

Navier-Stokes Equations

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{U} &= 0 \\ \frac{\partial \rho \vec{U}}{\partial t} + \nabla \cdot \rho \vec{U} \times \vec{U} + \nabla \cdot (\tau - PI) &= 0 \\ \frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho e + P) \vec{U} - \nabla \cdot \tau \vec{U} - \nabla \cdot k \nabla T &= 0 \\ P &= \rho RT \\ e &= \frac{R}{\gamma - 1} T \\ \mu &= \mu_0 \frac{T_0 + C}{T + C} \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \\ k &= \frac{\gamma R \mu}{Pr(\gamma - 1)}\end{aligned}$$

Definitions

$$\vec{U} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

ρ - density [kg/m³]

u, v, w - velocity in x, y, z in Cartesian coordinates [m/s]

P - pressure [Pa]

e - internal energy [J/kg]

k - thermal conductivity [$\frac{W}{m \cdot K}$]

T - temperature [K]

τ - viscous stress tensor

μ - dynamic viscosity [$\frac{N \cdot s}{m^2}$]