : \text{dup}; \\ \lceil s + 2, \mathcal{E}^{src}, e - 1 \rceil; \\ e : \text{invoke } \text{constr}(\mathbf{Class}); \end{array} \end{aligned}$$

- this instance

$$\lceil s, \mathbf{this}, s \rceil = s : \text{load } \mathbf{reg}_0$$

3.4 Compiling control statements in bytecode instructions

- compositional statement

$$\begin{aligned} \lceil s, \mathcal{S}T\mathcal{M}T_1; \mathcal{S}T\mathcal{M}T_2, e \rceil &= \\ \lceil s, \mathcal{S}T\mathcal{M}T_1, e' \rceil; & \\ \lceil e' + 1, \mathcal{S}T\mathcal{M}T_2, e \rceil & \end{aligned}$$

about the compilation of exception handlers

a redundant jump added in the compilation in order to see explicitly the relation between stmt1 and stmt2

- if statement

$$\begin{aligned} & \lceil s, \text{if } (\mathcal{E}^R) \text{ then } \{STM T_1\} \text{ else } \{STM T_2\}, e \rceil = \\ & \lceil s, \mathcal{E}^R, e' \rceil; \\ & e' + 1 : \text{if } e'' + 2; \\ & \lceil e' + 2, STM T_2 \rangle, e'' \rceil \\ & e'' + 1 : \text{goto } e + 1; \\ & \lceil e'' + 2, STM T_1, e \rceil; \end{aligned}$$

- assignment statement. We consider the case for instance field assignment as well as assignments to method local variables and parameters.

– field assignment.

$$\begin{aligned} & \lceil s, \mathcal{E}_1^{src}.f = \mathcal{E}_2^{src}, e \rceil = \\ & \lceil s, \mathcal{E}_1^{src}, e' \rceil; \\ & \lceil e' + 1, \mathcal{E}_2^{src}, e - 1 \rceil; \\ & e : \text{putfield } f; \end{aligned}$$

– method local variable or parameter update

$$\begin{aligned} & \lceil s, \text{var} = \mathcal{E}^{src}, e \rceil = \\ & \lceil s, \mathcal{E}^{src}, e - 1 \rceil \\ & e : \text{store reg}_i; \end{aligned}$$

- try catch statement

$$\begin{aligned} & \lceil s, \text{try } \{STM T_1\} \text{ catch } (\text{ExcClass var}) \{STM T_2\}, e \rceil = \\ & \lceil s, STM T_1, e' \rceil; \\ & e' + 1 : \text{goto } e + 1; \\ & \lceil e' + 2, STM T_2, e \rceil; \end{aligned}$$

$\text{addExceptionHandler}(s, e', e' + 2, \text{ExcClass}, m)$

The compiler compiles the normal statement $STM T_1$ and the exception handler $STM T_2$.

- try finally statement

$$\begin{aligned} & \lceil s, \text{try } \{STM T_1\} \text{ finally } \{STM T_2\}, e \rceil = \\ & \lceil s, STM T_1, e' \rceil; \\ & \lceil e' + 1, STM T_2, e'' \rceil; \\ & e'' + 1 : \text{goto } e + 1; \end{aligned}$$

$$\begin{aligned} & \{ \text{default exception handler} \} \\ & e'' + 2 : \text{store } l; \\ & \lceil e'' + 3, STM T_2, e - 2 \rceil; \\ & e - 1 : \text{load } l; \\ & e : \text{athrow}; \end{aligned}$$

$\text{addExceptionHandler}(s, e', e'' + 2, \text{Exception}, m)$

As you can notice, we compile the finally statement STM_2 by inlining, it is first compiled as a code executed after STM_1 and then it is compiled as part of the default exception handler. The default exception handler which starts at index $e''+2$ and which will handle any exception thrown by $\lceil s, STM_1, e' \rceil$. The exception handler first stores the thrown exception in the local variable at index l , then executed the subroutine code and after the execution rethrows the exception stored in the local variable l .

This compilation differs from the compilation scheme in the JVM specification for finally statements, which requires that the subroutines must be compiled using `jsr` and `ret` instructions. However, the semantics of the programs produced by the compiler presented here and a compiler which follows closely the JVM specification is equivalent. In the following, we discuss informally why this is true. The semantics of a `jsr k` instruction is to jump to the first instruction of the compiled subroutine which starts at index k and pushes on the operand stack the index of the next instruction of the `jsr` that caused the execution of the subroutine. The first instruction of the compilation of the subroutine stores the stack top element in the local variable at index k (i.e. stores in the local variable at index k the index of the instruction following the `jsr` instruction). Thus, after the code of the subroutine is executed, the `ret k` instruction jumps to the instruction following the corresponding `jsr`. This behaviour can be actually simulated by programs without `jsr` and `ret` but which inline the subroutine code at the places where a `jsr` to the subroutine is done.

Note:

1. we assume that the local variable l is not used in the compilation of the statement STM_2 , which guarantees that after any execution which terminates normally of $\lceil e''+3, STM_2, e-2 \rceil$ the local variable l will still hold the thrown object
2. here we also assume that the statement STM_1 does not contain a `return` instruction

The last remark that we would like to make is that subroutines and their compilation via `ret` and `jsr` has always presented a problem for Java. First, they slow down the performance of the JVM because of the special way `ret` and `jsr` work. Second, the analysis performed by the bytecode verifier in the JVM becomes rather complex because (and its first version was erroneous) of the Java subroutines. In the last version of Java compiler, subroutines has become obsolete where they are compiled by inlining. Thus, the compiler presented here represents a realistic approximation of the Java Sun compiler.

verify this

say the name
of the com-
piler

- throw exception statement

$$\lceil s, \text{athrow } \mathcal{E}^{src}, e \rceil = \begin{array}{l} \lceil s, \mathcal{E}^{src}, e-1 \rceil; \\ e : \text{athrow}; \end{array}$$

- loop statement

$$\begin{aligned}
&\lceil s, \text{while } (\mathcal{E}^{\mathcal{R}})[\text{INV}, \text{modif}] \{ \text{STMT} \}, e \rceil = \\
&\quad s : \text{goto } e' + 1; \\
&\quad \lceil s + 1, \text{STMT}, e' \rceil; \\
&\quad \lceil e' + 1, \mathcal{E}^{\mathcal{R}}, e - 1 \rceil; \\
&\quad e : \text{if } s + 1;
\end{aligned}$$

addLoopSpec($e' + 1$, INV, modif, m)

- return statement

$$\begin{aligned}
&\lceil s, \text{return } \mathcal{E}^{src}, e \rceil = \lceil s, \mathcal{E}^{src}, e - 1 \rceil; \\
&\quad e : \text{return}
\end{aligned}$$

3.5 Properties of the compiler function

In this last subsection, we will look at the properties of the compiled statements. In the following, we use the notation \mathcal{S} when we refer both to statements STMT and \mathcal{E}^{src} .

The first property that we observe is that the last instruction ins_e in the compilation $\lceil s, \text{STMT}, e \rceil$ of a statement STMT is always in execution relation with the instruction ins_{e+1}

Property 3.5.1 (Compilation of statements and expressions) *For any statement or expression \mathcal{S} , start label s and end label e , the compiler will produce a list of bytecode instruction $\lceil s, \text{STMT}, e \rceil$ such that*

$$ins_e \rightarrow ins_{e+1}$$

The proof is trivial by case analysis of the compiled statements.

Informally, the following property states that if there are instructions inside a compiled statement or expression $\lceil s, \mathcal{S}, e \rceil$ which are in execution relation¹ with an instruction ins_k which is not the start of an exception handler and which is outside $\lceil s, \mathcal{S}, e \rceil$ then $k = e + 1$. The conditions $\neg(inList(\lceil s, \mathcal{S}, e \rceil, ins_k))$ and $\neg isExceptionHandlerStart(ins_k)$ eliminate the case when the execution relation is between an instruction inside $\lceil s, \mathcal{S}, e \rceil$ which may throw an exception and the start instruction of the proper handler exception handler.

Property 3.5.2 (Compilation of statements and expressions) *For any statement or expression \mathcal{S} , start label s and end label e , the compiler will produce a list of bytecode instruction $\lceil s, \mathcal{S}, e \rceil$ such that:*

$$\begin{aligned}
&\forall i, (inList(\lceil s, \mathcal{S}, e \rceil, ins_i)) \wedge \\
&\quad (ins_i \rightarrow ins_k) \wedge \\
&\quad \neg(inList(\lceil s, \mathcal{S}, e \rceil, ins_k)) \\
&\quad \neg isExceptionHandlerStart(ins_k) \Rightarrow \\
&\quad k = e + 1 \wedge
\end{aligned}$$

¹see Def. ??

The proof is done by induction on the structure of the compiled statement. We sketch the proof for the compilation of the if statement, the rest of the cases being similar *Proof*:

$$\begin{aligned}
& \{ \text{ Assume that } \exists, s \leq i \leq e, \exists k, k \notin [s \dots e + 1] \wedge ins_i \rightarrow ins_k \} \\
& \{ \text{ by definition of the compiler function for if statements in section ?? } \\
& \quad \ulcorner s, \text{if } (\mathcal{E}^R) \text{ then } \{STMT_1\} \text{ else } \{STMT_2\}, e^\neg = \\
& \quad \ulcorner s, \mathcal{E}^R, e'^\neg; \\
& \quad e' + 1 \text{ if } e'' + 2; \\
& \quad \ulcorner e' + 2, STMT_2, e''^\neg \\
& \quad e'' + 1 \text{ goto } e + 1; \\
& \quad \ulcorner e'' + 2, STMT_1, e^\neg; \\
& (1) \{ \text{ Assume that } s \leq i \leq e' \} \\
& \{ \text{ by induction hypothesis for } \mathcal{E}^R \text{ we get } \} \\
& (2) \forall i, s \leq i \leq e', ins_i \rightarrow ins_k) \wedge \\
& \quad \neg(inList(\ulcorner s, STMT, e^\neg, ins_k)) \\
& \quad \neg isExceptionHandlerStart(ins_k) \Rightarrow \\
& \quad k = e' + 1 \\
& \{ \text{ From (1) and (2) we get a contradiction in this case} \} \\
& (3) \{ \text{ Assume that } e' + 2 \leq i \leq e'' \} \\
& (4) \forall i, e' + 2 \geq i \leq e'', ins_i \rightarrow ins_k) \wedge \\
& \quad \neg(inList(\ulcorner s, STMT, e^\neg, ins_k)) \\
& \quad \neg isExceptionHandlerStart(ins_k) \Rightarrow \\
& \quad k = e'' + 1 \\
& \{ \text{ From (3) and (4) we get a contradiction in this case} \} \\
& (5) \{ \text{ Assume that } e'' + 2 \leq i \leq e \} \\
& (6) \forall i, e'' + 2 \geq i \leq e, ins_i \rightarrow ins_k) \wedge \\
& \quad \neg(inList(\ulcorner s, STMT, e^\neg, ins_k)) \\
& \quad \neg isExceptionHandlerStart(ins_k) \Rightarrow \\
& \quad k = e + 1 \\
& \{ \text{ From (5) and (6) we get a contradiction in this case} \} \\
& (7) \{ \text{ Assume that } s = e' + 1 \} \\
& \{ \text{ as } e' + 1 \text{ if } e'' + 2 \text{ and } e'' + 2 \text{ and } e' + 2 \text{ are labels in} \\
& \text{ the compilation of the statement, we get a contradiction in this case} \} \\
& (8) \{ \text{ Assume that } s = e'' + 1 \} \\
& \{ \text{ as } e'' + 1 \text{ goto } e + 1 \text{ we get a contradiction once again} \}
\end{aligned}$$

Another property of the compiler is that any statement or expression is compiled in a list of bytecode instructions such that there could not be jumps from outside inside the list, i.e. the control flow can reach the instructions representing the compilation $\ulcorner s, \mathcal{S}, e^\neg$ of statement \mathcal{S} only by passing through the beginning of the compilation i_s .

Property 3.5.3 (Compilation of statements and expressions) *For all statements S' and S , such that*

- *S is such that it has as substatement S' , which we denote with $S[S']$*
- *their compilations are $\ulcorner s, S, e \urcorner$ and $\ulcorner s', S', e' \urcorner$*

then :

$$\neg(\exists i_j, \exists i_k, \\ \text{inList}(\ulcorner s, S[S'], e \urcorner, i_j) \wedge \\ \neg \text{inList}(\ulcorner s', S', e' \urcorner, i_j) \wedge \\ \text{inList}(\ulcorner s', S', e' \urcorner, i_k) \wedge \\ s' \neq k \wedge i_j \rightarrow i_k)$$

The proof is done by induction over the structure of statements and expressions and uses the previous lemma 3.5.2

The following property establishes that the compiler preserves the substatement relation between statements and expressions.

Property 3.5.4 (Substatement and subexpression relation preserved)

For all statements S_1 and S_2 such that their compilations are $\ulcorner s_1, S_1, e_1 \urcorner$ and that $\ulcorner s_2, S_2, e_2 \urcorner$, if we have that $\exists k, s_1 \leq k \leq e_1 \wedge s_2 \leq k \leq e_2$ then the following holds:

$$\begin{aligned} \ulcorner s_2, S_2, e_2 \urcorner &\in \ulcorner s_1, S_1, e_1 \urcorner \\ \vee \\ \ulcorner s_1, S_1, e_1 \urcorner &\in \ulcorner s_2, S_2, e_2 \urcorner \end{aligned}$$

Now, we give a definition for a set of instructions such that they execute sequentially

Definition 3.5.1 (Block of instructions) *If the list of instructions $l = [ins_{i_1} \dots ins_{i_n}]$ in the compilation of method m is such that*

- *none of the instructions is a target of an instruction ins_{i_j} which does not belong to l except for ins_{i_1}*
- *none of the instructions in the set is a jump instruction, i.e. $\forall m, m = 1..k \Rightarrow \neg(ins_{i_m} \in \{ \text{goto}, \text{if} \})$*
- $\forall j, i_1 < j \leq i_n, \neg \exists k \in m.\text{body}, ins_k \rightarrow^l ins_j$

We denote such a list of instructions with $ins_{i_1}; \dots; ins_{i_k}$

The next lemma states that the compilation of an expression \mathcal{E}^{src} results in a block of bytecode instructions.

Property 3.5.5 (Compilation of expressions) *For any expression \mathcal{E}^{src} , starting label s and end label e , the compilation $\ulcorner s, \mathcal{E}^{src}, e \urcorner$ is a block of bytecode instruction in the sense of Def. 3.5.1*

Following the definition 3.5.1 of block of bytecode instructions, the property states that the compilation of an expression results in a list of instructions that cannot be jumped from outside its compilation except for the first instruction of the compilation. This follows from lemma 3.5.3.

Definition 3.5.1 also requires that there are no jump instructions in the list of instructions representing the compilation of an expression. This is established by induction over the structure of the expression.

The third condition in Def. 3.5.1 states that the compilation $\lceil s, \mathcal{E}^{src}, e \rceil$ is such that no instruction except ins_s may be a loop entry in the sense of Def. ?? in Chapter ??, Section ?. This is the case, as there are no jump instructions inside the compiled expression and all the instructions inside an expression are sequential.

Our source language supports also exception handling. As we saw in the previous section, the compiler will keep track of the exception handlers by adding them in the exception handler table. We now establish that the elements in the exception handler table correspond to a statement in the source language.

this is not well explained

Property 3.5.6 (Exception handler element corresponds to a statement)

Every element (s, e, eH, Exc) in the exception handler table $m.excHndls$ is such that exists a statement $STMT$ such that $\lceil s, STMT, e \rceil$

Proof: The proof is done by contradiction. From the compiler definition, we get that elements are added in $m.excHndls$ only in the cases of try catch and try finally statement compilation and that the guarded regions in the added elements correspond to statements.

In the rest of this subsection we will look at properties of the compiler which are a consequence of the upper properties. We can establish also that the compilation of a statement $STMT$ may contain cycles only if $STMT$ is a cycle or contains a substatement which is a cycle.

Property 3.5.7 (Cycles in the control flow graph) *The compilation*

$\lceil s, STMT, e \rceil$ of a $STMT$ may contain an instruction ins_k which is a loop entry in the sense of definition ?? (i.e. there exists ins_j such that $ins_j \rightarrow^l ins_k$) only if $STMT$ is a loop or contains a substatement $STMT'$ which is a loop statement and the following holds:

alternative formulation: say that the only loop entries are the first instructions of a loop body

$$\begin{aligned}
& STMT = \\
& \text{while } (\mathcal{E}^R)[INV, \text{modif}] \{STMT\} \\
& \vee \\
& STMT[STMT'] \wedge \\
& STMT' = \\
& \text{while } (\mathcal{E}^R)[INV, \text{modif}] \{STMT\} \\
& \lceil s, \text{while } (\mathcal{E}^R)[INV, \text{modif}] \{STMT\}, e \rceil = \\
& s : \text{goto } e' + 1; \\
& \lceil s + 1, STMT, e' \rceil; \\
& \lceil e' + 1, \mathcal{E}^R, e - 1 \rceil; \\
& e : \text{if } s + 1; \\
& \Rightarrow k = e' + 1 \wedge j = e'
\end{aligned}$$

Another property concerning cycles in the control flow graph is that all the instructions in a compilation $\lceil s, STMT, e \rceil$ of a statement $STMT$ which target

the instruction ins_{e+1} are in the same execution relation with ins_{e+1} , i.e. if ins_{e+1} is a loop entry either all are loop ends or none of them is:

Property 3.5.8 (Cycles in the control flow graph) *For every statement $STMT$ its compilation $\lceil s, STMT, e \rceil$ is such that $(\exists k, s \leq k \leq e, ins_k \rightarrow^l ins_{e+1}) \iff (\forall k, s \leq k \leq e, ins_k \rightarrow ins_{e+1} \Rightarrow ins_k \rightarrow^l ins_{e+1})$*

Property 3.5.9 (Exception handler property for statements) *For every statement $STMT$ which is not a try catch in method m and its substatement $STMT'$, which we denote with $STMT[STMT']$ such that $\neg (\exists STMT'', STMT[STMT''] \wedge STMT''[STMT'])$ we have that if their respective compilations are $\lceil s, STMT, e \rceil$ and $\lceil s', STMT', e' \rceil$ then the following holds:*

give the definition of the function $isExceptionHandlerStart(ins_k)$

$$\forall \text{Exc}, findExceptionHandler(\text{Exc}, e, m.\text{excHndIS}) = findExceptionHandler(\text{Exc}, e', m.\text{excHndIS})$$

Proof: The proof is by contradiction. Assume this is not true, i.e.

- $\exists STMT, STMT',$
- (1) $STMT \neq \text{try}\{\dots\}\text{catch}\{\dots\}\wedge$
 - (2) $STMT[STMT'] \wedge \neg (\exists STMT'', STMT[STMT''] \wedge STMT''[STMT']) \wedge$
 - (3) $\lceil s, STMT, e \rceil \wedge$
 - (4) $\lceil s', STMT', e' \rceil \wedge$
 - (5) $\exists \text{Exc}, findExceptionHandler(\text{Exc}, e, m.\text{excHndIS}) \neq findExceptionHandler(\text{Exc}, e', m.\text{excHndIS})$

This means that there exists two elements $(s_1, e_1, eH_1, \text{Exc})$ and $(s_2, e_2, eH_2, \text{Exc})$ in the exception handler table of method m $m.\text{excHndIS}$ such that :

$$(6) \quad eH_1 \neq eH_2$$

From lemma 3.5.6 we get that :

$$(7) \quad \begin{aligned} & \exists STMT_1, s_1, e_1, \lceil s_1, STMT_1, e_1 \rceil \\ & \wedge \\ & \exists STMT_2, s_2, e_2, \lceil s_2, STMT_2, e_2 \rceil \end{aligned}$$

From the initial condition (2) we conclude that

$$(8) \quad \lceil s_1, STMT_1, e_1 \rceil \notin \lceil s, STMT, e \rceil$$

Because $e \in [s_2 \dots e_2] \wedge e' \in [s_1 \dots e_1] \wedge e, e' \in [s \dots e]$, (8) by applying Lemma 3.5.4 we can conclude that

$$\begin{aligned} & \lceil s, STMT, e \rceil \in \lceil s_1, STMT_1, e_1 \rceil \in \lceil s_2, STMT_2, e_2 \rceil \\ & \vee \\ & \lceil s, STMT, e \rceil \in \lceil s_2, STMT_2, e_2 \rceil \in \lceil s_1, STMT_1, e_1 \rceil \end{aligned}$$

In both of the cases and of the definition of $findExceptionHandler$ in Chapter ??, Section ?? this means that

$$findExceptionHandler(\text{Exc}, e, m.\text{excHndIS}) = findExceptionHandler(\text{Exc}, e', m.\text{excHndIS})$$

which is in contradiction with (6).

A similar property can be established about expressions

Property 3.5.10 (Exception handler property for expressions) *For every statement $STMT$ which is not a try catch in method m and its subexpression \mathcal{E}^{src} , which we denote with $STMT[\mathcal{E}^{src}]$ such that $\neg (\exists STMT'', STMT''[\mathcal{E}^{src}] \wedge STMT[STMT''])$ we have that if their respective compilations are $\lceil s, STMT, e \rceil$ and $\lceil s', \mathcal{E}^{src}, e' \rceil$ and (2) then the following holds:*

$$\forall \text{Exc}, \forall i, s' \leq i \leq e', \text{findExceptionHandler}(\text{Exc}, e, m.\text{excHndls}) = \text{findExceptionHandler}(\text{Exc}, i, m.\text{excHndls})$$

The last property concerns try catch statements

Property 3.5.11 (Exception handlers and try catch statements) *For every try catch statement $\text{try } \{STMT_1\} \text{ catch } (\text{ExcClass var}) \{STMT_2\}$ its compilation*

$$\begin{aligned} &\lceil s, STMT_1, e' \rceil; \\ &e' + 1 : \text{goto } e + 1; \\ &\lceil e' + 2, STMT_2, e' \rceil; \end{aligned}$$

is such that the following holds

$$\begin{aligned} \forall \neg(\text{Exc} <: \text{ExcClass}) \Rightarrow & \text{findExceptionHandler}(\text{Exc}, e', m.\text{excHndls}) = \\ & \text{findExceptionHandler}(\text{Exc}, e, m.\text{excHndls}) \\ \wedge \text{findExceptionHandler}(\text{ExcClass}, e', m.\text{excHndls}) = & e' + 2 \end{aligned}$$

4 Weakest precondition calculus for source programs

4.1 Source assertion language

The properties that our predicate calculus treats are from first order predicate logic. In the following, we give the formal definition of the assertion language into which the properties are encoded. Note that the language is the same to the bytecode assertion language presented earlier modulo the stack expressions $\text{st}(\text{cntr} + - \dots)$ and cntr .

Formulas 1 (Definition) *The set of formulas is defined inductively as follows*

$$\begin{aligned} \mathcal{F} ::= & \quad \psi(\mathcal{E}^{spec}, \mathcal{E}^{spec}) \\ & | \text{instances}(\mathcal{E}^{spec}) \\ & | T \\ & | \perp \\ & | \mathcal{F} \wedge \mathcal{F} \\ & | \mathcal{F} \vee \mathcal{F} \\ & | \mathcal{F} \Rightarrow \mathcal{F} \\ & | \forall x(\mathcal{F}(x)) \\ & | \exists x(\mathcal{F}(x)) \end{aligned}$$

$$\mathbb{P} ::= \quad == | \neq | \leq | \geq | > | < :$$

$$\begin{aligned} \mathcal{E}^{spec} ::= & \quad \text{constInt} \\ & | \text{true} \\ & | \text{false} \\ & | \text{boundVar} \\ & | \mathcal{E}^{spec} \text{ op } \mathcal{E}^{spec} \\ & | \mathcal{E}^{spec}.f \\ & | \text{var} \\ & | \text{null} \\ & | \text{this} \\ & | \backslash \text{typeof}(\mathcal{E}^{spec}) \\ & | \backslash \text{result} \end{aligned}$$

4.2 Weakest Predicate Transformer for the Source Language

The weakest precondition calculates for every statement $STMT$ in method m from our source language, for any normal postcondition $Post$ and exceptional postcondition function $\text{excPost}^{src} (Exc \rightarrow STMT \rightarrow \mathcal{F})$, the predicate Pre such that if it holds in the pre state of $STMT$ and if $STMT$ terminates normally then $Post$ holds in the poststate and if $STMT$ terminates on exception Exc then $\text{excPost}^{src}(Exc, STMT)$ holds. In the following, in subsection 4.2.1 we discuss how the exceptional postcondition function is defined. Subsections 4.2.2 and 4.2.3 present respectively the definition of wp function for expressions and for statements.

4.2.1 Exceptional Postcondition Function

As we stated earlier the weakest predicate transformer manages both the normal and exceptional termination of an expression(statement). In both cases the expression(statement) has to satisfy some condition : the normal postcondition in case of normal termination and the exceptional postcondition for exception Exc if it terminates on exception Exc

We introduce a function excPost^{src} which maps exception types to predicates

$$\text{excPost}^{src} : \text{ETypes} \longrightarrow \text{Predicate}$$

The function excPost^{src} returns the predicate $\text{excPost}^{src}(\text{Exc})$ that must hold in a particular program point if at this point an exception of type Exc is thrown.

We also use function updates for excPost^{src} which are defined in the usual way

$$\text{excPost}^{src}[\oplus \text{Exc}' \rightarrow P](\text{Exc}, \text{exp}) = \begin{cases} P & \text{if } \text{Exc} <: \text{Exc}' \\ \text{excPost}^{src}(\text{Exc}, \text{exp}) & \text{else} \end{cases}$$

4.2.2 Expressions

The weakest precondition function for expressions has the following signature:

$$wp^{src} : \mathcal{E}^{src} \rightarrow \mathcal{F} \rightarrow (\text{Exc} \rightarrow \mathcal{F}) \rightarrow \text{Method} \rightarrow \mathcal{E}^{spec} \rightarrow \mathcal{F}$$

For calculating the wp^{src} predicate of an expression \mathcal{E}^{src} declared in method \mathbf{m} , the function wp^{src} takes as arguments \mathcal{E}^{src} , a postcondition nPost^{src} , an exceptional postcondition function excPost^{src} and an \mathcal{E}^{spec} and returns the formula $wp^{src}(\mathcal{E}^{src}, \psi, \text{excPost}^{src}, \mathbf{m})_{\mathcal{E}^{spec}}$ which is the wp precondition of \mathcal{E}^{src} if the its value is represented by \mathcal{E}^{spec} .

In the following, we give the rules of wp^{src} .

- integer and boolean constant access
($\text{const} \in \{\text{constInt}, \text{true}, \text{false}, \text{constRef}\}$)

$$wp^{src}(\text{const}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_{\text{const}} = \text{nPost}^{src}$$

- field access expression

$$\begin{aligned} & wp^{src}(\mathcal{E}_1^{src}.f, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_{v.f} = \\ & wp^{src}(\mathcal{E}_1^{src}, \\ & \quad v \neq \text{null} \Rightarrow \text{nPost}^{src} \\ & \quad \wedge \\ & \quad v = \text{null} \Rightarrow \text{excPost}^{src}(\text{NullPointerExc}, \mathcal{E}_1^{src}) \\ & \quad \text{excPost}^{src}, \mathbf{m})_v \end{aligned},$$

- arithmetic expressions

$$\begin{aligned} & wp^{src}(\mathcal{E}_1^{src} \text{ op } \mathcal{E}_2^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_{v_1+v_2} = \\ & wp^{src}(\mathcal{E}_1^{src}, wp^{src}(\mathcal{E}_2^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_{v_2}, \text{excPost}^{src}, \mathbf{m})_{v_1} \end{aligned}$$

- method invocation

$$\begin{aligned}
& \text{wp}^{src}(\mathcal{E}^{src}.m(), \text{nPost}^{src}, \text{excPost}^{src}, \mathfrak{m})_v = \\
& \text{wp}^{src}(\mathcal{E}^{src}, \left\{ \begin{array}{l} v' \neq \text{null} \Rightarrow \\ m.\text{Pre}^{src}[\text{this} \leftarrow v'] \\ \wedge \\ \forall \text{boundVar}, \forall m \in m.\text{modif}^{src} \\ \left\{ \begin{array}{l} \backslash \text{typeof}(\text{boundVar}) <: m.\text{retType} \wedge \\ m.\text{nPost}^{src} [\backslash \text{result} \leftarrow \text{boundVar}] \\ \quad [\text{this} \leftarrow v'] \\ \Rightarrow \text{nPost}^{src}[v \leftarrow \text{boundVar}] \end{array} \right. \\ \wedge \\ \forall E \in m.\text{exceptions}^{src}, \\ \forall m \in m.\text{modif}^{src} \\ m.\text{excPostSpec}^{src}(E) \Rightarrow \text{excPost}^{src}(E) \\ \mathcal{E}_1^{src} = \text{null} \Rightarrow \text{excPost}^{src}(\text{NullPtrExc}) \end{array} \right. , \\
& \text{excPost}^{src}, \mathfrak{m})_{v'}
\end{aligned}$$

- Cast expression

$$\begin{aligned}
& \text{wp}^{src}((\text{Class}) \mathcal{E}^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathfrak{m})_v = \\
& \text{wp}^{src}(\mathcal{E}^{src}, \\
& \quad \backslash \text{typeof}(v) <: \text{Class} \Rightarrow \\
& \quad \quad \text{nPost}^{src} \\
& \quad \wedge \\
& \quad \neg \backslash \text{typeof}(v) <: \text{Class} \Rightarrow \\
& \quad \quad \text{excPost}^{src}(\text{CastExc}) \\
& \text{excPost}^{src}, \mathfrak{m})_v
\end{aligned}$$

may be give
an example

- Null expression

$$\text{wp}^{src}(\text{null}, \text{nPost}^{src}, \text{excPost}^{src}, \mathfrak{m})_{\text{null}} = \text{nPost}^{src}$$

- this

$$\text{wp}^{src}(\text{this}, \text{nPost}^{src}, \text{excPost}^{src}, \mathfrak{m})_{\text{this}} = \text{nPost}^{src}$$

- instance creation

$$\begin{aligned}
& \text{wp}^{src}(\text{new } \text{Class } (\mathcal{E}^{src}), \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_v = \\
& \text{wp}^{src}(\mathcal{E}^{src}, \\
& \quad \left\{ \begin{array}{l}
\forall \text{boundVar}, \\
\quad \text{not instances}(\text{boundVar}) \wedge \\
\quad \text{boundVar} \neq \text{null} \Rightarrow \\
\quad \quad \text{constr}(\text{Class}).\text{Pre}^{src} [arg \leftarrow v'] \\
\quad \wedge \\
\quad \forall m \in \text{constr}(\text{Class}).\text{modif}^{src}, \\
\quad \quad \text{constr}(\text{Class}).\text{nPost}^{src} \left[\begin{array}{l} \text{this} \leftarrow \text{boundVar} \\ arg \leftarrow v' \end{array} \right] \\
\quad \quad \quad [\backslash \text{typeof}(\text{boundVar}) \leftarrow \text{Class}] \\
\quad \Rightarrow \text{nPost}^{src}[v \leftarrow \text{boundVar}] \quad \wedge \\
\quad \forall \text{Exc} \in \text{constr}(\text{Class}).\text{exceptions}^{src}, \\
\quad \forall m \in \text{constr}(\text{Class}).\text{modif}^{src}, \\
\quad \quad \text{constr}(\text{Class}).\text{excPostSpec}^{src}(\text{Exc}) \Rightarrow \text{excPost}^{src}(\text{Exc})
\end{array} \right\}, \\
& \quad \text{excPost}^{src}, \mathbf{m})_{v'}
\end{aligned}$$

Let us see the relational expressions supported in the source programming language

- Instanceof expression

$$\begin{aligned}
& \text{wp}^{src}(\mathcal{E}^{src} \text{ instanceof } \text{Class}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) \backslash \text{typeof}(v) <: \text{Class} \wedge v \neq \text{null} = \\
& \text{wp}^{src}(\mathcal{E}^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_v
\end{aligned}$$

- Binary relation over expressions

$$\begin{aligned}
& \text{wp}^{src}(\mathcal{E}_1^{src} \mathcal{R} \mathcal{E}_2^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_{v_1 \mathcal{R} v_2} = \\
& \text{wp}^{src}(\mathcal{E}_1^{src}, \text{wp}^{src}(\mathcal{E}_2^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})_{v_2}, \text{excPost}^{src}, \mathbf{m})_{v_1}
\end{aligned}$$

4.2.3 Statements

In the following, we present the rules of the predicate transformer for control statements.

- integer and boolean constant access

$$\begin{aligned}
& \text{wp}^{src}(\text{STMT}_1; \text{STMT}_2, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\
& \text{wp}^{src}(\text{STMT}_1, \text{wp}^{src}(\text{STMT}_2, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}), \text{excPost}^{src}, \mathbf{m})
\end{aligned}$$

- assignment

– local variable assignemnt

$$\begin{aligned}
& \text{wp}^{src}(\mathcal{E}_1^{src} = \mathcal{E}_2^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\
& \text{wp}^{src}(\mathcal{E}_2^{src}, \\
& \quad \text{nPost}^{src}[\mathcal{E}_1^{src} \leftarrow v_2], \\
& \quad \text{excPost}^{src}, \mathbf{m})_{v_2}
\end{aligned}$$

- instance field assignemnt

$$\begin{aligned}
& \text{wp}^{src}(\mathcal{E}_1^{src}.f = \mathcal{E}_2^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\
& \text{wp}^{src}(\mathcal{E}_1^{src}, \\
& \quad \text{wp}^{src}(\mathcal{E}_2^{src}, \\
& \quad \quad v_1 \neq \mathbf{null} \Rightarrow \\
& \quad \quad \quad \text{nPost}^{src}[f \leftarrow f[\oplus v_1 \rightarrow v_2]] \\
& \quad \quad \quad \wedge \\
& \quad \quad v_1 = \mathbf{null} \Rightarrow \\
& \quad \quad \quad \text{excPost}^{src}(\text{NullPointerExc}) \\
& \quad \quad \text{excPost}^{src}, \mathbf{m})_{v_2}, \\
& \quad \text{excPost}^{src}, \mathbf{m})_{v_1}
\end{aligned}$$

- if statement

$$\begin{aligned}
& \text{wp}^{src}(\text{if } (\mathcal{E}^{\mathcal{R}}) \\
& \quad \text{then}\{STM_1\} \text{ , nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\
& \quad \text{else } \{STM_2\} \\
& \text{wp}^{src}(\mathcal{E}^{\mathcal{R}}, \\
& \quad v = \mathbf{true} \Rightarrow \text{wp}^{src}(STM_1, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) \\
& \quad \wedge \\
& \quad v = \mathbf{false} \Rightarrow \text{wp}^{src}(STM_2, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) \\
& \quad \text{excPost}^{src}, \mathbf{m})_v
\end{aligned}$$

- throw exceptions

$$\begin{aligned}
& \text{wp}^{src}(\text{throw } \mathcal{E}^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\
& \text{wp}^{src}(\mathcal{E}^{src}, \\
& \quad v = \mathbf{null} \Rightarrow \text{excPost}^{src}(\text{NullPointerExc}) \\
& \quad \wedge \\
& \quad v \neq \mathbf{null} \Rightarrow \\
& \quad \quad \forall \text{Exc} \text{ ,} \\
& \quad \quad \quad \backslash \text{typeof}(\mathcal{E}^{src}) <: \text{Exc} \Rightarrow \\
& \quad \quad \quad \mathbf{m}.\text{excPost}^{src}(\text{Exc}) \\
& \quad \text{excPost}^{src}, \mathbf{m})_v
\end{aligned}$$

- try catch statement

$$\begin{aligned}
& \text{wp}^{src}(\text{try } \{STM_1\} \text{ catch}(\text{Exc } c) \{STM_2\}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\
& \text{wp}^{src}(STM_1, \\
& \quad \text{nPost}^{src}, \\
& \quad \text{excPost}^{src} \oplus [\text{Exc} \longrightarrow \text{wp}^{src}(STM_2, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m})], \mathbf{m})
\end{aligned}$$

- try finally

$$\text{wp}^{src}(\text{try } \{STM T_1\} \text{ finally } \{STM T_2\}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) =$$

$$\text{wp}^{src}(STM T_1, \text{wp}^{src}(STM T_2, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}), \text{excPost}^{src} \oplus [\text{Exception} \longrightarrow \text{wp}^{src}(STM T_2, \text{excPost}^{src}(\text{Exception}), \text{excPost}^{src}, \mathbf{m})], \mathbf{m})$$

where exc is the exception object thrown by $STM T_1$.

- loop statement

$$\text{wp}^{src}(\text{while } (\mathcal{E}^R) [\text{INV}, \text{modif}] \{STM T\}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) =$$

$$\begin{aligned} & \text{INV} \wedge \\ & \forall m, m \in \text{modif}, \\ & \text{INV} \Rightarrow \\ & \quad \text{wp}^{src}(\mathcal{E}^R, \\ & \quad \quad v = \text{true} \Rightarrow \text{wp}^{src}(STM T, \text{INV}, \text{excPost}^{src}, \mathbf{m}), \\ & \quad \quad v = \text{false} \Rightarrow \text{nPost}^{src} \\ & \quad \quad \text{excPost}^{src}, \mathbf{m})_v, \end{aligned}$$

- return statement

$$\begin{aligned} & \text{wp}^{src}(\text{return } \mathcal{E}^{src}, \text{nPost}^{src}, \text{excPost}^{src}, \mathbf{m}) = \\ & \text{wp}^{src}(\mathcal{E}^{src}, \text{nPost}^{src}[\backslash \text{result} \leftarrow v], \text{excPost}^{src}, \mathbf{m})_v \end{aligned}$$

where $\backslash \text{result}$ is a specification variable that can be met in the postcondition and denotes to the value returned of a non void method

5 Weakest precondition calculus for bytecode programs

5.1 Weakest predicate transformer over compiled statements and expressions

In the following, we introduce a new formulation of the wp function which will be based on the compiler from source to bytecode language. The motivation for this new definition is that it will allow to reason about the relation between source and bytecode proof obligations. Of course, it is also important to see what is the relation between the new definition of the wp introduced here and the definition given earlier in Chapter ??, section ?. We will argue under what conditions both formulations of the wp function produce the same formulas.

We give now a definition of the wp function for a single sequential instruction (instructions different from `goto`, `if`, `jsr`, `ret`) which takes explicitly the postcondition and the exceptional postcondition function upon which the precondition will be calculated. Its signature is the following:

$$\text{wp}^{bc} : \mathbb{I} \setminus \{ \text{goto}, \text{if} \} \rightarrow \mathcal{F}^{bc} \rightarrow (\text{ExcType} \rightarrow \mathcal{F}^{bc}) \rightarrow \mathcal{F}^{bc}$$

For instance, the wp definition for `getfield` is :

$$\begin{aligned} wp^{bc}(\text{getfield } f, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \\ \text{st}(\text{cntr}) \neq \text{null} \Rightarrow \psi[\text{st}(\text{cntr}) \leftarrow f(\text{st}(\text{cntr}))] \wedge \\ \text{st}(\text{cntr}) = \text{null} \Rightarrow \text{excPost}^{src2bc}(\text{NullPtrExc}) \end{aligned}$$

Note that this differs from the definition of the wp given in Chapter ??, section ?? where the postcondition is a function of the successor of the current instruction. We do not give the rest of the rules because they are the same as the rules presented in ?? except for the fact that the local postconditions are given explicitly.

We also define the weakest predicate transformer function for a block of instruction that always execute sequentially as follows:

Definition 5.1.1 (*wp for a block of instructions*)

$$\begin{aligned} wp_{seq}^{bc}(ins_1; \dots; ins_k, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \\ wp_{seq}^{bc}(ins_1; \dots; ins_{k-1}, wp^{bc}(ins_k, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \text{excPost}^{src2bc}, \mathbf{m}) \end{aligned}$$

We turn now to the rules for compiled expressions. Note that from Property 3.5.3 it follows that the compilation of any expression is a sequence of instructions of instructions that execute sequentially and there is no jump from outside inside the sequence. Thus, we use the predicate transformer for a sequence of bytecode instructions defined above in order to define the predicate transformer for expressions.

Definition 5.1.2 (*wp for compiled expressions*) *For any expression \mathcal{E}^{src} , postcondition ψ and exceptional postcondition function excPost^{src2bc} the wp function for the compilation $\lceil \mathcal{E} \rceil$ is $wp_{seq}^{bc}(\lceil \mathcal{E} \rceil, \psi, \text{excPost}^{src2bc}, \mathbf{m})$*

For instance, the rule of the wp for the compilation of access field expression $\mathcal{E}^{src}.f$ where its compilation is

$$\begin{array}{l} \lceil s, \mathcal{E}_1^{src}; e - 1 \rceil \\ e \text{ getfield } f \end{array}$$

produce the following formula

$$wp_{seq}^{bc}(\lceil s, \mathcal{E}^{src}, e - 1 \rceil; e \text{ getfield } f, \psi, \text{excPost}^{src2bc}, \mathbf{m})$$

This is equivalent to :

$$wp_{seq}^{bc}(\lceil s, \mathcal{E}^{src}, e - 1 \rceil, wp^{bc}(\text{getfield } f, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \text{excPost}^{src2bc}, \mathbf{m})$$

The function which calculates the wp predicate of a compiled statement is called wp_{stmt}^{bc} and has the following signature :

$$wp_{stmt}^{bc} : \text{Set}(\mathbf{I}) \rightarrow \mathcal{F}^{bc} \rightarrow (\text{Exc} \rightarrow \mathcal{F}^{bc}) \rightarrow \mathcal{F}^{bc}$$

The definition of wp_{stmt}^{bc} uses the compiler function defined in Section ??

- sequential statement compilation $\ulcorner s, \mathcal{S}T\mathcal{M}T_1; \mathcal{S}T\mathcal{M}T_2, e^\neg$ which by definition is

$$\begin{aligned} wp_{stmt}^{bc}(\ulcorner s, \mathcal{S}T\mathcal{M}T_1; \mathcal{S}T\mathcal{M}T_2, e^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m}) &=_{def} \\ wp_{stmt}^{bc}(\ulcorner s, \mathcal{S}T\mathcal{M}T_1, e'^\neg; , \\ wp_{stmt}^{bc}(\ulcorner e' + 1, \mathcal{S}T\mathcal{M}T_2, s^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \\ \text{excPost}^{src2bc}, \mathbf{m}) \end{aligned}$$

- if statement compilation $\ulcorner s, \text{if } (\mathcal{E}^{\mathcal{R}}) \text{ then } \{\mathcal{S}T\mathcal{M}T_1\} \text{ else } \{\mathcal{S}T\mathcal{M}T_2\}, e^\neg$

$$wp_{stmt}^{bc}(\ulcorner s, \begin{array}{l} \text{if } (\mathcal{E}^{\mathcal{R}}) \\ \text{then } \{\mathcal{S}T\mathcal{M}T_1\} \\ \text{else } \{\mathcal{S}T\mathcal{M}T_2\} \end{array}, e^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m}) =_{def}$$

$$\begin{aligned} &wp_{seq}^{bc}(\ulcorner s, \mathcal{E}^{\mathcal{R}}, e'^\neg; \\ &, \\ &\mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \\ &\quad wp_{stmt}^{bc}(\ulcorner e'' + 2, \mathcal{S}T\mathcal{M}T_1, e^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m})[t \leftarrow t - 2] \\ &\wedge \\ &\neg \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \\ &\quad wp_{stmt}^{bc}(\ulcorner e' + 2, \mathcal{S}T\mathcal{M}T_2, e''^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m})[t \leftarrow t - 2] \\ &\text{excPost}^{src2bc}, \mathbf{m}) \end{aligned}$$

- assignment expression. We will look only at the case for compiled field assignment expressions $\ulcorner s, \mathcal{E}_1^{src}.f = \mathcal{E}_2^{src}, e^\neg$.

$$\begin{aligned} wp_{stmt}^{bc}(\ulcorner s, \mathcal{E}_1^{src}.f = \mathcal{E}_2^{src}, e^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m}) &=_{def} \\ wp_{seq}^{bc}(\ulcorner s, \mathcal{E}_1^{src}, e'^\neg, \\ wp_{seq}^{bc}(\ulcorner e' + 1, \mathcal{E}_2^{src}, e - 1^\neg; \\ e \text{ putfield } f, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \\ \text{excPost}^{src2bc}, \mathbf{m}) \end{aligned}$$

- try catch statement compilation

$$wp_{stmt}^{bc}(\ulcorner s, \begin{array}{l} \text{try } \{\mathcal{S}T\mathcal{M}T_1\} \\ \text{catch } (\text{ExcClass } \text{var}) \{\mathcal{S}T\mathcal{M}T_2\} \end{array}, e^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m}) =_{def}$$

$$wp_{stmt}^{bc}(\ulcorner s, \mathcal{S}T\mathcal{M}T_1, e'^\neg; \\ e' + 1 : \text{goto } e + 1; ,$$

$$\psi,$$

$$\text{excPost}^{src2bc}[\oplus \text{ExcClass} \rightarrow wp_{stmt}^{bc}(\ulcorner e' + 2, \mathcal{S}T\mathcal{M}T_2, e^\neg, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \mathbf{m})$$

- try finally statement compilation $\ulcorner s, \text{try } \{STM T_1\} \text{ finally } \{STM T_2\}, e \urcorner$

$$\begin{aligned}
wp_{stmt}^{bc}(\ulcorner s, \text{try } \{STM T_1\} \text{ finally } \{STM T_2\}, e \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \text{def} \\
wp_{stmt}^{bc}(\ulcorner s, STM T_1, e' \urcorner, \\
, \\
wp_{stmt}^{bc}(\ulcorner e' + 1, STM T_2, e'' \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \\
\text{excPost}^{src2bc}[\oplus \text{ExcClass} \rightarrow wp_{stmt}^{bc}(\ulcorner e'' + 2 : \text{store } l; \\
\ulcorner e'' + 3, STM T_2, e - 2 \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \mathbf{m}) \\
e - 1 : \text{load } l; \\
e : \text{athrow}
\end{aligned}$$

- throw exception compilation $\ulcorner s, \text{throw } \mathcal{E}^{src}, e \urcorner$

$$\begin{aligned}
wp_{stmt}^{bc}(\ulcorner s, \text{throw } \mathcal{E}^{src}, e \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \text{def} \\
wp_{seq}^{bc}(\ulcorner s, \mathcal{E}^{src}, e - 1 \urcorner, wp^{bc}(e / \text{athrow}, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \text{excPost}^{src2bc}, \mathbf{m})
\end{aligned}$$

- loop statement

$$\begin{aligned}
& \text{while } (\mathcal{E}^R) \\
wp_{stmt}^{bc}(\ulcorner s, [\text{INV}, \text{modif}], e \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \text{def} \\
& \{STM T\} \\
& \text{INV} \\
& \wedge \\
& \forall \text{mod} \in \text{modif}, \\
& \text{INV} \Rightarrow \\
& wp_{seq}^{bc}(\ulcorner e' + 1, \mathcal{E}^R, e - 1 \urcorner, \\
& \quad \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \\
& \quad wp_{stmt}^{bc}(\ulcorner s + 1, STM T, e' \urcorner, \text{INV}, \text{excPost}^{src2bc}, \mathbf{m}) \\
& \quad \wedge \\
& \quad \neg \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \psi \\
& \text{excPost}^{src2bc}, \mathbf{m})
\end{aligned}$$

5.2 Properties of the wp functions

The previous subsection introduced a new formulation of the wp function for bytecode which takes into account the source statement from which it is compiled. However, it is important to establish a relation between this new definition and the wp formulation given in Chapter wp??. The following statements establish the relation between the two versions of the wp calculus.

Lemma 5.2.1 (Equivalence of the formulations for single instructions)

For all instructions $i_j \in \mathbf{I} \setminus \{\text{goto}, \text{if}, \text{jsr}, \text{ret}\}$ and i_k which belong to method \mathbf{m} , formula ψ , and function $\text{excPost}^{src2bc} : \text{ExcType} \rightarrow \mathcal{F}^{bc}$ such that

- $i_j \rightarrow i_k$
- $\psi = \text{inter}(i_j, i_k)$
- $\forall \text{Exc}, \text{excPost}^{\text{src2bc}}(\text{Exc}) = \mathbf{m}.\text{excPost}(\text{Exc}, j)$

the following holds

$$\text{wp}^{bc}(i_j, \psi, \text{excPost}^{\text{src2bc}}, \mathbf{m}) = \text{wp}(i_j, \mathbf{m})$$

The proof is done by case analysis on the instruction i_j . In the following, we sketch the case for a `getfield` instruction. *Proof:*

{ by hypothesis }
 $i_j = \text{getfield } f$
 { by definition of the wp function }
 (1) $\text{wp}^{bc}(\text{getfield } f, \psi, \text{excPost}^{\text{src2bc}}, \mathbf{m}) =$
 $\text{st}(\text{cntr}) \neq \text{null} \Rightarrow \psi[\text{st}(\text{cntr}) \leftarrow f(\text{st}(\text{cntr}))]$
 \wedge
 $\text{st}(\text{cntr}) = \text{null} \Rightarrow \text{excPost}^{\text{src2bc}}(\text{NullPtrExc})$

 { by definition of the wp function }
 (2) $\text{wp}(\text{getfield } f, \mathbf{m}) =$
 $\text{st}(\text{cntr}) \neq \text{null} \Rightarrow \text{inter}(i_j, i_k)[\text{st}(\text{cntr}) \leftarrow f(\text{st}(\text{cntr}))]$
 \wedge
 $\text{st}(\text{cntr}) = \text{null} \Rightarrow \text{excPost}(\text{NullPtrExc}, j)$

 { from the initial hypothesis, (1) and (2) the lemma holds in that case }

Lemma 5.2.2 (wp for a block of instructions) For every block of instructions $\text{ins}_1; \dots; \text{ins}_j$ and instruction ins_k in method \mathbf{m} , formula ψ and function $\text{excPost}^{\text{src2bc}} : \text{ExcType} \rightarrow \mathcal{F}^{bc}$ such that

- $\text{ins}_j \rightarrow \text{ins}_k$
- $\psi = \text{inter}(\text{ins}_j, \text{ins}_{j+1})$
- $\forall \text{Exc}, \forall i, 1 \leq i \leq j, \text{excPost}^{\text{src2bc}}(\text{Exc}) = \mathbf{m}.\text{excPost}(\text{Exc}, i)$

then the following holds

$$\text{wp}_{seq}^{bc}(\text{ins}_1; \dots; \text{ins}_j, \psi, \text{excPost}^{\text{src2bc}}, \mathbf{m}) = \text{wp}(\text{ins}_1, \mathbf{m})$$

The proof is done by induction on the length of the sequence of instructions.
Proof:

(1) $wp_{seq}^{bc}(ins_1; \dots; ins_j, \psi, \text{excPost}^{src2bc}, \mathbf{m}) =$
 { by Def. 5.1.1 for wp of block of instructions }
 $wp_{seq}^{bc}(ins_1; \dots; ins_{j-1}, wp^{bc}(ins_j, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \text{excPost}^{src2bc}, \mathbf{m})$
 { by initial hypothesis we can apply lemma 5.2.1 from which it follows }
 $wp^{bc}(ins_j, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = wp(ins_j, \mathbf{m})$
 { by definition 3.5.1, ins_j is not a loop entry
 and from Def. ?? }
 (2) $inter(j, j+1) = wp(ins_j, \mathbf{m})$
 { apply the induction hypothesis over $ins_1; \dots; ins_{j-1}$,
 (2) and the initial hypothesis for exception handlers }
 (3) $wp_{seq}^{bc}(ins_1; \dots; ins_{j-1}, wp^{bc}(ins_j, \psi, \text{excPost}^{src2bc}, \mathbf{m}), \text{excPost}^{src2bc}, \mathbf{m}) =$
 $wp(ins_1, \mathbf{m})$
 { from (1) and (3) the proposition holds }

Lemma 5.2.3 (*wp for compiled expressions*) For every compiled expression $\lceil s, \mathcal{E}^{src}, e \rceil$ in method \mathbf{m} formula ψ and function $\text{excPost}^{src2bc} : \text{ExcType} \rightarrow \mathcal{F}^{bc}$ such that

- $\psi = inter(e, e+1)$
- $\forall \text{Exc}, \forall 1 \leq i, k \leq j, \text{excPost}^{src2bc}(\text{Exc}) = \mathbf{m}.\text{excPost}(\text{Exc}, i)$

then the following holds:

$$wp_{seq}^{bc}(\lceil s, \mathcal{E}^{src}, e \rceil, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = wp(ins_s, \mathbf{m})$$

Proof: From Property 3.5.5 of the compiler it follows that for every expression \mathcal{E}^{src} , start label s and end label e , the resulting compilation $\lceil s, \mathcal{E}^{src}, e \rceil$ is a block of instructions. We can apply the previous lemma 5.2.2 and we get the result.

Lemma 5.2.4 (*wp for compiled statements*) For every compiled statement $\lceil s, \text{STMT}, e \rceil$ in method \mathbf{m} , formula ψ and function $\text{excPost}^{src2bc} : \text{ExcType} \rightarrow \mathcal{F}^{bc}$ such that

- $\psi = inter(e, e+1)$
- $\forall \text{Exc}, \text{excPost}^{src2bc}(\text{Exc}) = \mathbf{m}.\text{excPost}(\text{Exc}, e)$

then the following holds: if $\psi = inter(e, e+1)$ then the following holds

$$wp_{stmt}^{bc}(\lceil s, \text{STMT}, e \rceil, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = wp(ins_s, \mathbf{m})$$

Proof : the proof is by induction on the compilation of a statement. We scatch here the proof of few cases

- if statement

$$\begin{aligned}
& \text{if } (\mathcal{E}^{\mathcal{R}}) \\
wp_{stmt}^{bc}(\ulcorner s, & \text{ then } \{STM_1\}, e^{\top}, \psi, \text{excPost}^{src2bc}, \mathfrak{m}) =_{def} \\
& \text{else } \{STM_2\} \\
\\
wp_{seq}^{bc}(\ulcorner s, \mathcal{E}^{\mathcal{R}}, e'^{\top}; & \\
, & \\
& \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \\
& wp_{stmt}^{bc}(\ulcorner e'' + 2, STM_1, e^{\top}, \psi, \text{excPost}^{src2bc}, \mathfrak{m})[t \leftarrow t - 2] \\
& \wedge \\
& \neg \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \\
& wp_{stmt}^{bc}(\ulcorner e' + 2, STM_2, e''^{\top}, \psi, \text{excPost}^{src2bc}, \mathfrak{m})[t \leftarrow t - 2] \\
& \text{excPost}^{src2bc}, \mathfrak{m})
\end{aligned}$$

{ by initial hypothesis, we have }

(1.1) $\psi = \text{inter}(e, e + 1)$

{ by the compiler definition $ins_{e''+1} = \text{goto } e + 1;$ }

(1.2) $ins_{e''+1} \rightarrow ins_{e+1}$

{ from Property 3.5.8 which states that every instruction inside
the compilation of $\ulcorner s, STM, e^{\top}$,
is in the same execution relation with ins_{e+1} ,

Def. ?? for *inter*, (1.1) and (1.2) we conclude that }
 (1.3) $\text{inter}(e'' + 1, e + 1) = \text{inter}(e, e + 1) = \psi$
 { From property 3.5.7 the edge between e'' and $e'' + 1$
 is not a loop edge and from Def. ?? }
 (1.4) $\text{inter}(e'', e'' + 1) = \text{wp}(e'' + 1, m)$
 { From the definition of the wp for goto instructions and (1.4) }
 (1.5) $\text{wp}(e'' + 1, m) = \text{inter}(e'' + 1, e + 1)$
 { From (1.4), (1.3) and (1.5) }
 (1.6) $\text{inter}(e'', e'' + 1) = \text{inter}(e, e + 1) = \psi$
 { From 3.5.9 }
 (1.7) $\text{m.excPost}(\text{Exc}, e) = \text{m.excPost}(\text{Exc}, e'')$
 { apply the induction hypothesis over (1.6) and (1.7) }
 (1.8) $\text{wp}_{\text{stmt}}^{bc}(\ulcorner e' + 2, \text{STMT}_2, e'' \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \text{wp}(e' + 2, m)$
 { apply the induction hypothesis over the initial hypothesis }
 (1.9) $\text{wp}_{\text{stmt}}^{bc}(\ulcorner e'' + 2, \text{STMT}_1, e \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = \text{wp}(e'' + 2, m)$
 { by definition of if function }
 (1.10) $\text{wp}(e' + 1 : \text{if } e'' + 2; , m) =$
 $\mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \text{wp}(e'' + 2, m)$
 \wedge
 $\neg \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow \text{wp}(e' + 2, m)$
 { from (1.8), (1.9) and (1.10) }
 (1.11) $\text{wp}(e' + 1 : \text{if } e'' + 2; , m) =$
 $\mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow$
 $\text{wp}_{\text{stmt}}^{bc}(\ulcorner e'' + 2, \text{STMT}_1, e \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m})$
 \wedge
 $\neg \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow$
 $\text{wp}_{\text{stmt}}^{bc}(\ulcorner e' + 2, \text{STMT}_2, e'' \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m})$
 { From Property 3.5.8 }
 (1.12) $\text{inter}(e', e' + 1) = \text{wp}(e' + 1 : \text{if } e'' + 2; , m)$
 { From the initial hypothesis }
 (1.13) $\text{m.excPost}(\text{Exc}, e) = \text{excPost}^{src2bc}(\text{Exc})$
 { From Property 3.5.10 and (1.13) }
 (1.14) $\forall \text{Exc}, \forall s \leq i \leq e', \text{excPost}^{src2bc}(\text{Exc}) =$
 $\text{m.excPost}(\text{Exc}, i)$
 { apply Lemma 5.2.3 over hypothesis (1.11), (1.12), (1.14) }
 $\text{wp}_{\text{seq}}^{bc}(\ulcorner s, \mathcal{E}^{\mathcal{R}}, e' \urcorner;$
 ,
 $\mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow$
 $\text{wp}_{\text{stmt}}^{bc}(\ulcorner e'' + 2, \text{STMT}_1, e \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m})[t \leftarrow t - 2]$
 \wedge
 $\neg \mathcal{R}(\text{st}(\text{cntr}), \text{st}(\text{cntr} - 1)) \Rightarrow$
 $\text{wp}_{\text{stmt}}^{bc}(\ulcorner e' + 2, \text{STMT}_2, e'' \urcorner, \psi, \text{excPost}^{src2bc}, \mathbf{m})[t \leftarrow t - 2]$
 $\text{excPost}^{src2bc}, \mathbf{m}) = \text{wp}(s, m)$
 { and this case holds }

- try catch statement

$$\begin{aligned}
(1) \quad & wp_{stmt}^{bc}(\ulcorner s, \text{try } \{STM T_1\} \\
& \quad \text{catch } (\text{ExcClass } \text{var}) \{STM T_2\} \urcorner, e^\top, \psi, \text{excPost}^{src2bc}, \mathbf{m}) =_{def} \\
& \{ \text{Def. of } wp_{stmt}^{bc} \text{ in the previous Section 5.1} \} \\
& wp_{stmt}^{bc}(\ulcorner s, STM T_1, e'^\top; , \\
& \quad \psi, \\
& \quad \text{excPost}^{src2bc}[\oplus \text{ExcClass} \rightarrow wp_{stmt}^{bc}(\ulcorner e' + 2, STM T_2, e^\top, \psi, \text{excPost}^{src2bc}, \mathbf{m})], \mathbf{m}) \\
& \{ \text{apply the induction hypothesis over } STM T_2 \text{ and the initial hypothesis} \} \\
(2) \quad & wp_{stmt}^{bc}(\ulcorner e' + 2, STM T_2, e^\top, \psi, \text{excPost}^{src2bc}, \mathbf{m}) = wp(e' + 2, \mathbf{m}) \\
& \{ \text{from the definition of excPost and property 3.5.11} \} \\
(3) \quad & \forall \text{Exc}, \text{excPost}^{src2bc}[\oplus \text{ExcClass} \rightarrow wp(e' + 2, \mathbf{m})] = \\
& \quad \mathbf{m}.\text{excPost}(\text{Exc}, e') \\
& \{ \text{we can also conclude from Prop. 3.5.8 and 3.5.7} \} \\
(4) \quad & inter(e', e' + 1) = inter(e' + 1, e + 1) = inter(e, e + 1) \\
& \{ \text{apply induction hypothesis over (1), (3) and (4)} \} \\
& wp_{stmt}^{bc}(\ulcorner s, STM T_1, e'^\top; , \\
& \quad \psi, \\
& \quad \text{excPost}^{src2bc}[\oplus \text{ExcClass} \rightarrow wp_{stmt}^{bc}(\ulcorner e' + 2, STM T_2, e^\top, \psi, \text{excPost}^{src2bc}, \mathbf{m})], \mathbf{m}) = \\
& wp(s, \mathbf{m})
\end{aligned}$$

[section] [section] [section]

6 Proof obligation equivalence on source and bytecode level

In the following, we will argue that the proof obligations generated by the weakest precondition over programs of our source language are equivalent to the proof obligations generated over bytecode.

First, we remark that a wp^{src} defined over source expressions generates formulas which may have two possible normal forms.

Lemma 6.1 (Normal form of wp^{src}) *for any source expression \mathcal{E}^{src} normal form of wp^{src} is such that*

$$\begin{aligned}
& \text{either there exists } Q, R : \mathcal{F} \\
& wp^{src}(\mathcal{E}^{src}, \psi, \text{excPost}, \mathbf{m})_v \equiv \\
& Q \Rightarrow \psi \\
& \wedge \\
& R
\end{aligned}$$

$$\text{or exists } Q, R : \mathcal{F}, v_1 \dots v_k : \mathcal{E}^{src}$$

$$\begin{aligned}
& wp^{src}(\mathcal{E}^{src}, \psi, \text{excPost}, \mathbf{m})_v \equiv \\
& \forall \text{boundVar}_1, \dots \text{boundVar}_k, Q \Rightarrow \psi \dots \wedge \\
& R \quad \begin{array}{l} [v_1 \leftarrow \text{boundVar}_1] \\ [v_k \leftarrow \text{boundVar}_k] \end{array}
\end{aligned}$$

This follows from the definition of wp^{src} in Section 4.2.2. The second form in which the part of the calculated weakest precondition predicate is quantified appears in the cases for instance creation and method invocation expressions.

We now turn to see what is the relation between the weakest precondition formulas for an expression \mathcal{E}^{src} and its compilation $\lceil s, \mathcal{E}^{src}, e \rceil$. Depending on the form of the weakest precondition (as we saw with the previous lemma there are two possible forms of the weakest precondition for source expressions) of a source expression \mathcal{E}^{src} obtained from wp^{src} we will state the relation between wp^{src} and wp_{seq}^{bc} in the following two lemmas. Informally, both lemmas express the fact that

- the hypothesis of the part of the weakest preconditions on source and bytecode level concerning the normal termination of the expression are the same
- the part of the weakest preconditions on source and bytecode level concerning the exceptional termination of the expression are the same

Lemma 6.2 (Wp of a compiled expression) *For any expression \mathcal{E}^{src} from our source language, for any formula $\psi : \mathcal{F}$ of the source assertion language and any formula $\phi : \mathcal{F}^{bc}$ such that ϕ may only contain stack expressions of the form $st(cnt - k)$, $k \geq 0$, the following holds*

There exist $Q, R : \mathcal{F}$ such that

$$\begin{aligned}
& wp^{src}(\mathcal{E}^{src}, \psi, \text{excPost}, \mathfrak{m})_v \equiv \\
& Q \Rightarrow \psi \\
& \wedge \\
& R \\
& \Rightarrow \\
& wp_{seq}^{bc}(\lceil s, \mathcal{E}^{src}, e \rceil, \phi, \text{excPost}, \mathfrak{m}) \equiv \\
& Q \Rightarrow \phi \begin{array}{l} [cnt \leftarrow cnt + 1] \\ [st(cnt + 1) \leftarrow v] \end{array} \\
& \wedge \\
& R
\end{aligned}$$

We proceed with several cases of the proof, which is done by induction over the structure of the formula

Proof :

1. $\mathcal{E}^{src} = \text{const}, \text{const} \in \text{constInt}, \text{true}, \text{false}$

$\{ \text{ source case } \}$
 $(1)wp^{src}(const, \psi, \text{excPost}, \mathfrak{m})_{const}$
 $\{ \text{ following the definition of the wp function for source expressions in subsection 4.2.2 } \}$
 $\equiv \psi$

$\{ \text{ bytecode case } \}$
 $(2)wp_{seq}^{bc}(\ulcorner s, const, s^\neg, \phi, \text{excPost}, \mathfrak{m} \urcorner)$
 $\{ \text{ following the definition of the compiler function in subsection 3.3 } \}$
 $\equiv wp^{bc}(s \text{ pushconst}, \phi, \ulcorner \text{excPost}^\neg, \mathfrak{m} \urcorner)$
 $\{ \text{ following the definition of the wp function for bytecode in subsection 4.2.2 } \}$
 $\equiv \phi \begin{array}{l} [\text{cntr} \leftarrow \text{cntr} + 1] \\ [\text{st}(\text{cntr} + 1) \leftarrow const] \end{array}$

$\{ \text{ from (1) and (2) and } Q, R = T \text{ this case holds } \}$

$$2. \mathcal{E}^{src} = \mathcal{E}^{src}.f$$

$$\begin{aligned}
& \{ \text{source case} \} \\
& (1) \text{wp}^{src}(\mathcal{E}^{src}.f, \psi, \text{excPost}, \mathbf{m})_{v.f} \\
& \{ \text{following the definition of the wp function} \\
& \text{for source expressions in subsection 4.2.2} \} \\
& v \neq \mathbf{null} \Rightarrow \psi \\
& \equiv \text{wp}^{src}(\mathcal{E}^{src}, \wedge \quad \quad \quad, \text{excPost}, \mathbf{m})_v \\
& \quad v = \mathbf{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \\
\\
& \{ \text{bytecode case} \} \\
& (2) \text{wp}_{seq}^{bc}(\ulcorner s, \mathcal{E}^{src}.f, e \urcorner, \phi, \ulcorner \text{excPost} \urcorner, \mathbf{m}) \\
& \{ \text{following the definition of the compiler function in subsection 3.3} \} \\
& \equiv \text{wp}_{seq}^{bc}(\ulcorner s, \mathcal{E}^{src}, e - 1 \urcorner, \phi, \text{excPost}, \mathbf{m}) \\
& \{ \text{following the definition of the wp function for bytecode} \\
& \text{in subsection 5.1} \} \\
& \equiv \text{wp}_{seq}^{bc}(\ulcorner s, \mathcal{E}^{src}, e - 1 \urcorner, \\
& \quad \text{st}(\text{cntr}) \neq \mathbf{null} \Rightarrow \\
& \quad \phi[\text{st}(\text{cntr}) \leftarrow \text{st}(\text{cntr}).f] \quad \quad \quad, \\
& \quad \wedge \\
& \quad \text{st}(\text{cntr}) = \mathbf{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \\
& \quad \ulcorner \text{excPost} \urcorner, \mathbf{m}) \\
& \{ \text{From (1) and (2) we apply the induction hypothesis} \} \\
& \exists Q', R' : \mathcal{F}, \\
& \quad v \neq \mathbf{null} \Rightarrow \psi \\
& (3) \text{wp}^{src}(\mathcal{E}^{src}, \wedge \quad \quad \quad, \text{excPost}, \mathbf{m})_v \\
& \quad v = \mathbf{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \\
& \equiv \\
& \quad v \neq \mathbf{null} \Rightarrow \psi \\
& Q' \Rightarrow \wedge \\
& \quad v \neq \mathbf{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \\
& \wedge \\
& R' \\
& \Rightarrow \\
& \text{wp}_{seq}^{bc}(\ulcorner \mathcal{E}^{src} \urcorner, \\
& \quad \text{st}(\text{cntr}) \neq \mathbf{null} \Rightarrow \\
& \quad \phi[\text{st}(\text{cntr}) \leftarrow \text{st}(\text{cntr}).f] \quad \quad \quad, \\
& \quad \wedge \\
& \quad \text{st}(\text{cntr}) = \mathbf{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \\
& \quad \ulcorner \text{excPost} \urcorner, \mathbf{m}) \\
& \equiv
\end{aligned}$$

$$\begin{aligned}
Q' &\Rightarrow \text{st}(\text{cntr}) \neq \text{null} \Rightarrow \phi[\text{st}(\text{cntr}) \leftarrow \text{st}(\text{cntr}).f] \quad [\text{cntr} \leftarrow \text{cntr} + 1] \\
&\quad \wedge \quad \text{st}(\text{cntr}) = \text{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \quad [\text{st}(\text{cntr} + 1) \leftarrow v] \\
&\quad \wedge \\
&\quad R' \\
&\quad \equiv \\
&\quad \{ \phi \text{ contains only stack expressions } \text{st}(\text{cntr} - k), k \geq 0 \text{ and properties of substitution} \} \\
Q' &\Rightarrow v \neq \text{null} \Rightarrow \phi \quad [\text{cntr} \leftarrow \text{cntr} + 1] \\
&\quad \wedge \quad [\text{st}(\text{cntr} + 1) \leftarrow v.f] \\
&\quad v = \text{null} \Rightarrow \text{excPost}(\text{NullPtrExc}) \\
&\quad \wedge \\
&\quad R' \\
&\quad \{ \text{from (3) this case holds} \}
\end{aligned}$$

Lemma 6.3 (Wp of a compiled expression) *For any expression \mathcal{E}^{src} from our source language, for any formula $\psi : \mathcal{F}$ of the source assertion language and any formula $\phi : \mathcal{F}^{bc}$ such that ϕ may only contain stack expressions of the form $\text{st}(\text{cntr} - k)$, $k \geq 0$, the following holds there exist $Q, R : \mathcal{F}, v_1 \dots v_k : \mathcal{E}^{src}$*

$$\begin{aligned}
&\forall \text{boundVar}_1, \dots, \text{boundVar}_k, Q \Rightarrow \\
&\quad [v_1 \leftarrow \text{boundVar}_1] \\
&\quad \psi \dots \quad \wedge \\
&\quad [v_k \leftarrow \text{boundVar}_k] \\
&\quad R \\
&\Rightarrow \\
&\quad wp_{seq}^{bc}(\ulcorner s, \mathcal{E}^{src}, e^\top, \psi, \text{excPost}, \mathfrak{m} \rceil \equiv \\
&\quad \forall \text{boundVar}_1, \dots, \text{boundVar}_k, Q \Rightarrow \\
&\quad \quad [\text{cntr} \leftarrow \text{cntr} + 1] \\
&\quad \quad [\text{st}(\text{cntr} + 1) \leftarrow v] \\
&\quad \quad \phi [v_1 \leftarrow \text{boundVar}_1] \\
&\quad \quad \dots \\
&\quad \quad [v_k \leftarrow \text{boundVar}_k] \\
&\quad \wedge \\
&\quad R
\end{aligned}$$

The next lemma establishes a relation between the source and bytecode wp w.r.t. to the same weakest precondition.

excPost

Lemma 6.4 (Proof obligation equivalence on statements)

$$\begin{aligned}
&\forall \mathcal{STM}, \psi, \text{excPost}^{src}, \\
&\quad wp_{stmt}^{bc}(\ulcorner s, \mathcal{STM}, e^\top, \psi, \text{excPost}^{src}, \mathfrak{m} \rceil = \\
&\quad wp^{src}(\mathcal{STM}, \psi, \text{excPost}^{src}, \mathfrak{m})
\end{aligned}$$

This follows from the previous lemma (which establishes the relation between the wp over source and the wp over bytecode that takes into account the

compilation structure) and lemma 5.2.4 (which establishes the relation between the wp over bytecode that takes into account the compilation structure and the original wp which does not consider the properties of the compiler)

The next statement establishes under what conditions the wp over source and the wp defined in the previous Chapter are equivalent.

Lemma 6.5 (Proof obligation equivalence on statements) *For every STM compilation $\lceil s, STM, e \rceil$, formula ψ and exceptional postcondition function excPost^{src2bc} such that :*

- $\psi = \text{inter}(e, e + 1)$
- $\forall \text{Exc}, \text{excPost}^{src2bc}(\text{Exc}) = \mathbf{m}.\text{excPost}(\text{Exc}, e)$

then it holds

$$wp^{src}(STM, \psi, \text{excPost}^{src}, \mathbf{m}) = wp(\text{ins}_s, \mathbf{m})$$

From the last lemma follows that if the body of the method \mathbf{m} is STM , and its compilation is $\lceil s, STM, e \rceil$, then the proof obligations generated over STM upon postcondition $\mathbf{m}.\psi^{postN}$ and exceptional postcondition function $\mathbf{m}.\text{exc}^{bc}$ and its compilation $\lceil s, STM, e \rceil$ are the same

$$wp^{src}(STM, \psi^{postN}, \text{excPost}^{src}, \mathbf{m}) = wp(\text{ins}_s, \mathbf{m})$$

for the next lemma we have to say also : if $\lceil s, STM, e \rceil$ and $\text{inter}(e, e+1)$ does not contain stack expr. then $wp(s, \mathbf{m})$ does not contain stack expr

References

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