1 Syntax

$$
\begin{aligned}
S = \ & \textbf{pure}\ E \\
\mid\ & x = S_1; S_2 \\
\mid\ & f\ E^* \\
\\
\mid\ & \textbf{fail} \\
\mid\ & S_1 \mathbin{[]} S_2 \\
\mid\ & S_1 \lhd S_2 \\
\\
\mid\ & \textbf{get} E \\
\mid\ & \textbf{peek} \\
\\
\mid\ & \textbf{parse}\ S\ E \\
\mid\ & \textbf{case}\ E\ \textbf{of}\ (P \to S)^+
\end{aligned}
$$

Figure 1: The Core language

2 Semantics

The semantics of a parser is a set of triples (v, X, Y), where v is a semantic value, and X and Y are strings. If (v, X, Y) is in the semantics of S, then when applied to input $X +\!\!+ Y$, S will consume X and produce result v. This formulation allows us to talk about parsers in context. We'll say that a parser *accepts* an input if it doesn't fail on it:

$$\text{accepts}\ S\ (X +\!\!+ Y) = \exists v.(v, X, Y) \in [\![S]\!]$$

$$
\begin{aligned}
[\![\textbf{pure}\ E]\!] &= \{([\![E]\!], [], X)\} \\
[\![x = S_1; S_2]\!] &= \{(v, X +\!\!+ Y, Z) \mid (u, X, Y +\!\!+ Z) \leftarrow [\![S_1]\!], (v, Y, Z) \leftarrow [\![S_2]\!]^{x=u}\} \\
[\![\textbf{fail}]\!] &= \{\} \\
[\![S_1 \mathbin{[]} S_2]\!] &= [\![S_1]\!] \cup [\![S_2]\!] \\
[\![S_1 \lhd S_2]\!] &= [\![S_1]\!] \cup \{(v, X, Y) \mid (v, X, Y) \leftarrow [\![S_2]\!], \text{not (accepts } S_1\ (X +\!\!+ Y))\} \\
[\![\textbf{get} E]\!] &= \{(X, X, Y) \mid |X| = [\![E]\!]\} \\
[\![\textbf{peek}]\!] &= \{(X, [], X)\} \\
[\![\textbf{parse}\ S\ E]\!] &= \{(v, [], Z) \mid (v, X, Y) \leftarrow [\![S]\!], [\![E]\!] = X +\!\!+ Y\} \\
[\![\textbf{case}\ E\ \textbf{of}\ A]\!] &= [\![\text{select}\ [\![E]\!]\ A]\!]
\end{aligned}
$$

Figure 2: Set semantics of Core parsers.

Example of a parser that depends on context:

$$S = x = \textbf{peek}; \textbf{case}\ x\ \textbf{of}\ \{[] \to \textbf{fail}; \_ \to \textbf{pure}\ ()\}$$

This parser accepts the empty string, but only if is not at the end of the input.

2.1 Semantics as a Relation

Figure 3: $\Gamma \vdash S \ \to\ v \triangleright X \cdot Y$ describes the behavior or parser S in dynamic environment Γ. When applied to the input $X +\!\!+ Y$, S will consume X and produce semantic value v.

$$\textsc{Pure}\quad \frac{\Gamma \vdash E \ \to\ v}{\Gamma \vdash \textbf{pure}\ E \ \to\ v \triangleright [] \cdot X} \qquad \textsc{Advance}\quad \frac{\Gamma \vdash E \ \to\ |X|}{\Gamma \vdash \textbf{get} E \ \to\ X \triangleright X \cdot Y}$$

$$\textsc{Look-Ahead}\quad \frac{}{\Gamma \vdash \textbf{peek} \ \to\ X \triangleright [] \cdot X}$$

$$\textsc{Sequnce}\quad \frac{\Gamma \vdash S_1 \ \to\ u \triangleright X \cdot Y +\!\!+ Z \qquad \Gamma, x = u \vdash S_2 \ \to\ v \triangleright Y \cdot Z}{\Gamma \vdash x = S_1; S_2 \ \to\ v \triangleright X +\!\!+ Y \cdot Z}$$

$$\textsc{Unbiased-Choice-Left}\quad \frac{\Gamma \vdash S_1 \ \to\ v \triangleright X \cdot Y}{\Gamma \vdash S_1 \mathbin{[\!]} S_2 \ \to\ v \triangleright X \cdot Y} \qquad \textsc{Unbiased-Choice-Right}\quad \frac{\Gamma \vdash S_2 \ \to\ v \triangleright X \cdot Y}{\Gamma \vdash S_1 \mathbin{[\!]} S_2 \ \to\ v \triangleright X \cdot Y}$$

$$\textsc{Biased-Choice-Left}\quad \frac{\Gamma \vdash S_1 \ \to\ v \triangleright X \cdot Y}{\Gamma \vdash S_1 \triangleleft S_2 \ \to\ v \triangleright X \cdot Y} \qquad \textsc{Biased-Choice-Right}\quad \frac{\Gamma \vdash S_2 \ \to\ v \triangleright X \cdot Y \qquad \Gamma \vdash (X +\!\!+ Y) \notin S_1}{\Gamma \vdash S_1 \triangleleft S_2 \ \to\ v \triangleright X \cdot Y}$$

$$\textsc{Nested-Parser}\quad \frac{\Gamma \vdash E \ \to\ (X +\!\!+ Y) \qquad \Gamma \vdash S \ \to\ v \triangleright X \cdot Y}{\Gamma \vdash \textbf{parse}\ S\ E \ \to\ v \triangleright [] \cdot Z}$$

$$\textsc{Case}\quad \frac{\Gamma \vdash E \ \to\ u \qquad \Gamma \vdash \text{select}\ u\ A \ \to\ v \triangleright X \cdot Y}{\Gamma \vdash \textbf{case}\ E\ \textbf{of}\ A \ \to\ v \triangleright X \cdot Y}$$

$$\textsc{Empty}\quad \frac{}{\Gamma \vdash X \notin \mathbf{fail}} \qquad \textsc{Too-Short}\quad \frac{\Gamma \vdash E \;\rightarrow\; v \qquad |X| < v}{\Gamma \vdash X \notin \mathbf{get} E}$$

$$\textsc{Unbiased-Mismatch}\quad \frac{\Gamma \vdash X \notin S_1 \qquad \Gamma \vdash X \notin S_2}{\Gamma \vdash X \notin S_1 \mathbin{[\!]} S_2} \qquad \textsc{Biased-Mismatch}\quad \frac{\Gamma \vdash X \notin S_1 \qquad \Gamma \vdash X \notin S_2}{\Gamma \vdash X \notin S_1 \triangleleft S_2}$$

$$\textsc{Not-Front}\quad \frac{\Gamma \vdash X \notin S_1}{\Gamma \vdash X \notin x = S_1; S_2} \qquad \textsc{Not-Back}\quad \frac{\Gamma \vdash S_1 \;\rightarrow\; v \triangleright X \cdot Y \qquad \Gamma, x = v \vdash Y \notin S_2}{\Gamma \vdash (X \mathbin{+\!\!+} Y) \notin x = S_1; S_2}$$

$$\textsc{Not-Nested}\quad \frac{\Gamma \vdash E \;\rightarrow\; v \qquad \Gamma \vdash v \notin P}{\Gamma \vdash X \notin \mathbf{parse}\ P\ E} \qquad \textsc{No-Case}\quad \frac{\Gamma \vdash E \;\rightarrow\; v \qquad \Gamma \vdash X \notin \text{select } v\ A}{\Gamma \vdash X \notin \mathbf{case}\ E\ \mathbf{of}\ A}$$

Figure 4: $\Gamma \vdash X \notin S$ asserts that X is not accepted by S in the sense described before.