Navier-Stokes Equations

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \rho\vec{U} = 0 \qquad \text{mass}$$

$$\frac{\partial \rho\vec{U}}{\partial t} + \boldsymbol{\nabla} \cdot \rho\vec{U} \times \vec{U} + \boldsymbol{\nabla} \cdot (\tau - PI) = 0 \qquad \text{momentum}$$

$$\frac{\partial \rho e}{\partial t} + \boldsymbol{\nabla} \cdot (\rho e + P)\vec{U} - \boldsymbol{\nabla} \cdot \tau\vec{U} - \boldsymbol{\nabla} \cdot k\boldsymbol{\nabla}T = 0 \qquad \text{energy}$$

$$\vec{U} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

ρ - density [kg/m^3]

u, v, w - velocity in x, y, z in Cartesian coordinates [m/s]

P - pressure [Pa]

e - internal energy [J/kg]

k - thermal conductivity [$\frac{W}{m \cdot k}$]

T - temperature [K]

τ - viscous stress tensor

μ - dynamic viscosity [$\frac{N \cdot s}{m^2}$]

Navier-Stokes equations are a set of equations that are solved simultaneously. For a 2D fluids domain, we need to solve 4 equations: Density (ρ), X-momentum (ρu), Y-momentum (ρv), and Energy (ρe). Once these equations are solved, the temperature gradient normal to the wall can be computed using e (Energy). The temperature gradient is not directly calculated from the Navier-Stokes equations. T is computed for a given e through the ideal gas law (thermodynamics equation below), which is used to calculate the gradient of T. As a reminder, in the challenge problem, q_w is transferred from the fluid to the solid as a Neumann boundary condition, and the new T from the solid is fed back into the fluid domain. To get q_w from T, take the dot product of the gradient of T with the wall normal vector and multiply by thermal conductivity (k):

$$q_w = k\boldsymbol{\nabla}T \cdot \vec{n}$$

Equation of State (closure equations)

$$e = \frac{R}{\gamma - 1} T \qquad \text{thermodynamics}$$

$$\mu = \mu_0 \frac{T_0 + C}{T + C} (\frac{T}{T_0})^{\frac{3}{2}} \qquad \text{dynamic viscosity}$$

$$k = \frac{\gamma R \mu}{P_r(\gamma - 1)} \qquad \text{heat conduction}$$